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THE NEUTRALIZATION ON AN EMPTY NUMBER LINE MODEL FOR INTEGER ADDITIONS AND SUBTRACTIONS: IS IT HELPFUL?

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Abstract

The number line and the neutralization model have been used very extensively in teaching integer additions and subtractions for decades. Despite their advantages, issues concerning subtractions on these models are still debatable. Therefore, the neutralization on an empty number line (NNL) model is proposed in this research to help students better understand the meaning of integer subtractions as well as additions. This report is a part of a design research study conducted in a classroom of 28 elementary school students at the fifth grade. Data were gathered by collecting students' written work, conducting interviews and observations during the teaching experiment. This paper focuses on students' perceptions of the NNL model and what factors that might contribute to students' achievements in understanding integer additions and subtractions. The analysis revealed that most students overemphasized on the use of the NNL model as a procedural method instead of as a model for thinking. Moreover, students' mathematical beliefs and conceptions on the use of the column strategy and the absence of a discussion on the need of using the model are found to be some factors that could cause students' misunderstandings. However, with a thorough planning, the NNL model has a potential to help students developing a meaning of integer additions and subtractions.

Keywords: Addition, Subtraction, Negative, Neutralization on an Empty Number Line (NNL) Model

Abstrak

Model garis bilangan dan model netralisasi telah digunakan secara luas dalam pembelajaran konsep penjumlahan dan pengurangan bilangan bulat. Terlepas dari kelebihan pada masing-masing model, permasalahan pada operasi pengurangan yang melibatkan bilangan bulat negatif masih menjadi perdebatan yang hangat. Oleh karena itu, model netralisasi pada garis bilangan kosong (NNL) digunakan dalam penelitian ini untuk membantu siswa lebih memahami makna operasi pengurangan bilangan bulat serta penjumlahan. Makalah ini merupakan bagian dari penelitian *design research* yang dilakukan di kelas V suatu Sekolah Dasar dengan 28 siswa. Data diperoleh dari hasil jawaban siswa, observasi dan interview selama pembelajaran di kelas berlangsung. Makalah ini fokus pada persepsi siswa tentang model NNL dan faktor-faktor apa yang mungkin berkontribusi terhadap kesuksesan siswa dalam memahami penjumlahan dan pengurangan bilangan bulat. Analisis mengungkapkan bahwa sebagian besar siswa terlalu menekankan pada penggunaan model NNL sebagai metode prosedural daripada sebagai model untuk berpikir. Selain itu, keyakinan dan konsepsi matematika siswa tentang penggunaan strategi susun ke bawah dan tidak adanya diskusi tentang kebutuhan menggunakan model tersebut ditemukan menjadi beberapa faktor yang dapat menyebabkan kesalahpahaman siswa. Namun, dengan perencanaan yang matang, model NNL memiliki potensi untuk membantu siswa mengembangkan makna penjumlahan dan pengurangan bilangan bulat..

Kata kunci: Penjumlahan, Pengurangan, Negatif, Model Netralisasi Pada Garis Bilangan (NNL)

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Studies indicated that the use of models and contexts could promote students' thinking in performing addition and subtraction of integers (Liebeck, 1990; Stephan & Akyuz, 2012; Bofferding, 2014; Teppo & van den Heuvel-Panhuizen, 2014; Sahat, Tengah, & Prahmana, 2018). Basically, there are two models for teaching addition and subtraction of integers, i.e. the neutralization model and the number line model (Van de Walle, 2004). In dealing with integer addition and subtraction involving negative numbers, the neutralization model or the cancellation model applies the principle of cancellation, where the sum of every number and its opposite is always zero. Whereas the number line model uses the idea of direction to represent the operation of addition and subtraction.

The neutralization model is usually introduced by using counters of two different colors representing positive and negative integers. If blue counter represents positive integer and red counter represents negative integer, then each pair of blue and red counter is equal to zero, and thus implies "$a + (-a) = 0$". When working with this model, integer addition and subtraction can be interpreted as "adding" and "taking away" respectively. As an example, to model the addition of "6 + (-2)", students first put six blue counters and then 'add' two red counters. Since two red counters cancel out two blue counters, then four blue counters remain as the sum. However, the problem happens when students are dealing with subtraction problem such as "6 – (-2)". First, students put six blue counters and then they must 'take away' two red counters. In this case, the red counter does not exist, so they have to 'add' two pairs of red and blue counters or zero pairs so they can 'take away' two red counters and 8 blue counters remain as the result (see Figure 1). Hence, students must understand that when pairs of opposite colors of counters are added to a quantity, the value of the original counters remains unchanged (Van de Walle, 2004).

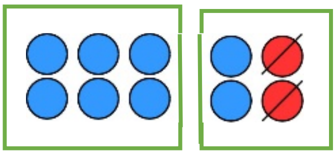


Figure 1. The neutralization model for solving "6 – (-2)"

On the other hand, the number line model is depicted as a horizontal line in which positive integers are located to the right of zero and negative integers are located to the left of zero. NCTM (2000) recommends students to use the number line model to explore negative numbers as extensions of positive numbers through familiar applications. The operation of addition and subtraction can be interpreted as "walking in the same direction" and "walking in the opposite direction". For example, to solve "6 + (-2)", students can start at zero facing right and move forward six steps (+6), then move two steps backwards representing negative two. Although the use of number line model is helpful for supporting integer addition, but students tend to have problems when dealing with subtraction with negative numbers because the procedure is more complicated. For example, to solve "6 – (-2)" using the number line model, students can start at zero facing right and then move forward six steps (+6), afterwards students have to face in the opposite direction (turn around) before they move two steps backward (-2) on the number line to represent the subtraction operation (see Figure 2).

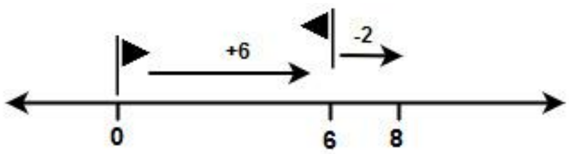


Figure 2. The number line model for solving "6 – (-2)"

Freudenthal (1973) showed that the number line model could be helpful for supporting integer addition, and Stephan & Akyuz (2012) indicated that the number line model together with a financial context (assets and debts) could support students' development of integer addition and subtraction, both procedurally and conceptually. However, Küchemann (1981) pointed out that the number line model should be avoided in teaching subtraction of integers, thus a discrete model or the neutralization model was offered as a better solution. Liebeck (1990) also emphasized that the existing concept of addition and subtraction are related to "adding" and "taking away" objects. When students use the number line model in subtracting a negative number, for example "6 – (-2)" as in Figure 2, there is no relevance between the intuitive concepts of subtraction as "taking away" and the "turn around" movement on the number line that represent the subtraction operation. On the other hand, the neutralization model could also be confusing for students, since the subtraction operation involves both addition and subtraction – that is, adding the zero pair first and then taking away the number indicated in the problem (Bofferding, 2014).

Some studies on the use of the combination of the neutralization and the number line model were identified. Steiner (2009) in his dissertation used the novel model with a context of money to spend (as a positive integer) and debt (as a negative integer). By placing red bills (debts) to the left side of zero and white bills (money to spend) to the right side of zero on a number line, the operation of addition and subtraction of integers were carried out using the cancellation principle. Similarly, Shutler (2017) utilized the banking model as a development of hills and dales model. Positive and negative integers represented as stacks of black circles and wells of white circles respectively. However, the situation of assets and debts is not suitable for Indonesian context, since the smallest amount of money in Indonesia is a hundred rupiah.

Despite the fact that some studies have been conducted in Indonesia on developing students' understanding of integer additions and subtractions (Aris, et al. 2019; Prahmana, 2017; Shanty, 2016; Muslimin, et al. 2012), research on developing a combination model of the neutralization and the number line model has not been done. Therefore, this study offers the neutralization on the number line (NNL) model to promote students' understanding in performing addition and subtraction of integers. The NNL model applies the procedure of neutralization model where the representations of positive and negative integers are located on an empty number line. Sari, et al. (2019) suggested that the NNL model could give meaning to students that subtracting a negative means adding a positive and subtracting a positive from a negative means adding two negatives, and it also allows us to work with big numbers.

Lesh & Doerr (2000) defined model as a system consisting of elements; relationships among elements; operations that describe how the elements interact; and patterns or rules, such as symmetry, commutativity, and transitivity that apply to the preceding relationships and operations. In the NNL model, there are three main elements, (i) a mound (a curve above the empty number line) to represent a positive integer, (ii) a hollow (a curve below the empty number line) to represent negative integers, and (iii) an empty number line to record and track students' strategies in performing the operation. The relationship between the mound and hollow is that every pair of mound and hollow counts as zero, meaning that “1 + (-1) = 0”. The operations on this model constitute addition as ‘adding’ a number of mounds or hollows and subtraction as ‘taking away’ a number of mounds or hollows as indicated in the problem. Furthermore, the rule that applies to this model is that when pairs of a mound and a hollow are added to the existed mounds or hollows, the value of the initial mounds or hollows remains unchanged. Figure 3 illustrates how the NNL model is used in a subtraction problem.

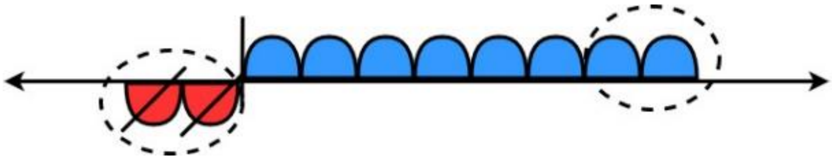


Figure 3. The NNL model for solving “6 – (-2)”

In the present research, we used a theory of RME (Realistic Mathematics Education) which was developed in the 1970s by Hans Freudenthal who perceived mathematics as a *human activity*. Opposite to traditional mathematics education, RME emphasizes mathematics education as a process of doing mathematics in reality that leads to a result, mathematics as a product (Gravemeijer & Terwel, 2000). The main features of RME are the use of contexts and models, students’ own constructions and productions, interactivity, and intertwinement (Treffers, 1987). The use of contexts and models are the two fundamental elements in RME to support students’ progressive mathematizing. Whitacre et al. (2012) suggested that reasoning about opposite magnitudes could serve as a basis for integer reasoning. Therefore, the scoring context which reveals two opposite magnitudes that are positive and negative scores has a potential to develop students’ reasoning with integers. Thus, the context of scoring was chosen in the present research as a situation that students can discuss within it and as a basis for the development of the NNL model.

Hence, aiming at developing the local instructional theory on integer addition and subtraction as well as improving practice, this paper seeks to address the following questions, first, how do students perceive the NNL model in solving integer additions and subtractions? Second, what factors that might contribute to students‘ achievement in solving integer additions and subtractions?

METHOD

This report is a part of the retrospective analysis conducted in a design research study. Design research is also known as developmental research (Freudenthal, 1991), design-based research (Cobb, et al. 2016), educational design research (McKenney & Reeves, 2014), or classroom-based design research (Stephan & Cobb, 2013). There are three phases in a design research study, first, the design and preparation, second, the implementation of the design, and third, the retrospective analysis (Gravemeijer & Cobb, 2006; Cobb, et al. 2003).

In the first phase, a design of an instructional sequence was developed based on the local instructional theory about integer addition and subtraction. The use of models and contexts in developing the concept of integer addition and subtraction was explored. A hypothetical learning trajectory (Simon, 1995) was elaborated to describe both a sequence of learning activities and conjectures of how students engage in the activities. In the second phase, the hypothetical learning trajectory (HLT) was tried out in a classroom of 28 students in SD Nurul Islam, Jakarta. Data such as video recordings of classroom activities, photos of students' activities, students' written works, classroom observations and interviews were collected. Observations took place on the classroom level, group level, and individual level, while interviews were made occasionally during the lesson or purposively after the lesson.

After the lessons took place, the data from different sources were gathered, selected, and analyzed by comparing the actual learning process and the hypothetical learning trajectory. Students' written works were chosen, examined and analyzed in accordance with other sources of data to improve the triangulation. The retrospective analysis reported in this paper will focus on how students' perceptions of the NNL model in solving integer additions and subtractions, do they find it helpful or not. Moreover, we will also find out what factors that might contribute to students' achievement in solving integer additions and subtractions, related to the classroom practices in the classroom.

RESULT AND DISCUSSION

This section summarizes the findings of the teaching experiment in conjunction with the hypothetical learning trajectory (HLT), followed by the retrospective analysis focusing on students‘ perception of the NNL model.

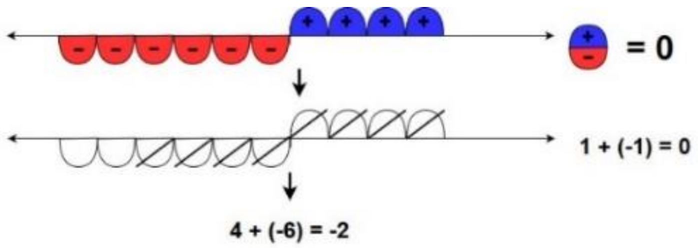
The Comparison between the HLT and the Implementation of HLT

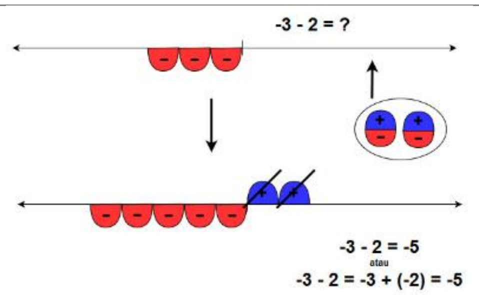
Based on Simon (1995), a hypothetical learning trajectory (HLT) is made up of three components: learning goals that defines the direction, learning activities, and hypothetical learning processes–a prediction of how students' thinking and understanding will evolve in the context of the learning activities. Similar to the present design research, Simon (2018) conducted a Learning Through Activity research program aiming at the construction of a HLT for a specific topic and the elaboration of theory and instructional design to promote students' conceptual understanding. In this design research, HLT serves as a guideline in the teaching experiment and also as a framework in the retrospective analysis where the actual learning process is compared to the HLT (Bakker, 2004). In the present research, a sequence of activities was designed aiming at developing students understanding of negative numbers and constructing a meaning of integer addition and subtraction by means of the NNL model and the scoring context.

Table 1 shows how the actual learning process took place in a classroom compared to the designed HLT. The designed sequence of activities in this research is basically divided into three parts. First, understanding negative numbers by exploring various contexts. Second, developing the context of scoring to introduce integer addition and the NNL model. Third, exploring integer subtraction on the NNL model. It was expected that by experiencing the scoring context, students will develop a meaning of addition and subtraction as ‘adding‘ and ‘taking away‘ positive or negative scores. However, it was also hypothesized that a problem might occur when students are dealing with subtraction, that is, when students have to take away scores or numbers that does not exist before. Thus, a term ‘neutral pairs‘ or ‘zero pairs‘ might be introduced to help students solve the problem.

Table 1. The comparison between the HLT and the implementation of HLT

The HLT	The Implementation of HLT
Understanding Negative Numbers Goals: Students develop a meaning of negative integer. Activities: A classroom discussion about the existence of negative numbers as the extension of positive numbers on the number line by using the context of temperature below zero. A further discussion about negative numbers as the opposite of positive numbers by using the context of height and depth, assets and debts, positive and negative scores. Hypotheses: Various contexts could help students build the meaning of negative integers. The number line model emerges naturally from the temperature context. Hence, students can compare positive and negative integers by using the number line. The number line can also be used to show that any two integers having the same distance from zero in opposite directions are called opposites. Other contexts might be introduced to students to enrich their understanding of negative numbers.	The temperature context can introduce the concept of negative numbers very well. The number line emerges from the picture of a thermometer with negative and positive temperatures on it. However, the students did not have any experience related to negative numbers. Thus, other contexts were introduced to students, including the scoring context. There was less attention in developing the idea of opposite numbers.
The Scoring Context and Integer Addition Goals: Students can perform integer additions involving negative numbers. Activities: The scoring context is introduced to students by playing a game. One positive score is signified as a mound, and one negative score is signified as a hollow. Students are asked to record their scores and find the total score after playing each game. A discussion	Students were very excited when playing the game. They can record their scores correctly and find the total score by the neutralization principle. For those who already have knowledge in performing integer addition involving negative numbers,

about the idea of one positive cancels out one negative, i.e. " 1+(-1)=0" precedes the game. Hypotheses: It was conjectured that the game will promote students' participation in the classroom. The idea of canceling out every pair of positive and negative scores will help them to find the total score. There might be also students who will use subtraction instead of the neutralization principle. Figure 4 shows how the NNL model emerges from the activity.  **Figure 4.** The emergence of the NNL model for integer addition	they can perform the addition as a subtraction: $\boldsymbol{a + (-b) = a - b}$**.** If the number of negative scores is bigger than the positive scores, then they find the positive difference between the two numbers and put a negative sign in front of the result. However, some students failed in determining the sum of two negative numbers, because they did not go back to the scoring context in adding the two negative numbers.
The Scoring Context and Integer Subtraction Goals: Students can perform integer subtractions involving negative numbers. Activities: The rule of the game in scoring context is modified: If they win they could take away three negative scores, and if they loose they have to take away two positive scores. Hypotheses: The rule was modified to provoke students' understanding that taking away a negative number means adding the opposite, that is $\boldsymbol{a - (-b) = a + b}$ and taking away a positive number means adding the opposite, $\boldsymbol{-a - b = (-a) + (-b)}$**.** To develop those idea, the problem that they might encounter is taking away a score that does not exist before, for example, to solve " -3 – 2", they have to take away two positive scores or two mounds from three negative scores or three hollows. If there is none of them come up with a solution, the teacher may introduce the idea of adding 'neutral pairs' or 'zero pairs' that counts as zero and will not change the initial score. Figure 5 shows how the NNL model can be used in representing the idea:	 There were some evidences that most of the students had difficulties in performing integer subtractions, so the teacher had to introduce the idea of neutral pairs and showed the students how to do the subtractions. It was observed that the difficulty in grasping the idea of subtraction involving negative numbers made them frustrated. The scoring context was no longer predominant in the classroom discussion since they over emphasized the idea of neutral pairs. It happened that the students keep adding the neutral pair though in some

 **Figure 5.** The emergence of the NNL model for integer subtraction	cases it was not always needed in solving a problem. Some evidences also showed that their previous knowledge in applying the algorithm strategy were quite dominant.

The Retrospective Analysis on the NNL Model

Table 2 describes an interpretative framework that underpins our analyses of students' mathematical learning both as individuals and as a community in a classroom mathematical practice. This framework combines two different perspectives in learning, the psychological or individual perspective and the social perspective. In the view of constructivism theory, the learning process evolves as a result of the contribution of an individual students' reasoning to the classroom community and reflectively the influence of the classroom community to the development of students learning as an individual.

Table 2. An interpretative framework for analyzing communal and individual mathematical activity and learning (Cobb et al, 2001)

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Socio-mathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical interpretations and reasoning

Cobb et al. (2001) explained that while the psychological perspective emphasizes on students' various ways of reasoning during their participations in mathematical practices, the social perspective brings out the development of mathematical practices as a result of the classroom discourse. More specifically, the classroom social norms include explaining and justifying solutions, indicating agreement or disagreement, and trying to understand others' reasoning. The socio-mathematical norms are more specific to mathematical activity in which the classroom community come to an agreement of what counts as an efficient mathematical solution and an acceptable mathematical reasoning. Furthermore, classroom mathematical practices focus on particular mathematical ideas.

In this research, the classroom mathematical practices that were expected to evolve in the classroom discussion are about the ideas of (i) zero pairs or neutral pairs, in other words, the sum of any integer and its opposite is zero; (ii) adding two numbers means putting together the two numbers and then finding what is left over after cancelling out the zero pairs; (iii) subtracting means taking away a certain quantity from another quantity; and (iv) if zero pairs are added to a quantity then the value of the original quantity remains unchanged

Therefore, to answer our questions about students' perception of the NNL model in the teaching experiment, we will look at students' perception of the NNL model as an individual and how this contribute to the classroom mathematical practices. Similarly, we will also analyze how the three aspects of the social perspective contribute to the development of students' perception of the NNL model in performing integer addition and subtraction.

To begin with, we will expound how the scoring activity was developed as a context for the emergence of the NNL model. A game was introduced to the students and they played the game in pairs. Every student had to record their score along the game and found the sum at the end of the game. If they win, they earn a positive score, and if they lose, they earn a negative score. The teacher demonstrated the game in front of the classroom by using blue cards and red cards representing positive and negative scores respectively, as it is shown in Figure 3. The students were very excited when playing the scoring game, they came up with different representations when finding the sum of positive and negative integers (see Figure 6).

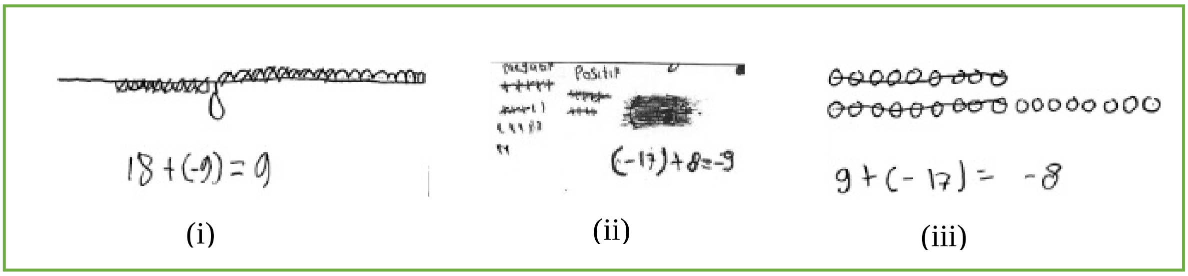


Figure 6. students' representations when solving integer addition

There were three different models identified in solving addition of positive and negative integers. Basically, they were doing the cancellation or neutralization model but with different representations. Based on the observation, some students even already knew the relation of " $\boldsymbol{a} + (-\boldsymbol{b}) = \boldsymbol{a} - \boldsymbol{b}$**".** The scoring activity can help them understand how to find the sum of a positive number and a negative number. However, there was an absence of a classroom discourse in explaining why the relation "$\boldsymbol{a} + (-\boldsymbol{b}) = \boldsymbol{a} - \boldsymbol{b}$" applies.

The next activity was the scoring activity with a modified rule, that is, if they win then they can take away three negative scores, and if they lose then they must take away two positive scores. By modifying the rules, they were challenged to come up with the idea of 'neutral pair'. However, almost all of the students in the classroom did not come up with the idea of adding a neutral pair when they had to take away a quantity that did not exist before. It might happen because the students did not have enough support to find the idea of adding the neutral pairs to their problem, they need physical objects to work with instead of just drawing mounds and hollows.

Dialog 1. A transcription of a classroom discussion on the idea of a neutral pair

The teacher is introducing the subtraction problem involving negative numbers:

T	*: Now I only have one positive score, and then I win, so I have to take away three negative scores.*
S	*: you can't*
T	*: Ok, how should I find out my score now if I only have one positive score and I have to take away three negative scores? Please discuss this problem with your friend.*

About 15 minutes later… (the whole classroom discussion began and one of the students came forward to explain the solution as shown in Figure 7)

T	*: Now you have one positive score.*
S1	*: I add negative three and positive three*
T	*: Why? You only had one positive in the beginning. Why did you add three negative scores and three positive scores?*
S1	*: To make a neutral pair*
T	*: To make a neutral pair (emphasizing)* *Ok, now let's look at an example, if this is positive one and this is negative one, then what is it?*
S1	*: A neutral pair*
T	*: A neutral pair means…? (asking)*
S1	*: zero*
T	*: zero,,, this is a neutral pair which equals to …? (asking)*
S1	*: zero*
T	*: and for this one, how many neutral pairs are there? (pointing to their solution)*
S1	*: three*
T	*: Three neutral pairs equal to zero … Is that what you mean?* *Now, I want to clarify once again, because your friends might want to know, why did you add three neutral pairs, three positives and three negatives?*
S1	*: neutral pair*

Dialog 1 shows the idea of neutral pair was too dominant in the discussion, but the need of adding the neutral pair in the subtraction problem has not been discussed. Thus, there was a missing discussion about the third and the fourth classroom mathematical practices that were expected to evolve during the discussion. It was also found later that the idea of 'neutral pairs' was very powerful that cause students' misunderstandings in handling problems. Once the teacher emphasized the idea of neutral pairs, they started to build their understanding that a neutral pair is a very important idea in solving such problems. As a result, during a small group discussion, they always think of adding a neutral pair to any kind of problems although it was not needed.

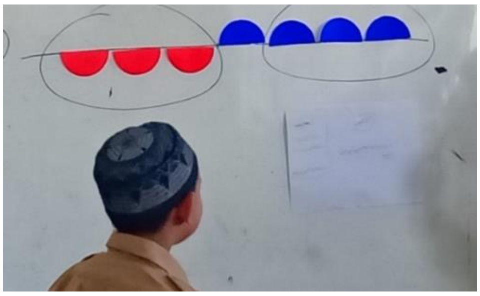
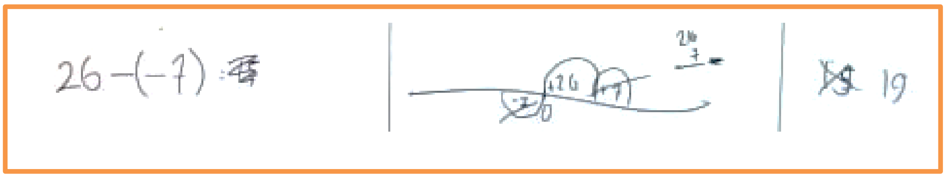


Figure 7. A student is explaining his solution in front of the classroom

Figure 8 (i) shows us one common perception of the NNL model among the students. To solve the problem "46 – 20", the student drew a mound representing 46 together with a neutral pair of positive 20 and negative 20, then she/he took away positive 20 from positive 46. But, there is no need to add a neutral pair of 20 and (-20) to solve this problem, as she/he already had positive 20 that could be subtracted from positive 46. This suggests us that they have beliefs in the classroom that they must add a neutral pair for every subtraction problem. Moreover, it seems that they have to do exactly what the teacher told them to do so, though they could have done it using different strategies. Somehow, this belief could impede their creative and critical thinking in observing and solving a problem situation flexibly.

(i)



(ii)

Figure 8. Students' perception of the NNL model

In another case (Figure 8(ii)), a student was doing a subtraction of minus 7 from positive 26. The student should have crossed only the hollow of negative 7 after adding a pair of 7 and (-7), but in fact she crossed both positive seven and negative seven that did not represent the actual problem. There was also an indication from the picture that the student did a column strategy to find the difference between 26 and 7 which means that she ignored the negative sign of seven, thus she found positive 19 as the result. If the student were going back to the context of scoring, then she might have performed the operation more meaningfully.

Moreover, another student kept repeating the same procedure in modeling the problem using the NNL model. She always adds a neutral pair to every problem, but she did not give meaning to the model. In Figure 9 (ii), the instruction of the problems was to find the difference between -26 and 19. Although she was successful in modeling the problem into the NNL model, but she did not use the representation of two hollows meaning that she had to add the two negative numbers. What she did was subtracted 19 from 26 with an algorithm strategy or column procedure that was not successful. Furthermore, the student's modeling in Figure 9(i) and 9(iii) do not represent the problems whatsoever. It shows how their previous knowledge on the use of the column strategy influenced them in performing integer addition and subtraction. This result is in line with Fuson, et al (1997), that students' concetenated single-digit conception of numbers while proceeding the column strategy could lead to various errors. From the Figure 9, we could see that the student ignored the negative sign of the number and proceed as if the negative sign did not exist.

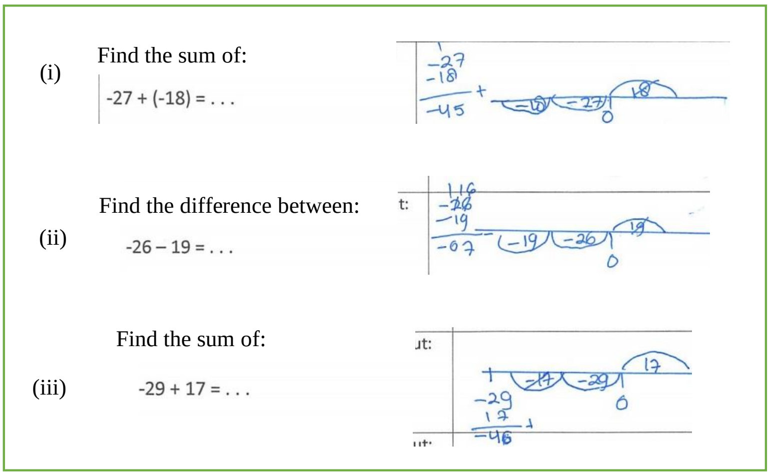


Figure 9. Another student‘s perception of the NNL model

On the other hand, there were also some evidences that show us the benefit of using the NNL model in performing integer addition involving negative integers. As an example, in Figure 10, the student can correct his mistake when finding the sum of 4 and (-6). By cancelling out the same number of mounds and hollows, the student realized that his first answer was a mistake, then he wrote down the correct answer below the first one. The benefit of using the NNL model together with scoring context is that students can always refer back to the contextual situation when they have diifficulties. They can record their strategy on an empty number line and see how many mounds or hollows must be added or taken away. The NNL model allows students to work with big numbers with mounds and hollows signify positive and negative quantities respectively. While in the neutralization model, both positive and negative numbers are signified with circle with similar shape, they differ only when the circles are filled with different colors or signs.

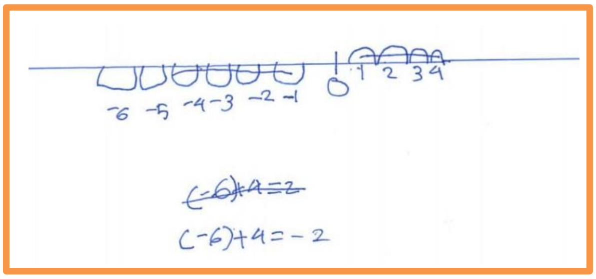


Figure 10. The NNL model in an addition problem

Some factors that might contribute to students‘ achievement were also the lack of classroom discussion where the three aspects in the social perspective of learning did not establish in the classroom. In most of our classrooms in Indonesia, students do not get used to reasoning and justifying their opinions (Prahmana, Zulkardi, & Hartono, 2012). Most of the students seems to put themselves as a listener instead of as a speaker in sharing their ideas. This might happen because they are afraid of making mistakes. The students need a strong encouragement to change their beliefs and to develop positive classroom cultures such as sharing ideas, understanding each other, and proposing different ideas, as students' contributions and interactivity are important aspects in the theory of RME and other constructivism approaches. Therefore, it needs a serious attention for teachers and researchers to build positive classroom cultures that can support the development of students‘ learning both as an individu and as a classroom community.

The findings suggest that the designed activity, the instructions given, and the tools provided must be taken into consideration in developing the hypothetical learning trajectory. The problem happened in the subtraction has been conjectured, but the anticipations were not made clearly. If it seems that the students have not developed the need of adding neutral pairs on the NNL model for solving subtraction problems, then the teacher must give an ample space for students to really get involved in the problem and come up with meaningful ideas. As it was stated in Lesh & Doerr (2000) that the teacher plays a critical role in a classroom to create the need for students in sharing their tools and representations, creating and nurturing diverse approaches, also creating meaningful and powerful models through classroom discourse. Moreover, modeling involves the interaction among three types of systems: (a) internal conceptual system, (b) representational systems that function both as externalization of internal conceptual systems and as internalizations of external systems, and (c) external systems that are experienced in nature, or that are artifacts that were constructed by humans. (Lesh & Doerr, 2000). If the interaction among the systems is absent, then the externalization of internal conceptual system will not emerge.

CONCLUSION

The purpose of the present study was to determine students' perceptions of the neutralization on an empty number line 'NNL' model when they are dealing with additions and subtractions of integers. The finding has shown that students found it helpful when they were working with addition problems, although some difficulties were apparent on the subtraction problems. The classroom mathematical practice about the need of adding a neutral pair to a subtraction problem was not developed very well in the classroom. Therefore, some adjustments and revisions must be made related to the hypothetical learning trajectory, particularly on developing the idea of adding a neutral pair in a subtraction problem.

The findings of this study suggest some considerations should be made. First, the need of adding a neutral pair in solving subtraction problems must be clear to students. Although the context has provided a meaningful situation, the absence of classroom discussion on the need of using a neutral pair could be misguided. Second, there must be an ample space for students to develop their thinking, manipulate tools, and collaborate with others to come up with a meaningful representation for them. Third, a teacher should continuously build constructive classroom cultures to improve students' contribution, responsibility, and understanding of their own roles for the development of classroom mathematical practices.

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PROSPECTIVE MATHEMATICS TEACHERS' COGNITIVE COMPETENCIES ON REALISTIC MATHEMATICS EDUCATION

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Abstract

Realistic Mathematics Education (RME) is based on the idea that mathematics is a human activity; and its main principle is to ensure the transition from informal knowledge to formal knowledge through contextual problems. This study aims at revealing how RME is configured in the minds of prospective mathematics teachers and their cognitive competency in that sense. For that purpose, at the end of the process, in which the approaches used in mathematical education including RME are examined and interpreted, 32 prospective teachers were asked various open-ended questions. Moreover, they were expected to pose contextual problems that could be used in RME. After analysing the obtained data via qualitative research techniques, it is seen that the majority of the prospective teachers possesses theoretical knowledge on RME. However, it is also observed that their ability to present its differences and similarities with other approaches and to pose contextual problems suitable to RME has been decreased.

Keywords: Realistic Mathematics Education, Prospective Mathematics Teacher, Cognitive Competency, Contextual Problem

Abstrak

Pendidikan Matematika Realistis (PMR) didasarkan pada gagasan bahwa matematika adalah aktivitas manusia; dan prinsip utamanya adalah memastikan transisi dari pengetahuan informal ke pengetahuan formal melalui masalah kontekstual. Penelitian ini bertujuan untuk mengungkapkan bagaimana PMR dikonfigurasi dalam pikiran calon guru matematika dan kompetensi kognitif mereka dalam pengertian itu. Untuk tujuan itu, pada akhir proses, 32 calon guru ditanyai berbagai pertanyaan terbuka, yang mana pendekatan yang digunakan dalam pendidikan matematika termasuk PMR diujikan dan ditafsirkan. Selain itu, mereka diharapkan untuk dapat mengemukakan masalah kontekstual yang dapat digunakan dalam pendekatan PMR. Setelah menganalisis data yang diperoleh melalui teknik penelitian kualitatif; terlihat bahwa mayoritas calon guru memiliki pengetahuan teoritis tentang PMR. Namun, pengamatan lebih jauh terkait kemampuan mereka untuk menyajikan perbedaan dan persamaan antara pendekatan PMR dengan pendekatan yang lain dan pengajuan masalah kontekstual yang sesuai dengan PMR, telah berkurang.

Kata kunci: Pendidikan Matematika Realistik, Calon Guru Matematika, Kompetensi Kognitif, Masalah Kontekstual

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The studies conducted in mathematical education, which is highly impacted by theoretical and methodological framework of psychology (Sriraman & English, 2010), have paved the way for the innovative ideas on learning including new learning outcomes, new types of learning processes, and new instructional methods that are both wanted by society and currently stressed on in psychological and educational theory (Simons, Van der Linden, & Duffy, 2000; Yackel, Gravemeijer, & Sfard, 2011). These ideas occur in a manner that is totally different from behavioral approach studies (Aubrey & Riley, 2016) which traditional education methods are based on. And their approaches can be fundamentally categorized as cognitive approaches and constructivism. Cognitive theories focus on the conceptualization of one's learning process and emphasize how information is received, organized, stored and retrieved in one's mind (Davis, 1990; English, 1995; Ertmer & Newby, 1993; Jonassen, 1991; OECD, 2003). Despite being viewed as a branch of cognitivism as a result of expressing learning as a cognitive activity (Ertmer & Newby, 1993), constructivism is based on the idea that the mind filters the input received from life in order to create its own reality (Jonassen, 1991a) and the information is not simply received but actively constructed by the person himself/herself (Driscoll, 2005).

The different perspectives in education led to radical changes in objectives and nature of mathematics as well as many other areas. This change in mathematics education particularly emphasizes that mathematics learning can occur only when students discover things through actual experience and structured problem-solving procedures and finally by means of interaction process among students and/or teachers (Kwon, 2002). In addition to this, many countries aiming at raising productive and innovative individuals in the 21st century made radical changes in their curriculum with the purpose of attaching much more importance to the quality of their mathematics education. Essentially, various countries such as UK, USA, Singapore, Finland and Australia believe that mathematics enables the improvement of thinking capacity as well as its use in real life in a critical, creative and logical manner. These countries have begun to pursue the goal of raising individuals with problem solving, reasoning, and connecting skills, as well as productive disposition and conceptual comprehension (Cai & Howson, 2013; Stacey, 2005). It is believed that among these skills, the problem solving is the key skill in the 21st century (Barell, 2010). Moreover, it is asserted that the teaching process can be implemented for students through case-based or problem-based learning methods by presenting authentic problem situations called as real-world problem (Barell, 2010; Barrows, 1986). Conceptual comprehension, on the other hand, is a process that takes place in coordination with such skills (Wu, 1999) and involves the understanding of concept, relation and operation (National Research Council, 2001).

This new perspective shares many common points with the theoretical perspective of Realistic Mathematics Education (RME) (Bray & Tangney, 2016; Kwon, 2002) as discussed by Freudenthal (1973). RME focuses on mathematization that is actualized through the re-invention of formal mathematics. In RME, the student is informally guided by the teacher in a class-interaction, thus is encouraged to utilize self-developed models in order to solve and interpret the experientially real contextual problems (Dawkins, 2015; Gravemeijer, 1994, 1999; Treffers, 1991). In this regard, it is believed that RME contributes to the formation of the targeted skills such as problem solving, reasoning and connecting; as well as to conceptual comprehension (Borko & Putnam, 1996; Hadi, 2002).

Teachers' Pedagogical Knowledge (PK), which includes knowledge the teachers have on curriculum and teaching methodology as well as on how to teach them (Lianghuo, 2014), and Pedagogical Content Knowledge (PCK) that is referred to as "special amalgam of content and pedagogy" (Shulman, 1987), are important determinants of instructional quality that impact students' learning gains and motivational development (Baumert & Kunter, 2013). And also they have vital importance in gaining the targeted skills (Borko & Putnam, 1996). This situation causes many results, which could affect teacher education and causes major changes in prospective teachers' knowledge (Battista, 1994; Putnam & Borko, 2000; Szydlik, Szydlik, & Benson, 2003). However, teachers' efforts to change their PK and practices are usually not reflected on their tendencies at skill-based applications in the targeted curriculum (Anderson, White, & Wong, 2012). Although, it is difficult to evaluate teachers' PCK (Beswick & Goos, 2012), understanding the pedagogical theories underlying the radical changes about mathematics education and detecting effective teaching strategies are significant in terms of being able to professionally assessing teachers (Sowder, 2007). Furthermore, besides prospective teachers' knowledge of general and specific approaches; what is essentially important is whether they comprehend when, where, how, and why they should use these approaches, rather than their variety (Feiman-Nemser, 2001; Tsamir, 2008). However, many prospective teachers are not ready to use them the way they should (Herman & Gomez, 2009). Therefore, it is believed that it is important to know to what extent prospective teachers have cognitive competences about these approaches, in order to be able to choose the approach that will be used depending on place and time and to implement them in a manner that is suitable for the purpose.

In a general sense, the study aims at identifying prospective teachers' cognitive competency regarding the approaches before moving onto practice from theory. In this regard, the study mainly focuses on RME in terms of the approach and the answer to the question of "How are the cognitive competencies of prospective mathematics teachers related to RME?" has been sought. Thus, the sub-problems of "How do prospective mathematics teachers explain RME and its implementation? How do they interpret the similarities and differences with other approaches? How a contextual problem do they pose in compliance with the theoretical structure of RME?" have been discussed.

In the below-mentioned theoretical framework, firstly the definition and primary elements of RME have been explained, and information related to its implementation has been presented. Subsequently, the criteria used in determining the cognitive competency and the levels of cognitive competency have also been explained.

THEORETICAL FRAMEWORK

Realistic Mathematics Education (RME)

RME, which is based on the idea that mathematics is a human activity (Freudenthal, 1973) and the idea that student achieves formal mathematics knowledge by using his/her informal knowledge by means of re-inventing under the guidance of a teacher (Treffers, 1991) has a significant place in the studies conducted in the field of mathematical education (e.g. Barnes, 2004; Beswick, 2011; Bray & Tangney, 2016; Makonye, 2014; Rasmussen & King, 2000; Streefland, 1991). In this approach, the informal knowledge in real life is transformed into formal knowledge after being abstracted and is again associated with real life as a result of mathematical implementations (De Lange, 1996). Transformation is believed to be achieved by contextual problems that are experientially real to the students (Gravemeijer, 1999). The process of conceptual mathematization in RME has been given in Figure 1. In this process, mathematical concepts start to develop from the real world and ends with the reflection of the solution back to the real world.

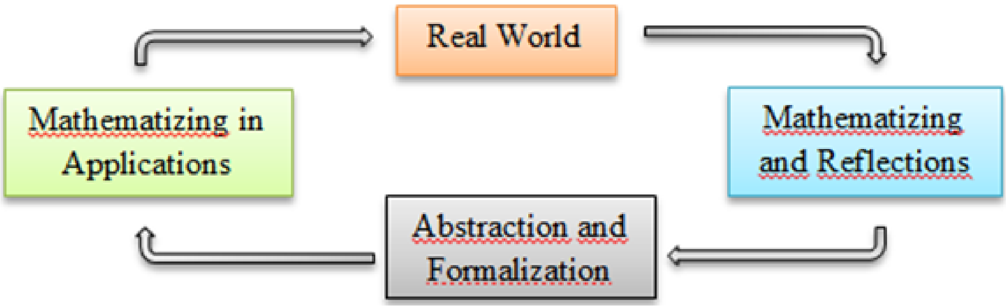

Figure 1. Conceptual mathematization (De Lange, 1999)

The contextual problems used in RME are mathematical problems presented in the real life situations that children are familiar with, through stories that are fictionalized from the real world. These problems can be a word problem, a game, a drawing, a newspaper clipping, a graph or the combinations of such elements. At the same time, a pure mathematical problem can also be a contextual problem. However, the main point here is to what extend the problem would fulfill the criteria of being experientially real or authentic; and thus would provide a concrete orientation towards a new concept/skill; and would also allow utilization of prior knowledge (De Corte, 1995; Doorman, Drijvers, Dekker, van den Heuvel-Panhuizen, de Lange, & Wijers; 2007; Gravemeijer, 1999).

Gravemeijer (1994, 2001) emphasizes the necessity for three main elements while designing education in RME, which includes the following:

1. Guided reinvention through progressive mathematization
2. Didactical phenomenology
3. Self-developed or emergent models

Guided reinvention is based on configuring and organizing problems in order to discover rules by revealing mathematical factors in a problem. This research, which is conducted with a strong intuitional component, is considered to be the discovery or reinvention of mathematical conception (De Lange, 1987; Freudenthal, 1973, 1991). In this process, the teacher should design the roadmap to enable students to learn correctly and should provide the students with the opportunity to experience a process that is similar to the discovery process of mathematicians (Gravemeijer, 1994; 2001). By doing so, students have the opportunity to obtain knowledge by themselves (Freudenthal, 1991;

Gravemeijer, 1999; Yackel & Cobb, 1996) and they mathematize the contextual problems by solving them (Treffers, 1987; 1991).

Guided reinvention through progressive mathematization can be considered as a two-stage process: horizontal and vertical mathematization. Horizontal mathematization is where one uses informal strategies by schematization in order to define and solve contextual problems, in other words transforming real life problems into mathematical problems. Vertical mathematization is to abstract the conception in the world of symbols and solve the problem by adopting different models or find the relevant algorithm by using mathematical language in the light of informal strategies (Freudenthal, 1991; Gravemeijer, 1994; Treffers, 1987, 1991; Van den Heuvel-Panhuzen, 2003). In Figure 2, horizontal and vertical mathematization is described.

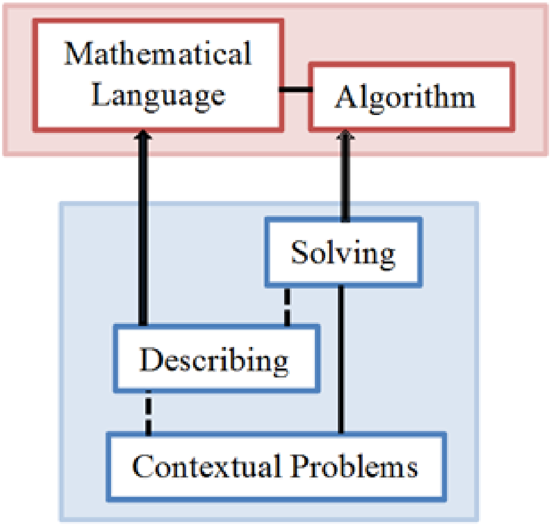

Figure 2. Horizontal and vertical mathematizations (adapted from Gravemeijer, (1994))

Didactical phenomenology requires working with phenomenon that are meaningful to students in the process of learning mathematics, can be organized by students, are stimulating for the learning process and meet four functions including concept formation, model formation, applicability and practice (Gravemeijer, 1994, 2001; Treffers & Goffree, 1985).

Self-developed or emergent model bridges informal knowledge of students with formal knowledge while solving a problem. At the beginning, the student develops a model, which gradually becomes a dynamic and holistic model compatible with his or her own mathematical thinking after generalizing and formalizing processes (Gravemeijer, 2001; Treffers, 1991). Thereby, at the end of this process, which is named as the transformation from 'model-of' to 'model-for', the student obtains a model that enables him/her to achieve mathematical reasoning (Gravemeijer, 1999).

Cognitive Competency

Cognition refers to the variables with respect to the kind and quantity of information, and the classification of relations among the variables of information (Kraiger, Ford, & Salas, 1993).

Cognitive domain in learning contains learnings, in which person's mental sides are in the foreground (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956; Bloom, 1994). Cognitive competency is a psychological construction which cannot be directly observed but can be inferred from the behavior or performance of an individual on content-relevant tasks (Wang, 1990). Researchers have asserted various taxonomies in order to evaluate this kind of learnings. Some of these taxonomies are Bloom's Taxonomy (Bloom, et al. 1956), Revised Bloom Taxonomy (Anderson & Krathwohl, 2001), Barrett's Taxonomy (Barrett, 1976), Pearson-Johnson Taxonomy (Pearson & Johnson, 1978), Webb's Depth of Knowledge Levels (Webb, 1997, 1999) and Program for International Student Assessment (PISA)'s Competency Levels (OECD, 2003). Bloom's Taxonomy among such taxonomies, is the first and most widely accepted classification (Granello, 1995) in the subject of cognitive abilities and educational objectives used in education. In a similar way, PISA is implemented and reported in many countries and regarded as a new approach in national and international evaluation (Sadler & Zeidler, 2009).

Categories are classified in Bloom Taxonomy from simple to complex and from concrete to abstract acting as prerequisite for one another: Knowledge refers to a person's acts of remembering such as recognizing, expressing when asked, or repeating the characteristics of any object or event from his/her memory; Comprehension means interpreting, assimilating, and expressing the obtained targets without losing their meanings at knowledge level; Application, is when a person implements knowledge by solving the problem in a new situation by making use of his/her learning at knowledge and comprehension level; Analysis refers to cognitively differentiating among the items of a pattern or knowledge in terms of their relationships and organizations; Synthesis means bringing together and creating a whole of the items in such a way that they would bare characteristics such as innovation, originality, and creativity based on certain relationships and rules. Evaluation is when a person decides whether the products created are competent enough by stating justifications (Bloom, et al. 1956; Bloom, 1994; Krathwohl, Bloom, & Bertram, 1973; Krathwohl, 2002).

In PISA, OECD (2003) has determined three levels for detecting the competency levels of students in order to define their cognitive activities. These levels include reproduction, connection, and reflection. In reproduction level, already known contents, previously used knowledge, standard algorithms, and elementary formula are used and basic operations are conducted. In connection level, less commonly known contents are interpreted and explained; systems, representation of which are different, are obtained through association; and necessary strategies are chosen and used for extraordinary problem solving. In reflection level, in which comprehension is required; reflection, creativity, and knowledge necessary for solving complex problems are associated; observed results are generalized and justified; and abstraction is carried out.

The association between Bloom's Taxonomy and PISA's Competency Levels is indicated in Figure 3.

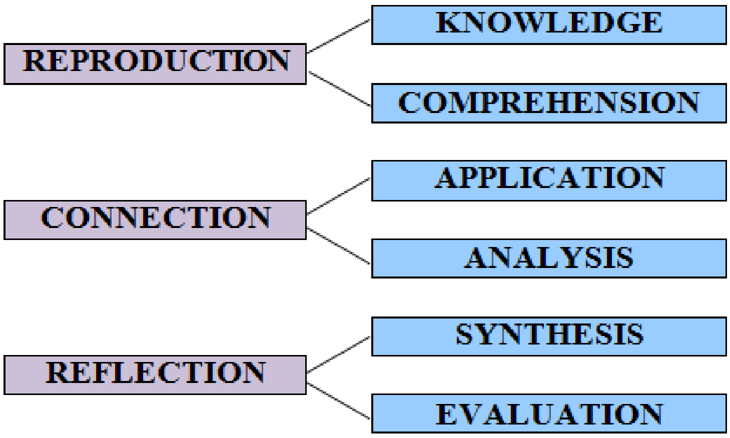

Figure 3. The relations of cognitive competency (OECD, 2003; Bloom, et al. 1956)

METHOD

This research is a qualitative study that has been conducted with the purpose of researching the cognitive competencies of prospective mathematics teachers related to the approaches used in mathematics education. Research design is a case study, which has been evaluated as a case of RME.

This study has been conducted in the faculty of education of a state university in the Black Sea region of Turkey with 32 prospective mathematics teachers, 20 of which are female and 12 of which are male. These prospective mathematics teachers are all senior students studying their fourth year of their 5-year education plan. The majority of these prospective mathematics teachers have successfully completed fundamental education courses such as introduction to education, educational psychology, guidance, theories and approaches of learning and teaching, curriculum development and instruction besides pure mathematical courses. All participants were informed about the process of the research. They volunteered to attend the research and gave the researcher the permission to use the data acquired from their interpretations and their posed problems in the manuscript.

The prospective mathematics teachers had been taking Methods of Teaching Mathematics lesson during the time this study was conducted. In the lesson instructed by the researcher and lasted for four hours a week, some approaches about mathematical education (e.g. Ausubel's meaningful learning approach (Ausubel, 1963), Freudenthal's RME approach (Freudenthal, 1973), Bruner's discovery learning approach (Bruner, 1961) have been elaborately examined and their implementation in teaching has been interpreted and discussed. In this research, it has been aimed to reveal cognitive status of prospective teachers about RME at the end of their 12-hour experience.

In this study, two sessions that were suggested by Selter (1997, 2001) and grounded on the study of Zulkardi (2002) about how preservice teachers have developed the RME learning environment, were taken into consideration: understanding the new approach by providing a theoretical overview and by actually doing mathematics-the mathematical component, designing instructional materials-the didactical component. Thus, cognitive competencies of prospective teachers in the theoretical sense were focused on and the components of the sessions were discussed as "overview of RME theory, doing mathematics and designing contextual problems" (Zulkardi, 2002).

During 'overview of RME theory' session, firstly, the prospective teachers were provided with theoretical information on RME, and then they conducted activities about the approach. Afterwards, the implementation process and effectiveness of the activities were discussed. The prospective teachers stated their comments on the elements of the transition of RME application from real life to mathematics such as the characteristics of contextual problems applicable to horizontal mathematization, the modelings created during its solution, and roles of teacher and students. Then, they continued to discuss about vertical mathematization. During doing mathematics session, prospective teachers were treated as learners while the researcher performed as a teacher and it was aimed that they learn how to teach using with RME. In Appendix, there are some examples of the contextual problems used in the applied activities during these sessions (Altun, 2011; Fauzan, 2002; Feijs, 2005; Wubbels, Korthagen, & Broekman, 1997). During the second session, for instance, Feijs's (2005) Grand Canyon Problem (problem 4 in appendix) developed in line with the construction of RME learning environment was implemented with prospective teachers. The prospective teachers formed 3- person student groups in real classroom environment and two of the prospective teachers from the group sat on one desk, whereas the other one sat on the desk that is parallel to the other desk. On the paper, which was hung down these parallel desks in order to create an imaginary river in the gap between the two desks, the points where the vision lines of these two people have been drawn with the help of a third person and canyon tables activity have been implemented. In the practice process, discussions were held on how they identify situations that can be seen or not seen by a person and how they perceive the vision lines from these situations. After the implementation, the situation of treating steepness of vision line as phenomena was examined according to the didactical phenomenology criterion where the main idea is familiarity and appeal for them. At this point, the mathematical content, which requires the ratio of a right triangle formed by an angle, was associated with the concept of steepness, and the state of abstraction resulting in the concept of tangent was also discussed. In addition, the self-developed or emergent models' criterion of the approach was also discussed, considering the models that they can suggest for solution of the problem.

During designing contextual problem session, it was aimed to relate the context and the concept in a learning environment. In this manner, prospective teachers could learn how to design contextual problems to use in RME. So, they were asked to pose a contextual problem, for which they were given an adequate period of time and which would enable them to conduct horizontal mathematization.

After these sessions, they were asked some open-ended questions about RME and the problems they posed; moreover, they were expected to write down their opinions about these questions.

The questions asked to the prospective teachers are as follows:

1. What is RME? Please explain.
2. How is RME applied? How is this approach different from the other approaches you have learned? Please explain.
3. Pose a (contextual) problem that you could apply in RME, and explain for which conception's mathematization, this problem will be used. Please, evaluate the approach, on which the problem you posed will be used, by considering different approaches.

The written answers received from the prospective teachers and their contextual problems were analyzed, through the descriptive analysis method so as to determine their cognitive competencies about RME. In descriptive analysis, primarily a framework is established for analysis; the data are processed in accordance with thematic framework; findings are described and interpreted (Patton, 1990). Due to that reason; PISA Competency Clusters (OECD, 2003) and Cognitive Domain Taxonomy (Bloom, et al. 1956), which are displayed in Figure 3 below in order to reveal the cognitive competencies of prospective mathematics teachers, were used as the foundation for the data analysis. In this process, the relevant categories and data related to sub-categories that belong to each category have been processed and common themes were determined (Creswell, 1998; Patton, 1990). In the process of analyzing the data, the data collected from the first part of the question asked to the prospective teachers have been processed in the reproduction-knowledge category; the data collected from the second part of the first question have been processed in the reproduction-comprehension category; the data obtained from the first part of the second question have been processed in the connection-application category; the data collected from the second part of the second question have been processed in the connection-analysis category; the data retrieved from the first part of the third question have been processed in the reflection-synthesis category; and the data collected from the second part of the third question have been processed in the reflection-evaluation category. Furthermore, the sub-categories within each category have been composed as adequate ones in and deficient (or incorrect) ones in the related category.

In order to ensure the reliability of the study, two researchers, who have completed their PhD in the field of mathematics education and are experts in qualitative studies, firstly individually analyzed the data then it was discussed among the members until they reached a consensus on overall categories, sub-categories and themes. So, the use of multiple experts, as well as the use of their evaluations has led to conformability of the data. The credibility has been increased through the data obtained from the contextual problems consisting of the written answers given by the participants to the open-ended questions. For transferability, in order to help applying the findings to the other contexts, the description of the context was delivered in a clear and detailed manner. In other words, the participants, the approaches examined and the activities conducted in the classroom were elaborated. Besides, in order to ensure that the results can be conveyed into similar media, the obtained findings have been supported with the quotations and detailed descriptions have been made (Berg, 2001; Lincoln & Guba, 1985; Yıldırım & Şimşek, 2005).

RESULTS AND DISCUSSION

The contextual problems and answers given by 32 prospective teachers participating in the study were examined and evaluated; furthermore, common themes were created based on their cognitive competencies. The categories and sub-categories created for determining common themes are displayed in Table 1. Moreover, frequencies of categories and sub-categories are also demonstrated in that table. The findings on cognitive competencies of the prospective teachers have been explained within each category by directly quoting them.

Table 1. Frequencies and percentages of categories and sub-categories of prospective teachers' competencies

Category		Sub-category	Frequency	%
Reproduction	**Knowledge**	Having knowledge about RME	28	87.5
		Deficient or incorrect knowledge about RME	4	12.5
	Comprehension	Making explanation or interpretation about RME	27	84.375
		Deficient or incorrect comprehension about RME	5	15.625
Connection	**Application**	Applying knowledge about RME	26	81.25
		Deficient or incorrect application about RME	6	18.75
	Analysis	Analyzing knowledge about RME	18	56.25
		Deficient or incorrect analysis of RME	14	43.75
Reflection	**Synthesis**	Posing an original (or an unfamiliar) contextual problem	11	34.375
		Deficient or incorrect posing about contextual problem	21	65.625
	Evaluation	Making valuation by passing a judgment	5	15.625
		No answer or deficient evaluation	27	84.375

Reproduction

Knowledge

In the first part of the 1st question, prospective teachers were asked to answer to define RME approach. Thereby, it was aimed to reveal the level of knowledge of the prospective teachers. The 28 (%-87.5) of 32 prospective teachers have correctly answered to the question 'what is RME?'. An example from the answers given by the participants is stated below:

> *"We come across with mathematics in many parts of our everyday lives and we have to do the math. RME starts with this idea. In RME, a problem about real life are presented to students before teaching them the subject, and students solve this problem by making use of their previous knowledge through their own models. Thus, the student achieves a mathematical expression"* (3rd PT).

It was observed that 4 (12.5%) of the participants have deficiently or incorrectly defined RME. There were no participants, who did not answer to the question. One example from one of the prospective teachers, who deficiently answered to the question, is as follows:

> *"Enabling them comprehend mathematics by giving them examples from life. How can we make life easier by using mathematics in our life. This is based on this"* (13th PT).

The prospective teacher, who deficiently answered the question, mentioned the association of the approach with real life and said that mathematics could be learned through real life examples. However, the fact that she did not mention contextual problems, which are considered to be necessary for the transition from real life to mathematics and is the focus of the approach, demonstrates that the prospective teacher does not have adequate knowledge about the approach.

The answer received from one of the prospective teachers, who incorrectly answered the question, is as follows:

> *"RME is an approach that could help us solve the problems we may face in life. It is teaching students a subject by giving examples of the events we may experience in our lives about the subject"* (31th PT).

Even though the prospective teacher talks about real life problems, his/her '...teaching students a subject by giving examples of events from life...' statement indicates that he/she has inaccurate knowledge on the roles of the student and teacher and also on the application of the approach.

Recalling the knowledge is vital in terms of making learning meaningful and solving a problem, thereby this knowledge can be used in more complex assignments (Anderson & Krathwohl, 2001). Moreover, Ayer (1936) discusses that when something is defined, it should contain the definition itself or its synonymous or expressions with equivalent meanings. Examining the answers received by the prospective teachers; majority of the prospective teachers have knowledge on RME.

Comprehension

In the 1st question, which was asked in order to understand whether the prospective teachers comprehended RME or not, they were expected to explain the approach. Their explanations suggest that 27 (84.375%) of the participants have correctly comprehended RME. One of the teachers, who correctly comprehended RME, explained it as the following:

> *"The mathematical conception, which is intended to be taught to the student, is taught through a problem that could be experienced in real life. The student tries to solve this problem, which is created by preparing the necessary setting, through his/her own knowledge and modelings. In a sense, how mathematicians discover mathematics in the events, which they come across with in real life, is experienced by the student. They are introduced to mathematical knowledge by solving the problem. Thus, this problem should be compatible with real life and should enable students reach mathematical conceptions"* (24th PT).

The explanations suggest that 5 (15.625%) of the prospective teachers didn't comprehend RME. One of these prospective teachers (16th PT) could not adequately comprehend it even though he/she correctly defined it. The other four prospective teachers are in the category of those who incorrectly or deficiently answered the question at the knowledge level. The prospective teacher, who could not comprehend the conception, explained it as the following:

> *"In this approach, real life problems are asked to the student. We solve the problem and teach them the subject by saying this problem is its application in mathematics"* (16th PT).

The statements of the prospective teacher, suggest that it should be begin with real life problems. According to the approach, the problem should be solved through a model by the student himself/herself by discussing it with his/her group friends. However, his/her statements about the teachers' solving the problem for students and conducting the mathematization indicate that the prospective teacher incorrectly comprehended the approach.

If we consider explaining as conveying the meaning to the other person and interpretation as realizing broad understandings (OECD, 2009); it is possible to say that both actions refer to comprehension. Thus, the answers given via explanations or interpretations indicate that the majority of the prospective teachers could significantly comprehend the approach.

Connection

Application

Examining the answers received by the prospective teachers for the first part of the 2nd question about the application of RME, 26 (81.25%) of them correctly answered to the question about the application of this approach. One of the prospective teachers explains the application of approach as follows:

> *"Firstly, a real life problem, which could enable student to reach a conception at the end, is given to the student. The student tries to actively solve this problem through the model he/she desires by discussing. The teacher helps them as a guide. At the end, the*

> *student reaches mathematics through the model he/she created and is introduced to the desired conceptions. This part, in other words transition from real life to mathematics is called horizontal mathematization. Afterwards, the operations conduced within mathematics are called vertical mathematization. For example, everybody solved the snake problem we applied in classroom through their own desired models and discussion; and everybody was introduced to geometrical progression without even realizing. While doing so, we discussed and made associations in the classroom; and the example was modelled through expressions such as exponential numbers or common factors. This part was horizontal mathematization. Then we continued with vertical mathematization through conducting operations about geometrical progression. What is important in the approach is that students create their own models, discover mathematics through teachers' guidance, and obtain knowledge"* (18th PT).

The above-quoted prospective teacher pointed out the three main factors, which are required in RME applications and stated by Gravemeijer (1994, 2001) including reinvention through progressive mathematization, didactical phenomenology, and self-developed or emergent models. The explanations suggest that he/she correctly evaluated the RME applications conducted in classroom; and coordinated between the approach and its application.

It was observed that 6 (18.75%) of prospective teachers incorrectly evaluated the application of RME. 5 of these prospective teachers fall under the category of those who incorrectly comprehended the approach; while 1 of them is in the category of those who correctly comprehended the approach. The prospective teacher, who correctly comprehended the approach but made incorrect statements about its application, made the following statement:

> *"The student is asked real life problems. Afterwards, the student tries to solve this. For example, in the problem about Canyon, various lines were drawn in order to see the river. From what perspective these lines should be coming and the distance between the lines and the river were discussed and tackled. We thought of their ratios and associated them with tangent conception. In conclusion, the teacher makes associations in order to enable the student to solve the problem; and the student solves the problem by bringing them together"* (30th PT).

In the process of transition from real life to mathematics which is called mathematization, full process of transmitting to mathematical model from original problem situation is referred as modelling (Blum & Niss, 1991). However, in RME, modelling should be conducted by the student (Gravemeijer, 2001; Treffers, 1991). The statements made by the prospective teacher, which expressed that associating should be done by the teacher while solving the problem, suggest that he/she is unable (or incorrectly) coordinate between the approach and its in-class application.

Analysis

In the second question asked to prospective teachers, they were asked to compare RME to other approaches besides discussing about its application. 18 (56.25%) of the prospective teachers correctly analyzed the approach and evaluated its similarities and differences with other approaches. Some of the comments they have made are as follows:

> *"The student reaches at a mathematical conclusion through his/her own knowledge by himself/herself. Since student is in the center of learning and the teacher guides; it is similar to discovery learning. However, in discovery learning, it is not necessary to begin with real life problems. In other words, a student can learn through discovery in a setting, which could enable generalizing a conception, and conduct applications after discovering the conceptions. In fact, in general, this application and constructivist approaches are quite different from traditional education. All of them are focused on the process and student-centered. The students themselves make sense of it. The main difference in RME is the environment is a stimulant and real life problems are what we begin with"* (3rd PT).

Their explanations suggest that 14 (43.75%) of the prospective teachers are inadequate at analyzing the approach, thus their comparisons of approach to other approaches are sometimes incorrect and sometimes deficient. 6 of these prospective teachers are those, who already incorrectly applied the approach, and other 8 are among those who correctly applied it. One of the students, whose comparisons of other approaches are deficiently, explains it as:

> *"I think there is not a huge difference between them. All of them are student-centered. Some of them create a problem while some of them do the same through an activity instead of a problem. In conclusion, they all end the same"* (14th PT).

Constructivism, which deals with how knowledge emerges and is based on cognitive psychology, takes the relationships among complex problem solving and cognitive structures and behaviours as basis (Noddings, 1990) and is based on the idea that knowledge is not directly taken from the teacher but structured actively by the learner (Lesh, Doerr, Carmona, & Hjalmarson, 2003; Von Glasersfeld, 1987). RME fundamentally share similarities with various learning and teaching theories of constructivism, which can be considered to be a theory of knowledge. In discovery learning, which is founded by Bruner (1961) and is one of the constructivist approaches, the learner is not informed of target knowledge or the conception, and learning setting is prepared by ensuring proper circumstances (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011). Learning occurs through the assumptions made by organizing patterns and examples, which are based on conceptualizations and generalizations from simple to complex; and conducting researches in an intuitive and systematic manner (Jacobsen, Eggen, & Kauchak, 1993) furthermore, in this process teacher's duty is to guide students (Hammer, 1997; Svinicki, 1998). In RME, even though the student is not informed of target knowledge; basic differences in main principles and learning setting play a major role in differentiating approaches form one another. Primarily, basic differences in RME are based on the idea that human knowledge structures the knowledge and mathematical intuitions and procedures are invented and not discovered (Freudenthal, 1973). Thus, even though 'guided reinvention', which is one of the main principles of RME, stresses on guidance in the process; it is still different from the other approaches from that perspective. Moreover, the meaning of guidance in RME refers to the facilitative role of teacher for reinvention while enabling scaffolding instead of making explanations to the students (Hamzah & Bustang, 2014).

Essentially, RME and social constructivism, which focuses on the impact of the setting, are concerned with whether student is active; creativity; problem solving; reality of contexts; mathematical reality; and emergence of mathematical objects. However, in RME it is necessary to not only motivate students with everyday life contexts; but also to associate with experimentally real contexts and use them as the starting point for progressive mathematization (Gravemeijer, 2001).

Examining the views of the prospective teachers on differences of RME approach from other approaches; the fact that more than half of them mentioned these basic differences suggests that they can analyze at connection-building level in terms of cognitive competency.

Reflection

Synthesis

The prospective teachers were asked to pose a contextual problem (4th question) that they could use in RME, in order elaborately examine the cognitive status of the prospective teachers about RME. Thereby, above the theoretical and practical knowledge they have about the approach; it was aimed to find out how they bring this knowledge together in a different manner and integrate it with associations. Examining the problem they posed, 11 (34.35%) of the prospective teachers produced a contextual problem, in which they could apply the approach; whereas 21 (65.625%) of them did not pose such a problem.

Some of the contextual problems are stated below: When these problems are examined, it becomes visible that the students can solve these problems and mathematize new conceptions through the models they created.

> *"Ayşe will meet her friends and go to a movie this weekend. However, she cannot decide what to wear. When she looks at her closet she sees alternatively blouses, skirts, pairs of shoes, and socks with different colors. How many different outfit combinations can she create by doing these pieces in her closet?"* Conception: Counting rule-factorial (2nd PT).

"Uncle Ahmet wants to cover his rectangular garage with square tiles. He measures the size of his garage and goes to buy the tiles from a construction market. However, Uncle Ahmet believes that the less number of tiles, the less it will cost him. For that job, what dimension of tiles Uncle Ahmet should buy"? Conception: The greatest common factor (6th PT).

"A medical aid helicopter taking off from the emergency aid base brings aid to those injured in a traffic accident. If the pilot knows the distance from emergency aid base to the hospital, the distance from the scene of accident to the hospital and the angle between them, how can he find the distance between the emergency aid base and the scene of accident?" Conception: Cosine Law (12th PT).

The two problems stated below, which were posed on the mathematization of proportion conception by the prospective teachers, are inappropriate for the approach. Although these problems exist in real life, the modeling that will be conducted in the process of solution will not help the mathematization of any conception. Moreover, it is believed that these problems can be an exercise question, which could be applied on taught proportion-ratio conception.

"Ezgi's watch loses time 5 minutes every hour. At 10.00 o'clock Ezgi agreed to meet her friend Mehmet at 17.00. What time Ezgi should be there according to her time, in order to meet Mehmet punctually at the time they agreed to meet?" (26th PT).

"6 workers worked at a construction and completed the building in 20 days. These workers want to make a planning for their next job. According to this, if 6 workers work together again; in how many days can they finish 5 buildings?" (8th PT).

Similarly, even though the problem stated below can exist in real life; it cannot be qualified as a contextual problem.

"How many pieces can we cut the cake by 5 moves?" (29th PT).

It is believed that the below-mentioned problem cannot be evaluated as a contextual problem, which could serve as a real life problem and help teaching a conception; it rather aims at achieving a mathematical relation by making students generalize through different cases.

"Four children play with one hoop each. Later, the children pile hoops over one another and start to examine their junction points. What is the maximum number of junction points for these hoops?" (23th PT).

Posing a problem is a significant component for problem solving and good mathematical problems can be posed by good mathematics teachers (Kilpatrick, 1987). The person in the process of posing a problem is actively engaged in challenging situations that involve them in exploring, questioning, constructing, and refining mathematical ideas and relationships (English, 2003). In light of this view, it is believed that most of the prospective teachers have carried out the activities mentioned in the process of problem posing. However, a problem should point out the types of realistic thinking, which characterizes out-of-classroom problem solving, in order to be realistic (Verschaffel, Greer, & De Corte, 2000). Also, it should bring out a variety of mathematical interpretations and solution strategies which serve as a basis for progression to a more formal and sophisticated mathematics, and should support students' mathematisation process (Widjaja, 2013). For example, even though a problem such as "Do the medians of a triangle intersect at a single point" is concrete; it is still far from everyday life problems, thus cannot be considered realistic. Hence, all contextual problems are not realistic. In order to evaluate a problem as realistic, it has to be real or experienced by the person in an interesting manner (Wubbels, Korthagen, & Broekman, 1997). Similarly, if a person wants to pose a problem about a farm, using sentences such as 'imagine a cow in a sphere form' will not make the problem realistic (Greer, 1997).

Taking into account of all of these ideas, it is observed that more than half of the problems posed by the prospective teachers are not qualified enough to serve for RME and only 34% of them fit for the purpose. Thus, it can be accepted that only the students in this percentage have cognitively reached the synthesis stage at reflection-construction level.

Evaluation

In the 4th question asked to the prospective teachers, they were expected to evaluate the application of the problem they posed and interpret it according to the other approaches. The 5 (15.625%) of the prospective teachers, who posed a relevant problem, have carried out an evaluation; whereas the other 27 (84.375%) of them failed to either pose a relevant problem or to carry out an evaluation.

The evaluation made by a prospective teacher, who is believed to have posed a relevant problem to RME is as follows:

> *"I aimed to enable the students to achieve the greatest common factor in my problem. The teacher could try to make student comprehend the conception through explaining its meaning. In other words, if it were given through presentation, the student would be provided with prepared knowledge. Furthermore, since the student would not be active; he/she may be bored and mathematics would be far from real life for him/her. I believe that the things a person does or achieves on his/her own is always more valuable and not forgotten. Therefore, for a student to solve a problem, a real problem, and to meet a new conception will be more meaningful for him/her. We could also give this knowledge through discovering numbers and the correlations among factors, without using a problem. This may be effective, too. However, I believe that it will be more permanent for students to reach at knowledge through real life events, discussion and creating their own models"* (6th PT).

According to Hiebert and Carpenter (1992), understanding is defined as making connections between ideas, facts or procedures, and occurs through recognizing relationships between pieces of knowledge. Consolidated knowledge enables the use of this knowledge in various cases in a proper and confident manner (Wubbels, Korthagen, & Broekman; 1997). Also, if knowledge is consolidated; the evaluations about this knowledge will be conducted in a meaningful manner. In that sense, it can be concluded that the evaluations of prospective teachers through the problems they posed are insufficient.

CONCLUSION

Enabling students to learn mathematics in a meaningful manner and to acquire various targeted skills during the process can be only made possible by placing education of mathematics teachers in the center. Therefore, it is of vital importance for prospective teachers to be able to prepare appropriate learning environments and to correctly plan the roadmaps for that purpose (Brousseau, 1987; Richards, 1991). In order to achieve that, the prospective teachers must have enough knowledge on the key principles and fundamental philosophy of the approach that they plan on using while preparing such environments, moreover, they must also be able to implement and interpret instructional activities (Gravemeijer & Cobb, 2006). In this study, where prospective teachers' cognitive structures about the approaches have been examined, RME has been the focused approach and prospective teachers' cognitive structures about this approach have been evaluated. The study suggests that the majority of the prospective teachers have theoretical knowledge about RME and can generally interpret that knowledge, however, their ability to associate that knowledge with other approaches falls by half. Moreover, the prospective teachers' skills fell even shorter when they were asked to pose a contextual problem suitable with RME, which was asked of them in order to see their high cognitive competencies. Even though it is necessary to place mathematical connections in the relevant social situations for achieving meaningful learning in mathematics; creating authentic activities is naturally quite difficult (Ainley, Pratt, & Hansen, 2006) and also designing lesson materials, especially finding real life examples that match with the mathematics concepts to be taught is difficult (Zulkadri, 2002). Also, preparing a learning environment that allows for reinvention process, is quite complicated (Gravemeijer, 2008). Furthermore, some of the reasons why these problems emerged may be due to the fact that they do not have adequate contextual knowledge about the nature of some of the conceptions they were supposed to learn; or they could not detach from the constructions in the learning settings, in which they were raised. Also, as Widjaja (2008) mentioned, their prior knowledge about the concept and the nature of their knowledge may cause this result. For that reason, prospective teachers should be trained in a way that reflects what is expected from their teachings (Gravemeijer, 2008).

Since this study aims to reveal the cognitive competency of prospective teachers on RME; it is believed that the obtained results suggest ideas on the construction about RME only in the minds of prospective teachers. Hence, reflections of prospective teachers on RME can be conducted through a phenomenological study so as to obtain more elaborated knowledge. Moreover, in the school experience and school practice stages; the application processes of prospective teachers on RME can be examined and in the light of the retrieved results, necessary adaptations can be conducted for real classroom situations.

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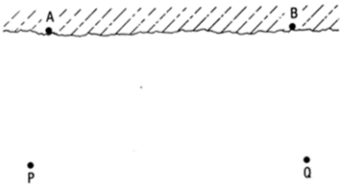
APPENDIX

Problem 1

When the sea snake in the picture becomes 1 month old, a black ring emerges around its body. Each month, a yellow ring emerges in the middle of the black ring; and thereby, two black and one yellow rings appear. In the following months, this continues in the same way. In other words, each black ring is cut in the middle with a yellow ring. How many rings does a 12 month old sea snake have? How can we find the age of a sea snake according to its number of rings; or the number of black and yellow rings of a sea snake of a certain age? (Altun, 2011).

Comment: It was aimed to reach geometrical sequence conception through this problem.

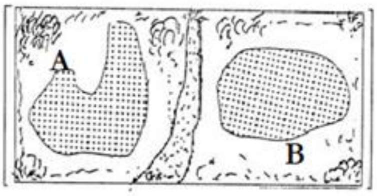
Problem 2



Ship P is going in a straight line toward point B on the shore, at a constant speed. Ship Q is going in the direction of A at double the speed of ship P. How close do the two ships get? (Wubbels, Korthagen, & Broekman, 1997).

Comment: It was aimed to reach vector conception through this problem.

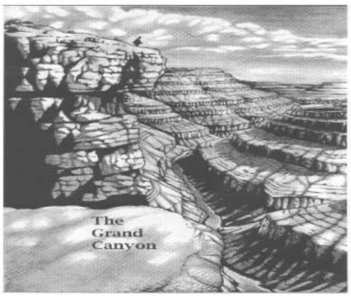
Problem 3



The figure below shows two rice fields separated by a road. Both rice fields are planted with the same rice and they are given the same fertilizer. The dots on the figure represent rice clusters. Which rice field produces more rice? (Fauzan, 2002).

Comment: It was aimed to reach reallotment-congruent area conception through this problem.

Problem 4



This is a photograph of a hiker on the rim of the Grand Canyon looking down trying to see the Colorado River at the bottom of the canyon. Can the hiker see the river below? From which points and perspective should the hiker look in order to see the river? (Feijs, 2005).

Comment: It was aimed to discuss tangent conception through this problem.

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GEOMETRY REPRESENTATION TO DEVELOP ALGEBRAIC THINKING: A RECOMMENDATION FOR A PATTERN INVESTIGATION IN PRE-ALGEBRA CLASS

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Abstract

The present study is a part of design research in local instructional theory in a pre-algebraic lesson using the Realistic Mathematics Education (RME) approach. The article will focus on recommendations for the type of pre-algebra class that supports elementary school students' algebraic thinking. As design research study, it followed the three steps of preliminary studies, teaching experiment and retrospective analysis. The subject of the study is 32 fifth grade students of MIN 2 Palembang during the teaching experiment phase. The data were gathered from students' worksheets, lesson observation and interviews with the students. Data analysis was done using a constant comparative qualitative method. The results from the study indicate that pattern investigation in pre-algebra class that visualized geometrically supports the students to identify the form of the pattern and construct generalization.

Keywords: Pre-Algebra, Algebraic Thinking, Geometry Representation, Pattern Investigation, Realistic Mathematics Education

Abstrak

Penelitian ini adalah bagian dari *design research* untuk menghasilkan teori pembelajaran lokal di bidang pra-aljabar dengan menggunakan pendekatan Pendidikan Matematika Realistik Indonesia (PMRI). Artikel ini akan memfokuskan pada pembahasan tipe kelas pra-aljabar yang dapat membantu pengembangan kemampuan aljabar siswa sekolah dasar. Sebagai suatu penelitian desain, tahapan yang digunakan dalam penelitian ini adalah studi pendahuluan, uji coba lapangan, dan analisis retrospektif. Subjek penelitian ini adalah 32 orang siswa kelas V MIN 2 Palembang pada saat tahapan uji coba lapangan. Data dikumpulkan melalui lembar jawaban siswa pada lembar kegiatan, observasi dan wawancara selama kegiatan diskusi berlangsung. Data kemudian dianalisis secara kualitatif dengan menggunakan metode komparasi konstan. Dari hasil analisis, diketahui bahwa penelusuran pola pada kelas pra-aljabar yang direpresentasi secara geometri dapat membantu siswa untuk mengidentifikasi pola dan membuat generalisasi.

Kata kunci: Pra-aljabar, Berpikir Aljabar, Representasi Geometri, Penelusuran Pola, Pendidikan Matematika Realistik

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Back to the origin of algebra, Abu Ja'far Muhammad Ibnu Musa al-Khwarizmi defined algebra as "the whole discipline dealing with 'equations'"(Kvasz, 2006). It becomes one of the large mathematical topics which is taught in school. Drijvers, Goddijn, & Kindt (2011) emphasized the role of algebra to develop an algebraic thinking.

Algebraic thinking is the skill to focus on relations among the numbers (Kieran, 2004; Kieran, 2018; Venkat, et al. 2018; Widodo, et al. 2018). It involves the generalization of arithmetic and provides reasoning related to it, the development of mathematical models (mental and formal) in solving algebraic problems, formulate and visualize pattern and the construction of the algebraic language (Dekker & Dolk, 2011; Hendroanto, et al. 2018). It will be essential point for the students in all advance mathematical knowledge (Brawner, 2012). The lack of algebra may contribute to the difficulties in further study such as calculus (Müller, Cury, & de Lima, 2014) and other mathematical skills such as mathematical proof (Güler, 2016) and problem solving (Ferryansyah, Widyawati, & Rahayu, 2018). It is also can become obstacle for the students who pursue higher education and career that use mathematics, even though not mathematics department; such as engineering (Sazhin, 1998).

In spite of the significant role of algebraic thinking, it still be considered as an arduous skill in teaching and learning (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014; Capraro & Joffrion, 2006). Booth, Barbieri, Eyer, & Paré-Blagoev (2014) found that many students have misconception in this topic. The current problem in the teaching and learning algebra is the abstract introduction of algebra in which there is a gap between students' prior knowledge and the emerging algebraic symbols (Dekker & Dolk, 2011; Julius, Abdullah, & Suhairom, 2018). Recently, Wijaya, Retnawati, Setyaningrum, Aoyama, & Sugiman (2019) reported that the students' low understanding of algebra is one of the three most prominent obstacles for students in learning mathematics. The other two problems are still related to the mastery of algebra, which are lack on calculation and represent the problem into mathematical models. Therefore, we may take a look back on how algebra should be introduced for younger students.

Usiskin (1988) proposed four point of view to introduce algebra in school including the arithmetic generalization, strategies in solving problem, relationships between numbers and the structures of the pattern. Therefore, the classroom activities should promote the students' awareness towards patterns, relationships, problem solving abilities and the concept of making generalization. One recommendation to accommodate all of these needs is by having a pre-algebra class (Carraher, Schlieman, Brizuel, & Earnes, 2006).

Pre-algebra class is not aimed to bring the high school mathematics for group of students in their early age (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Indeed, its function is to familiarize the students with the structure (Ball, 2003) and is supposed to provide connections between arithmetic and algebra (Jacobs, et al. 2007).

Lannin (2005) recommend the investigation of the pattern as the center of early algebra activities since it provides "dynamical representation of variables" (p. 233). It can be used as an initial point in which the students will do investigation and identification towards the structure and relation between mathematical objects (Drijvers, Dekker, & Wijers, 2011).

Despite of many benefits of using patterns as initial activity to introduce algebra in earlier ages, its implementation encounters some difficulties. The most common problem is how to foster the students' movement from seeing the pattern as its unit to its generalization (Quinlan, 2001; Lannin, 2005) and how the students making connection between the ideas in mathematics (Kenedi, Helsa, Ariani, Zainil, & Hendri, 2019). Hence, in this study, the pattern investigation was supported by the visual representation embodied in geometrical objects to help the students seeing the structure and make generalization. The research question addressed in this paper is what is the function of the geometry representation to enhance the students' algebraic thinking?

METHOD

This is a design research study with three steps of preliminary study, teaching experiment and retrospective analysis (Bakker & van Eerde, 2015). The subject of the study is 32 fifth grade students of a state junior high school in Palembang who participated in the second cycle of teaching experiment phase on the study. The teacher is the homeroom teacher of the classroom who already teach for more than twenty years.

The researchers discuss the activity with the teacher before, during and after the implementation to adjust the lesson plan with the teacher's experiences based on the students' general condition. The discussion with the teacher also support the retrospective analysis phase in which the researchers classified the strategies employed by the students during the lesson.

The data were gathered from classroom observation during the learning process and students' written work and interview. During retrospective analysis phase, the data was analyzed qualitatively using constant comparative method by continuously testing the data with the conjectures and finding its counterexample.

The instrument of this study is a set of learning trajectory in pre-algebraic class using pattern investigation activities. It was developed under the design heuristic of Realistic Mathematics Education (RME). According to Gravemeijer & Bakker (2006), the design heuristic of RME are including the following items.

1. Guided Reinvention, which refers to the role of students who actively construct their own conceptual scheme.
2. Didactical Phenomenology, which refers to the context that encourage the students to do meaningful learning.
3. Emergent Modelling, which refers to the use of models to connect phenomenon and mathematical concept.

RESULTS AND DISCUSSION

The pre-algebra class using pattern investigation for elementary school students is divided into several topics, including: (1) constant pattern, (2) growing pattern with constant difference and (3) growing pattern with growing difference. This study will focus on the second topic which is growing pattern with constant difference.

The context of the lesson is a dance formation. It takes the form of *V* which is one type of growing pattern with constant difference. The discussion will be narrowed to the plan of doing a flash-mob, which means in every certain amount of time the number of dancers will be added without changing the *V* shape (Figure 1). The learning goals of the activity is to enable students in predicting the number of the dancers in every next formation and assessing the conjecture for generalization of the *V* pattern.

(Source: https://www.travelerbase.com/)

Figure 1. *V* Formation in Balinese Traditional Dance

Growing Pattern with Constant Difference

The number pattern used in this lesson is the odd number start from 3. Hence, the difference is constant, the next term will have two more than the previous one. Mathematically, this pattern can be written as 3, 5, 7, 9, etc. The students were asked to generate the general characteristics of the number that can be listed in the pattern. To begin with, the dance formation in the Figure 1, is elaborated more in the Figure 2, by symbolizing each dancer with a dot. A grid is given to help the students see the position of the dancer.

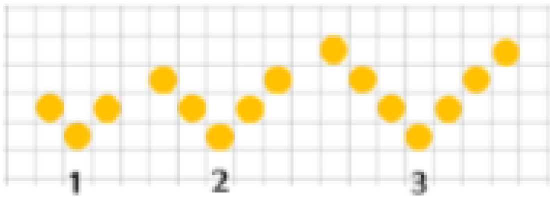


Figure 2. The first-three Number of Dancers of *V* Formation

The students' were working in pair to solve the following questions number 1 to 3; and in a group of four to solve the questions number 4 to 7. The last question was given during the posttest to check the students' transferability in seeing the structure of the number pattern.

1. Make a draw to represent the 4^{th} formation!
2. Calculate the number of dancers in the 6^{th} formation?
3. Make a draw to represent the 17^{th} dancers!
4. Calculate the number of pairs of dancers in the 45^{th} formation?
5. How you find the number of dancers in the 100^{th} formation!
6. Is it possible if a *V* formation has 92 dancers? Explain!
7. Is it possible to combine two *V* formation to get one *V* formation? Explain!

8. Figure 2 is the first-three of *V* formation, while the following Figure 3 is the first-three of *W* formation.

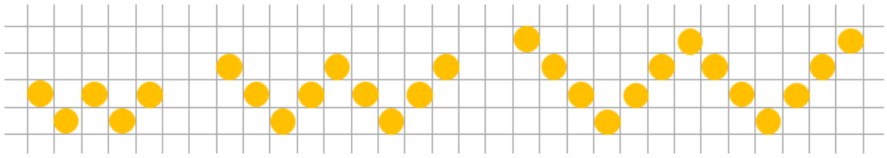


Figure 3. The first-three Number of Dancers of *W* Formation

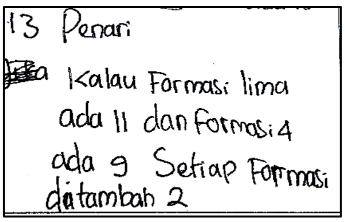
A student concluded that the *W* formation can be obtained from the combination of two *V* formations. Do you agree with the student? Explain your reason!

The strategies and obstacles encountered by the students in solving the pattern investigation problem and how the geometry representative of the pattern help them build the algebraic thinking is discussed in the following section.

Emerging Strategies

Adding Two Strategy

The first-three questions are likely to be solved using the adding two strategy. Most of the students were using "adding two strategy" which is adding two dots (represent dancers) to the previous formation). The following Figure 4 showed the example of the students' work to figure out the number of dancers performed in the 6th formation.



Translation:
There are 13 dancers.
So, there are 11 dancers in the 5th formation and the 9 dancers in the 4th formation.
We add two for each formation.

Figure 4. Adding Two Strategies

Considering Rows

To find the number of dancers using the row consideration methods, the students develop the idea of multiplication by two or doubling. In the same time, they also become able to use the reverse operation to find the number of formations when the total number of dancers are given.

The following Fragment 1 showed students' discussion about the number of dancers in each sides and in the middle to solve the second question. From now and the rest of the paper, the use of initials in the Fragment will be as follows: S refers to student, R to researcher and T to teacher.

Fragment 1. Sides and Middle

[1] S1 & S2 : There are 6 dancers in the right and the other 6 in the left. There is one in the middle.

[2] R : How many dancers will be in each side of the seventh formation?

[4] S1 & S2 : Seven dancers.

[5] R : How about the dancers in the middle?

[6] S1 & S2 : One dancer.

To determine the number of dancers in the 17th formation, the students applied the same approach to solve the third question. However, in the middle of discussion one of the student got confused with the number represented in the *V* formation, especially the single person in the middle. See Fragment 2 to observe the discussion.

Fragment 2. How to Find It?

[1] R : Explain your strategy to find the number of dancers in each side of the formation.

[2] S2 : We can divide the total number of dancers by two and then subtract by one.

[3] Eh, divide it by two and plus one.

[4] Eh.

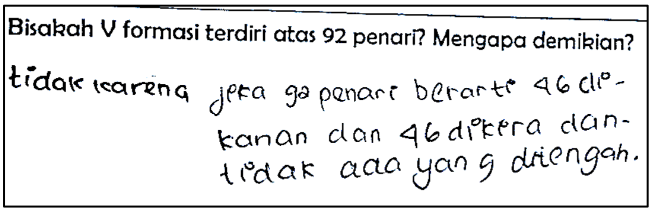
[5] R : What do you mean by divide it by two and then subtract or add by one?

[6] S1 : If we divide the total number of dancers by two, you will have one more dancer left.

[7] R : Where will the left dancer stand?

[8] S1 : The dancer will be in the middle of the formation.

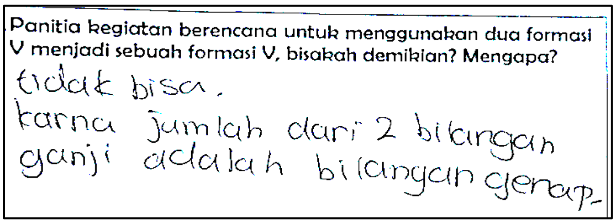
The notion of "someone should be in the middle" play an important role of students' generalization on *V* formation. It can be seen in the example reason the student used to solve the 6th problem as shown in Figure 5.


Bisakah V formasi terdiri atas 92 penari? Mengapa demikian?
tidak karena jeka 92 penari berarti 46 di-kanan dan 46 dikiera dan-tidak ada yang g ditengah.

Translation:
It cannot be. If 92 dancers, 46 dancers will be in the right and 46 dancers in the left. No dancer will left to be in the middle.

Figure 5. Someone should be in the Middle

Also, the students were able to state the fact that the addition of two odd numbers will always be an even number and not an odd (see Figure 6). In other words, it cannot perform a *V* formation. Hence they refused the hypothesis given in the 7th problem that two *V* formations can constructed a *V* formation.



Translation:
It cannot be. The addition of two odd numbers is an even number.

Figure 6. Two odd numbers cannot produce odd

The Function of the Geometry Representation

The students' strategy development in solving pattern investigation related problems showed that the geometry representation played a valuable role in students' algebraic thinking. We can distinguish three major roles of the geometry representation in the pre-algebra classroom.

1. **Context**

   The geometry representation is used as the context of the problem. Mathematically, the *V* formation has the general form of $n = 2m + 1$, where n and m represent the number of dancers and formation respectively. However, the formal algebraic expression is too abstract for the student. In the first prototype of the learning trajectory, we tried a lesson with problem in general form of $n = 2m + 3$. The problem was state descriptively using words, but not be represented in certain geometrical shape. From the observation, the students were hardly find the general relation between n and m and work exhaustedly to count three more from the previous term and so on.

   Differently, pattern investigation activities with geometrical representation helps the students to see the structure of the pattern in more realistic way. It is a good start to set the students' intention in exploring what aspect of the pattern remains constant and what changes. Later, the students can use the support of the geometry visualization to elaborate the change of the mathematical objects and construct the general idea. The similar finding also found by Rivera (2011) that visual representation helps students to establish personal inferences in seeing particular pattern.

2. **Model of and model for situation**

   Besides of the use of context, Realistic Mathematics Education (RME) also well-known for the use of vertical instrument that enable students to perform a guided-reinvention process and construct their own understanding. According to Treffers (1987) models are the most useful bridge to help students to shift from reality to mathematical objects and ideas. Therefore, a mathematical model should give a sense of visualization to the actual condition in phenomena.

   Based on Realistic Mathematics Education (RME) approach, there are two types of model, which are *model of* and *model for* situation. According to Gravemeijer & Doorman (1999), model of situation is used by the students to transfer the context into mathematical statement or object, meanwhile the *model for* situation is used by the students to work with the mathematical ideas.

   In this study, the geometry representation of the *V* dance formation become a bridge to translate the context of dance and the mathematical ideas of constant difference in a growing pattern. Figure 7 showed how the students use the geometry representation as a mathematical model.

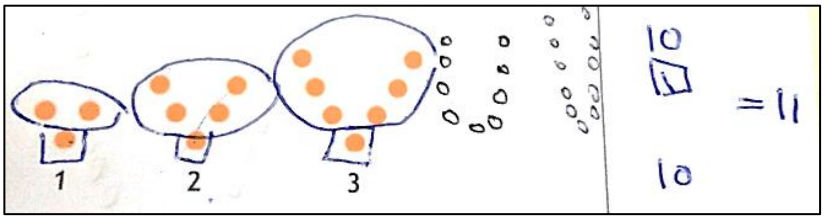


Figure 7. One is single, the rest with pairs

Previously, the pair of students whose written work showed on Figure 7, continue drawing the 4th and 5th formation. After realize that it is exhausted, they gave a mindful thought further and differentiate the formation into "has pair" and the "single" one. The students were using the representation given in the work sheet as a model to start their investigation by circling the "paired" part and squaring the "single" part. On the right side, they use the representation as a model for situation to write the number: the upper part is the number of dancer in pair and one left in the square.

This finding in line with the results of Kusumaningsih, Darhim, Herman, & Turmudi (2018) that emphasize the important of using multiple representation strategy with realistic approach to develop students' algebraic thinking. The mathematical model is used to express the generalization and to ensure that the students grasp the number of pattern, not merely got the answer by accident or from trial and error process (Radford, 2006).

3. Scaffolding

The use of geometry representation not only beneficial for the students. The teacher can use it as crossed question to enhance students' critical thinking in observing the pattern. Therefore, the teacher can provide a limited help for the students without directly give the answer and stop the students' thinking. Consider the following Fragment 3 as the example of the use of geometry representation as the scaffolding.

Fragment 3. What Does The Number Represents: Total Dancers or Formation?

[1] R	:	What is the information provided in the problem?
[2]		Are there 17 dancers or the dancers in the 17th formation?
[3] S4	:	There are 17 dancers.
[4] S5	:	So, the answer should be 35 dancers.
[5] R	:	Okay first, can you draw the picture of the formation which has 3 dancers?
[6]S6	:	Yes, it will be the first formation.
[7]S3,4,5,6	:	(Draw 3 dancers in the 1st V formation).
[8]S4 & S3	:	One in the right, one in the left and one in the middle.
[9]R	:	Okay, so you have 3 dancers. How if you have 17 dancers?

[10]S4 & S5 : 35 dancers in total.

[11]R : Can you show it in picture?

[12]S3 : Eh.

The researcher asked them to draw another formation with 5 dancers and compared to the number of dancers they have in the 5th formation. Then, asked them to re-read the question. Finally, they realize that 17 is not the number of formation but the number of dancers that should be in the formation. This finding can be a meaningful support for the teacher in teaching algebra in the classroom especially to cope with the students' struggles in mathematizing word problems (Jupri & Drijvers, 2016; Salemeh & Etchells, 2016).

4. Students' Mathematical Reasoning and Proof

The students also use the geometry representation to express their mathematical ideas. The example can be seen in the Figure 8 in which the students were working to solve the 8th problem: does two *V* formation can form a *W* formation.

Translation:
I don't think so, because there will be one left [dancer].

Figure 8. Two *V*s cannot be a *W* Formation

There also a student who said that it is possible for two *V* formation to be a *W* formation, but he drew it differently in which he has to remove a dancer. During the interview, he used illustration to support his argument as can be observed in the following Fragment 4.

Fragment 4. Take One

[1] S1 : So, *W* formation is actually the combination of 2 *V* formation.

[2] R : Can you show me how it can be combined?

[3] (*S1 showed his worksheet*)

[4] R : Do you find any strange in the picture?

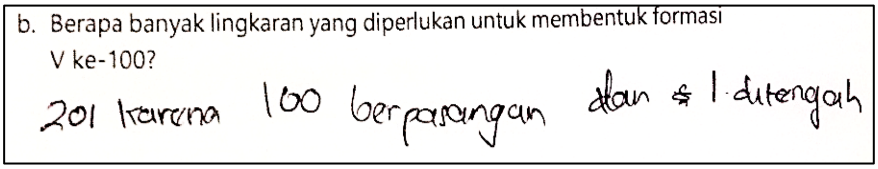
[5] S1 : Yes, the one in the middle (*pointed to the picture*)

[6] R : What do you think about the person in the middle?

[7] Do you have any specific rule to create a *W* from 2 *V* formations?

[8] S1 : Indeed. You have to remove one dancer.

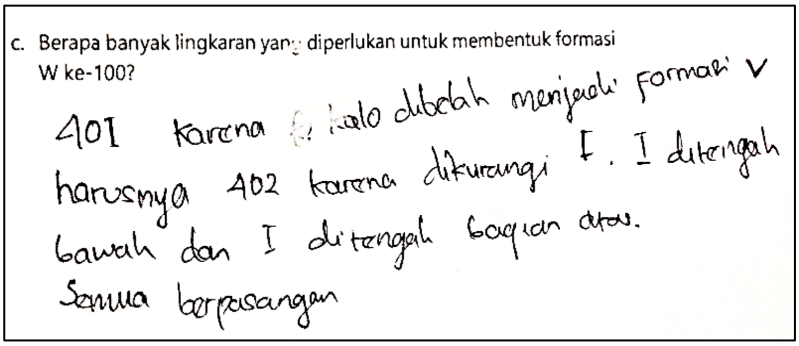
The similar why of thinking also showed in his idea for the next question to determine the total dancers in the 100th formation of *V*. He mentioned the notion of "dancers in pairs and one in the middle” structure and explained as in Figure 9.



Translation:
201 [dancers], 100 [dancers] have pairs and 1 in the middle

Figure 9. The 100^{th} *V* formation structure

To determine the number of dancer in the 100^{th} W formation, he connected the idea of the *V* and *W* structures. He employed his conclusion that the combination of 2 *V* formations can become a *W* formation when he removes one dancer. Since in the Figure 9 the dancers needed for 100^{th} formation of *V* are 201 then he said it should be doubled and subtract by one. The Figure 10 explained his argument.



Translation:
401. There are 402 [dancers] in 2 V formations, but you remove 1.

Figure 10. The 100^{th} W formation structure

The aforementioned functions of geometry representation enables the students to make connections between the problems, mathematical models, problem solving strategies and see the structure of the pattern algebraically. It provides a meaningful support for the students to work with mathematical objects which will be beneficial for the development of algebraic thinking especially the ability to generalize and reason within algebraic structure (Dekker & Dolk, 2011).

CONCLUSION

A geometry representation is an important support in students’ movement from recursive calculations to expressing general formulas. In other words, it helps the students’ transition from arithmetic to algebraic thinking. For practice in mathematics classroom, it is recommended for the teachers to conduct a pre-algebra lesson using number investigation embodied in geometry representation. The geometry representation provides an opportunity for the students to see the whole picture of the series of numbers by considering the characteristics of the geometrical object given. Hence, the students will not get lost too early. Also, it creates a challenge for the students to provide a general proof for the figure they started with. Therefore, the existence of the geometry representation of the number pattern is used as a context to start, a model to work with, a scaffolding to additional support and as a reasoning tool to express their mathematical ideas.

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ELEMENTARY PRESERVICE TEACHERS' KNOWLEDGE, PERCEPTIONS AND ATTITUDES TOWARDS FRACTIONS: A MIXED-ANALYSIS

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Abstract

Previous research has shown knowledge, perceptions, and attitudes are essential factors during mathematics classroom instruction. The current study examined the effects of a 3-week fraction instructional unit using concrete models, problem-solving, and problem-posing to improve elementary preservice teachers' knowledge, perceptions and attitudes towards fractions. A quasi-experiment design was implemented to gather data via closed-ended, open-ended, and essay tasks from a convenience sampling of 71 female elementary preservice teachers during pre- and post-assessments. The study discovered that the select preservice teachers were weak in the content knowledge specifically on unit-whole, part-whole, equivalent area, arithmetic operations, and ordering fractional values. In contrast, the incorporation of concrete models, problem-solving and problem-posing was effective in improving the preservice teachers' level of pedagogical content knowledge, perceptions and attitudes towards fractions. Implications of the results and suggestions are discussed.

Keywords: Elementary School, Problem Posing, Teacher Preparation Program, Preservice Teachers, Mixed Methods

Abstrak

Penelitian sebelumnya menunjukkan bahwa pengetahuan, persepsi, dan sikap merupakan faktor penting dalam pembelajaran matematika. Penelitian saat ini meneliti tentang pengaruh dari pembelajaran pecahan selama kurun waktu 3 minggu menggunakan model konkret, pemecahan masalah, dan *problem-posing* untuk meningkatkan pengetahuan, persepsi, dan sikap calon guru sekolah dasar terhadap materi pecahan. Sebuah desain eksperimen semu diimplementasikan untuk mengumpulkan data melalui permasalahan dalam bentuk soal *closed-ended*, *open-ended*, dan uraian dari pengambilan sampel yang representative dari 71 calon guru wanita sekolah dasar selama pra dan pasca penilaian. Studi ini menemukan bahwa calon guru terpilih lemah pada konten materi pecahan, khususnya pada *unit-whole*, *part-whole*, area yang sama, operasi aritmatika, dan nilai pecahan berurutan. Sebaliknya, penggabungan model konkret, pemecahan masalah, dan *problem-posing* adalah efektif dalam meningkatkan level pengetahuan konten pedagogis calon guru, persepsi, dan sikap terhadap materi pecahan. Implikasi hasil dan saran lebih lanjut dibahas dalam tulisan ini.

Kata kunci: Sekolah Dasar, *Problem Posing*, Program Persiapan Guru, Calon Guru, Metode Campuran

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Knowing numbers and operations is a cornerstone of mathematics education in the school curriculum (National Council of Teacher of Mathematics [NCTM], 2000). Young students should have acquired a conceptual understanding of number systems, their structures, and properties during classroom instruction (Lamon, 1999). Fractions are considered central concepts for school mathematics but have conventionally been challenging and cumbersome for teachers to deliver as well as for students to learn (Barnett-Clarke, Fisher, Marks, & Ross, 2010; Lin, 2010). An incomplete understanding of fractions can eventually affect individuals' learning in subsequent mathematical areas such as algebra, trigonometry, and calculus and related disciplines (Barnett-Clarke et al., 2010).

Numerous previous studies have emphasized the impact of diverse classroom instruction (e.g., traditional, concrete models, web-based, one-on-one) on student learning of fractions (Lin, 2010; Newton, 2008; Osana & Royea, 2011). However, there has been scant research measuring the potential benefit of an instructional practice that integrates concrete models for developing preservice teachers' conceptual understanding of fractions (Sarama & Clements, 2009). In this study, we focused on measuring the integration of Mathematics TEKS Connection module with concrete model instructional practices in facilitating preservice teacher knowledge construction, perceptions and attitudes towards fractions during the teacher preparation programs.

A body of literature has documented that a majority of in-service teachers and preservice teachers have limited profound knowledge for teaching mathematics (Ball, 1993; Hill, 2010; Ma, 1999; Newton, 2008; Timmerman, 2004). In addition, White, Way, Perry and Southwell (2005) and Llinares (2002) stated preservice teachers' belief or perceptions is reflected in their action or attitudes during teaching and learning that influence the classroom instructional practices. Despite this evidence, little is known about the nature of knowledge for teaching fractions, perceptions and attitudes that teachers should have nurtured during teacher preparation, the place where teachers should have acquired their teaching repertoire.

Teacher Knowledge

Ball (1993) and Ma (1999) stated that teachers' content knowledge could be a possible aspect affecting classroom instruction. Meanwhile, Shulman (1986) noted content knowledge alone is not sufficient; pedagogical content knowledge is also significant for making the learning of mathematical concepts understandable. Shulman (1986) pointed out that content knowledge is solely an amount of mathematics knowledge that one should have for teaching a particular concept. It includes the conceptual and procedural understanding of specific mathematical ideas (Shulman, 1986). Many researchers argue the importance of teacher knowledge in making teaching and learning of mathematical content meaningful. When teachers do not possess in-depth knowledge of a particular concept (Ball, Thames, & Phelps, 2008) and do not know how to represent the idea and to make it comprehensible and understandable (Shulman, 1986), they often fail to deliver the concept for students' understanding (Barnett-Clark et al., 2010). In contrast, pedagogical content knowledge includes the way a teacher represents fractions to facilitate student learning by using appropriate models, analogies, illustrations, examples, explanations, and demonstrations (Shulman, 1986). Also, teachers must be aware of students' knowledge of fractions and know the strategies for reorganizing students' understanding appropriately (Shulman, 1986).

Perceptions and Attitudes

Previous research showed preservice teachers entered teacher preparation programs with preexisting views grounded mainly on their background and positive or negative experiences in learning mathematics (Timmerman, 2004; White et al., 2005). These preexisting perceptions are resistant to change and difficult to break (Kane, Sandretto, & Heath, 2002). White et al. (2005) claimed an intervention is needed to stop a cycle of negativity perceptions and attitudes towards mathematics especially for preservice teachers who in turn influence the formation of student attitudes. According to White et al., negative perceptions could create negative teaching ways, that can successively affect students' beliefs, attitudes and learning outcomes. Thus, teacher educators should perceive preservice teacher perceptions and attitudes expressly for utilizing the potential ways for knowledge construction and stimulating learning.

Multiple Tasks and Concrete Models

Appropriate instructional materials and mathematical tasks in classrooms are indeed crucial for helping students grasp abstract concepts for knowledge construction (Cramer & Wyberg, 2009; Sarama & Clements, 2009). The incorporation of problem-solving and problem-posing tasks can help teachers to enhance students' understanding of fraction with the assistance of concrete models such as fraction strips, and paper-folding. For instance, when students are asked to generate a problem for this division of fractions, $\frac{3}{4} \div \frac{1}{8} = n$ (Sowder, Philipp, Armstrong, & Schappelle, 1998), they can use a concrete model to discover fraction size (Empson, 2002) and the relationships between the $\frac{1}{4}$ and the $\frac{1}{8}$. The teacher can examine students' understanding by asking students to explain the meaning of $\frac{3}{4} \div \frac{1}{8}$ as illustrated in Figure 1. Based on the representational model of Figure 1, students should be able to make connections to their prior knowledge about fractional numbers and find the answer to the problem.

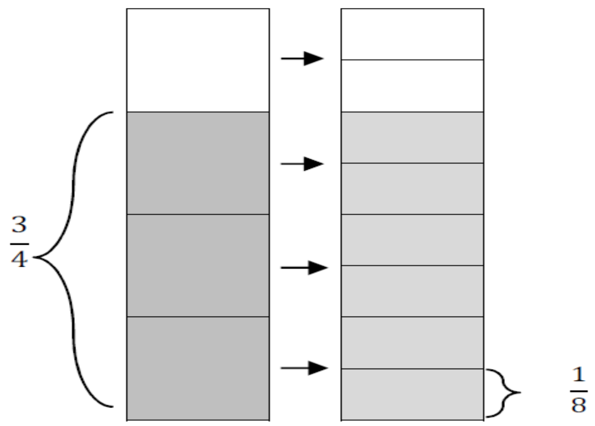

Figure 1. Models for Division of Fraction

Teachers may assist students' learning during the process of operating the models. A student must be able to grasp the concepts and ponder the underlying association on six sets of $\frac{1}{8}$ in $\frac{3}{4}$. Based on the necessary knowledge, the student could think of a fraction situation that fits the given equation and then teachers would ask the students to share their situations with others in the class.

METHOD

In utilizing both quantitative and qualitative data and rigorous methodological research techniques, we adapted close-ended and open-ended problem-solving and problem-posing tasks to shed light on the research question: What was the effect of the instructional unit of developing conceptual understanding of fractions with concrete models on the levels of elementary preservice teachers' content knowledge, pedagogical content knowledge, perceptions and attitudes towards learning and teaching fractions?

The pragmatist approach (Onwuegbuzie, Johnson, & Collins, 2009) drove the study that involved administering two pre- and post-assessments containing close-ended items and open-ended questions. Pragmatist researchers believe that there are many useful ways of seeking knowledge, including mixing quantitative and qualitative data and methods (Onwuegbuzie et al., 2009). A quasi-experimental research method was utilized for gathering both quantitative and qualitative data that focused on a one-group pretest-posttest design (Shadish, Cook, & Campbell, 2002). The qualitative component of the study was based on the constructivist-naturalistic paradigm (Lincoln & Guba, 1985). We attempted to assess the preservice teachers' understanding and attitudes about fractions by examining the open-ended responses and seek "individual and collective reconstructions that may unite around consensus" (Onwuegbuzie et al., 2009, p. 122). It was hoped that we could assess the treatment integrity (e.g., fidelity score) of the fraction instruction using both quantitative and qualitative instruments (Collins, Onwuegbuzie, & Sutton, 2006) to reduce any implementation biases (Onwuegbuzie, 2003). Besides, mixing quantitative and qualitative data can provide significant enhancement of the findings for generalizability purposes (Collins et al., 2006).

The study employed the use of a convenience sampling technique to select the participants who were willing to be part of the study (Collins, Onwuegbuzie & Jiao, 2007). The instructor, Dr X who is an experienced professor, has been teaching elementary methods courses for more than ten years at one of the public universities in Texas. The method course was compulsory for the completion of the undergraduate degree with certification in the EC-Grade 6 Core Subjects/Generalist program. The participants were 71 female preservice teachers who were pursuing a Texas teacher certification in elementary school classrooms. They were from two class sections of the experienced professor in the Fall semester. The course comprised of one hour and 15 minutes of 16-week face-to-face meetings together with an online module through the Blackboard Learning System (eLearning) of the university. Concerning ethical considerations, permission was granted to perform the study over the Institutional

Review Board (IRB) of the university and an information sheet was provided to the participants informing them that the study would run as regular class session. The times for both class sections were back to back and the professor taught the same content and used similar instructional strategies.

Close-ended items from the Learning Mathematics for Teaching [LMT] (2008) were utilized for examining content knowledge and pedagogical content knowledge. Only 11 close-ended were selected based on content-related validity to fractional concepts (Measure A) under investigation. In addition, we adopted five close-ended items on geometry (Measure B) to examine the non-equivalent dependent variable for improving the design. This instrument was made available to participants using an online survey tool, Qualtrics, and was piloted to preservice teachers in the other class sections of the methods course. The internal consistency scores (Cronbach's alpha) were .74 for fractions and .73 for geometry using the Statistical Program for Social Science (SPSS) software version 21.0 (IBM Corp., 2013).

Additionally, nine open-ended items were selected from Sowder et al., (1998), five questions were used to measure content knowledge and four were utilized to assess pedagogical content for teaching fractions. Two professors reviewed these open-ended questions to determine the content-related validity such as the relevancy and appropriateness of the tasks (Collins et al., 2006). The open-ended instrument was piloted and no changes were made to the instrument and the administration time of the present study.

The participants received three weeks of classroom instruction (each week for 4 hours) on basics fraction concepts including addition, subtraction, multiplication, and division. The classroom instruction was comprehensively concentrated on concrete models that helped preservice teachers discover and construct basics mathematics knowledge based on the constructivist theory. Four to five preservice teachers were grouped into a small station (table) and concrete models such as tangrams, fraction bars, fraction circles, fraction towers, fraction strips, and pattern blocks were made available in every class session. During class instruction, Dr. X adopted a meaningful teaching approach through demonstrations and problem-posing activities that involved preservice teachers' use of modeling to develop conceptual knowledge of fractions. Preservice teachers were given various fraction problems to create a scenario for and model with manipulatives or drawings with their partners. Problems were such as as $\frac{1}{2}+\frac{1}{4}$, $\frac{2}{3}-\frac{1}{4}$, $\frac{1}{2}\times\frac{3}{4}$, $\frac{3}{4}\times\frac{1}{2}$, $4\div\frac{1}{2}$, and $\frac{1}{2}\div 4$.

In addition to classroom instruction, preservice teachers engaged and participated actively in online modules, which utilized multiple instructional strategies including web-based activities, videos, and readings. The modules were developed and field-tested that followed the Mathematics Texas Essential Knowledge and Skills (TEKS) Connection [MTC Project]. It was a part of the Texas Math Initiative and represented a partnership between the Texas Education Agency (TEA) and the university. With three different bands (K-4, 5-8, 9-12), it covered some specific and vital mathematics concepts (e.g., place value, fractions) for each grade level. The modules were made available for future teachers use from http://mtc.tamu.edu/home.htm?intro-pre.htm.

The fraction assessments were administered sequentially online during class periods in October 2011. A week before the professor began the instruction on fractions, the preservice teachers were asked to access the Qualtrics survey tool and respond to 16 multiple-choice items by choosing their best options. Then, during one class period, a graduate assistant administered the open-ended tasks wherein participants were required to write their responses on an answer sheet. The fraction instruction took place during three weeks wherein unstructured observations were made and field notes were taken to capture all potentially relevant phenomena. The graduate assistant was an observer-as-participant (Johnson & Christensen, 2012) wherein the primary role was to collect related information (e.g., setting, interactions, and subtle factors) concerning fraction instruction (Erlandson, Harris, Skipper, & Allen, 1993). A week after Dr. X completed the fraction instruction, the preservice teachers were required to login to the Qualtrics instrument and answer the same items as on the pre-assessment. Similarly, the graduate assistant administered the same open-ended assessment to the participants.

The close-ended items were scored based on their correctness, one point for a exact answer or zero point for an inappropriate one. Data were stored in the SPSS software (SPSS Inc., 2007) and points for items were totaled up for content knowledge (i.e., Item 1, 3, 4, 6, 7, 10, and 11) and pedagogical content knowledge (Item 2, 5, 8, and 9). Next, the open-ended items were transformed quantitatively where each pre- and the post-written response was assessed and coded into a numerical value (Teddlie & Tashakkori, 2006) based on the degree of accomplishment, zero (lowest) - four points (highest). For inter-rater agreement percentage, two independent raters graded seven identical scripts and attained an 83.3% agreement. The raters discussed to resolve any disagreement.

Based on the data collection, a few statistical analyses were run and effect sizes for pre-post contrast were calculated. Then, we analyzed the written responses and observation data through a constant comparison analysis (Glaser, 1965) for a more in-depth and better understanding of the elementary preservice teachers' strategies (Lincoln & Guba, 1985) for solving fractional tasks. The essay parts were unitized using the QDA Miner 3.2 (Provalis Research, 2009) following the classical content analysis method (Onwuegbuzie & Teddlie, 2003). The written essays were examined, underlined into chunks, and unitized into smaller significant parts (codes). The codes that were frequently appear were counted to symbolize essential concepts of preservice teachers' perceptions and attitudes towards fractions.

RESULTS AND DISCUSSION

Based on the quantitative results and qualitative findings, we hoped to make external (statistical) generalizations regarding the level of fractional understanding to the entire population from where the participants were conveniently drawn. In the present study, the differences from pretest to posttest scores on the problem-solving and problem-posing tasks were utilized to observe the effect of the fraction instruction that focused on concrete models. These employed mixed data analyses to measure the change in the levels of elementary preservice teachers' content knowledge, pedagogical content knowledge, perceptions and attitudes regarding learning and teaching fractions.

Analysis of Quantitative Data

This quasi-experimental design utilized a one-group pretest-posttest design to assess the cause-and-effect relationship of fraction instruction (treatment) and preservice teachers' knowledge and belief about fractions. For reducing threats to validity of the experimental design, we adapted six geometry items as the nonequivalent dependent variable (measure B) in addition to 11 fraction items for measure A. The results showed the preservice teachers' scores of fraction knowledge (measure A) changed statistically significantly, $t(59) = 3.50$, $p = .001$ with a mean gain effect size of .32 from pretest to posttest possibly because of the treatment of fraction instruction. No statistically significant change for geometry knowledge (measure B) was noted from pretest ($M = 5.13$) to posttest ($M = 5.05$). According to Shadish et al. (2002), this indication is useful for ruling out the possibility of a cause-and-effect relationship in a research study.

The results of the close-ended assessment included 60 completed responses from the select preservice teachers. Eleven answer scripts were eliminated due to an invalid/incomplete pretest or posttest. The t-test value indicates that the mean score for fraction knowledge on the posttest was significantly greater than the mean score on the pretest. Specifically, the results from the close-ended items revealed statistically significant differences in means scores for both preservice teachers' content knowledge [$t(59) = 2.14$, $p = .037$] and pedagogical content knowledge [$t(59) = 2.87$, $p = .006$]. The standardized mean gain effect sizes were 0.19 and 0.38 respectively. The results indicate that the preservice teachers' content knowledge and pedagogical content knowledge improved during instruction as measured by using close-ended items.

A review of the item analysis showed that preservice teachers scored a lower percentage for content knowledge specifically on Items 4, 6, 7 10, and 11 with less than 40% answering correctly on both pretest and posttest. Also, we found that preservice teachers had the most difficulty on Item 4 with only 10% correct on the pretest and posttest. They were not able to interpret and to analyze different situations given in the question that related to the unit-whole of a fraction. The results supported Lamon's (1999) argument that most of the classroom instruction failed to present the unit-whole fraction that contributes to students' misunderstanding of the concept. However, the majority of preservice teachers were able to answer correctly the part of Items 2, 5, and 8 that involved their knowledge of problem posing. Besides, the most significant differences between the percentage of scores on the pretest and posttest were found on Items 6, 2 (item b only), 8 (all sub-items), and 9.

Analysis of Qualitative Data

As mentioned in the previous section, the qualitative data were mainly collected from preservice teachers' responses to the open-ended assessment consisting of five fraction content knowledge items and four fraction pedagogical content knowledge items. Based on 71 completed responses, each item was assessed and transformed into a numerical value (0-4 points) that can be analyzed statistically; then the subtotals were calculated for content knowledge and pedagogical content knowledge. Overall, the Wilcoxon signed-rank test revealed that a 3-week instruction did not elicit a significant improvement statistically in the level of elementary preservice teachers' content knowledge [Z = -2.25, p = .024, r =.21]. Indeed, the median content knowledge score was 2.0 both pre- and post-assessment. The statistical analyses of each item under content knowledge were utilized and the Bonferroni adjustment with $\alpha = .05/5 = .01$ was applied in controlling for the familywise error rate (Jaccard & Guilamo-Ramos, 2002). Even though scores mostly were higher on the post-assessment, no statistically significant difference was evident in respect to solving arithmetic operations of fractions (Item 2), recognizing a fractional part-whole (Item 3), when finding an equivalent area (Item 4), and ordering fractional values (Item 5). Instead, the study revealed an impressive result where the post-assessment score was statistically significantly lower than pre-assessment [Z = -3.54, p < .001] for identifying a unit whole (Item 1) with effect size, r = .30. In both assessments, about 58% to 77% preservice teachers did not conceptually understand unit wholes—they were not able to recognize that the shaded part of two pizzas eaten was $\frac{5}{8}$ (item 1). Instead, they believed that the two pizzas were a separated unit whole, supported Lamon's (1999) and Ball's (1993) argument that many individuals tended to refer to a single item (e.g., a single pizza) as a unit (one).

Similarly, the Wilcoxon test analyses were performed for multiple comparisons among items under pedagogical content knowledge for adjusted α values of .05/4 = .0125. The results indicated that the select participants gained much input from the 3-week fraction instruction as measured by their pedagogical content knowledge with all items showing statistically significant differences between the assessment results ($p \leq .001$). Specifically, the preservice teachers indicated statistically significantly greater improvement when recognizing the accurate representation, $2 \times \frac{3}{4}$ for Item 2 with Z = 4.33 and an effect size, r = .36. When they were asked, "A recipe calls for $\frac{3}{4}$ the cup of flour. How much flour is needed if the recipe is doubled?" during the pre-assessment, many mentioned $2 \times \frac{3}{4}$ and $\frac{3}{4} \times 2$ were the same representations and accurate. They stated that because of the commutative property, multiplication is reciprocal thus resulting in the same final answer. Nevertheless, after the instruction, they were able to differentiate that both representations were not identical.

Preservice teachers attained statistically significantly higher scores on the post-assessment for Item 1, 3, and 4 with similar effect size, r = .27. Through a closer examination of the written responses, rich information was revealed about preservice teachers' pedagogical content knowledge on different concepts. For instance, for Item 1 on problem posing, when creating a scenario problem on pre-assessment to match the division fraction $\frac{3}{4} \div \frac{1}{8} = n$, many confused it with $\frac{3}{4} \div 8 = n$. Here are the examples of the fraction scenarios created from the same preservice teacher in the study:

There are 24 students on the class roll. Monday at school, only ¾ of the class was present. The teacher needed to divide the class into eight groups. How many students are in each group? (Pre-assessment)
How many 1/8 ft long strips of ribbon can be cut from a ribbon that is ¾ ft long? (Post-assessment)

The responses were similar to the previous results (Item 2) that showed many preservice teachers in the present study could not differentiate the underlying fractional concepts between two arithmetic operations. To build preservice teachers' understanding of fractions, Dr. X allocated extra time presenting and using various concrete examples for creating scenarios during class instruction. Preservice teachers spent time practicing and generating their scenario problems based on the given fraction operation. As a result, they were able to pose meaningful and accurate scenarios during post-assessment.

For Item 3 and 4 that represented classroom situations for pedagogical analysis, the majority of preservice teachers were having some difficulties in analyzing the contexts and in providing brief descriptions supporting their arguments. For example, only a small number of preservice teachers could identify a missing unit from the children's responses in Item 3. For this reason, preservice teachers had difficulties deducing the size of chunks the children considered in the case of cola. The participants were unable to analyze the children's thinking and were difficult to make inferences based on the children's unlabeled responses. However, other participants considered students' thinking, understanding and misunderstanding when responding to this open-ended item and focused on the number that children provided in much detail. They attempted to predict children's thinking with some drawings, assuming a case with 12 or 24 colas, and working backward to solve the problem. Item 4 focused on the conversation among three children who were comparing two fractions $\frac{4}{4}$ and $\frac{4}{8}$ with their representation. Initially, preservice teachers' responses failed to include the vital concept of unit-whole and the role of a common unit. However, during post-assessment, they were able to point out that the children struggled to recognize which one was a more substantial value using the fraction model, failing to understand that the size of rectangles must be the same (equal unit-whole) before comparing the fractions. For instance, a preservice teacher responded:

Student 2 had the right idea that if student one had made the whole rectangles the same instead of the fractional part of a whole, he would have seen that 4/4≠4/8. However, student 3 is correct in that four parts out of 4 take up the entire rectangle while four parts of 8 take up half of a rectangle, which shows two different fractions. The students definitely could have used fraction bars to make the whole shapes even, but students 2 and 3 seemed to have understood without seeing the representation. They knew that the whole was represented with four parts for one fraction and 8 for the other. They knew *the parts of the whole were being compared.*

Observation during Fraction Instruction

From the classroom observation, most of the preservice teachers were not experiencing problems creating scenarios involving the addition and subtraction of fractions. They were able to pose scenarios and check each other for appropriate modeling and terminologies. However, when it came to multiplication and division, preservice teachers demonstrated the most considerable anxiety because they were not able to pose scenarios that fit the algorithmic operations. They struggled to understand the concept clearly, thus were not able to come up with appropriate word problems. For instance, in one of the class sessions preservice teachers were demonstrated and illustrated scenario problems for two fraction multiplications $2 \times \frac{1}{3}$ and $\frac{1}{3} \times 2$. They were guided in creating meaningful scenarios to differentiate the underlying concepts between these arithmetic operations that can produce the same answer, $\frac{2}{3}$ but have different meanings. Many were not capable of grasping the idea for the first time and showed their frustration by expressing their anxiety and anxiousness to their group members. Then, they asked Dr. X to show other examples that would hopefully help them build their conceptual understanding. After several explanations and practices with concrete models, most of them were able to create some meaningful scenarios involving multiplication and division even though some still did not fully understand the notion. We believed more time and effort would be necessary for them to build and understand these new concepts.

Overall, we found the majority of preservice teachers enjoyed and gained much knowledge from each session of their fraction instruction and the online modules. In class, they actively participated in exploring and experimenting with various manipulatives with group members or partners to build their fractional understanding. One preservice teacher stated, "I believe that manipulatives are the best learning tool for children because they are concrete objects that they can work out the math problems with". The response supports Piaget's (1964) argument that hands-on materials or concrete models can help younger children build mental sense and abstract ideas in mathematics.

Perceptions and Attitudes towards Fractions

After the semester end, sixty-six preservice teachers submitted short written responses that described their perceptions and attitudes towards learning and teaching fractions based on their involvements in the method course. Each written response was coded into meaningful units of data in the form of phrases, sentences, or paragraphs and then these data were grouped into similar emergent themes (Johnson & Christensen, 2012). Eighteen themes were developed from the coding process and were classified into three meta-themes that related to their positive, negative, or unchanged attitudes towards fractions. We categorized preservice teachers under positive attitudes meta-theme when they showed optimistic phrases, statements and words from the written essays. Otherwise, the negative responses noted all preservice teachers' unfavorable endorsement towards learning and teaching fractions.

The results from the coding process of the essays revealed that 62 of 66 selected preservice teachers showed some positive remarks about fractions based on their experiences from participating in the activities during the method course. Nine themes with 377 units of data were formed and considered from the responses demonstrating the change in the perceptions and attitudes. Among all the themes, the course material was frequently coded (100 units of data) indicating this might be the most critical concept noted by the majority of preservice teachers. They believed the incorporation of concrete manipulatives, online modules and videos, daily examples, and scenario problems during class instruction helped their learning about fractions. They discovered meaningful experiences in exploring and practicing different approaches to learning and teaching fractions from the method course. Also, most preservice teachers indicated that they felt comfortable with fractions (71 units of data) and their understanding of fractional concepts had increased (57 units of data) as compared to the beginning of the semester. However, they firmly believed that more time, practice, and research (39 units of data) were needed to understand fractional concepts profoundly and to be able to teach fractional concepts to young children effectively. For example, one preservice teacher stated:

> *I feel that my attitude towards fractions has changed for the better. Dr. X did a great job explaining fractions to the class in ways I could understand. She allowed us to use different manipulatives that helped in understanding fractions rather than just using a pencil and paper. Although I am no 100% confident about teaching fractions I feel so much better about it and I know that with a little more practice I will be able to teach others what I learned about fractions in Dr. X's class.*

Simultaneously, preservice teachers believed the instructor (23 units of data), Dr. X, facilitated the learning of fractional concepts through various teaching aids and scenario problems. Dr. X focused on building the conceptual understanding of fractions with hands-on activities that emphasized the 'why' instead of the 'how' behind addition, subtraction, multiplication, and division of fractions. Based on preservice teachers' past experiences, many argued that they were taught fractions with rules and memorization (40 units of data) in elementary and middle school and the learning process was extremely confusing. Also, they mentioned the 'rote learning' of fractions through a series of steps and formulas (i.e., algorithms) without an understanding of what was going on. A participant stated, "I have known how to get the correct answer when working with fractions, but until this class I didn't understand how I got the answer or why it was correct". It was an unlearning and relearning process for the majority of preservice teachers during the method course with Dr. X.

In addition, creating and practicing different scenarios (e.g., problem-solving problems, word problems) involving fractions with concrete models helped preservice teachers understand the concepts better. After the course, they could "see" how portions make sense in their daily lives and would be interested in incorporating the same teaching method into their future classroom (21 units of data). For instance, one preservice teacher discussed:

> *When we started with creating word problems I thought no big deal. However, it became difficult to word a problem to show if it was* $\frac{3}{4} \times \frac{1}{3}$ *versus* $\frac{1}{3} \times \frac{3}{4}$*, because the modeling was completely different even though the answer will be the same. We practiced a lot on fractions, and also though I do not feel like I am an expert in them, I feel confident enough to teach them. I may have to consult my notes from time to time, but isn't that what those are for? I would use my notes mainly in planning out my lessons.*

Contrary to the many positive responses, some preservice teachers had included an opposite perception throughout their essay. We found four themes that revealed their negative attitudes about the learning and teaching of fractions. Most preservice teachers still felt anxious (64 units of data) about fractions and intimidated by mathematics (14 units of data) as a subject in general. They were nervous about teaching fractions to young children (39 units of data) because the resistance towards learning fractions had been entrenched for years and it was not going to disappear overnight demonstrating their low self-esteem and confidence towards fractions. Therefore, they were still conscious and not sure how to explain the concepts in class. For them, it was difficult and challenging to fully understand fractions especially when creating scenarios for multiplication and division with correct terminologies. Also, we found several preservice teachers were overwhelmed with the methods class (14 units of data) but were able to rationalize the situation, thus tried to motivate themselves and be more positive towards the teaching and learning of fractions. Comment for a preservice teacher:

> *My view of fractions has changed slightly. Before this class, I did not even think about how difficult fractions would be to teach. I got nervous when I heard that it is one of the most challenging subjects to teach. When we started learning about them, I did find that fractions are tough for me to understand. Therefore, I was fearful of teaching them. I think the main thing I am concerned about is the ordering of fractions. For some reason, that gives me the most difficulty. I do believe that after this course, it has gotten a little easier to understand because of the hands-on activities that we did daily. However, I think I will always have fear when teaching fractions.*

The analysis of essay questions revealed two preservice teachers had entirely negative attitudes after instruction. They realized how complex working with fractions can be and hated fractions because they did not feel adequate in their teaching abilities even after taking a method course. Also, we noticed a preservice teacher claimed that she did not see any changes in her attitudes towards fractions after the instruction. She had been struggling with fractions and how to understand the concept before and after the methods course. She mentioned "It is hard for me to learn in a group setting. I am much better at learning things one on one when I cannot comprehend them".

The external generalization of the present study is limited to all elementary preservice teachers from a particular university where participants are conveniently drawn and are based on the results taken from the validated measures. However, we emphatically trust the outcomes because we incorporated both quantitative and qualitative research based on a pragmatist approach providing a perfect combination yielding "complementary strengths and nonoverlapping weaknesses" (Johnson & Turner, 2003, p. 299). Notably, the use of both closed and open-ended items uncovered several key findings from multiple perspectives that add to the existing literature on the preservice teachers' level of knowledge and attitudes towards fractions.

Results of the present study advance current understanding of the effect from a 3-week fraction instruction of a mathematics method course in several valuable ways. First, it provides useful insights into the potential benefits of integrating concrete models, problem-solving, and problem-posing activities on building prospective teachers' profound knowledge of teaching fractions. Second, the study was distinctive because it represents the first study using a mixed-methods research design to more deeply understand the complex phenomena of selected preservice teachers' content knowledge and pedagogical content knowledge of fractions, and attitudes towards fractions.

Seventy-one preservice teachers participated in the study where they complete pre- and post-assessments on content knowledge and pedagogical content knowledge of fractions. Also, they submitted an essay about attitudes towards fractions after the 3-week instructional unit. Importantly, the study showed that the incorporation of problem-solving, problem posing, and hands-on activities have the potential to assist instructors in making fraction instruction more meaningful and enjoyable thus eventually helping learners to develop conceptual knowledge of fractions (Thompson, 1994). Some other significant outcomes and issues need to be addressed aside from the inconclusive results as measured by the closed and open assessment tasks.

Related to the select preservice teachers' content knowledge of fractions, results from the descriptive statistics revealed low percentages of preservice teachers achieved the full points for most of the items on both pre-and post-assessments. The results might indicate the participants were weak on the particular fraction concepts being tested. Specifically, we noticed the select preservice teachers were not aware that the unit-whole may include more than one item or may consist of items packaged as one known as a composite unit. The behavior was consistently evident when they were identifying the unit-whole of Item 4 (closed-ended) and Item 1 (open-ended). On pre- and post-assessments, almost 90% of preservice teachers were not able to notice that children were considering a different set of unit-whole on Item 4 (close-ended). Similarly, the picture for Item 1 (open-ended) showed two pizzas with the same or different kinds, which is called a one two-unit and not two one-unit pizzas. However, many preservice teachers assumed a single object (i.e., a single pizza) as a unit (one) and answered $1\frac{1}{4}$ pizza instead of $\frac{5}{8}$ pizza. Lamon (1999) argued that the concept of the unit has frequently been neglected in classroom discussions. Majority teachers and textbook authors emphasized using the same unit-whole such as one cake in which making students assumed a unit was always a single object (Lamon, 1999). Lamon argued many classroom instructions failed to introduce the vital concept of unit-whole of fractions to students.

CONCLUSION

While this study revealed the preservice teachers' lack of content knowledge of fractions, the results showed compelling evidence of their understanding of teaching fractions supported previous research by Ball (1993), Hill (2010), Ma (1999), Newton (2008), and Timmerman (2004). It is noteworthy to see that they demonstrated statistically significant improvement in their level of pedagogical content knowledge even though they received only a three-week fraction instruction unit mainly focusing on problem posing and hands-on activities. We believe the close interaction with the instructor and quality of instruction during the mathematics method course possibly helped preservice teachers develop knowledge about pedagogy and nurture their fraction teaching repertoire (White et al., 2005). Our data warrant further investigation into the relationship among the duration of instruction, the role of instructor, quality of instruction, and level of fractional knowledge they gained. Therefore, a more thorough examination is critical to determine the factors that might affect the level of knowledge before and after instruction.

In the post-assessment, many preservice teachers successfully generated and identified meaningful and accurate fraction situations that matched the given arithmetic operations. When reviewing their fraction essays, they mentioned how problem posing and hands-on activities had an impact on their learning and understanding of fractions. The majority of preservice teachers showed positive attitudes towards fractions after receiving the instruction and believed it was an eye-opening session. Still, some expressed their concerns about additional practice and the time needed for developing confidence in teaching fractions.

Our results revealed that the select preservice teachers were able to reflect on students' work and to suggest basic teaching approaches for helping students' misunderstanding of fraction concepts. However, the responses showed they had limited practical experiences teaching fractions to children. Because the participants would be elementary school teachers later, more exposure working directly with students in a real classroom setting is needed. The experience responding to elementary students would develop their pedagogical skills and improve preservice teachers' understanding of educational components (Llinares, 2002; White et al., 2005).

Taken together, teacher education services are a place for teachers to build expertise, teach skills, values, and understanding in order to become effective teachers in mathematics (Llinares, 2002). Nonetheless, the prior knowledge and behaviors of service teachers have a significant impact on what and how they learn during their teacher training programs (Llinares, 2002). Teacher educators are responsible for providing potential teachers with the in-depth knowledge and experience required to effectively teach the basic concepts of fractions.

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GENERALIZATION STRATEGY OF LINEAR PATTERNS FROM FIELD-DEPENDENT COGNITIVE STYLE

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Abstract

Linear pattern is the primary material in learning number patterns in junior high schools, but there are still many students who fail to generalize the linear pattern. The students' failure in generalizing the pattern occurred when the students ended to view the problems globally without breaking them into the constructors' components such as the experience of field-dependent type students. For this reason, this study was carried out to explore the thinking process of students who fail and investigate the thinking processes of students who succeed in generalizing linear patterns. The results of this study provide an effective learning strategy solution for field-dependent students in generalizing linear patterns. This study employed a qualitative approach with a case study design to junior high school students. The results indicated that students in the field-dependent cognitive style looked at pattern questions represented in the form of geometric images globally without looking at the structure of the image. Two strategies for generalizing linear patterns used by field-dependent students were examined, namely recursive and different strategies.

Keywords: Generalization, Generalization Strategy, Cognitive Field-Dependent Style, Linear Pattern

Abstrak

Pola linear merupakan materi utama dalam pembelajaran pola bilangan di Sekolah Menengah Pertama, akan tetapi masih banyak siswa yang mengalami kegagalan dalam menggeneralisasi pola linear tersebut. Kegagalan siswa dalam menggeneralisasi pola diduga banyak dilakukan oleh siswa yang cenderung memandang masalah secara global tanpa memecah ke dalam komponen penyusunnya seperti siswa *field-dependent*. Untuk itu dalam penelitian ini bertujuan untuk mempelajari proses berpikir siswa yang gagal dan mempelajari proses berpikir siswa yang berhasil dalam menggeneralisasi pola linier. Hasil penelitian ini memberikan solusi strategi belajar yang efektif untuk siswa *field-dependent* dalam menggeneralisasi pola linier. Penelitian ini adalah penelitian kualitatif dengan pendekatan studi kasus terhadap siswa Sekolah Menengah Pertama. Hasil penelitian menunjukkan bahwa siswa dengan gaya kognitif *field-dependent* memandang soal pola yang direpresentasikan dalam bentuk gambar geometri secara global tanpa melihat struktur gambar. Pada penelitian ini mempelajari dua strategi generalisasi pola linearyang digunakan oleh siswa *field-dependent*, yaitu strategi rekursif dan strategi beda.

Kata kunci: Generalisasi, Strategi Generalisasi, Gaya Kognitif Field-Dependent, Pola Linier

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Pattern generalization is essential in mathematics learning since it can develop new knowledge (Ellis, Lockwood, Tillema, & Moore, 2017). Furthermore, pattern generalization is conceptually related to mathematical structures (Rivera, 2015), and pattern realization is the latest approach to studying algebra (Barbosa, Palhares, & Vale, 2007). The importance of pattern generalization has resulted in the curriculum shifts in Indonesia, teaching pattern generalization to students at the secondary school level (Kemendikbud, 2016). Likewise, the National Council of Teacher of Mathematics recommends the idea of algebraic reasoning through pattern generalization in elementary and secondary schools (NCTM, 2000). Therefore, pattern generalization would be one of the important supporting topics to learn mathematics in Indonesia.

One of the pattern materials is generalization of linear patterns. Generalization is defined as the process of finding similarities in each example or case, so that it applies in general (Brief, 2003; Kaput, 2008; Kaput, Blanton, & Moreno, 2008). While the linear pattern is a pattern that has the first difference that is constant and the terms are formulated with $U_n = an + b$ (Stacey, 1989). Thus, the generalization of linear patterns in this study is the process of finding similarities in each of the terms of a linear pattern, so the formula is used to determine the terms of the linear pattern.

A study carried out by Becker and Rivera (2005) yielded that junior high school students still failed to generalize linear patterns because they tend to start with numerical strategies. These students focused on numerical data which were trapped in recursive relationships (Hourigan & Leavy, 2015). In addition, students failed to generalize linear patterns because they used a trial and error strategy without understanding coefficients in linear patterns (Becker & Rivera, 2005). Furthermore, in many cases students who generalize linear patterns follow patterns recursively (Chua, 2009; Hourigan & Leavy, 2015). The results of Ellis' study (2007) also showed that students who recognize patterns or rules may not be able to generalize the pattern. Students' failures in generalizing linear patterns are due to the strategies students use.

Factors influencing students' failures in generalizing linear patterns are worth exploring. In their study, Lannin, Barker, and Townsend (2006) found that there were three factors influencing the selection and use of pattern generalization strategies, namely task factors, social factors, and cognitive factors. Task factors relate to tasks that are based on problem situations (Lannin, et al. 2006). For instance, students tend to use recursive strategies when solving generalization problems whose independent variables are implicitly stated. Social factors are social interactions when students are involved in the task of simultaneous generalization influenced by other students or teachers (Lannin, et al. 2006). Lastly, cognitive factors can be in the form of knowledge possessed by students (Lannin, et al. 2006) or cognitive factors can also be cognitive styles of students. Cognitive style is the tendency of individuals to understand, think, and store information (Hadfield & Maddux, 1988).

Recent research only focused on generalization strategies used by students, such as strategies to find differences, namely generalization strategies that focus on the differences between terms in number patterns (Montenegro, Costa, & Lopes, 2018; Becker & Rivera, 2005; Stacey, 1989), strategies for finding patterns, that is, generalization strategies focusing on the pattern of the formation of terms from a number pattern (Ellis, 2007), quantity relationship strategy, namely the strategy of generalizing patterns involving the relationship of input values with output values (Ellis, 2011), trial and error strategies (Becker & Rivera, 2005), linear pattern strategies, namely generalization strategies using linear pattern formulas (Stacey, 1989), visual strategies, namely visual grouping strategies, and visual growth strategies (Becker & Rivera, 2005). Even though students often used recursive strategy, namely the strategy to find the next n^{th} term using the previous n^{th} term (Becker & Rivera, 2005; Hourigan & Leavy, 2015; Lannin, et al. 2006). Through this recursive strategy, students often experienced failure in generalizing patterns, so identification of the strategy was required to know the layout of the error of the students in generalizing patterns.

Previous studies partly focused on strategies employed by students in generalizing patterns. This means that there is still a space to conduct research of pattern generalizations involving strategies to generalize cognitive patterns and factors. Lannin, et al. (2006) contended that when students work on tasks with different structures (i.e. tasks with increasing and decreasing structures), they have a tendency to use the same strategy. The results also portrayed that students who consciously use recursive strategies will change to use more effective strategies. For this reason, Lannin, et al. (2006) proposed for further research of pattern generalization involving cognitive factors. Based on this fact, the present study seeks to uncover effective strategies employed by students in generalizing linear patterns in terms of cognitive factors.

There are two kinds of cognitive factors, field-dependent and field-independent styles. Cognitive style is defined as the tendency to see the problem globally, while the cognitive style of field-independent is defined as the tendency to see the problem into constructor components (Onyekuru, 2015; Tinajero & Paramo, 1998; Loranger, et al. 1984; Coventry, 1989). Several studies captured that cognitive style influences students in learning. The results of the study Pithers (2002) indicated that the field-dependent and field-independent cognitive style approaches have implications on the effectiveness of individual and group learning. The results of the study by Karamaerouz, Abdi, and Laei (2013) found that field-dependent students are passive and dependent, while field-independent students are active and independent. The results of Al-Salameh's study (2011) indicated that individuals with field-dependent cognitive styles cannot handle objects that are perceived separately from the surrounding elements, while field-independent students can handle objects separately from the surrounding elements. This study concluded that cognitive style influences how a person learns, including solving problems of generalizing linear patterns.

This study focuses on field-dependent cognitive styles. The main reason for choosing this cognitive styles is due to the characteristic possessed by field-dependent students who view complex situations globally without identifying key elements of these complex situations (Coventry, 1989; Loranger, Gosselin, & Kaley, 1984; Onyekuru, 2015; Tinajero & Paramo, 1998). The second reason is because students with field-dependent cognitive styles experience more mathematical anxiety (Hadfield & Maddux, 1988); less active in classroom learning (Loranger, et al. 1984); lacking in accuracy (Ling & Salvendy, 2009); more errors in algebraic thinking (Agoestanto, Sukestiyarno, Isnarto, Rochmad, & Lestari, 2019); better for verbal information than acting analytically (Karamaerouz, et al. 2013). This study explores how students with characteristics looked at the situation globally when solving problems of generalization. In addition, recommendations from previous studies Lannin, et al. (2006) to conduct further research of cognitive factors in the selection and use of linear pattern generalization strategies were implemented.

This study provides more understanding that cognitive styles also influence the selection and use of strategies for generalizing linear patterns. The results of this study are also useful for teachers in giving strategies to generalize appropriate linear patterns to students with field-dependent cognitive styles. In addition, it also contributes to solving problems that field-dependent students have in learning to generalize linear patterns, especially mathematical anxiety.

METHOD

This research was conducted in two state secondary schools with "A" accreditation. This research implemented randomized sampling consisting of 40 students of grade 8th who has learned the materials about the patterns of numbers. A descriptive qualitative approach was employed in this study. Forty junior high schools were recruited to complete the Group Embedded Figure Test (GEFT) instrument. From this test, six students were found to have field-dependent characteristics, eight students have field-independent characteristics, and twenty-six students have the same characteristics between field-dependent and field-independent. From this data, six students who had field-dependent characteristics were invited as the participants. The data were analyzed by grouping the generalization strategies employed by the participants. Interviews were used to validate the descriptive results of the generalization strategies.

The first phase was to determine the research participants, namely students with field-dependent characteristic. Researchers used a copy of Group Embedded Figure Test (GEFT) developed by Witkin, Oltman, Raskin, and Karp (in Mykytyn, JR., 1989) to determine participant students with field-dependent characteristic. GEFT is a perceptual test that requires the students to find a picture that was seen before in more complex pictures. GEFT consists of 7 simple images and 18 complex pictures tested in 20 minutes. A copy of the instrument Group Embedded Figure Test (GEFT) was translated into Indonesian and given to forty students in grade eight of a secondary school. When the students finished working on a copy of the instrument Group Embedded Figure Test, the result was computed to classify the students into the field-dependent, field-independent, and ambiverts (neither field-dependent nor field-independent). This test consisted of three sessions, the first session with simple questions contained seven test items, the second session consisted of nine medium-difficult test items and the third session consisted of nine difficult test items. Witkin (in Morrison, 1988) reported that seven the first points did not have a discriminatory value with a population, or they were exercise session. Nine items in the second section and nine items in the third part of each were scored one for the correct answer and score zero for incorrect answers (Morrison, 1988). So, the students' minimum score was zero, and the maximum score was 18.

Categorization of the field-dependent, field-independent, and ambiverts in this research adopted the procedures of Maghsudi (2007). The reason researchers adopted Maghsudi' procedure (2007) was that Witkin, Oltman, Raskin, and Karp did not specify a clear score to categorize the individual of field dependent and field independent by their performance in the GEFT (Onyekuru,2015). Maghsudi's procedure (2007) was also adopted by Onyekuru's research (2015) and also the research of Ayvaz, Gündüz, Durmuş, and Dündar (2016) who utilized Mean and Standard Deviation. Maghsudi (2007) used Mean $\pm$ Standard Deviation as a limit point (see Figure 1) where the Mean and Standard deviation was determined from the students' score after they completed the Group Embedded Figure Test instrument (GEFT).

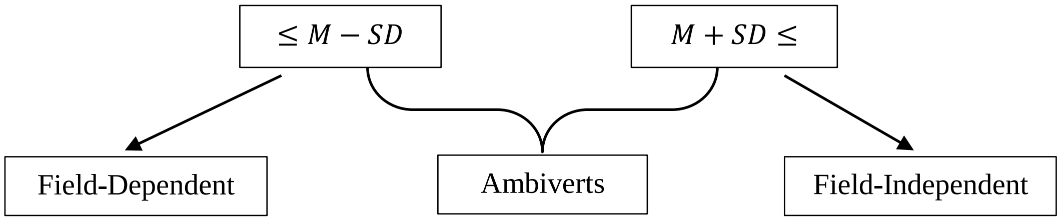


Figure 1. Category Cognitive by Implementing Style (M: Mean and SD: Standard Deviation)

From the Figure 1, it is clear that if the students obtained score $\leq \mathrm{M} - \mathrm{SD}$, they were in the category of field-dependent, but if the students got a score $\geq \mathrm{M} + SD$, then they were included in the field-independent category, and the rest were ambiverts (Maghsudi, 2007). Therefore, the students got less than or equal to the average minus Standard Deviation were classified into the field-dependent category while whose score were greater or equal to the average plus Standard Deviation were classified into the field-independent category (Onyekuru, 2015). The rest of the students were in the category of ambiverts. The students of the field-dependent category were selected as the participants of the research.

The second stage was to analyze generalization strategies used by the participants in the completion of the second instrument, namely the problem of the linear pattern (see Figure 2). The instrument generalization linear pattern is the test that requires the students to find a formula of the term nth term from the pattern of numbers that have a constant first difference. The second instrument was used to find out strategies for generalizing linear patterns used by the participants. This instrument consisted of three questions with different representations (see Figure 2). The first question used a representation of a number sequence. This question was used to find out how field-dependent students found input values to generalize linear patterns. Questions number 2 and number 3 were used to find out whether students with field-dependent cognitive styles viewed images globally or broke down the components of the image. This problem could be used to find out the pattern generalization strategies of students in a field-dependent cognitive style, whether students in a field-dependent cognitive style used visual strategies or numerical strategies, or changing the image to a number sequence.

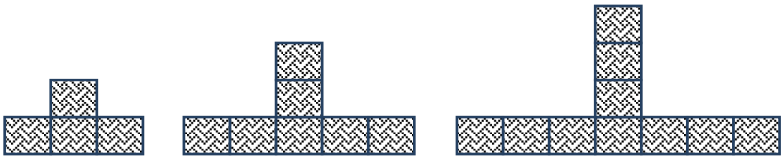
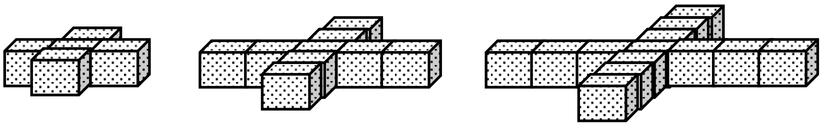
Questions of linear pattern using number representation	1. Number sequence is presented as follows: 6, 10, 14, 18, … Decide: a. Number of the 5th until the 10th term! Write down how you find the formula! b. General formula for term *n*! Write down your method! c. The 57th term! Write down your method!
Questions of linear pattern using plane figures	2. Pay attention to this linear patter:  Decide: a. How many squares appear in the 4th until 10th pattern! Write down your method! b. General formula to decide the number of squares in pattern *n*! Write down your method! c. How many squares in the 71th pattern! Write down your method!
Questions of linear pattern using solid figures	3. Pay attention to this linear pattern:  Decide: a. How many cubes appear in the 4th until the 10th pattern! Write down your method! b. General formula to decide the number of cubes in pattern *n*! Write down your method! c. How many cubes in the 83rd pattern! Write down your method!

Figure 2. Pattern Generalization Instrument

The second data obtained from the results of field-dependent students' work in solving the problem of generalizing patterns was analyzed descriptively to describe the strategies of generalizing linear patterns used by the students with field-dependent cognitive styles. The qualitative analysis for the students' generalization strategy consisted of three steps. The first step was to analyze the views of students about the given Mathematics questions. The results proved that the characteristics of the field-dependent students tended to view the complicated situation globally without identifying the elements of the constructors (Onyekuru, 2015; Tinajero & Paramo, 1998; Loranger, et al. 1984; Coventry, 1989).

The second step was to analyze the process of generalization that the field-dependent students did. Further, the analysis of the process of generalization in this research used the taxonomy of generalizations that consists of relating, searching, and extending (Ellis, 2007). The relating action that how the students form the relationship between the situations in the questions with the other situation in the act of searching was analyzed. Then, whether the students perform repetitive actions, find similarities, or find patterns was also analyzed. Meanwhile, the extending activities related to how the students extended the similarity or a more general pattern was analyzed later (Ellis, 2007). The third step was to analyze the results of how the pattern generalization that students wrote the formula of the n^{th} term (Ellis, 2007). Then, from the results of the analysis using the steps in Figure 2, it was obtained a descriptive process of the students field-dependent in generalizing the pattern began from paying attention to the questions globally. Then, they generalized the patterns and produced the formula generalizations of the n^{th} term.

RESULTS AND DISCUSSION

From the Group Embedded Figure Test (GEFT) filled out by the participants, scores and frequencies are presented in Table 1.

Table 1. GEFT Score and Frequency

Score	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Frequency	1	0	0	0	2	1	2	3	3	2	1	4	3	6	1	3	5	2	1

Table 1 shows the average and standard deviation of the student scores were determined using the formula (1).

$$\bar{x} = \frac{\sum_{i=1}^{n}(x_i \times f_i)}{n} \quad (1)$$

The average student score is 11.175, followed by determining the standard deviation (4.29) using the formula (2).

$$S = \sqrt{\frac{n\sum x_i^2 - (\sum x_i)^2}{n(n-1)}} \quad (2)$$

From this analysis, the average range was also determined (between the lower limit and the upper limit). The lower limit was obtained from the mean less standard deviation (M-SD), that is 11.175-4.29 = 6.885 and the upper limit was determined from the average added standard deviation (M + SD), which is 11.175 + 4.29 = 15.465.

From the average range, three student score categories were obtained, consisting of six students who received the GEFT score below the average range ($x_i \leq 6.885$), twenty-six students who obtained GEFT score between the average range ($6.885 < x_i < 15.465$), and eight students who achieved GEFT score above the average range ($x_i \geq 15.465$). From the student score category, it was found that there were six students having field-dependent characteristics (because they got a GEFT score below the average range), twenty-six students had the same characteristics between field-dependent and field-independent (because they got GEFT scores between ranges average), and eight students had field-independent characteristics (because they get a GEFT score above the average range). Consequently, the six students were opted as the research participants.

From the linear pattern questions answered by the participants, it was found that the field-dependent subjects employed two strategies to generalize linear patterns; these are recursive and difference strategies. This data were presented in Table 2.

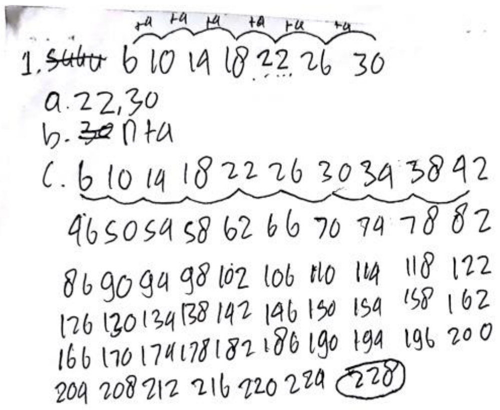
Table 2. Generalization Strategy Based on Question Number

No	Generalization Strategy		Failed
	Recursive	**Difference**	
1	1 students	2 students	3 students
2	2 students	2 students	2 students
3	1 students	2 students	3 students

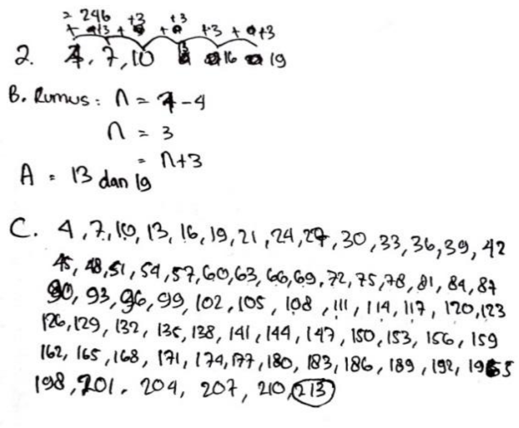
The results of the analysis on the strategy used by the participants and interviews to explore students' thinking process of the strategy are presented below.

Recursive Strategy

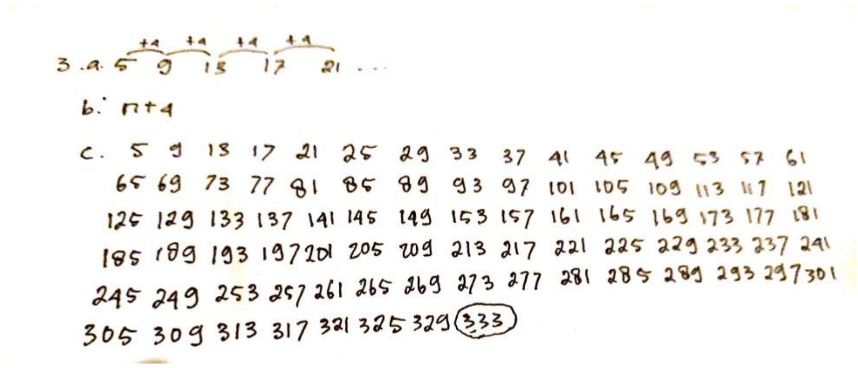
The first strategy used by field-dependent subjects is a recursive strategy. Recursive strategy is term-to-term reasoning (Lannin, et al. 2006) or in other words recursive strategies describe relationships that occur in situations between sequent values of independent variables. The results of this study indicate that participants use a recursive strategy by writing all of their terms to the terms in question. The results of the participants completing generalization questions using recursive strategies can be viewed in Figure 3. These results indicate that the participant's answer is correct, while the way to write the generalization formula is incorrect.



(a) Answer of Question 1



(b) Answer of Question 2



(c) Answer of Question 3

Figure 3. Solving Problem Using Recursive Strategy

From Figure 3 (a), it can be seen that in solving number generalization problems, participants focus on differences. Participants use differences to complete close generalizations, namely determining the 5th term, 6th term, and 7th term. In addition, participants write the results of the wrong generalization, which is number 1 is $U_n = n + 4$. The interview below was conducted to understand how participants obtained the formula shown in Figure 3.

Researcher	:	*in the generalization formula* $U_n = n + 4$ *(what is n? and 4?)*
Participant 1	:	*n is 6 Sir and 4 is the different, because* $6 + 4 = 10$.
Researcher	:	*Why did you write* $U_n = n + 4$*?*
Participant 1	:	*Because I could not find the generalization formula.*
Researcher	:	*Why did you write all terms until the 57th terms?!*
Participant 1	:	*Because I didn't know the formula, so I wrote it all.*
Researcher	:	*How did you know this formula?!*
Participant 1	:	*I did it by myself, Sir!*

From the interviews, it is unveiled that the field independent subjects do not view the input value, but the output value by saying n is 6. The participants cannot find the generalization formula. As the result, the participants write the formula to determine the second term with the first term plus difference. To determine the third term, the participants write the second term which is added differently, and to determine the fourth term, the participants write the third term that is added differently, and so on until the 57th term. Furthermore, because participants write all the terms until the questioned term, this strategy is called recursive strategy, which is to write all the terms until the term is asked.

In the question of plane figure (Figure 3 (b)) and in the question of solid figure (Figure 3 (c)), it can be seen that the field-dependent subjects in solving generalization problems in the form of visual images change the image into number sequence pattern. It means that participants view visual images globally and do not view the components that make up images that can be used as a basis for generalizing patterns. Then participants also write the formula for generalizing the wrong pattern for question number 2 with the formula $U_n = n + 3$. After finding this wrong generalization formula, participants write all of their terms until the 71st terms. Participants also write the formula for generalizing the wrong pattern for question number 3 with the formula $U_n = n + 4$. After finding this wrong generalization formula, participants write all of their terms to the 83rd term.

Based on the tasks and interviews with participants, the recursive strategy process of field-dependent subjects that causes failure of pattern generalization can be seen in Figure 4.

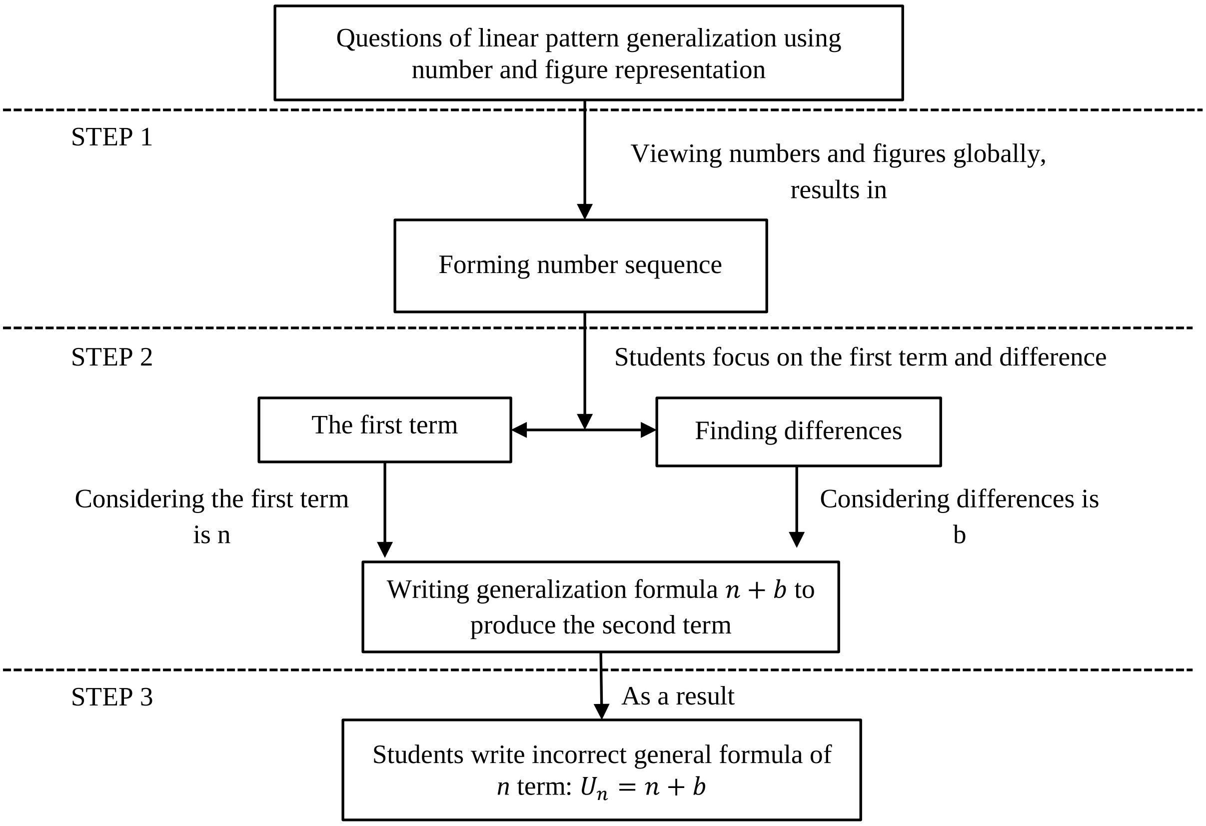


Figure 4. The Thinking Process of Incorrect Recursive Strategy

From Figure 4, there are three stages of the recursive strategy process carried out by field-dependent subjects. The first step undertaken by the field-dependent subjects looks at the question of generalization globally and is converted into a row of numbers. This is consistent with the characteristics of globally oriented field-dependent students and prefers external information structures (Onyekuru, 2015). Individuals with field-dependent cognitive styles tend to be global in analyzing learning situations and have difficulty breaking information into isolated parts (Onyekuru, 2015). Field-dependent subjects are those who have greater difficulty releasing part of the context (Tinajero & Paramo, 1998). Field-dependent subjects react to complex situations globally without identifying key elements and tend to be controlled by situations rather than themselves that make up the situation (Coventry, 1989; Loranger, et al. 1984). Field-dependent students are less able to think analytically (Karamaerouz, et al. 2013). Field-dependent students are not good at choosing numbers, pictures in detail (Sözcü, İpek, & Kınay, 2016).

In the second step, it was obtained that the participants connected with the situation of the question during the linear pattern learning at school. These actions resulted in the participants focusing on searching the variance from the sequence numbers pattern. The participants expanded the activities to search for a variance to find the solution for the nth term. The dependent field subjects were influenced by the characteristics of the environment, especially by their teachers. This means that field-dependent subjects are influenced by social factors, such as the teacher. It is depicted from an interview carried out to the participant: "Where did you get this method?". The participant answers "I got this method from the teacher". This is consistent with the theory that field-dependent students have higher social skills than field-independent students, in which field-dependent students prefer group learning, interact with teachers and peers (Rayner & Riding, 1997). Field-dependent students tend to be friendly, such as working with other people, influenced by others, being adaptive at solving problems related to social interaction, influenced by criticism, preferring jobs based on interaction with others, disliking tasks that require analysis (Witkin, Moore, Goodenough, & Cox, 1977), and field-dependent students tend to have sensitivity to others (Holmes, Liden, & Shin, 2013).

In the third step, participants form a generalization formula using the first and different terms. Participants assume the first term is n, so participants write the formula for generalizing $U_n = n + b$. This can be seen from the results of student interviews which state that n is 6 of the formula n + 4, so participants contend 6 + 4 = 10. It is written when participants find a term obtained from the second term plus 4, and so on. As a result of this formula, participants include in a recursive relationship, which uses the rule "add 4". This is in line with the opinion of Hourigan and Leavy (2015) arguing that if students focus on numerical data, students are at risk for recursive relationships. Recursively participants write all their terms to the questioned term (this is seen in the results of participants who write all of their terms up to the 57th term). Hourigan and Leavy (2015) said that this recursive rule approach cannot be said as a generalization strategy.

Different Strategy

Generalization strategies that are also used by field-dependent subjects are different strategies. Different is defined by the result of the reduction in the *n-th* term by the term (n-1). Mathematically, the difference is written in the formula $b = U_n - U_{(n-1)}$. Stacey (1989) contended that different strategies are assumed to be repeated additions. The different strategies in this study are defined as generalization strategies using the results of the reduction in the *n-th* term by the (n-1) term. The results of participants completing the generalization problem using a different strategy can be seen in Figure 5.

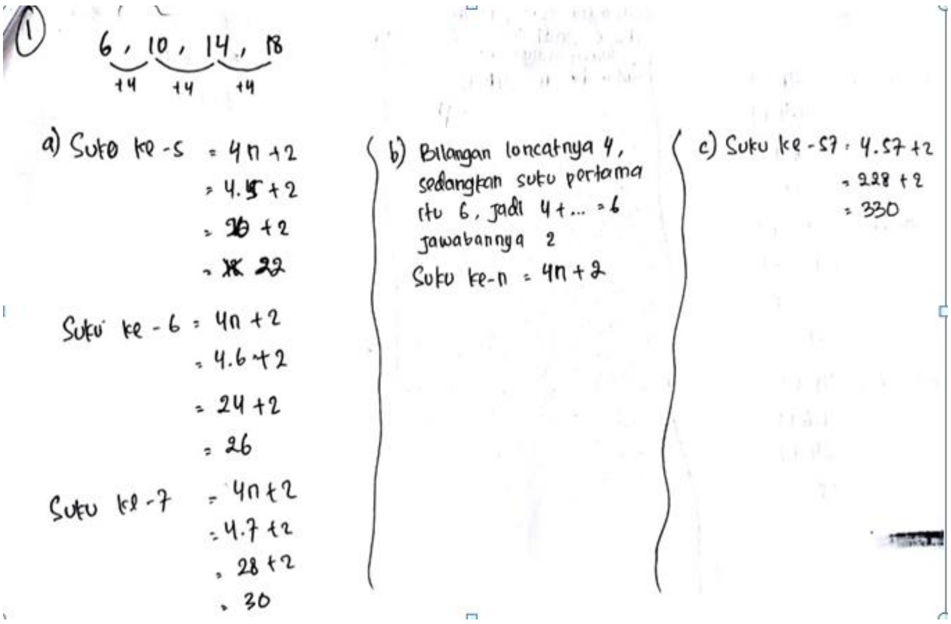


Figure 5. Generalization Done by Participants Using Difference Strategy

From Figure 5, participants have completed problem number 1 by finding a difference. First, participants look for differences from the number sequence. Second, participants use differences to find the general formula for the sequence. Participants argued that difference is a jump number (problem number 1 has a difference of 4, so participants proceed to number 4). Afterwards, participants connect the difference with the first term, which is 6. Different relations with the first term written by participants in the form of $4 + \cdots = 6$ (participants write the answer is 2). By combining jump numbers and jump number relationships with the first term, participants write the first generalization formula, namely $U_n = 4n + 2$. Participants complete questions number 2, 3, and 4 in the same way when solving question number 1. These steps are confirmed to participants through the interview as follows.

Researcher	:	*For what purposes did you find a difference?*
Participant 2	:	*To decide the generalization formula, Sir*
Researcher	:	*What did you think difference as jump number!*
Participant 2	:	*Yes Sir, because it has equal difference between the terms, so jump to 4!*
Researcher	:	*Where did you obtain the generalization formula* $U_n = 4n + 2$
Participant 2	:	*4n was obtained from jump number, Sir. If jump one time, so it is* $4(1) = 4$, *two times is* $4(2) = 8$, *and so on.* *2 was obtained from 4 added 2 equals 6. So, I wrote the formula as follows:* $U_n = 4n + 2$
Researcher	:	*Why did you complete number 2, 3, and 4 using the same way?*
Participant 2	:	*Yes, Sir. Because this is the best way*
Researcher	:	*How did you get this way?!*
Participant 2	;	*I am thinking by myself, Sir..*

Based on the results of the interview above, it was found that participants determine the difference to find the generalization formula. Participants interpret differences as jump numbers, and write jump numbers in the form of 4n. Afterwards, participants look for relationships 4 with 6 (4 + ... = 6), namely plus 2. Participants write their generalization formula in the form of $U_n = 4n + 2$. Participants solve the generalization questions number 2 and number 3 in the same way, namely finding differences. The results of the interview indicate that participants share how to find differences that can be done and it is easier to solve the problem of generalizing linear patterns. This strategy is obtained from the participants themselves.

The process of finding strategy differs from field-dependent subjects can be seen in Figure 6. From Figure 6 it is found that there are 3 steps in field-dependent subjects completing generalizations of linear patterns using strategies to find differences. The first step taken by field-dependent subjects is to look at the problem globally and fertilize it into a row of numbers. This stage is the same as students who use recursive strategies. This means that individuals with cognitive style have similarities in information processing, where field-dependent individuals process information globally (Coventry, 1989; Loranger, et al. 1984; Onyekuru, 2015; Tinajero & Paramo, 1998). In addition field-dependent students find it difficult to break information into isolated parts (Onyekuru, 2015).

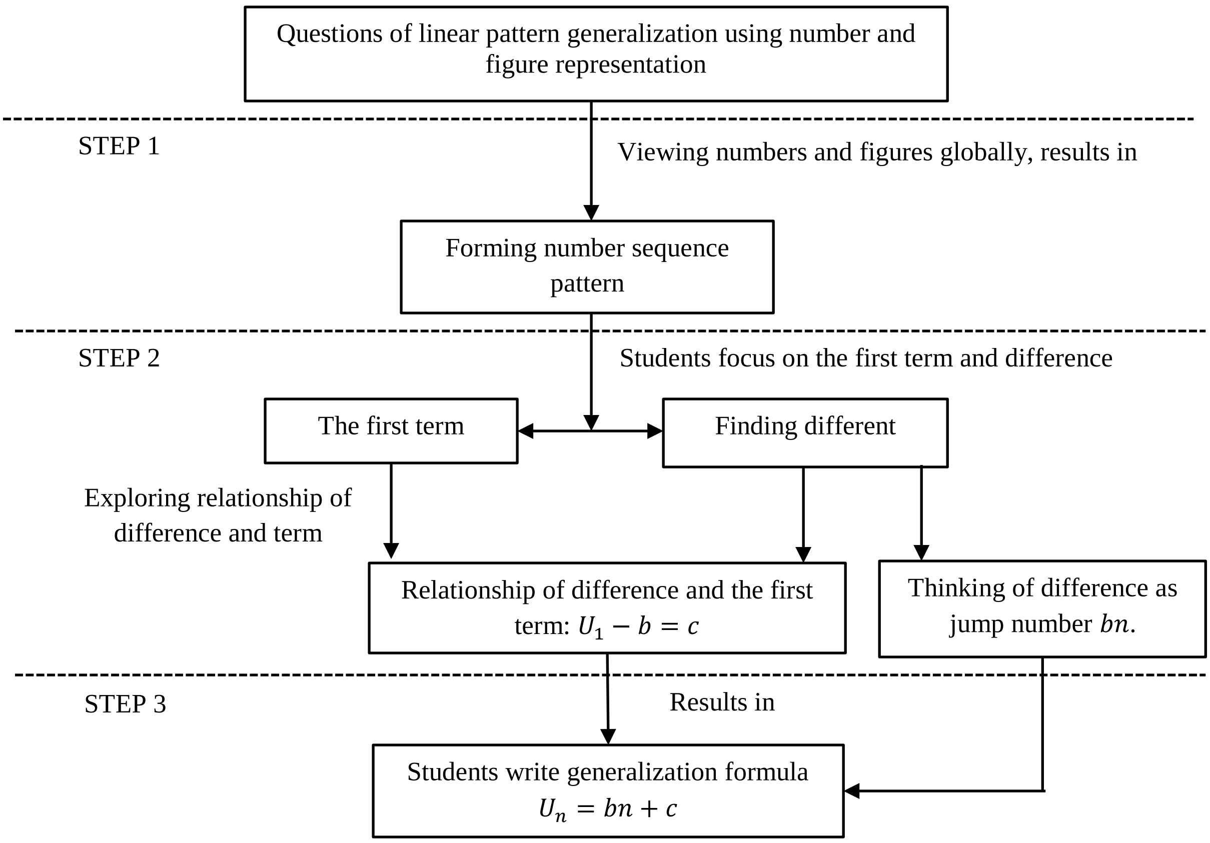


Figure 6. The Thinking Process of the Presence of Difference Strategy

The second step, this second stage has similarities and differences with participants who use recursive strategies. Its similarity was to connect with the current situation of teaching in schools. In addition, the similarity of the recursive strategy with the strategy of finding differences at this stage is that participants focus on differences and the first term. The difference in recursive strategy with the strategy of finding differences at this stage is the way to kick different and the first term. Participants with recursive strategies view the first term plus different to get the second term and view the second term plus the difference to get the third term, and so on. Meanwhile, participants with different find strategies view differences as jump numbers written with bn and then look for different relationships with the first term, namely $U_1 - b = c$. Then the participants expanded the relationship between the jump numbers bn with c to obtain more general rules. So, the participants successfully wrote the generalization linier formula, namely $U_n = bn + c$.. This means that field-dependent subjects can see the structure of information that is explicitly visible (Tinajero & Paramo, 1998), which are the first and different terms. At this stage, it is also expected that learning for field-dependent subjects sees the relationship of the visible information structure (i.e. the first and different terms). Participants are also asked to find the relationship of the first term with different (6 with 4, obtained 4 (1) + 2 = 6), the relationship of the second term with different (10 with 4, obtained 4 (2) + 2 = 10), the third term relationship with different (14 with 4, obtained 4 (3) + 2 = 14), so that the relationship is generally obtained between the terms with different, namely 4n + 2.

In the third step, there is a tangible difference between a recursive strategy and a strategy for finding differences. The recursive strategy is done by writing all the terms until questioned term, while the strategy of finding differences succeeded in generalizing linear patterns by writing the results of generalizing linear patterns, namely $U_n = 4n + 2$. Afterwards, participants use this generalization formula to find the next terms. For instance, participants find the 57th term with this formula, namely $U_{57} = 4(57) + 2 = 230$. The strategy of finding this difference is ultimately successful in making generalizations. Thus, strategies to find differences include strategies for generalizing linear patterns.

Based on the resolution of the problem of linear pattern generalization, it was found that there were two strategies used by field-dependent subjects in generalizing patterns, namely recursive and difference strategies. The recursive strategy starts with subjects changing the image pattern into a row of numbers. Second, subjects look for differences from the number sequence. Third, subjects use the difference and the first term to determine the second term, use the difference and the second term to determine the third term, and so on which results in the subjects writing all the terms to the terms that is questioned. While the strategy of finding differences starts with students changing the image pattern into a row of numbers. Second, subjects look for differences from the number sequence. Third, subjects interpret differences as jump numbers (written bn) and look for the first term relationship with different ($U_1 - b = c$). Subjects who use the find difference strategy succeed in generalizing a linear pattern by writing the general form of the nth term generalization, namely $U_n = bn + c$.

The critical findings of this research are that the participants considered the first term is n, and the vary is b, which led them to the failure in generalizing the term of a linear pattern, so that the participants wrote an error generalization formula, $U_n = n + b$. As a result, recursively, the participants rendered all the term terms of the pattern of numbers until the term questioned term. This failure is resolved by using a strategy of difference, which the students look different as the jump numbers written in “bn” and seek the relationship of varies with the first term, $U_1 - b = c$. So, they could obtain an appropriate generalization of the linear pattern, $U_n = bn + c$.

CONCLUSION

The results of this study contribute to generalization learning of linear patterns for students with field-dependent categories. The limitation of the present study was that the researcher selected 40 students of grade 8 from two state secondary schools as the participants. Nevertheless, the findings of the present research have significant implications for the mathematics teachers in teaching linear patterns, especially in resolving the problem of recursive strategy that caused the students to fail to generalize linear pattern. From the results of this study, it is recommended that teachers teach linear patterns to field-dependent students by changing the pattern of images into number patterns since field-dependent students prefer to generalize geometric patterns by changing into number patterns rather than finding image structures that can be generalized. For this reason, it is also necessary that field-dependent students recognize the relationship between different terms until a general formula from the nth term is found. Further, future studies can employ a larger sample to obtain strategies used by the field-dependent students and the errors they make to generalize the linear pattern. In addition, further studies are worth-conducting to examine generalization strategies of students who have field-independent cognitive styles. By examining the generalization strategies of students in a field-dependent cognitive style, it impacts for learning betterment of field-independent students.

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SEMIOTIC REASONING EMERGES IN CONSTRUCTING PROPERTIES OF A RECTANGLE: A STUDY OF ADVERSITY QUOTIENT

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Abstract

Semiotics is simply defined as the sign-using to represent a mathematical concept in a problem-solving. Semiotic reasoning of constructing concept is a process of drawing a conclusion based on object, representamen (sign), and interpretant. This paper aims to describe the phases of semiotic reasoning of elementary students in constructing the properties of a rectangle. The participants of the present qualitative study are three elementary students classified into three levels of Adversity Quotient (AQ): quitter/AQ low, champer/AQ medium, and climber/AQ high. The results show three participants identify object by observing objects around them. In creating sign stage, they made the same sign that was a rectangular image. However, in three last stages, namely interpret sign, find out properties of sign, and discover properties of a rectangle, they made different ways. The quitter found two characteristics of rectangular objects then derived it to be a rectangle's properties. The champer found four characteristics of the objects then it was derived to be two properties of a rectangle. By contrast, Climber found six characteristics of the sign and derived all of these to be four properties of a rectangle. In addition, Climber could determine the properties of a rectangle correctly.

Keywords: Reasoning, Semiotic, Semiotic Reasoning, Construction Concept, Adversity Quotient (AQ)

Abstrak

Semiotik didefinisikan sebagai penggunaan tanda untuk mewakili konsep matematika dalam menyelesaikan masalah. Penalaran semiotik dalam mengonstruksi konsep adalah proses penarikan kesimpulan berdasarkan objek, tanda dan interpretasi. Makalah ini bertujuan untuk menggambarkan tahap penalaran semiotik siswa sekolah dasar dalam membangun sifat-sifat persegi panjang. Subjek penelitian kualitatif ini adalah tiga siswa sekolah dasar yang dikelompokkan menjadi tiga tingkat yaitu Adversity Quotient (AQ): quitter/AQ rendah, champer/AQ sedang, dan climber/AQ tinggi. Hasil penelitian menunjukkan tiga subyek mengidentifikasi objek dengan mengamati objek di sekitar mereka. Dalam tahapan membuat tanda, mereka membuat tanda yang sama yaitu gambar persegi panjang. Namun, dalam tiga tahap terakhir, yaitu menginterpretasikan tanda, menemukan sifat tanda, dan menemukan sifat persegi panjang, mereka membuat cara yang berbeda. Quitter menemukan dua karakteristik objek persegi kemudian menganggapnya sebagai sifat persegi panjang. Champer menemukan empat karakteristik objek kemudian diturunkan menjadi dua sifat persegi panjang. Namun, Climber menemukan enam karakteristik dari tanda dan menjadikan semua ini sebagai empat sifat persegi panjang. Selain itu, Climber dapat menentukan sifat-sifat persegi panjang dengan benar.

Kata kunci: Penalaran, Semiotik, Penalaran Semiotik, Konstruksi Konsep, *Adversity Quotient* (AQ)

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Reasoning is the ability of students to analyze a problem, solve a problem, draw conclusions and express their ideas logically. Panchal (2013) stated that reasoning has been determined as a process of attainment conclusions based on relevant facts and sources. The process requires high-level thinking skills. Rapanta (2018) also argues that reasoning is the process of drawing a conclusion based on the premises, the facts of the investigation results. The reasoning related to symbols is called semiotic reasoning.

Semiotics in mathematics is defined as the use of symbols to represent mathematical concepts in problem-solving (Radford, et al. 2008; Ostler, 2011). Peirce (1931) and Turkcan (2013) believe that semiotics is identical to the logical concepts focusing on the knowledge of the process of human thought. Peirce (1931) also offers an idea related to signs, the use of logic and metaphysics, and a more complete theoretical context for culture called social semiotics. Someone thinks through signs which lead one to communicate with each other and gives any meaning to their environment (West, 2015). Moreover, Peirce asserts a theory of signs focusing on triad dimensions or trichotomy systems. Furthermore, Peirce classifies his theory into three aspects, namely sign, object and interpretant (Yang & Hsu, 2015; Metro-Roland, 2009; Presmeg, 2016).

In semiotics, signs are given meanings from observers to objects to achieve interpretation (Parcell & Parcell, 2009). Furthermore, Deledalle (2013) states that the relationships between sign and its object can be classified into three categories: icons, index, and symbols. An icon refers to the sign which has the same characteristics as the territory object (Miller, 2015). An index stands for the sign that represents a considered object which is the effect of causative relation. A symbol refers to the objects by mutual agreement or law. Peirce (1931) introduces three types of common icons, namely images, diagrams, and metaphors. An image stands for the properties of the icon as indicated by their own simple nature. A diagram is an icon that represents a relational nature. A metaphor refers to the icon representing their original semiotic nature and its certain semiotic properties in an unusual way (Uslucan, 2010).

In recent years, several researchers have studied semiotics, namely Campos (2010), Bjuland (2012), Ng & Sinclair (2015), Turgut (2017). Campos (2010) discussed Peirce's philosophy in mathematics education to foster the development of student semiotic abilities by doing such activities as imagination, concentration, and generalizations to do mathematical investigations on diagrams. Furthermore, Bjuland (2012) elaborated the semiotic sources used by a teacher of sixth-grade students whose students complete their mathematical assignments that provide written text and two inscriptions, namely images and diagrams. On the other hands, Ng & Sinclair (2015) investigated students' learning of reflectional symmetry in a dynamic geometry environment. Students' reasoning for linear transformation when using Dynamic Geometry System (DGS) from the perspective of semiotic mediation (Turgut, 2014). However, the study of semiotic reasoning in constructing the concept of a rectangle has never been done by other researchers.

Semiotic reasoning in constructing concept is a process of deducing a conclusion based on the object, sign (representamen), and interpretant. A concept, sign, and object can be unstoppably interpreted. Every interpretation can add new knowledge. A sign can be configured as verbal, visual, gestural, and musical. A sign is a representation of an object (Eco, 1976). Furthermore, a sign can represent anything that represents an object (Inna, 2013; Stjernfelt, 2015). An interpretant refers to a notion or notation to represent an object (Sarbo & Yang, 2015). The sign is only a sign whether it considers a subject as a sign while the interpretant exists if and only if there is a sign. On the other hand, nothing can stands for a sign if there is no interpretant. This relation is an indispensable element of Pierce's semiotic triad. A sign can emerge an interpretant which is another sign that is similar to that which is in someone's mind (Inna, 2013). Therefore, each sign can turn out into an object or an interpretant of other signs (Parcell & Parcell, 2009). Mathematical concept which its representation use a lot of signs is geometry.

Geometry is a fundamental concept since geometry can be applied to solve problems in daily life (Ahamad, et al. 2018). It is in line with the opinion Usiskin (1982) revealing that several reasons for teaching geometry, namely geometrical concept can associate mathematics with daily life activities, visualize abstract mathematical ideas by drawing, and exemplify mathematical systems by using representations. One of geometrical concept taught in elementary school is plane figure (2 dimensional shape) as it can improve students' reasoning abilities. This statement is in accordance with the opinion of Fujita and Jones (2007) stating that learning geometry material especially rectangular plane figure can help students in developing their ability to prove and give reason deductively. In addition, learning materials related to plane figure is widely used in daily life. As Hardiarti's research results (2017) prove that the concept of a rectangular plane figure has been found in the structure of the Muaro Jambi temple. This research will be focused on rectangular plane figure.

Constructing a concept is a particular way in which students attempt to understand available information and make connections using the entire cognitive structures to build concept. A student, in his/her own way, learns concepts specifically related to their prerequisite concepts. In constructing the concept of a rectangle, students are involved in observation, investigation, interpretation and drawing a conclusion. One of embedded factors for students in learning mathematics is Adversity Quotient (AQ). AQ is the persistence of a person when dealing with obstacles to obtain success. AQ is expected to contribute a strong motivation of a person to solve encountered problems so that AQ can support a person in achieving success (Stoltz, 2004; Syah, 2010). Stoltz grouped individuals based on their fighting power into three, namely quitter, camper, and climber. Moreover, he stated that people who give up easily are called quitters, people who feel satisfied with certain achievements are called camper, and someone who continues to want to achieve success is called a climber.

This study highlight the semiotic reasoning in constructing properties of rectangle based on students' AQ level. The related studies are still very limited. This study is important because students should understand the mathematical concepts for understanding the mathematical concept. Concepts can be embedded in students' memories (long term memory) if students have a strong motivation to learn.

METHOD

The methodological underpinnings of this study were established through a qualitative approach and explorative descriptive type. The researcher investigated the students' semiotic reasoning in constructing the concept of a rectangle. The term semiotic reasoning, in this study, means the process of drawing conclusions based on identified objects, making signs and interpretations to interpret the students' made-signs. Constructing a concept is a students' way to understand available information and connect their previous concepts to form a new concept.

The participants were three elementary school students who met the criteria: 1) Fourth-grade students aged 10-11 years, 2) 28 students participated in learning rectangular topics by solving items about rectangular. Students determined rectangular objects based on their experience and their knowledge in determining the properties of rectangular objects, 3) Based on these facts, students were classified into three types who have different levels of Adversity Quotient (AQ), namely one student with quitter/AQ low, one student with champer/AQ medium, and 1 student with climber/AQ high, 4) can communicate their reasoning well (verbal or written), and 5) students carry out semiotic reasoning in constructing the concept of a rectangle.

The data of this study were compiled from video recordings of each lesson, students' notes, and interview results. The videotaping depicted students' activities in constructing the concept of a rectangle. Students' notes described students' identification of objects, students' representamens based on the objects, and students' interpretants in constructing the concept of a rectangle. The interview was employed to explore the students' semiotic reasoning in constructing the concept of a rectangle and complete the unclear data from videos and students' notes.

The learning process carried out by the teacher on 2D shapes. During the study, the researcher focused on following the lesson unit of rectangular shape for 8 meetings. In each lesson, the students' activities in constructing a rectangle concept were videotaped. The researcher, then, selected three students with different levels of abilities based on the results of the video recordings, student's notes, and teacher's consideration. The participants were chosen to triangulate data whether the data were saturated (Creswell, 2015).

The collected data was analyzed to describe students' semiotic reasoning in constructing the concept of a rectangle. The analysis was carried out through 3 steps: 1) selecting appropriate data and excluding the unused one; 2) presenting data by grouping the data based on Pierce's semiotic triad namely object, representamen, and interpretant (Eco, 1976); and 3) drawing conclusion based on the research findings.

RESULTS AND DISCUSSION

The present study observed the fourth-grade learning process on two dimensional (2D) shapes. The subtopic of that material was triangle, square, and rectangle. The researcher attended the lessons as much as 8 meetings. Of the 8 lessons, there were 3 lessons – the 1st, 3rd, and 6th lesson – where the learning process focused on how students find the characteristics of rectangular shape. In the first lesson, the students observed the surrounding objects that were related to 2D shapes.

In this lesson, students collected all figures of 2D shapes. A student (says N) asked the teacher (G):

N: Does this 2D shapes, Ma'am? (see Figure 1)
G: Try to draw a picture on your book!
N: N rushed to sketch the picture on his book (see Figure 2)
G: what is the shape?
N: hemmmmmm ... I don't know ma'am (with a confused face)

Figure 1. N Collected Object

By looking at the object (Figure 1), the student N sketched the object as seen in Figure 2.

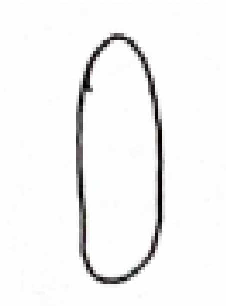


Figure 2. N Sketched Object

The teacher asked all students in the class to pay attention to the N-made figure. The following is the conversation between the teacher (G) and some students (says S) to find out the concept of 2D.

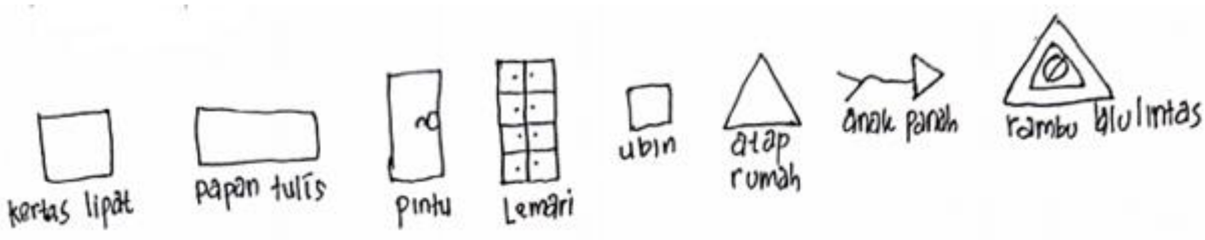
G: Is this picture of 2D?
S : ... (none of the students answered the teacher question)
G: Can we fill this shape with something?
S: No Ma'am .... (all students answered the question loudly)
G: If we cannot insert an object into a figure, then what is that called?
S : 2D shape Ma'am ... (one student answered doubtfully)
G: That's right ...
G: If we cannot insert an object into a figure, then we called 2D shape/figure (the teacher attempted to confirm students' understanding)
G: The 2D shape can be bounded by straight lines or curved lines. The N-made figure is a 2D shape that is bounded by a curved line
S: So is the circle, Ma'am... (one student asked the teacher)
G: That's right ...
G: Do you understand N?
N: Yes, I understand, Ma'am ...
M: Do others understand?
S : Yes, we understand, Ma'am ... (students answered simultaneously)

The teacher, then, invited students to classify the 2D shapes by its boundary lines, namely straight lines and curved lines (see Figures 3 and 4).

Translation:
plane figure

Figure 3. 2D Shape with Its Curved Lines

Figure 3 shows the 2d shape drawn by the students as they classified it into an object with curved lines. Meanwhile, Figure 4 shows the shapes which are surrounded by straight lines.



Translation:
Folding paper Whiteboard Door Cupboard Tile Roof Arrow Traffic signs

Figure 4. 2D Shapes with Its Straight Lines

In the third lesson, the teacher asked students to observe objects bounded by 4 sides. In this activity, the students began to determine the properties of rectangles. The following is the conversation between the teacher and students.

G: What kind of objects are bounded by 4 sides?
S: Tables, doors, blackboards, tiles, windows, creative boards, folding paper
G: yes ... all answers are right
S: Ma'am ... but the form of folding papers and the tiles are square (what he meant is the four sides are similar), but the form of table, door, blackboard, window, and board are not square (the right and left side are similar, the top and bottom side are similar)
G: yes, you are right ... now, please all of you, separate all objects who four sides with its shape have formed square and not
S: students rushed to group them

In the sixth lesson, the teacher invited students to observe the objects that have four sides but the shape was not a square, and identify its characteristics (each object is at least two characteristics). The following were the answered of three participants.

First Participant

The first participant (S1) is a quitter who has a low Adversity Quotient. In determining the properties of the rectangle, the first step taken by S1 was to collect rectangular objects, such as

Blackboard, pencil case, and door (**identify object**). After the objects were collected, the S1drew pictures (**create signs/representamens**) of the founded object (see Figure 5). From these pictures, S1 discovered the properties of the objects (**interpret sign**), then S1 could mentioned the properties of each object, namely blackboard has two equal sides and have four equal angles, the characteristics of a pencil case are the same as a blackboard, the characteristics of the door are the same as the blackboard (**find out properties of sign**). From each of the properties of the objects, S1 **determined the properties of a rectangle**, namely (1) the length of two sides are equal, (2) the measure of each angle is 90^0. Figure 5 shows that the results of S1 in constructing the properties of rectangles.

Translation:

Whiteboard	Pencil case	Door
Characteristics:	1. It has 2 equal sides	1. It has 2 equal sides
1. It has 2 equal sides	2. It has 4 angles with equal measure	2. It has 2 angles with equal measure
2. It has 2 angles with equal measure		

Figure 5. S1 Notes in Constructing the Properties of a Rectangle

The interview results between researcher (P) and first participant (S1) in constructing the properties of rectangles are as follows.

P: what activities do you do to construct the properties of a rectangle?
S1: I collect rectangular objects
P: What objects do you collect?
S1: blackboard, pencil case, and door
P: How do you know that the objects are the rectangular shape?
S1: It is because the shape is like this (while pointing at figure 5)
P: after collecting rectangular objects, what do you do then?
S1: I draw pictures of each object
P: You draw a picture? What for?
S1: To find the characteristics of the object
P: What are the characteristics?
S1: the characteristics of the blackboard are had two equal sides (this side and this side, this side and this side, while pointing at Figure 5), have four equal angles (this angle, this, this and this, while pointing at Figure 5), the characteristics of a pencil case are

the same as a blackboard, the characteristics of the door are the same as the blackboard

Q: *What characteristics of the object for?*

S1: *to find the characteristics of a rectangle (in a soft voice)*

Q: *What are the characteristics of a rectangle?*

S1: *a rectangle has two equal sides (this side and this side, this side, and this side, while pointing to a figure of a rectangular object) and an angle of 90^0 (this angle, this, this and this, pointing to the figure of one rectangular object).*

P: *In the properties of objects, you don't say the measure of the angle, 90^0, but in properties of the rectangle you say the measure of the angle, 90^0. Why can it be like that?*

S1: *yes ... there are 4 angles, the measure of the angle is 90^0*

Considering the aforementioned interview results, S1 identified the same characteristics of three rectangular objects. S1 derived the characteristics of the three objects to determine the properties of rectangles: (1) two sides are equal, (2) the measure of each angle is 90^0. The scheme of S1 in determining the properties of rectangle depicts as Figure 6.

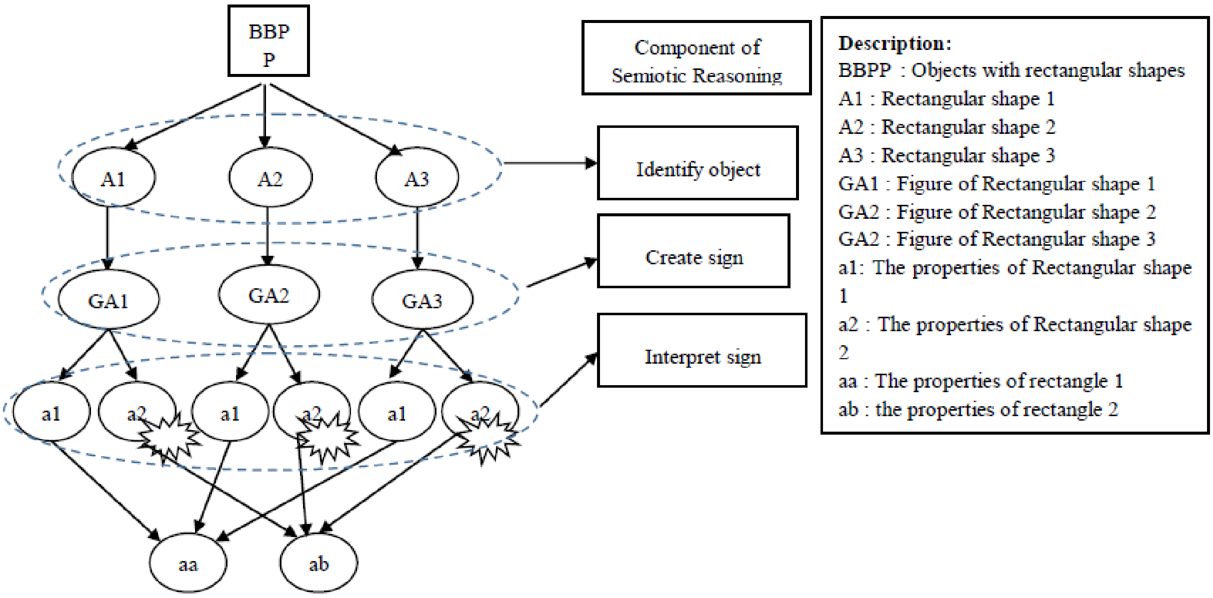
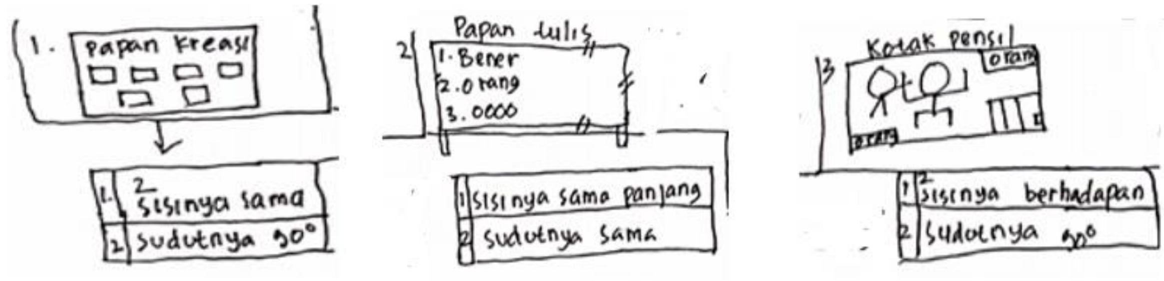


Figure 6. S1 Stages in Determining the Properties of Rectangle

Second Participant

The second participant (S2) is a participant with a medium level of AQ, champer. The first step taken by S2 in determining the properties of a rectangle was the same as S1 steps, namely collecting rectangular objects, such as the creative board, blackboard, and pencil case (**identify objects**). Based on Pierce's semiotic, the activity was called the identifying object. After collecting the objects, S2 drew a figure (**made a sign**) of the determined object (see Figure 7). From these figures, S2 identified the characteristics of the objects (**interpret sign**), namely the creative board has two equal sides and has angles which its measurements are 90^0, the blackboard has two equal sides and similar angle, the pencil case has two sides and the measure of its angle is 90^0 (**find out properties of sign**). From each property of the three objects, then, S2 **constructed the properties of a rectangle**, namely two sides are equal, the measure of its angle is 90^0. However, the properties of rectangles determined by S2 differed from

S1 identification. Figure 7 shows that S2 steps in constructing the properties of rectangles.



Translation:

A Creative Board	A whiteboard	A pencil case
1. It has two equal sides	1. It has equal sides	1. Its sides are opposite
2. Its angles are 90^0	2. The measure of its angles are equal	2. Its angles are 90^0

Figure 7. S2 Notes in Constructing the Properties of Rectangle

The interview results between researcher (P) and second participant (S2) in constructing the properties of rectangles described as follows.

P: what activities do you do to find the properties of rectangle?
S2: I am looking for a rectangular object
P: What objects do you collect?
S2: creative board, blackboard, and pencil case.
Q: What do you do after collecting rectangular objects?
S2: I draw a picture
Q: What are the pictures for?
S: to find out the properties of the figure
Q: What properties did you find from the figure?
S: the creative board has two equal sides, 90^0 angles,
the blackboard has the equal two sides, the similar angle
the pencil case has two sides, the measure of the angle is 90^0
Q: After discovering the properties of the figure, what other activities did you do?
S2: determine the properties of a rectangle
Q: What are the properties of rectangles?
S2: the properties of a rectangle are two equal sides in length, the angle is 90^0
Q: You already mentioned the number of characteristics of a rectangular object is six, but then you mention the properties of a rectangle, only two characteristics. Why?
S2: hemmmmm ... (with a confused face), two opposite sides has equal length and the measure of the angles is equal to 90^0

Based on the results of the interview above, S2 found six characteristics of three rectangular objects. As S2 determined the properties of a rectangle, S2 only identified two properties of a rectangle: (1) two sides are equal, (2) the measure of the angle is 90^0. The properties of the rectangle were derived from several characteristics of 2D shapes. The S2 stages in determining the properties of rectangles can be seen in the following scheme.

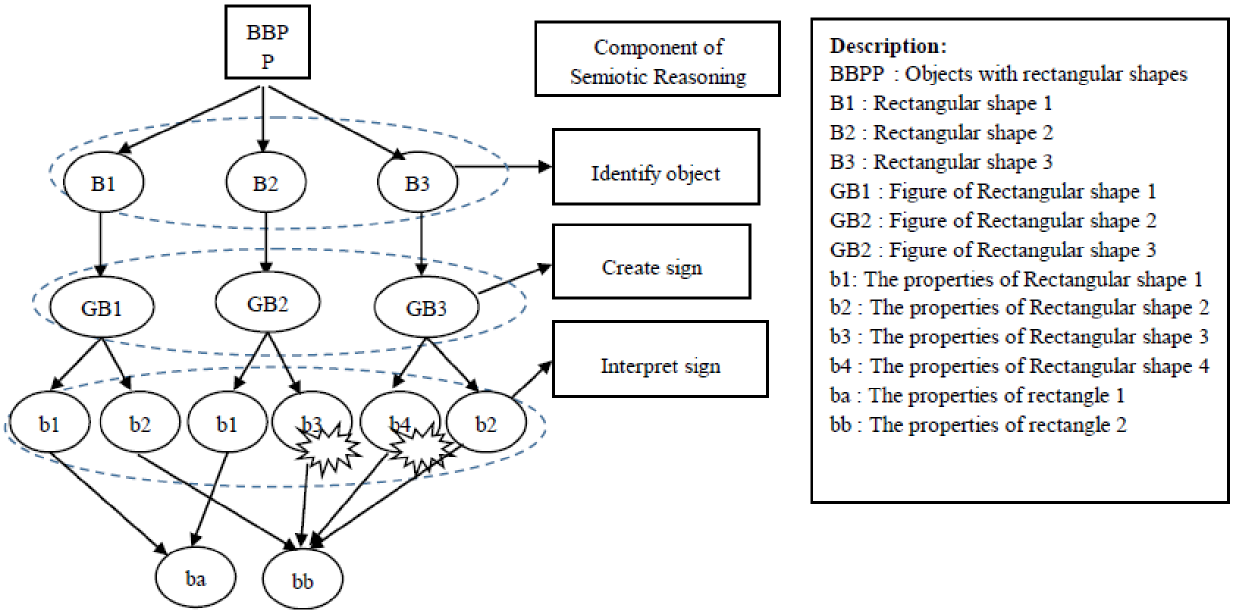
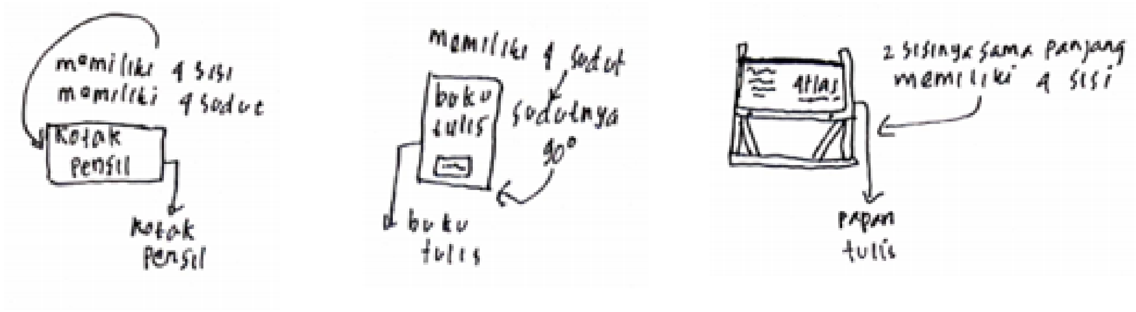


Figure 8. S2 Stages in Determining the Properties of Rectangle

Third Participant

The third participant, called S3, is a student with a high level of AQ, climber. The steps of S3 in determining the properties of rectangles are the same as S1 and S2 did, namely collecting rectangular objects including pencil boxes, notebooks, and blackboards (**identify objects**). After collecting, S3 also made an image (**making a sign**) of the object that had been found (see Figure 9). From these images, S3 discovered the characteristics of objects (**interpret sign**). From each of the properties of the three objects, S3 found that the pencil box has four sides and four angles, the notebook has four 90^0 angles, blackboard has four sides and two sides are equal in length (**find out properties of sign**). Based on these findings, S3 **constructed the properties of rectangles**, but the properties of the rectangles found by S3 differ from the properties of rectangles found by S1 and S2. The properties of rectangles founded by S3 are the measure of each angle is 90^0, has four sides, has four angles, its two sides have equal length (Figure 9).



Translation:

It has 4 sides	It has 4 angles	It has 2 equal sides
It has 4 angles	Its angles are 90^0	It has 4 sides
A pencil case	A book	A whiteboard

Figure 9. S3 Notes in Constructing the Properties of Rectangle

The following dialogs are the results of interview between researcher (P) and third participant (S3) in constructing the properties of rectangles.

Q: *How do you find the properties of a rectangle?*
S3: *I am looking for a rectangular object*
P: *What objects do you collect?*
S3: *pencil box, notebook, and blackboard*
Q: *What do you do after finding rectangular objects?*
S3: *I made pictures of these objects*
P: *what are you making pictures for?*
S3: *to look for the properties of that object*
Q: *What properties do you find from each of these objects?*
S3: *pencil box, has four sides and four corners*
notebook, has four 90^0 angles and angles
blackboard, has four sides and two sides are equal in length
Q: *After knowing the properties of that objects, what do you do?*
S3: *I write the properties of a rectangle which its angle is 90^0 angles, has four sides, has four angles, and two equal sides*

Based on the results of the interview above, S3 found five characteristics of three rectangular objects. When S3 determined the properties of a rectangle, only two traits can be identified, but in the interview process S3 was aware that an error occurred while finding the properties of a rectangle. Therefore, S3 found the properties of a rectangle as (1) the measure of each angle is 90^0, (2) has four sides, (3) has four angles, (4) the two sides are equal. The properties of the rectangle were taken from all the characteristics of 2D objects. The steps of S3 in determining the properties of rectangles can be seen in the following scheme.

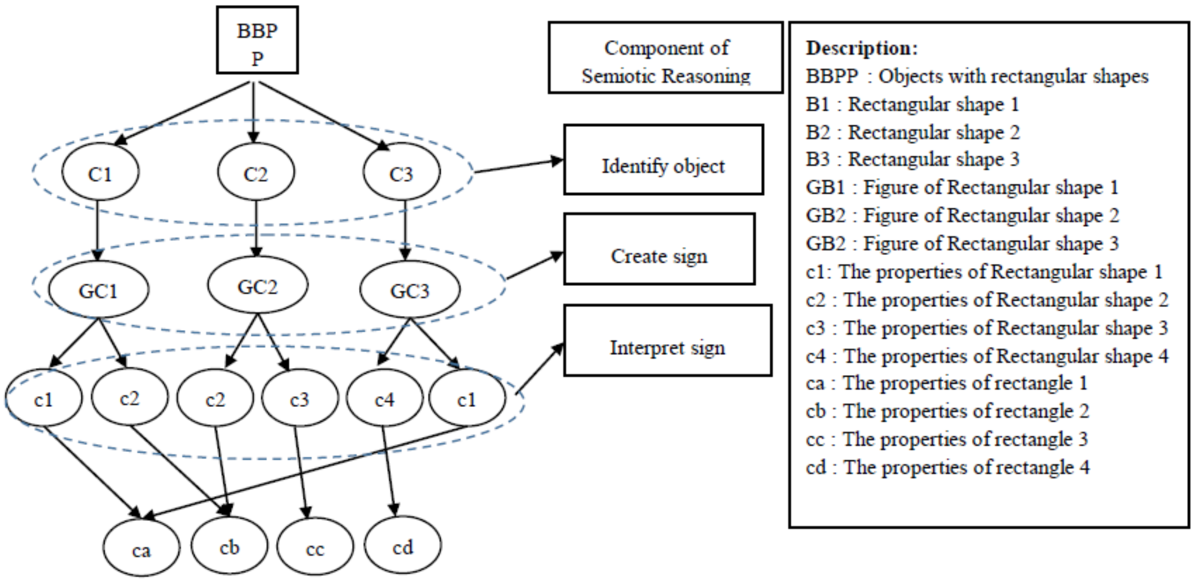


Figure 10. S3 Stages in Determining the Properties of Rectangle

First activity of discovering the properties of rectangle is identifying rectangular objects. The activity was carried out by three participants by observing objects around them. These activities, in Peirce semiotics are called identifying objects. Objects are the result of observing signs from external students (Schreiber, 2013).

Based on the phases of the three research participants, it was found that there were differences in the objects found. It can be seen from the objects collected by the three participants. The objects found by the subject of research are influenced by the interpretation of the participants in observing objects around the subject. It is in accordance with the opinion of Brier (2015) and Priss (2016) which states that an object is something that represents the interpretant produced. Interpretation is a response to an object through the interpretation of signs.

In creating a sign, the three participants made the same sign that was a rectangular image. Pictures made, in the semiotic theory of Peirce is called a sign. It is in line with the opinion of Arzarello and Sabena (2011) which states that images are signs that represent their original properties with their own simple characteristics. The simple characteristics means that these properties are relevant to their original nature. The relationship of images to other properties does not play a role in developing iconic representational relationship (Kralemann & Lattmann, 2013; Hendroanto, et al. 2018).

The participants carried out the activity of interpreting the sign by identifying the properties of each object found in the object identification activity. The interpretation of the participants to the sign made is different. For instance, in determining the properties of blackboard, S1 mentioned its characteristics such as having two equal sides and having four equal angles. S2 mentioned the characteristics of the blackboard, namely having two equal sides and having 90^0 angles of each measure. Whereas S3 mentioned the characteristics of the blackboard were having two equal sides and having four sides. It can be seen that one's interpretation of an object can vary. A person can have different interpretations related to the image. It depends on how the person interpreted the image (Sáenz-Ludlow & Kadunz, 2016; Godino, et al. 2011).

In determining the properties of a rectangle, there are differences between S1, S2 and S3 findings. S1 took from one of the characteristics of a rectangular object and found a new properties interpreted by S1 in interpreting one of the characteristics of a square object. S1's interpretation of the sign did not change when S1 found rectangular properties. It is in line with the opinion of Kralemann and Lattmann (2013) and Brier (2015) stating that each sign can act as an object or as an interpreter of other signs that are part of a single set of interpretations which a person has regarding certain specific matters. Meanwhile, the properties of the rectangle discovered by S2 were obtained from one of the characteristics of a rectangular object and there were two characteristics of a rectangular object which merged into one properties of rectangle. S3 derived all the characteristics of rectangular objects in determining the properties of rectangles. The interpretation of research subjects to the sign varies. The differences are influenced by the experience and knowledge possessed by the subject. It is consistent with the opinion of Schreiber (2013) who states the interpretation of each individual based on their previous experience and background. Each sign can be interpreted differently by other subjects. Moreover, the interpretation of each sign can trace the subject's judgment (Kralemann & Lattmann 2013; Ali, 2016).

CONCLUSION

The steps of semiotic reasoning conducted by the three participants (quitter, campers, and climber) of this research in determining the properties of rectangles are identifying objects, is the activity of collecting objects that are relevant to a rectangle; creating a sign, is an activity of making pictures based on identified objects; interpreting the sign, is an activity to interpret the sign based on the relationship between objects; determining the characteristics of the sign; and determining the properties of a rectangle by deriving the characteristics of a rectangular object. The results show that the three participants identify object by observing objects around them. In creating sign stage, three participants made the same sign that was a rectangular image. However, in three last stages, namely interpret sign, find out properties of sign, and discover properties of the rectangle, they did different ways.

In interpreting the sign, quitter found only two characteristics, namely the objects have four sides and four angles. Based on these characteristics, the quitter determined that a rectangle has properties such as it has four sides and four angles. On the other hands, the properties are not specific since the properties also belong to other rectangular objects. The champer, in interpreting the sign, found four characteristics of the sign namely the objects have equal sides, the measure of its angle is 90^0, the opposite sides and angles are the same. However, the properties of a rectangle which is derived from the properties of the sign by the champer is only two, that are, its two sides are equal and the measure of its angle is 90^0. Two properties of the sign merge into one rectangular property. Consequently, the properties of rectangle found by the quitter are not specific, because these properties are also possessed by right trapezoid.

In contrast, Climber, in interpreting the sign, found six characteristics of the sign that are the objects have four sides, four angles, 90^0 angles and two equal sides. Of the four sign characteristics, the Climber derived all of these properties so that the subject can find the properties of a rectangle, namely a rectangle has four sides, has four angles, 90^0 angles, and two opposite sides are equal. In the activity of finding the properties of a rectangle, Climber determined the properties of a rectangle correctly.

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STUDENTS' POSITIONING AND EMOTIONS IN LEARNING GEOMETRIC DEFINITION

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Abstract

The purpose of the present paper is to study the positions and emotions of grade 7 students who work with technology to learn geometry. This consideration of students' emotions is socially based, which makes it necessary to use a socially-based theoretical framework in order to study them. One such theory is the discursive analysis framework suggested by Evans, Morgan, and Tsatsarony, which is utilized in the present paper to analyze the positioning and emotions of fifteen groups of grade seven students who utilized technology to investigate the circle topic. The findings show that the group leaders took their positions through knowledge, action, initiation, persistence and meta-processes, while the followers of directions took their positions by accepting the group leader's requests. What most distinguished the collaborator was the communication with the other members of the group. Furthermore, the insiders used pronouns that indicated their inclusion. The results show that technology nurtured students' positive emotions as a result of nurturing their positioning throughout the investigation of the circle topic.

Keywords: Discursive Analysis, Students' Emotions, Students' Positions, Geometry, Technology

Abstrak

Tujuan dari makalah ini adalah untuk mempelajari posisi dan emosi siswa kelas 7 yang bekerja dengan teknologi untuk belajar geometri. Pertimbangan emosi siswa ini berbasis sosial, yang membuatnya perlu menggunakan kerangka teori berbasis sosial untuk mempelajarinya. Salah satu teori tersebut adalah kerangka kerja analisis diskursif yang ditemukan oleh Evans, Morgan, dan Tsatsarony, yang digunakan dalam makalah ini untuk menganalisis posisi dan emosi lima belas kelompok siswa kelas tujuh yang menggunakan teknologi untuk menyelidiki topik lingkaran. Hasil penelitian menunjukkan bahwa pemimpin kelompok mengambil posisi mereka melalui pengetahuan, tindakan, inisiasi, kegigihan, dan meta-proses, sedangkan pengikut arahan mengambil posisi mereka dengan menerima permintaan pemimpin kelompok. Yang paling membedakan kolaborator adalah komunikasi dengan anggota kelompok lainnya. Selanjutnya, anggota tim menggunakan kata ganti yang menunjukkan inklusi mereka. Temuan ini menunjukkan bahwa teknologi memelihara emosi positif siswa sebagai hasil dari memelihara posisi mereka selama penyelidikan topik lingkaran.

Keywords: Analisis Diskursif, Emosi Siswa, Posisi Siswa, Geometri, Teknologi

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Students' positions and emotions in educational contexts are attracting the attention of educational researchers in the last two decades (Evans, Morgan, & Tsatsaroni, 2006; Harré & van Langenhove, 1999), where the interest in these two educational aspects is due to the positions' and emotions' influence on other aspects of students' learning. For example, students' positions influence their educational identity (Hickey, 2011), levels of their learning engagement (Hickey, 2011), their interpersonal, intergroup and intrapersonal actions and interactions (Harré & van Langenhove, 1999), and, as a component of the social aspect of learning, students' cognition and behavior (Daher, 2013). On the other hand, students' emotions attract the attention of mathematics educators because of their relation with students' achievement in mathematics (Kleine, Goetz, Pekrun, & Hall, 2005), students' experiences in learning mathematics (Sumpter, 2010), mathematical sophistication in problem solving (Bradford & Carifio, 2007) and the carrying out of assignments with tools (Thiel, 2008). In the following, we describe in more detail the roles of positioning and emotions in mathematics education.

Harré and van Langenhove (1999) suggested positioning as a more dynamic and flexible form of social role than identity. Conversations are on-going discursive practices in which participants' roles are subject to change as conversations evolve. Participants in conversations take on certain roles, such as speaker, active or passive listener, opponent of the issue being discussed, and so on. However, the participants may not keep the same role from the beginning to the end. So, the participants assume different kinds of roles during the conversation, where these roles change according to the context and to power relations inside the conversation. Considering changes in participants' roles, it seems quite relevant to use "positioning" in order to describe the dynamics of the discursive practice.

Positioning has been used in mathematics education as a tool to analyze teaching as well as learning processes and identities of students and teachers. Esmonde (2009) considers access to discourse practices as related to access to positional identities, where this access places mathematics students as authoritative and competent members of the group as learners of mathematics. The two previous discursive aspects; access to discourse practices and access to positional identities, constitute students' opportunities to learn mathematics.

Langer-Osuna (2015) investigated a 9th grader positioning within his group and patterns of his engagement in mathematics project-based learning. In addition, the student's shift of positioning was examined across the academic year. This shift was performed through the utilization of classroom resources to serve both project-related and social functions in the process of interaction with peers. Yamakawa, Forman and Ansell (2005) found that the teacher, through revoicing and through commenting on students' characteristics - as courage, persistence, and flexibility, positioned specific students as active members of the classroom community. On the other hand, Skog and Andersson (2015) used positioning as an analytical tool for investigating teachers' identities. They found that institutional conditions may constrain the possibilities to deepen and develop identities of becoming mathematics teachers. They concluded that power relations and the becoming teacher's positioning in social settings are critical for understanding becoming teachers' identities.

Emotion is a fundamental element of the affective domain (Hannula, 2004; McLeod, 1992). McLeod (1992) identified three constructs of affect in mathematics education: beliefs, attitudes and emotions. He described emotions as the most intense and least stable. Hannula (2004) pointed at emotions as related to personal goals, which means that the achievement or the success or failure to achieve personal goals affects emotions positively or negatively. Moreover, the appropriate management of emotions could result in effective thinking rather than constrain it (Antognazza, Di Martino, Pellandini, & Sbaragli, 2015). Furthermore, Bradford and Carifio (2007) say, based on the literature that emotions can organize, focus, disrupt, distract or energize problem solving which would influence one's mathematical powers. The previous description indicates that emotions can serve cognitive as well as meta-functions in managing students' problem solving.

Different theoretical qualitative frameworks were suggested to analyze students' emotional expressions during their mathematical learning. DeBellis and Goldin (2006) suggested considering affect as an internal representational system. Roth and Radford (2011) suggested considering emotions as produced through the system of activity. A third framework utilizes the reversal theory approach (e.g., Lewis, 2013) that connects emotions with motivation. A fourth framework is the discursive framework as named by Evans, Morgan and Tsatsaroni (2006). This framework relates emotions to positions taken by students in the learning group, and is grounded in social semiotics and discourse theory with sociological, semiotic and psychoanalytic perspectives. We adopt in the present research the fourth framework, as we assume that emotions are socially based. Doing so, we consider emotions as connected with power relations in the group of learners. Here, students' discursive positions and emotions are claimed dialectically during the classroom discourse when they study geometric concepts in the presence of technology, specifically GeoGebra. Understanding students' social and emotional activity enables to better understand the cognitive and meta-cognitive aspects of their learning of mathematics (Daher, 2013). One way to get engaged in this understanding is through the discursive analysis framework.

The discursive analysis framework considers emotions and cognition as related, and emotions as socially organized phenomenon that participates in constructing and maintaining students' social identity (Evans, 2000; Evans, Morgan & Tsatsaroni, 2006). The analysis of learners' mathematical emotions and positioning, according to this framework, takes into consideration positions available to the mathematics learners through their learning practices, where those positions enable and constrain learners' emotions, and thus these emotions are considered as shaped by power relations in a small group or the whole-class situation. Evans, Morgan and Tsatsaroni (2006) describe two phases in the analysis method of students' positioning and emotions: the structural and the textual phases. The structural phase identifies, using Bernstein's sociological approach, pedagogic discourse positions available to mathematical learners in a specific educational setting. The textual phase examines the use of language and other signs in learners' interaction and describes the positions taken up by them. At this phase, the analysis focuses on indicators of emotions, like excitement and anxiety, linked to participants' positions. We describe below the discursive analysis basis and phases.

Discursive analysis is related to discourse. A discourse is a system of signs with which the participants can construct social meanings and identities, experience emotions, and relate actions. The first phase of discursive analysis, as described above, is the structural phase that examines the positions taken by the learners in the discourse. Positioning is defined as a process where a participant claims one of the positions made available by the discourse at a specific context. This results in the mutual influence of the individual and the social, where the social setting constrains the individual, but the individual keeps a degree of agency that enables the positioning in available or produced positions. According to this framework, a person's identity comes from repetitions of positions and related emotion (Evans, 2006).

Evans, Morgan and Tsatsaroni (2006) describe the positions taken care of in the structural analysis: Helper and seeker of help; where the helper is positioned as more powerful, collaborator and single worker, director of activity and follower of directions; where the follower is less powerful, evaluator and evaluated, and insider and outsider. Moreover, thee participant can claim more than one position, if in one discourse or in different discourses.

Evans (2003) says that it is not clear to what extent the above positions are associated with the criteria of the official classroom discourse or with discourses in which the students participate outside the classroom. The present research assumes that these positions could be found in groups' learning of mathematics, and in particular, when students learn mathematics using technology. It is so because technology is a tool that could empower students and help them position themselves as independent learners. This empowerment is the essence of students' positioning, as them become insiders to the group learning and also expected to be accompanied by positive emotions as pride and content.

The structural analysis helps examine the participants' emotions since positioning affects these emotions. Moreover, positioning is not permanent, not completely determined, nor freely chosen, where participants are constrained and enabled by their personal histories and the discursive resources available to them (Evans, Morgan & Tsatsaroni, 2006). Arriving at the participants' positions is the first step towards looking for the characteristics of these positions using different indicators. It is done in the textual phase of the discursive analysis which has two functions (Evans, 2006): (a) showing how positions in social interaction are actually taken up by the participants, and (b) providing indicators of emotional experience. Indicators of emotional experience can be divided into (I) those understood within the institutional and/or wider culture, drawing on the everyday culture of participants (verbal expression of feeling); behavioral indicators (facial expression, tone of voice); use of particular metaphors (e.g. a student claiming to be “coasting” in mathematics), and (II) indicators suggested by psychoanalytic theory, as indicators of defense against strong emotions like anxiety, surprising error in problem solving, behaving strangely (as laughing nervously), and denial (e.g. of anxiety).

Few attempts have been made to study students’ emotions as related to their positioning. These attempts were made mainly by those who developed the framework and by the author of this paper and colleagues (e.g., Daher, Swidan & Masarwa, 2017). The present study is concerned with examining students' positioning and emotions in various groups of students. In Daher (2015), we analyzed only one group's learning when they utilized technology to investigate the circle topic. In the present study, we analyze the positions and emotions of fifteen groups of grade seven students who utilized technology to investigate the same topic. Doing that, we keep in mind the results of our previous studies, especially the factors that affect students’ positioning and emotions when coming to learn mathematical topics. Two such factors are the type and functions of the group leaders, as well as the type of their processes (cognitive, meta-cognitive, etc.).

Becta (2009) identified six major ICT potentialities for learners to utilize in learning mathematics:

learning from feedback, observing patterns, seeing connections, exploring data, teaching the computer, and developing visual imagery. Researchers elaborated how technology assists students in building mathematical understanding as they interact with its tools (Noss & Hoyles, 1996; Nisiyatussani, et al. 2018; Nurwijayanti, et al. 2019). These tools facilitate the development of different and sophisticated mathematical ideas than when working with static representations (Heid & Blume, 2008; Noss & Hoyles, 1996). There are many different technological tools that students can use to learn mathematics, and each of these tools is likely to shape students' learning in a slightly different way. Assuming that technology offers new ways for students to understand mathematics, it is necessary to examine how students' learning is shaped by their uses of technology. In the present research, we examine how technology affects students' positioning and emotions. In addition to the previous potentialities of technology, it can provide opportunities for students to collaborate in learning mathematics, where students' collaboration in groups has attracted the attention of researchers in mathematics education (Hoyles & Sutherland, 1989; Noss & Hoyles, 1996, Septia, et al. 2018).

Working in groups provides students with assistance from each other, where stronger students can help those who are struggling (Webel, 2010). This assistance could indicate asymmetric relations between students, where some students take on an "expert" role, while their peers accept their authority (Esmonde, 2009). Moreover, dialogue and interchange in groups encourage mathematical critical thinking (Meyers, 1986) and the development of understanding (Schoenfeld, 1991). Furthermore, Students' experiences with collaborative learning formats, as part of their experiences in the mathematics class, can affect their identities as learners of mathematics (Boaler, 2002).

In addition to the above, technology assists the collaborative learning of mathematics. Baya'a, Daher & Mahagna (2017) reported positive effects of the collaborative use of GeoGebra on the development of seven graders' concept images of the angle, especially the dynamic one. Moreover, Naftaliev (2017) reported that 13- and 14-year-old students explored collaboratively mathematical models in electronic diagrams and developed shared knowledge of the abstract representations regarding the phenomena. In the present study we analyze students' positioning and related emotions when they collaborate to learn geometric definitions by utilizing technology.

The positioning and emotions phenomenon is a complex one which has different relationships that we tried to verify in previous research but on a small scale. Here we look at a larger scale that involves various groups in order to examine the relationships that Figure 1. The present research tries to verify this relations (Figure 1) and to shed light on them.

Emotion is distinct, but inseparable from cognition (Evans, Morgan, & Tsatsaroni, 2006). Furthermore, positions are one aspect of the social climate of learning, which influence students' emotional behavior (Daher, 2013). This influence is little studied from a discursive standpoint that combines the social and the emotional using different methodological methods. This standpoint will be used in the present research to analyze the properties of the mathematical discourse, in terms of positions and related emotions, when middle school students consolidate their knowledge about the circle topic in the presence of technology. It will be done looking at the learning of fifteen groups, which would shed more light on power and other social relations between mathematics students learning in a group, as well as their associated emotions, when they learn a geometric topic in the presence of technology.

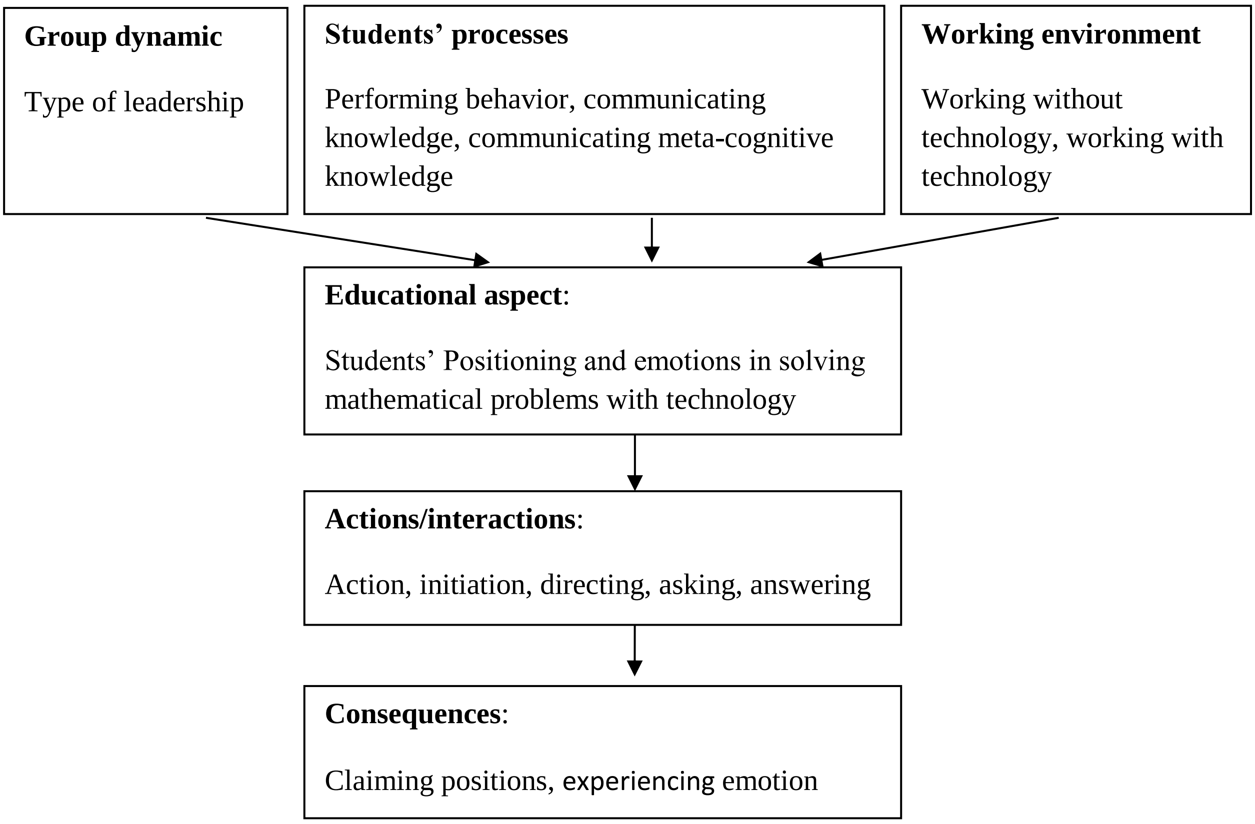

Figure 1. Conjectured relationships associated with the positioning and emotions phenomenon

This goal meets calls from mathematics education researchers to move beyond distinguishing between positive and negative emotions (Martínez-Sierra & García González, 2014). Martínez-Sierra and García González (2014) argue, depending on Lewis (2013), that this has not been done because it is more difficult to have a solid basis for qualitative results concerning emotions than for other affective constructs. At the same time, it is methodologically easier to study affective phenomena quantitatively, especially emotions phenomena; what lessened the study of this phenomena qualitatively. Here we attempt to study qualitatively students' emotions by associating their positive and negative emotions with the positions that they claim in their group learning. In previous studies, we studied one group's participants' positioning and emotions while using GeoGebra to learn geometric definitions (Daher, Swidan, & Shahbari, 2015; Daher, Swidan, & Masarwa, 2017). Here we look at the positioning and emotions of various groups, which makes it possible to reach more general conclusions than in our previous attempts. In addition, this would enable arriving at the comparison of the positions and emotions in groups of different characteristics; in the present study, these characteristics are related to the leadership style in the group. Therefore, this study describe the positions characterize that students undertake when they learn geometry with technology and the characteristics of students' emotions in each of the positions that they undertake when they learn geometry with technology.

METHOD

The present research is part of an ongoing research that examines middle school students' positions and emotions, when they study mathematics with technology. These students work in groups of 2-4 students each, where the participation in the research, the number of members in a group, as well as its formation, were decided by the students themselves. The only condition was that the group consists of 2-4 students each. Our use of the 2-4 member groups agree with group settings described by previous researchers. These researchers described 2-4 member groups to be appropriate for group setting in the mathematics classroom (See, for example Gillies, 2003; Webb, Nemer, Chizhik, & Sugrue, 1998). Furthermore, this agrees with other researchers' report about the size of groups that participated in their research (See for example Lan, 2007; Swidan & Yerushalmy, 2014).

The present research reports students' positions and emotions in fifteen groups of seventh grade students who used GeoGebra to improve their knowledge of the circle topic, where each group consisted of 2-4 students. These students were between 12 and 13 years old, coming from different socio-economic status and different mathematics ability. Some of the groups were all males, all females or mixed.

Examining the group leaders, some groups had one leader, while other groups had two leaders. Having one or two leaders depended on the characteristics of the members themselves, where, as described above, the formation of the group was decided by the students themselves. Moreover, analyzing the leaders' discourse in the group, five types were found: the one leader collaborates with the other members, the one leader does not collaborate with the other members, the two leaders collaborate with the other members, one leader collaborates and the other does not, and neither of the two leaders collaborates with the other members. In addition to the previous description, according to the theoretical framework utilized in this research, positions are not fixed, but regarding the mathematical context in which the students were engaged, these positions did not change, but the students could have other positions in other groups. In addition, the learning experiences of the students could result in change in their positions in other mathematical contexts.

The theoretical saturation of sampling, according to the grounded theory (Glaser, 2001), was followed to ensure the appropriateness of the sample for the present research goals. This theoretical saturation is "the conceptualization of comparisons of these incidents which yield different properties of the pattern, until no new properties of the pattern emerge" (ibid, p.191). In our case, the pattern was the leadership types.

We used two collecting tools (observations and interviews) to examine the group learning of grade seven students of the circle topic. The fifteen groups, who participated in the research, were videoed and at the end of each lesson, the members of each group were interviewed regarding their positions and emotions during learning. The learning of each group was videoed using a computer program that captured the footage in two different windows, one each for the computer screen and for the student. The interviews were semi-structured and held with each student individually. Examples on the interviews' questions are: What was your role in the group during your learning the circle topic? What did you feel while collaborating in the group to define the center of the circle? Why did you have this feeling?

We analyzed the two types of collected data using the discursive analysis suggested by Evans, Morgan, and Tsatsaroni (2006), and which was presented above. First the videos transcripts were analyzed, and then the interviews transcripts and afterwards the two analyses were combined. Using this method, all the transcribed data were coded independently by two coders; one of them is the author and the other is a colleague in the educational department at the university where the author works.

The validity of the research analyzing procedure was guaranteed by our analysis method which ensured the theoretical saturation. This theoretical saturation was due to our taking account of all the possible values of leader's type in the group, which ensured that no new group with different leader's type (category) would appear (Strauss & Corbin, 1998).

Lincoln and Guba (1985) say that no validity exists without reliability, so the ensuring of validity also ensures reliability. This means that following theoretical saturation maintains not only the validity of the research procedure but also its reliability. Further, two experienced coders (one of them the author) coded the participants' positions and emotions, searching for the participants' position and emotion types. The agreement between the coders (Cohen's Kappa coefficient) (when satisfied) ensures the reliability of the qualitative coding.

Cohen's kappa (Cohen, 1960) was used as a measure of agreement between the two raters. Inter-rater reliability for the observation data was 90.7% for positions and 89.1% for emotions. For the interviews data, it was 88.8% for positions and 89.3% for emotions. The high percentages of agreement suggest a satisfactory reliability for each type of data. Following is a detailed description of the categorization made in the present research of students' positions and their emotions.

We describe the categorization of students' positions in the learning groups, which utilizes the description in Esmonde (2009).

1. Leaders and followers: group members were coded leaders when they were frequently deferred to (mathematically) and who were often granted authority to decide whether the mathematical work of the group was correct. A leader participant is accompanied by a follower participant. With no follower, there is no one to defer to the leader and to take up his or her ideas. A participant was coded as a follower when he or she was instructed by a leader and accepted these instructions.
2. Solitary workers: group members were coded solitary workers when they went through periods of individual work during their work to define geometric concepts. The distinguishing characteristics for solitary workers were a tendency for working individually at least part of the learning, not asking for help when needed, and refraining from help to other members who needed help with or without requesting assistance.

3. Collaborating participant: group members were coded collaborative participants when group members worked together, as for example completing an idea together. Collaborative participants contributed together to arrive at a geometric definition. Evidence for claiming the collaborative participant position was the group member's putting more than one idea forward for discussion, or if several people jointly arrived at a geometric definition.
4. The ‘helper’ participant: members were coded as helpers when the mathematical talk was asymmetrically organized, and a participant instructed other participants about what to do. In contrast with the collaborative participant, a 'helper' was characterized by the uncritical uptake of ideas. Evidence for claiming the 'asking for help' position was when the participant oriented his/her actions towards obtaining an answer from a more expert participant.

The categorization made in the present research of students' emotions utilized the multimodal analysis described in Sakr, Jewitt and Price (2016). This analysis attended to a range of multimodal aspects of the students’ emotions, as the use of language explicitly referring to an emotion (e.g., “happy,” “sad”); the use of a gesture or movement associated with an emotion (e.g., a hand raised in the air to indicate accomplishment or content); the presence of a facial expression that suggests an intensity of feeling (e.g., a furrowed brow). These three means of evaluating emotions; language, gestures and facial expressions, lessened the possibility of mistakenly recognizing a behavior as related to an emotion or an action as related to more than one emotion.

In the analysis, students' positioning were connected with their related emotions. We did that utilizing the categories described above of the two previous categorizations of positions and emotions. The connection was done utilizing the form represented in Table 1.

Table 1. The form for analyzing students' positioning

Raw	Participant	Conversation/ [action]	Position	Emotion
25	Haya	How can we set the radius of the circle? [Her facial expressions showed she was worried]	Asking for technical help	Worry
26	Janan	We extend a line from the center to the line of the circle. [She was sitting comfortably. Her voice tone showed she was happy to help]	Helper	Comfort, Happiness
28	Haya	[Pointing at the circumference on GeoGebra interface] To the circumference. [Her facial expressions showed she was content to correct Haya, while her voice tone showed she was enjoying doing so]	Leader	Content Enjoyment

RESULTS AND DISCUSSION

Grade seven students worked with GeoGebra to improve their knowledge about the circle and its components. Doing that, the members of each group took different discursive positions: Group leader, follower of directions, collaborator, insider and outsider. Students' positions and emotions associated with them are described below, with examples from the groups' work.

Group Leader

In nine of the fifteen groups, one student took the position of the group leader, where in three of them, the leader was authoritative; not a collaborator, while in the other six groups, the leaders were collaborators. Moreover, in six of the groups, two students took the position of the group leader. Table 2 details the discursive functioning of these two leaders in terms of their collaboration with the other group members, as well as the collaboration between them.

Table 2. Discursive positioning of the leaders in the two-leader groups

The group	Collaborating with the other group members	Collaborating with themselves
First group	Two of them	Collaborating
Second group	Two of them	Not collaborating
Third group	One of them	Collaborating
Fourth group	One of them	Not collaborating
Fifth group	Neither of them	Collaborating
Sixth group	Neither of them	Not collaborating

In all the groups, the group leader who collaborated with the other members of the group directed the work of the group towards performing specific mathematical actions. In addition, he/she manipulated the applet, initiated mathematical actions and participated actively in the mathematical discussions. Moreover, in all the groups, the group leader who did not collaborate with the other group members tried to solve the problem alone, manipulating the applet and writing down the answers to the activity questions.

In one of the collaborating-leader groups, the leader (Let's call her Haya), as Transcript 1 shows, initiated the exploration of the group (1), by telling the group's members (she and two other members) that they should follow the directions of the activity (1 and 5), and by using GeoGebra to drag the circle. Then she addressed Janan and Rana (the other two members of the group), and started to discuss the circle's center, but soon the conversation turned to be about the chord (6-11), the diameter (6-11), the secant (12-17) and the tangent (12-17).

Note: When describing the learning events, silence for m moments is denoted by [..m..].

1	Haya	The circle's center is ….
2	Janan	It is the point lying in the middle of the circle.
3	Haya	The middle …
4	Janan	It is the center.

5	Haya	(again): The circle's center is ….
6	Janan	Every chord that passes through it becomes a diameter.
7	Haya	A diameter? [..15..] What is a diameter?
8	Janan	It is this that passes through the circle.
9	Haya	It is this that passes through the center and the circle.
10	Janan	It is a line that passes through any part of the circle.
11	Haya	If it passes through the center it becomes a diameter [Haya uses the mouse to drag the circle and watch how the diameter and radius change] … the secant is like … it intersects the circle in two points.
12	Rana	The tangent surrounds the entire circle [Rana and Janan were looking at GeoGebra interface).
13	Janan	(Vehemently): Yeh [Haya dragged the tangent repeatedly].
14	Rana	The tangent is this that touches the circle.
15	Janan	(Again vehemently): It does not intersect the circle. It touches the outer line. [Haya continues to drag the tangent].
16	Janan	(Looking at GeoGebra interface with interest): When the secant touches the circle it becomes a tangent.
17	Haya	The secant is like … it intersects the circle in two points.

Transcript 1: The leader functioning

Haya claimed the position of the group leader, but generally the group worked as collaborators more than a leader and two followers. The collaboration occurred through a asking questions and answering them, in an attempt to agree on the definitions of terms associated with the circle. Haya directed the activity by persisting in asking questions and the utilization of GeoGebra to generate new examples of the circle and its elements. Haya's questions and actions led the group to improve their discourse regarding the concepts associated with the circle. These positions were accompanied by positive emotions, where the facial expressions of the group members showed that they enjoyed learning with GeoGebra on their own. This learning enabled them to improve, on their own, their knowledge of the circle topic, which empowered them, making them content and happy [interview]. Table 3 describes the functions of the collaborating leaders in the groups.

In the two-leader group, where the two leaders collaborated between themselves and collaborated with the other group members, the two leaders performed the functions in Table 3, where these functions were interchangeable between them. Following are some leadership styles, which give us a picture of how the individual leaders functioned and claimed leadership. Haya, a group leader, claimed authority mainly by action and knowledge through working with the applet, asking questions about the definition of concepts related to the circle topic, correcting the other two members' mathematical statements or actions and writing the 'assumed correct mathematical statements'. Wafa, another group leader, claimed authority by initiation (as asking question), action (as reading the activity requirement), requesting action (as requesting to manipulate the applet), and knowledge. Yara, a third group leader, claimed authority mainly by action and persistence (going back again and again to define a concept related to the circle topic). Sewar, a fourth group leader, claimed authority mainly through action and giving orders (as requesting to answer a question in the activity), as well as through demonstrating knowledge of geometry.

Table 3. Functions of a collaborating leader-Haya, Wafa, and Yare as leaders

The function type	The function in the specific context	Examples
Directing the learning	Giving orders	Haya: Wait till I finish writing.
Planning	Stating the next step in the group's work	Haya: Now we should work with the applet, and follow the directions in the activity.
Demonstrating knowledge	Correcting another member's mathematical statements or action	Haya: to draw the tangent, we do not draw a straight line totally outside the circle. It should touch it.
Evaluating through asking questions	Asking mathematical questions	Haya: What is a diameter?
Evaluating through rejecting other's statements	Rejecting mathematical statements	The same example in 'Demonstrating knowledge'
Requesting action	Requesting to manipulate the applet	Wafa, to Halim: Can you please drag the end point of the diameter to see what happens to it.
Initiation	Starting the discussion, summarizing the group work or discussion	Wafa: this dragging means that the diameter is the longest chord in the circle.
Doing	Controlling the work with the applet	Haya dragged the circle to get new circles and new measures of the diameter and radius.
Discussing	Participating and directing the discussion of the group's members	Haya: How can we set the circle radius? Janan: we draw a line from the center. Janan: no, a radius. Haya: a radius, how can we set a radius? Janan: we draw a line from the center of the circle to the line. Haya (pointing at the circumference in the applet): to the circumference
Writing mathematical statements	Writing the properties of geometric objects	After asking the group about the definition of the circle's center, and not getting an answer that satisfied her, Haya wrote on the paper what she thought the definition of the center: a point located in the middle of the circle
Persisting	Yara requested again and again that the group should define the circle center	Yara: We should go back to define the center, the first time we didn't finish to do that.

The work of Haya, as a leader of her group, made her enjoy the learning of the circle topic, as she declared in the interview: "I enjoyed that my questions led the group to understand the circle topic". This is the case of the rest of collaborating group leaders. The collaborating leaders had also negative emotions as a result of their discursive positioning. This is the case Sewar who said that she was uncomfortable because the other group's members did not collaborate with her [interview]. Moreover, the collaborating leaders had negative emotions because of personal characteristics or previous experience; i.e. as learners and not as a result of their discursive positioning. For example, the work of Haya with the computer made her a little uncomfortable, because she was used to stand and walk while she studied, as she admitted [interview]. In addition, Wafa; another collaborating leader, was a little uncomfortable with the activity as a result of being uneasy with geometric activities [interview]. The previous description substantiates that students' processes, the type of leadership and the working environment (here with technology) impact students' positioning and emotions.

Follower of Directions

The follower of directions generally followed the directions of the group leader towards performing specific mathematical actions. In the group of Haya, Rana and Janan, the last two took the position of the follower of directions. Doing so, as Transcript 2 shows, they answered the questions of the leader of activity (35, 36, 38), repeated her statements (40, 41), and nodded with their head as a sign of agreeing with the leader of the activity, or with each other (41).

34	Haya	What is the radius of the circle?
35	Rana	A line from the center of the circle …
36	Janan	The radius is a line that goes from the center to the circumference.
37	Haya	[Reads the next question]: What is the diameter of the circle? [The group members looked at the diameter that they drew].
38	Janan	It is a straight line that passes through the circle's center.
39	Haya	It is the chord that passes through the circle's center.
40	Janan	Yes, a chord that passes through the circle's center.
41	Rana	[Nodded with her head, also repeating Haya's statement]

Transcript 2: The follower functioning

Rana behaved as a follower. At the same time, Janan not only tried to answer the leader's questions and agree with her (40), but also contributed to building the definitions (36, 38). Table 4 describes the functions of the followers in the groups.

Sometimes the followers of directions expressed a negative emotion for being called back to engage in learning, as when the follower of directions did not enjoy engaging in geometric problems. This call was performed by the leader of the group, which shows that leader's actions impact the other group members' positioning and emotions. In addition, all the members' processes impact their positioning and, as a result, their emotions, which agrees with the theoretical framework assertion of the relationships between students' processes, positions and emotions.

Table 4. Functions of the followers of directions in Haya's group- Rana and Janan as followers

The function	The function in the specific context	Examples
Answering questions	Answering mathematical questions	Haya: what is the center of the circle? Janan: It's the point in the middle of the circle, and each chord that goes through it becomes a diameter.
Performing requested actions	Performing geometric calculations	After Haya explained how to find the circumference of the circle, she asked Rana to do the calculation for a circle in the applet on the paper, which Rana did.
Repeating statements	Repeating statements related to the influence of dragging the circle on its circumference and area	Haya: what happens to the circle when we drag it? Janan: the center does not stay the center. Haya: sometimes it gets larger and sometimes smaller. Janan: sometimes larger and sometimes smaller. Rana: when you enlarge it gets bigger.
Nodding with the head	Nodding to agree with a mathematical claim	Haya: there are infinite number of radii Rana: (nodding with her head): Yes.

Collaborator, Insider, and Outsider

The collaborating members in a group participated to internalize the topic of the circle. To do that, as Transcript 3 shows, they tried to refer to each other sayings (40), building upon each other sayings, where each one built on the precedent's statement (41-45)

39	Haya	[Reads the next question]: What is the relation between the chord and the diameter?
40	Rana	The relation is ... [..3..] the diameter is a chord.
41	Haya	The diameter is in origin a chord passing through the circle's center.
42	Janan	If it does not pass through the center it will be a chord, but if it passes through the center it will be a diameter.
43	Haya	The diameter is a straight line that starts at the circle's circumference and passes through the circle's center.
44	Rana	It ends at the second side of the circle. [Rana's and Janan's facial expressions showed satisfaction]
45	Haya	[Wrote]: the diameter is a straight line that starts at the circle's circumference, passes through the center and continues to the other side of the circle.

Transcript 3: The collaborator functioning

Working as collaborators led the group to agree on one of the definitions of diameter of the circle. Here, Haya was the one to ask questions (39) and write answers (45). Rana and Janan, as contributors to the knowledge building of the group, increased their power, which resulted in their satisfaction, as appearing in their facial expressions (44).

Table 5 describes the collaborator's functions in the groups. It includes the type of the collaborator's function, the function in the specific context, and an example from the groups' work. These functions were specifically fulfilled in two groups with collaborating leaders.

Table 5. Functions of the collaborator

The function	The function in the specific context	Examples
Asking and answering questions	Asking and answering a question about a geometric object that could be related to another	Wafa: the diameter is the segment that passes through the circle's center. What about the chord? Yara: The chord passes in any part of the circle.
Requesting and agreeing to do actions	Requesting and agreeing to investigate the consequences of changing a geometric object	Wafa: change the center to see what happens. [Yara dragged the center of the circle, while the group members watched the consequences].
Agreeing with another member's statement, with or without withdrawal/ modification	Improving a geometric statement about a geometric object	Haya reads: what can you say about the radius? [Silence] Rana: it is half of the diameter. Haya: it is half the length of the diameter. Janan: it is half the length of the whole diameter.
Looking together at the computer screen	Watching the consequences of a geometric action	Haya: the question asks what happens to the circle when we drag it….. Let us drag it. [The three girls looked at the computer screen, watching what happens to the circle.

The collaborating group leader, as in the case of Haya, referred to the sayings of the other group members less than they referred to hers, which could mean that she attempted to encourage the participation of her group members in the geometric activity. Moreover, the collaboration of the group members varied. For example, one member of Haya's group (Janan) collaborated more than the other member (Rana), indicating that she (Janan) was more an insider than the other member. Different reasons influenced the participation of the collaborators, where the educational setting, personal characteristics, previous experience and leadership styles were the most apparent reasons behind the collaborators' discursive positioning and associated emotions. More elaboration on the issue follows.

The relatively little collaboration of Rana could have occurred due to two reasons. First, she had some difficulty in her learning due to previous experience (coming newly to her present school from a school that did not teach enough geometry), and second due to personal characteristics (being weak in English; the language of the applet). When interviewed, Rana said: "it is uncomfortable to work with applets in English, because my English is poor". This means that the educational setting (applets in English) affected negatively Rana's learning emotions. From the other side, Rana felt content and happy because her participation in the activity was facilitated by the group work and by the use of technology, which made her understand the topic. In the interview, she said: "I am content because our work together and GeoGebra helped us understand all about the circle, while we do not understand geometry when the teacher teaches us. Here we knew what to do". Rana's use of the pronoun 'we' indicates that she considered herself an insider. This probably suggests that Rana will become more of an insider in future group work than she was here for she liked this work. Regarding the use of technology, the work with the applet made Rana curious to study the circle topic. In the interview, she said: "the work with the applet made me curious to learn the circle topic … the applet enabled us to experiment with the circle". The use of the pronoun 'me' indicates that Rana wanted to describe the curiosity as her own experience, while her use of the pronoun 'we' indicates that the applet facilitated the work of the whole group, which supported them being collaborators. Thus, technology potentiality to support students' experimenting encouraged the inclusion of the group members in the activity, which resulted in their positive emotions as content. This conclusion is supported by the declaration of Sana (an insider) in the interview: "in the sixth grade I did not understand the circle topic. In this activity, I felt content because the applet made us understand it". The use of the pronoun 'I' probably indicates Yara's intention to talk about a special experience she went through. All the previous argument proves the influence of the educational setting on available students' positioning and related emotions.

In one group, the three members were not collaborators, probably because the leader of the group was not a collaborating leader, which was represented in her little reference to the sayings of the other two members, trying to answer the activity questions alone. This situation could indicate two possibilities: (1) the leader of the activity was not used to working in a group, or/and (2) the leader of the activity pre-evaluated negatively the mathematical work of the other two members. In the described group, the leader used only singular first and second person pronouns, when communicating in the group, to refer to herself and each of the other members respectively. This little collaboration made the members feel uncomfortable. When interviewed, Salim (a member of the described group) said: "I did not feel comfortable in the group … Sewar was dominant and Sandra did not participate". Probably, Sewar and Sandra, being outsiders, made Salim feel an outsider too. This positioning resulted in a negative feeling; specifically being uncomfortable.

Here too, the previous description substantiates the hypothesis that students' processes and the type of leadership impact students' positioning and emotions. Furthermore, students' processes were, in this case, in mutual relationship with the learning environment that included technology.

It was our intention in the present research to characterize the positions and related emotions of seven grade students who work with technology to learn geometric definitions. Below, we discuss the research results through discussing what impacted each position taken by the members of the various groups, as well as the emotions associated with each position.

Group Leaders

The research results show that action was the main means to claim authority in all the groups, while knowledge was so in twelve out of the fifteen groups. Knowledge was acknowledged as means of claiming authority in other studies (see for example Evans, Morgan & Tsatsaroni, 2006), but in our case action was as important as demonstrating knowledge. A distinguished part of the group's action was done utilizing GeoGebra to notice geometric objects' behavior, geometric patterns and geometric relations. So, the presence of technology could explain the phenomenon that action turned to be the main means to claim authority in the present research. These findings point at technology as a main participant in students' social as well as cognitive learning of mathematics, where the behavioral aspect (students' work with technology) influenced the social aspect (their positioning), and as a result the affective aspect. Here, we can say that the work with technology (the behavioral aspect) affected students mathematical processes (the cognitive aspect), which affected their positioning (the social aspect) and, as a result, their emotions.

In addition to action and knowledge, initiation and persistence also helped maintain the position of the group's leader, where these behaviors support maintaining leadership (Bachiochi, Rogelberg, O'Connor, & Elder, 2000) and a successful student's life in general (Hirschy & Wilson, 2002). Furthermore, the two behaviors could be related to the 'task oriented skills' described as part of effective leadership (Bachiochi, et al. 2000). Other studies reported metacognitive processes as impacting the leaders' claim of their position (e.g., Daher, Anabousy, & Jabarin, 2018).

Only part of the group leaders admitted that they enjoyed being group leaders. The other leaders did not express emotions that could be related to their position as leaders. From the other side, some group leaders expressed negative emotions that could be related to this positioning, like being uncomfortable in leading the group's learning. In some cases, this lack of comfort is due to the other group members being not collaborators. Thus it could be claimed that the leading processes or events determined the emotions of the leader.

Other causes of the lack of comfort of the group members were personal characteristics as learners, though these causes could have been overcome if the learner was not a group leader who needs to act in a specific way like staying in front of the computer to manipulate the applet. This learner, if she was not a leader, would have acted in a way that resulted in her comfort. For example Haya would have stood and walked while she studied. Thus, we can say that the leading functioning, when contradicting with the personal characteristics could result in the negative emotions of the leader. Furthermore, we can say that personal characteristics moderated the relation of positioning and emotions. For example the inability to stay in front of the computer made the leader's emotions negative; these emotions that could have been positive, due to the leader being in control.

It can be concluded, from the above, that students' behavioral processes (manipulating the circle with technology), their cognitive processes (observing and being attentive to the change in the circle and its elements), the type of leadership and the working environment (technology-based) impacted students' positioning and emotions. The research results also indicate that the personal characteristics could also impact students' emotions while learning mathematics.

Followers of Directions

To realize the position of the follower of directions, the group member accepted the group leader request for action or for answering geometric questions. Tsatsaroni, Evans, and Morgan (2007) say that the relation leader-follower of directions makes implicit the hierarchical nature of the relationship between transmitter and acquirer. This is true generally, but the collaborating leader lessens the hierarchical nature of the relationship between her and the other group members, towards a more equal relation. This lessening of the hierarchical nature of the relationship between the leader and the follower could result in positive emotions of the follower for it empowers the follower, causing him/her to feel confident and content (Daher, Swidan, & Masarwa, 2017).

Sometimes the follower of directions had a negative emotion for being called back to the group learning. In the present research, this happened for personal characteristics as not liking to engage in geometry learning; an emotion held by other school students (Sunzuma, Masocha & Zezekwa, 2013). Nevertheless, if the same problem was related to real life and specifically to students' life, there is greater chance that the learner, in our case follower of directions, would be willing to engage with the problem, as students are enthusiastic and curious to work with such problems (Daher, 2012). So, the present research joins the call of researchers (e.g., Treacy & O'Donoghue, 2012) for integrating real life and authentic activities into students' learning of mathematics.

Collaborator, Insider, and Outsider:

What mostly distinguished the collaborator were the communication with the other members of the activity and the mutuality of relations with them, where this mutuality was represented in students' engagement with dualities like asking – answering questions related to the definition of the circle concepts and requesting – performing actions related to working with GeoGebra. This communication regarding the geometric concepts resulted in the mutuality in relations between the group's members, which helped maintain common ground (e.g. mutual understanding of how to go on discovering the definition of the circle's concepts) necessary for the success of the group's work (Cornelius & Boos, 2003).

Collaborators expressed positive emotions (as being content) as well as negative emotions (as being uncomfortable), where these emotions were influenced by different factors related to previous mathematical history and learning processes. More specifically, what made the collaborators uncomfortable were: previous mathematical history (here not being strong in geometry), and one property of the educational setting (the applets being in English). From the other hand, another property of the educational setting (learning in a group) made the collaborators content and happy. Furthermore, a third property of the educational setting (working with an applet) made the members of each group curious to collaborate in investigating and thus internalizing the circle topic, and, as a result, made the collaborators content and happy. It seems that two properties of the educational setting; working with an applet and the characteristics of the activity (here, being clear and gradual), encouraged the collaborators to participate in investigating the circle topic, and as a result they understood this topic. These two aspects of students' learning (working with technological tools and the properties of the activity) are reported to have influence on students learning (Baya'a & Daher, 2010). Moreover, the work with technology supported the participants in their geometric processes, and as a result, at arriving at the geometric definitions caused the collaborators to be happy and content for their activity and achievement. Thus, it could be said that understanding the geometric concepts mediated positioning (as collaboration) and positive emotions, as content and satisfaction.

Pronouns and use of verbs, as imperatives, used by the group's member, indicated whether he/she was an insider. Being an insider or outsider influenced the dynamic interchange during the progress of the group's discourse, as pronouns are "one of the main factors in maintaining a good interchange in a conversation activity" (El Saj, 2012). The use of the singular first person pronouns by the leader of one of the groups in her communication with the other members reflected the slowness of the defining process of the circle concepts. The previous implies that the collaborator’s position and emotions were impacted by four factors: students’ processes, leadership type, the learning environment and personal characteristics.

CONCLUSION

The current research utilized the discursive analysis framework to analyze the positioning and emotions of groups of seventh grade students while working with technology to investigate the circle topic. The research findings indicate that the students took the following positions in the groups: group leader, follower of directions, collaborator and insider and outsider. The group leaders took these positions by demonstrating knowledge, performing actions, initiation and persistence, while the followers of directions took this position by following the request of the group leader to perform actions and by answering questions. What mostly distinguished the collaborator were the communication with the other members of the activity and the mutuality of relations with them, where this mutuality was represented in the students' engagement with dualities like asking/answering. Moreover, the insiders' language included pronouns that indicated their inclusion in the group. Furthermore, it can be claimed that technology generally nurtured the participants' positive emotions towards their learning of geometry, for it allowed the participants to investigate mathematical relations on their own. To be more specific, the group leaders utilized GeoGeobra to lead the group into understanding the mathematical concepts, and thus into claiming their positioning by manipulating the geometric object, which indicates the role of the use of technology in leading the learning of the group. At the same time, GeoGebra encouraged the participants to be collaborators. This happened due to the support of the technological tool for the group members towards discovering mathematical relations, which made the participants value their being collaborators and, at the same time, have positive emotions about geometric learning. The previous argument shows that students' positions and emotions are impacted by the group dynamics, by the group processes; including their geometric processes, by the learning environment - here technology-rich environment.

In light of the above, in some groups whose members work together to learn mathematics, leaders could impact negatively the equality between the members in the group, which could impact negatively the engagement of some group members because their opportunity to perform geometric processes lessens. This situation of inequality in the group could result when the leader is not a collaborating one. This happened in some of the groups that we studied. The teacher needs to observe the group's working in order to suggest specific suggestions that improve the collaboration of the group, as well as trying to move the less collaborating leader into a more collaborating one. Thus, it is the present research conclusion that teachers need to encourage students to show initiation and persistence, for they help maintain a successful student's life in general and a way of maintaining leadership, in our case mathematics education leadership. This could be done by making teachers aware, through workshops, of values as initiation and persistence. Moreover, teachers should plan and prepare educational environment that are full of potentialities for students' actions and interactions, where some of these potentialities are exploration activities and tools that support the exploration of mathematical concepts and relations. These actions and interactions help maintain positive positions for students, and as a result, positive emotions towards mathematics and doing mathematics. Furthermore, technology can be means of encouraging students' being collaborators and insiders and thus have positive emotions. In addition, teachers, who work with technology, need to pay attention to students' working with the technological tool, so to encourage every member to manipulate the mathematical objects which the tool addresses. This would encourage their being collaborators and insiders. Taking that into account, it is important that the teacher does not impose positions on the students, because this intervention could impact the group dynamics negatively, which could impact negatively all the group's learning of mathematics. Nevertheless, the teacher should try to encourage equality between the group's members through encouraging their collaboration. It needs to be done smoothly.

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DESIGNING PISA-LIKE MATHEMATICS TASK USING ASIAN GAMES CONTEXT

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Abstract

This study aimed to produce a set of valid, practice and had potential effects of PISA-like mathematics tasks using Asian Games context to support students learning. Design research and lesson study were used as the method both during the design and implementation stages. Target users are 15th years old middle school students from PMRI pilot schools in Palembang. Results show that a set of PISA-like problems on uncertainty and data content are valid, practical, and had a potential effect. Students were doing mathematics in a collaborative, and the learning process becomes meaningful and easily.

Keywords: PISA-like Mathematics Task, Design Research, Asian Games

Abstrak

Penelitian ini bertujuan untuk menghasilkan soal matematika tipe PISA menggunakan konteks Asian Games yang valid, praktis, dan memiliki efek potensial untuk mendukung pembelajaran siswa. Penelitian ini menggunakan metode *design* research dengan sistem *lesson study* selama tahap pendesainan dan implementasi. Subjek penelitian merupakan siswa sekolah menengah berusia 15 tahun yang tergabung didalam sekolah ujicoba PMRI di Palembang. Berdasarkan hasil penelitian menunjukkan bahwa soal matematika tipe PISA pada konten *uncertainty and data* telah valid, praktis, dan memiliki efek potensial. Selain itu melalui soal tipe PISA menggunakan konteks Asian siswa dapat belajar kolaboratif sehingga proses belajar menjadi bermakna dan mudah.

Kata kunci: Soal Matematika Tipe PISA, *Design Research*, Asian Games

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Several people use uncertainty and data content to solve the problems which are closely related to daily life (Johar, 2012; Permatasari, et al. 2018). Also, the content can be used to looking for the possibilities that can happen (Yanti, et al. 2016). The PISA's result in 2015, Indonesia was ranked 63 out of 70 countries in mathematics literacy (OECD, 2016).

Students were learning mathematics using textbooks that do not provide opportunities for students to learn mathematical procedures in solving context-based problems such as PISA (Wijaya, 2016). Also, Indonesia still using a low-level problem in the evaluation system, so that the student's abilities to solve non-routine problems become weak (Stacey, 2010; Novita, et al. 2012; Permatasari, et al. 2018; Nizar, et al. 2018; Pratiwi, et al. 2018).

In PISA, the problems were presented mostly in real-world situations, so it can feel the benefit of mathematics to solving the issues of daily life (Putri & Zulkardi, 2018; Jannah, et al. 2018). Furthermore, Zulkardi (2010) suggested to design the PISA-like mathematics problems and use them in the learning of mathematics in the class. However, in reality, the teachers have a problem when designing and implementing lessons so that the necessity of collaborating between math-educators and teachers using the lesson study community.

In lesson study, the activities group of teachers collaboratively and continuously carry out, observe, and report learning outcomes (Putri & Zulkardi, 2019). According to Sato (2014), lesson study for learning community will make teachers eager to improve the quality of teaching from within, so it will later continue to strengthen their professionalism.

In 2018 Indonesia hosted the Asian Games. Wulandari and Atmojo (2014) stated that Asian Games was a sporting event of Asia countries on everwhich is held every four years. The kind of the sport at the Asian Games was the games that most students did or watched like weightlifting, swimming competitions, football, table tennis, long jump, bike, aquatic, volleyball, taekwondo, karate, and bowling. Sport context can make the learning process more interesting because relate to students' daily activity (Nizar, et al. 2018; Yansen, et al. 2018; Pratiwi, et al. 2019; Jannah, et al. 2019; Rawani, et al. 2019; Efriani, et al. 2019). The purpose of this paper is to produce a set of valid, practice, and have the potential effect of PISA-like mathematics problems using bowling context in Asian Games through lesson study.

METHOD

This research used design research method with development studies type through two stages (Zulkardi, 2002). Firstly, the preliminary stage with focuses on the preparation and design, literature review, designed instruments such as lattices, question cards and rubric assessment together with teacher by the 2015 PISA framework. Secondly, formative evaluation that includes the stage of self-evaluation, expert reviews, on-to-one, small group and field test (Zulkardi, et al. 2019).

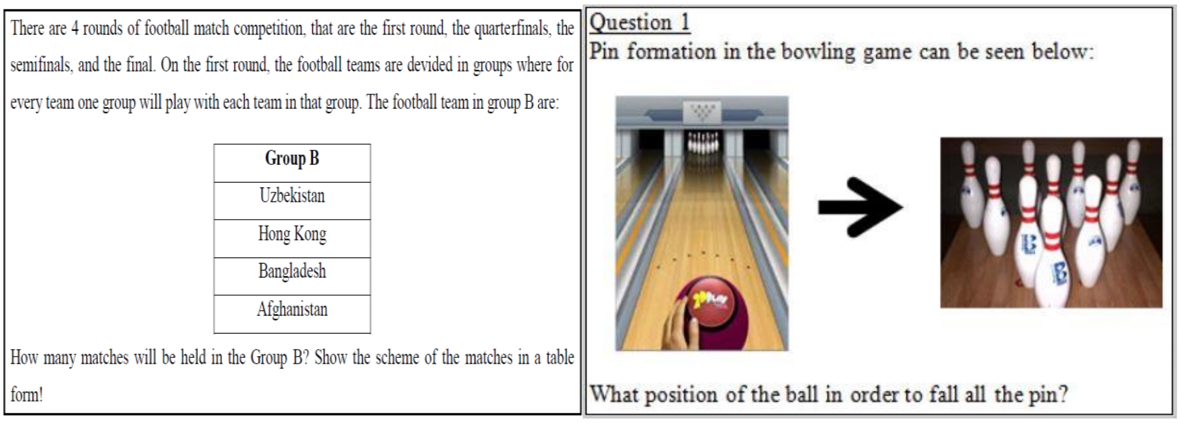
In self-evaluation the researchers have analyze the instrument by ourselves. After that, the prototype was validated by experts based on content, constructs and language. Along with validations with experts, a one-to-one stage performed. This stages involving three students with high, medium, and low-ability. From the expert reviews and one-to-one phase, the instruments was valid.

Small group stage was conducted to find out the practicality of problems developed involving six students with various abilities. Then, the last stage was the field test involving 15th years old students in junior and senior high school as PMRI or Indonesian version of Realistic Mathematics Education pilot school in Palembang, Indonesia. The results of field test were analyzed to see a potential effect emerging from PISA-like problems using bowling context through students' answer sheets.

The data collection techniques used walkthrough, documentation, observation, interview, and test. The data were analyzed by using the qualitative descriptive method to describe the result of each step of the development.

RESULTS AND DISCUSSION

This study produce a sharing and jumping task using Asian Games context. On the sharing task, PISA-like mathematics problem using football context and jumping task using bowling context. The stages of research implementation are preliminary involving lesson study socialization and plan stage. In this stage, the researchers analyzed PISA frameworks, curriculum 2013, designing a PISA-like using Asian Games context and making predictions of the students' answers with mathematics teachers choose a model teacher in the field test. Design result of sharing and jumping task can be seen in Figure 1.



There are 4 rounds of football match competition, that are the first round, the quarterfinals, the semifinals, and the final. On the first round, the football teams are devided in groups where for every team one group will play with each team in that group. The football team in group B are:

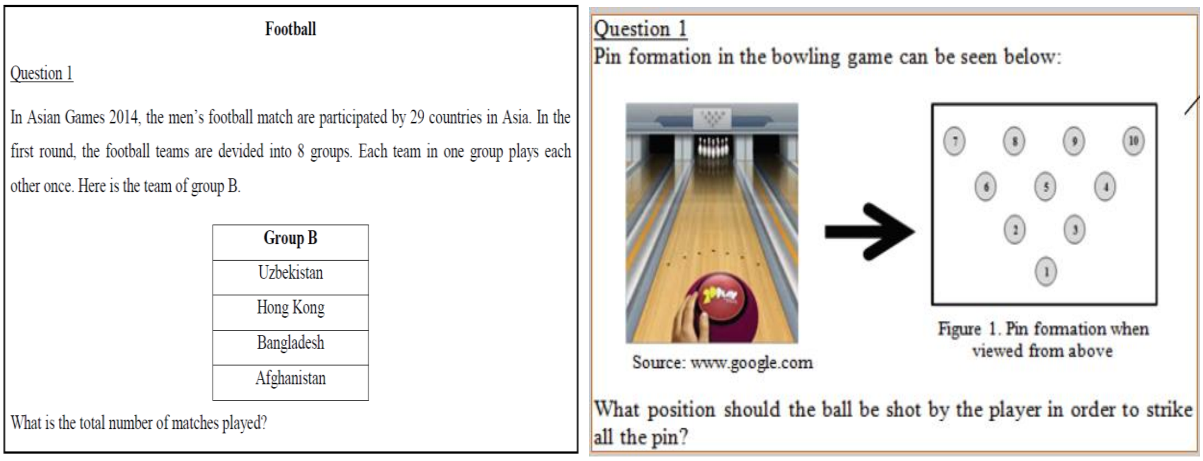
Group B
Uzbekistan
Hong Kong
Bangladesh
Afghanistan

How many matches will be held in the Group B? Show the scheme of the matches in a table form!

(a) Sharing task using football context (b) Jumping task using bowling context

Figure 1. Design result of sharing and jumping task

After plan stage, the task used in the expert reviews and one-to-one stage were conducted in parallel to see the validity of problems. The problems were validated by expert from Brunei Darussalam University, Universitas Sriwijaya, and mathematics teachers in the part of the content, constructs, and language. While the one-to-one and small group stage involved three students with high, medium, and low-ability. The revision results at that stage can be seen in Figure 2.



Football

Question 1

In Asian Games 2014, the men's football match are participated by 29 countries in Asia. In the first round, the football teams are devided into 8 groups. Each team in one group plays each other once. Here is the team of group B.

Group B
Uzbekistan
Hong Kong
Bangladesh
Afghanistan

What is the total number of matches played?

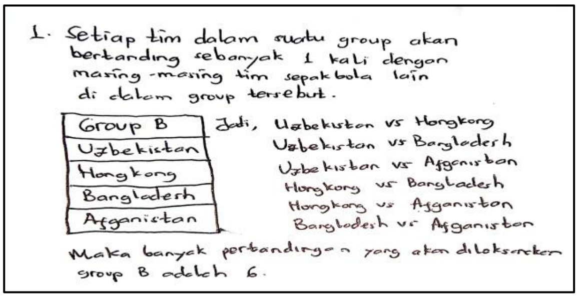
Figure 2. Revision results of the jumping task after validation

Based on the comment at the expert reviews and one-to-one stage, the researchers decide to revise the sharing and jumping task. On the sharing task the researchers change the name of country with the familiar on student's thinking. The reseachers also change the question , so the student can be explore their answer. On the jumping task the reseachers change the picture of pin formation and revise the question. The experts said that if the figure of pin formation was seen from above, it would make it easier for students to think and imagine the direction of the bowling ball.

In the small group stage, students were first asked to solve the problem individually before discuss with their friends in the group. Then, if they have difficulties to solve the problem, they might ask for help from their friends. The norm is by saying "Please Teach Me" and then the students who asked for help must teach it. The jumping tasks can create learning activities among students such as dialogue, interaction, and effective collaboration (Sato, 2014; Putri & Zulkardi, 2019).

Figure 3. Students ask for help with his friend

In the field test stage, almost students couldn't solve the problems well. It's reflected in Figure 2 that students are confused and scratching his heads while trying to solve the problem. But, after they collaborated and ask for help with their friend who understands, the student can explain it well. The analysis results of PISA-like mathematics task using football context is as follows in Figure 4.



Each team in a group will compete once with other team

Group B: So,

Uzbekistan — Uzbekistan VS Hong Kong

Hong Kong — Uzbekistan VS Bangladesh

Bangladesh — Uzbekistan VS Afghanistan

Afghanistan — Hong Kong VS Bangladesh

Hong Kong VS Afghanistan

Bangladesh VS Afghanistan

So, there are 6 matches in group A

	Hong Kong	Bangladesh	Afghanistan	Uzbekistan
Uzbekistan	Uzbekistan Hong Kong	Uzbekistan Bangladesh	Uzbekistan Afghanistan	Uzbekistan Uzbekistan ×
Hong Kong	Hong Kong Hong Kong ×	Hong Kong Bangladesh	Hong Kong Afghanistan	Hong Kong Uzbekistan
Bangladesh	Bangladesh Hong Kong	Bangladesh Bangladesh ×	Bangladesh Afghanistan	Bangladesh Uzbekistan
Afghanistan	Afghanistan Hong Kong	Afghanistan Bangladesh	Afghanistan Afghanistan ×	Afghanistan Uzbekistan

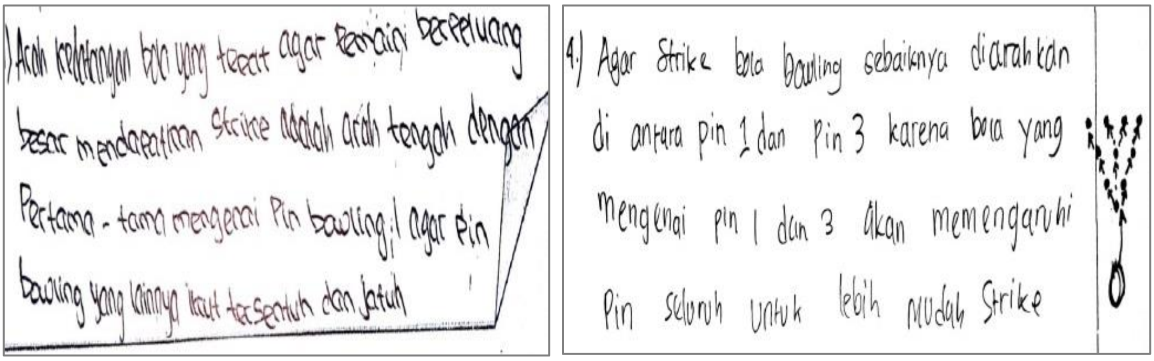
Total of matches: 6 times. Because every country will compete once with others country.

(a) (b)

Figure 4. Students' answer sheet of sharing task

Based on the Figure 4 (a), the student make the assumption that each country will compete once with other countries, so that there are 6 matches in group A. In the Figure 4 (b) the students make the prediction all the matches that will be held from each country, after that he eliminate the repetitive matches, so there are 6 matches. Accordance with Murtafiah and Lukitasari (2019), the learning should be emphasized in the development of student thinking.

Almost students can solve the sharing task using their communication skills. The students writing down the process of achieving a solution by making a list of teams that will compete based on existing rules to get 6 matches correctly and completely. Students can also make conclude of mathematical results accordance with the existing situation, namely the number of matches that will be held in group A. Meanwhile, the analysis result of jumping task using bowling context is as follows in Figure 5.



The direction of the ball is in the middle. First, the ball hits pin number 1 and then the others pin will fall. So the player can get strike.	The player can get the strike if the ball shoot between pin number 1 and 3 because that is can affects and hits all the pin, so the player easy to get strike.
(a)	(b)

Figure 5. Students' answer sheet of jumping task

Figure 5 (a) student makes a mistake possible next happen of the problem. Students make predictions that the ball should be thrown toward the center of the pin so that players get a strike. In Figure 5 (b), the student with high ability makes a reflection, argumentation, and reasoning when solving the problems. The student answer that the player must shot the ball between pin 1 and 3 so it will hit another pin on end. The students that have good reasoning ability can solve the problems correctly and adequately (Ahyan, et al. 2014; Permatasari, et al. 2018).

After the field test, the researchers and teachers doing a reflection about the lesson. This stage aims to find the advantages and disadvantages of the implementation of learning that has been carried out. The model teacher as an object starts the discussion by conveying her impressions, experiences, constraints, and opinions regarding the implementation of the learning (Nuraida & Putri, 2018). Furthermore, observers explain what they find during the lesson. From the reflection, it can be concluded that during solve the PISA-like problems, students working collaboratively in their groups although they still have a mistake. Students with low-ability were guided and connected to ask problems to their peers with high-ability (Putri & Zulkardi, 2018). Accordance with Sato (2014), the students have already learned in their groups.

Based on the result of the interviews with some students, they felt happy and interested to solve problems such as PISA-like mathematics problems using Asian Games contexts such as football and bowling. The students also said that developed problems could help them improve their mathematical thinking. The use of context in mathematics learning was very important because it could present the abstract mathematical problems to the form of representation that was easily understood by students (Permatasari, et al. 2018; Fajriyah, et al. 2017; Yansen, et al. 2019).

CONCLUSION

This research has produced mathematics PISA-like problems using the Asian Games context, which valid and practical. The validity reflected based on the comment of experts and students in the on-to-one stage, in terms of content has according to the domain of mathematics literacy in PISA. In terms of construct, the problem has been accorded with characteristics of the PISA problem level and abilities of the target group. In terms of language, the problems in accordance with enhanced spelling and didn't have a variety of meanings. The practically reflected from the small group stage, the problem could be understood as learning uncertainty and data, and easy to use. The potential effect of students' answers when they were solving PISA-like problems in uncertainty and data content. Especially in communication, representation, and mathematization. Also, through lesson study and design research can make students collaborate well so that mathematics learning becomes meaningful.

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IMPLEMENTATION OF REACT STRATEGY TO DEVELOP MATHEMATICAL REPRESENTATION, REASONING, AND DISPOSITION ABILITY

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Abstract

The purpose of this study was to describe how to implement the REACT strategy to develop students' mathematical representation, reasoning, and disposition ability. This research was a descriptive study with a qualitative approach. The subject of this study was grade 8 junior high school student in Bandung. Data collection techniques in this study with observations, interviews, and documentation. Based on data analysis results, it could be concluded that REACT strategies can be applied to develop a mathematical representation, reasoning, and disposition ability that engages students actively. Implementation of the REACT strategy runs smoothly and gets enthusiastic responses from students. The application of REACT strategies should be undertaken sustainably so that the learning objectives can be achieved by integrating various mathematical skills that were capable.

Keywords: REACT Strategy, Mathematical Representation, Reasoning, Disposition Ability.

Abstrak

Tujuan penelitian ini adalah untuk mendeskripsikan bagaimana implementasi strategi REACT dalam mengembangkan kemampuan representasi, penalaran, dan disposisi matematis. Penelitian ini adalah penelitian deskriptif dengan pendekatan kualitatif. Subjek penelitian adalah siswa SMP kelas VIII di Bandung. Teknik pemilihan subjek penelitian ini menggunakan teknik purposive sampling. Teknik pengumpulan data dalam penelitian ini dengan observasi, wawancara, dan dokumentasi. Berdasarkan hasil analisis data, dapat disimpulkan bahwa strategi REACT dapat diterapkan untuk mengembangkan kemampuan representasi, penalaran, dan disposisi matematis yang melibatkan siswa secara aktif. Implementasi strategi REACT berjalan dengan lancar dan mendapatkan respons antusias dari siswa. Penerapan strategi REACT hendaknya dilakukan secara berkelanjutan agar tujuan pembelajaran dapat tercapai dengan mengintegrasikan berbagai kemampuan matematis yang mumpuni.

Kata kunci: Strategi REACT, Kemampuan Representasi, Penalaran, Disposisi Matematis.

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Mathematics is one of the sciences that can improve the ability to think, argue, communicate, and contribute to solving daily problems and the world of work, as well as the development of science and technology (Sari & Mahendra, 2017; Muhtadi, et al. 2018; Kenedi, et al. 2019; Genc & Erbas, 2019). An understanding of mathematics prepares students to be able to survive in changing and competitive conditions in the future.

However, mathematics has a public image as a difficult, tedious lesson, and can only be accessed by a few people (Atallah, Bryant, & Dada, 2010; Laurens, Batlolona, Batlolona, & Leasa, 2018; Larkin & Jorgensen, 2016). It's because mathematics contains many abstract concepts (Mumu, Prahmana, & Tanujaya, 2017; Dreyfus, 2002). In general, learning in Indonesia only emphasizes memorization and is not accompanied by deep understanding that can be applied in real situation (Muslich, 2007; Nuari, et al. 2019). Besides that, generally the learning method used is conventional learning, without any variation in learning. So, students are passive in the learning process. As a result, students' mathematical abilities are of low quality. It can be seen from the achievements of mathematical literacy in the Programme for International Student Assessment (PISA). The PISA 2018 results, Indonesian students scored lower than the OECD average in mathematics (OECD, 2019).

PISA questions use non-routine problems that very often involve object representation and mathematical situations (OECD, 2014) and test high reasoning abilities. Based on the results of the PISA 2012 on mathematical representation and reasoning, it is illustrated that the ability of Indonesian students is still weak in both abilities. In fact, these abilities are interrelated and have become the goal of education in Indonesia.

Mathematical representation ability is the ability to restate a problem or mathematical object through things such as: selecting, interpreting, translating, and using graphics, tables, images, diagrams, formulas, equations, and concrete objects to express problems so that they are more clear (OECD, 2003). Representation does not only refer to the results or new construction products, but involves the process of thinking that is done to capture and understand the concept. In other words, mathematical representation ability is used as a reasoning tool to express mathematical concepts and ideas. Reasoning in mathematics can develop and reveal one's views about a problem. Reasoning is logical thinking that uses both induction and deduction techniques to get conclusions (Santrock, 2010).

Learning mathematics is also intended to develop affective domains such as mathematical disposition (Almerino, et al. 2019). Mathematical disposition is related to student attitudes that are flexible, confident, persistent in facing mathematical challenges and problems, enjoying mathematics, having high curiosity in learning mathematics, and appreciating the beauty of mathematics (Polking, 2019). Thus, fostering students' mathematical disposition is no less important than efforts to improve mathematical representation and reasoning ability.

Recognizing the importance of the mathematical representation, reasoning, and disposition ability in learning mathematics, it is necessary to use learning strategies that can provide opportunity and encourage students to practice these abilities. One of several learning strategies that can be applied in mathematics learning is REACT strategy. Where, REACT strategy is teaching based on contextual learning strategies arranged to encourage student involvement in the classroom (CORD, 2019). REACT is an acronym of Relating, Experiencing, Applying, Cooperating, and Transferring (Sari, Darhim, & Rosjanuardi, 2018). Learning with REACT strategies will provide many learning experiences to students because learning is more interpreted as learning throughout of life; students learn by actively exploring the information and technology needed, both individually and in groups to build knowledge; students not only master the contents of their subjects but they also learn how to learn (Crawford, 2001). REACT strategy can be used by teachers to practice students' mathematical representation and reasoning ability. According to the Center for Occupational Research and Development or CORD (2019), students enrich the basic understanding of the concept of Learning with hands-on activity (experiencing). Representation should be seen as an important element to (1) support students' mathematical understanding and reasoning, and understanding of relationships (NCTM, 2000). Then, students need confidence and persistence in facing every problem given in the learning process (CORD, 2019). It must always be maintained and developed through the creation of a learning atmosphere that interests students and tends to be challenging to explore. The REACT strategy is designed to foster students' mathematical dispositions.

Based on several studies, it is stated that REACT strategy have an influence and increase understanding of students' mathematical concepts (Novri, Zulfah, & Astuti, 2018; Anas & A, 2018; Junedi & Ayu, 2018). In another study showed REACT strategy was more effective than conventional learning from aspects of mathematics learning achievement, problem solving ability, connection ability, self efficacy, and motivation (Putri & Santosa, 2015; Safitri & Mahmudi, 2017). The study result showed that there was an increase in students' mathematical understanding and representation ability in learning with REACT strategy (Wulandari, Praja, & Aminah, 2018). Thus, the purpose of this study was to describe how to implementation REACT strategy to develop mathematical representation, reasoning, and disposition ability.

METHOD

This study was conducted as an effort to describe how to implementation REACT strategy to develop mathematical representation, reasoning, and disposition ability of junior high school students. This research is a descriptive study with a qualitative approach. The subject of this study were grade 8 junior high school students in Bandung. The technique sampling of this study was using a purposive technique sampling. The subject of this study obtained learning with REACT strategy. In this study, researchers play a direct role as a teacher in the learning process.

Data collection techniques in this study are by observation, interview and documentation. Observation is carried out during learning. The observation sheet is used to observe situations that occur during the learning process and are prepared based on the characteristics of REACT strategy. The characteristics of REACT strategy were Relating, Experiencing, Applying, Cooperating, and Transferring. The observation sheet is filled by observers. This observation sheet is in the form of observations about the course of ongoing learning, so that it can be evaluated and know what aspects must be improved. Interview is conducted to determine the difficulties experienced by students during the learning process. Furthermore, documentation is done with video recording while learning takes place. Learning tools is used in this study include syllabus, lesson plan, hand out, and student worksheets. Learning activities are designed with REACT strategy learning steps, so students can build their knowledge. Learning tools are emphasized to develop students' mathematical representation, reasoning, and disposition ability. In addition, the main material in this study is 3 dimentional shape.

RESULTS AND DISCUSSION

REACT strategy includes activities relating, experiencing, applying, cooperating, and transferring (CORD, 2019; Harwell, 2003; Crawford, 2001). Implementation of REACT strategy designed to foster mathematical representation, reasoning, and disposition runs smoothly and gets enthusiastic responses from students. It could be seen from the activity of students in every learning activity in the classroom. The activeness of students in the classroom could be seen from high motivation in learning, activeness of students in group discussions, asking the teacher, students are more enthusiastic when learning such as working on the questions on the board.

Related activities are carried out at the beginning of learning where the teacher asks questions that can be answered by almost all students from life experiences or knowledge that they already have (Crawford, 2001; CORD, 2019; Harwell, 2003). In this activity, the teacher can design responsive experiences and learning in building students' knowledge with familiar things, thus forming a deeper understanding. When the questions posed by the teacher can be answered by almost all students. It can be a motivation, interest, and lead to strong self-confidence for students at the beginning of learning (Furner & Berman, 2005).

In relating activities, the teacher provides an illustration of the concept of 3 Dimentional shape with a real model. The teacher introduces the concept of the cube by showing a cube model, namely dice. When dice are shown, students remember the cube shape. The activity of showing the cube model is intended as a first step to bring back the knowledge that students have (Kurniasih, 2012). Through this step, the activity of presenting the next problem can be carried out by accommodation, which raises new problems by considering the knowledge that students already have.

Furthermore, the relating activities were revealed in the submission of questions, such as: “Do you know to play snake ladders? Monopoly game? In snakes and monopoly games, players alternately move their pieces after throwing the dice. Related to 3 Dimentional shape, what shape is this dice?”. The question is intended to recall memory about the cube concept related to the problem presented. Students respond to questions enthusiastically about the dice and most students answer correctly, namely answering that dice is cube. A small number of other students answer dice shaped box. The shape of the box is considered a concept similar to a rectangular prism. Students often encounter box terms in daily life, such as first aid box, cellphone box, and others. It is an opportunity for the teacher to straighten the term box as a form of rectangular prism which in some cases can be a cube or cuboid.

Then, students are given a stimulus in the form of a problem, namely: "Mention the cube-shaped objects that you know! Cube objects have sides. What shape is the side of the cube? Is each side congruent?“ Students are able to answer with creative examples. Some of them mention that the object they know which is in the form of a cube is rubik. Rubik is the right answer regarding the problems given. There are also students who answer the cube form can be a birthday gift box. The birthday gift box answers get a variety of responses from other students. They revealed that a birthday gift box is not only a cube, but can also a cuboid. Furthermore, to support answers that are considered similar, some students answer that birthday gift boxes can be in the form of cube. It was revealed that there were students who said they had made a birthday gift box in the shape of a cube.

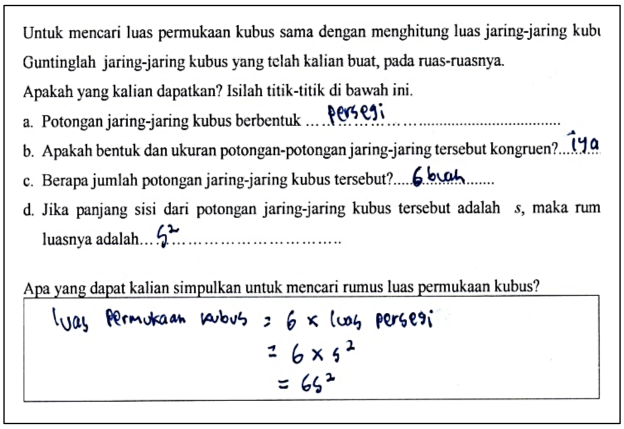
The questions in relating activities are specifically designed to train mathematical representation, reasoning, and disposition ability. The representation ability is raised in the question: "Do you know snake ladder games? Monopoly game? In snake ladder and monopoly games, players alternately move their pieces after throwing the dice. Related to 3 Dimentional shape, what shape is this dice? " The question presents real-world representation given by the teacher to students to bring what they already know with the concepts to be learned.

The teacher gives the question: "Mention the cube-shaped objects that you know!". These questions provide opportunities for students to come up with ideas regarding cube objects in daily life through words. Based on these questions also, students are able to communicate their ideas with different answers. The difference in views about the example of a cube shaped birthday gift box. On the one hand, there are students who say the birthday gift box is in the form of a cube. However, on the other hand there are students who refute the answer on the grounds that birthday gift boxes are not always cube shaped. Students maintain the answer by giving logical reasons why the shape of a birthday gift box is cube. The logical reason given by the student was that he had made a cube birthday gift box. It indicates that there is a reasoning process, namely the efforts of students to maintain their opinions as a truth (Sumarmo, 2010).

The involvement of students in responding to questions from the teacher is a form of mathematical disposition (Polking, 2019). Questions that can be answered by almost all students move from life experience or knowledge that they already have is an effort designed to trigger student involvement in learning. Student involvement is seen in a strong curiosity to know something or to solve a problem. For example, when the teacher provides a stimulus with the question: "Cube have sides. What shape is the side of the cube? Is each side congruent?" Students give a response that implies a strong curiosity by asking the teacher back, "What does congruent mean, mom?". Questions from students are answered with scaffolding in the form of questions that are close to students (Kurniasih, 2012). The teacher shows two sheets of A4 paper with the same shape and size, then the teacher asks the students, "These two A4 papers are examples of mutually congruent objects. Can you estimate what is congruent?" Then the students answer, "Objects are called congruent if the shape and size are the same". Based on this statements, students are able to state that the sides of the cube are congruent.

In experiencing activities, students are guided by the teacher when working on the worksheet so that it will be easier to understand a concept. Experiment is learning in the context of exploration, discovery, and invention (CORD, 2019). Experiencing activities in mathematics learning can also be illustrated by the involvement of students in every design of activities carried out in the classroom, including various instructions through tasks in hand out or worksheet.

Students are given a cube model in experiencing activities (Figure 1). The cube model is designed from six squares combined with the tape. Tape as an adhesive can be released easily, allowing students to obtain various cube nets. Then, the cube nets are drawn by students on the worksheet given. In addition, students are also asked to draw other possible cube nets. Based on the instructions on the worksheet, students are asked to open the tape on the net of the cube they have obtained, so that they get six squares (Figure 1).


Untuk mencari luas permukaan kubus sama dengan menghitung luas jaring-jaring kubus
Guntinglah jaring-jaring kubus yang telah kalian buat, pada ruas-ruasnya.
Apakah yang kalian dapatkan? Isilah titik-titik di bawah ini.
a. Potongan jaring-jaring kubus berbentuk … persegi …
b. Apakah bentuk dan ukuran potongan-potongan jaring-jaring tersebut kongruen?… iya
c. Berapa jumlah potongan jaring-jaring kubus tersebut?… 6 buah …
d. Jika panjang sisi dari potongan jaring-jaring kubus tersebut adalah s, maka rumus luasnya adalah… s^2 …

Apa yang dapat kalian simpulkan untuk mencari rumus luas permukaan kubus?

luas permukaan kubus = 6 × luas persegi
= 6 × s²
= 6s²

Translate:
To find the surface area of a cube is the same as calculating the area of cube nets. Cut out the cube nets that you have made, on the sections.
What did you get? Fill in the blank.

a. Pieces of cube nets shaped **square**
b. Are the shapes and sizes of the webs congruent? **Yes**
c. How many pieces of the cube net? **6 pieces**
d. If the side length of the cube nets is *s*, then the area formula is $\boldsymbol{s^2}$

What can you conclude to find the cube surface area formula?
Cube surface area $= \mathbf{6} \times \boldsymbol{Square\ Area} = \mathbf{6} \times \boldsymbol{s^2} = \mathbf{6}\boldsymbol{s^2}$

Figure 1. Result of Student Work on Student Worksheets

Previously, students had known the area formula of a square $= s^2$. Next, to obtain the formula the cube surface area is equal to the total area of the entire side of the cube. Because the cube has six congruent squares, students can conclude that the cube surface area is six times the square area. Next, the surface area of the cube $= 6 \times s^2$. Students learn through reasoning and show ideas for their thinking about the cube surface area with symbolic representation. The results of the reasoning are converted into simple mathematical sentences in the form of formulas. When students reason well, they become more understanding. Deep understanding leads students not to memorize.

Students reason well because the problems given are able to reach them cognitively. Students already know some of the knowledge they need to find the formula for the cube surface area. Problems that are close to the cognitive area guarantee involvement and make student learning more meaningful.

Students actively find the formula for the surface area of the cube. Students also actively ask the teacher about representation by drawing cube nets. Student questions are responded to by scaffolding (Kurniasih, 2012). The teacher gives feedback questions. High curiosity, persistence, and perseverance in solving mathematical problems show mathematical disposition in learning (Polking, 2019).

The applying activity includes the activities of applying mathematical concepts in solving problems on a worksheet. In this activity, the teacher designs the task in the form of questions that are diverse, interesting, challenging, and reasonable in terms of student abilities (CORD, 2019; Harwell, 2003; Crawford, 2001). It can be seen in the following example problem.

Syifa will make a cube box with edge length 20 cm. Then, She will coat the box with wrapping paper. How much wrapping paper is left if the available wrapping paper size is 60 cm x 50 cm?

The example above shows that the assignment given directs students to apply the formula for the cube surface area. First, students calculate the area of wrapping paper, that is 60 cm x 50 $cm = 3000\ cm^2$. Second, students calculate by applying the concept of the cube surface area, $6 \times s^2 = 6 \times (20\ cm)^2 = 2400\ cm^2$. Finally, students calculate the remaining wrapping paper area by calculating the difference in area of wrapping paper with the cube surface area, $3000\ cm^2 - 2400\ cm^2 = 600\ cm^2$. Through this problem, students are trained to reason, which is to conclude problem solving logically and communicate ideas through exposure to mathematical answers (Stacey, 2005; Sumarmo, 2010).

In applying activities, the teacher designs questions to train students' mathematical representation and reasoning ability. This activity certainly simultaneously trains students' mathematical disposition, which are characterized by active involvement of students in solving problems in the worksheet (Sumarmo, 2010).

Cooperating activities are one of a series of activities that students need to gain a deeper understanding (Crawford, 2001). Students build understanding through group discussions and compare each other's representations of problems, represent the steps of problem solving, and compare answers obtained. Students also gain broader knowledge about the discussion themes discussed. It is because through discussion, students can see mathematical problems from many perspectives (Anas & A, 2018; Junedi & Ayu, 2018; Novri, Zulfah, & Astuti, 2018; Wulandari, Praja, & Aminah, 2018; Chen, Yang, & Hsiao, 2016). Discussion friends can also act as controls for solving mathematical problems. For example, some students solve a mathematical problem together. One of them sometimes corrects the selected formula application, corrects troubleshooting steps, or corrects calculation errors that can be seen in Figure 2.

Figure 2. Cooperating and Applying Activities

Cooperating activities provide ample opportunities for students to get information (Crawford, 2001). Students can ask what they do not know to discussion friends without feeling awkward, or students can share information they have in solving problems. On the other hand, in a group sometimes there is only one student who understands the problem and the solving problem steps. When that happens, the student can explain to a discussion friend in his group about the mathematical problems faced. The diversity of acquisition of knowledge that does not always lead to the teacher is a sign of positive learning. Discussion provides space for students to learn more meaningfully because deep understanding is also obtained through communication with other students.

Students who answer correctly give logical arguments while explaining to friends their discussion. Some groups decide to believe the answers obtained without asking the teacher. Some other groups need teacher confirmation about the answers they get. On the other hand, there are also in a group that none of its members can provide the right solution. The teacher does scaffolding to groups who have difficulty with questions (Kurniasih, 2012; van de Pol, Mercer, & Volman, 2019). For example, "How many of one apple are added to one apple?" And followed by a statement “So, suppose s^2 equals one apple”. Students understand the scaffolding of the teacher, so that they finally get the right answers through their own minds. However, there is also another scaffolding, namely when interpreting $(AC)^2 = s^2 + s^2$ as $(AC)^2 = 1s^2 + 1s^2$. The teacher gives knowledge of s^2 as $1s^2$ as a fact. If only a few group members understand the teacher's intentions, then students who don't understand can ask their friends who understand.

Transferring activity is a conscious thinking activity in building concepts and solving problems (CORD, 2019; Harwell, 2003). Transferring activities occur in meaningful and natural learning. The teacher provides space for students to think and deduce their own concepts learned. Transferring activities do not occur in teacher-centered learning. The role of the teacher in transferring activities is as a motivator and facilitator. The teacher as a motivator means that the teacher brings students closer to the learning objectives, while the teacher as a facilitator means that the teacher provides guidance or scaffolding. Students know the concept of the cube surface area by summing the area of all sides of the cube, knowing the number of sides of the cube, and knowing the formula for the square area. On the logical reasoning of the transferring process (the problem above), students calculate the difference in area of wrapping paper with the cube surface area.

Students are involved in learning activities because the problems presented are still in the cognitive area. The problems presented are authentic problems that are presented in a realistic context, namely students are invited to practice applying the concepts through effective problems. Problems presented effectively means providing a context that is close to the lives of students. Observations are carried out as triangulation material to check the implementation of REACT strategy. Observations were carried out during learning in the class that had learning with REACT strategy at each meeting. The assessment results of the observers at each meeting showed that learning activities with REACT strategy were carried out in accordance with the planned learning steps. The assessment results and input by the observer become the improvement material for the teacher in carrying out the learning, so that they involve students actively in accordance with REACT strategy steps.

The meeting that received the attention of observers was the first meeting. At the first meeting, all REACT strategy steps were implemented. However, learning has not been maximized. It is because students are still adjusting to learning. Students are directed to study in groups with REACT strategy. Whereas previously students were accustomed to learning individually, so it was not optimal in solving questions on the worksheet. Valuable input based on discussions with observers to maximize learning, including (1) the teacher must be disciplined in managing the timing of learning steps according to the lesson plan, (2) when students conduct group discussions, the teacher should write down the number of questions to be presented students in front of the class to make time effective, (3) at the end of learning, the teacher must prepare a scaffolding to direct students to make conclusions from the material that has been studied, (4) the teacher must memorize the names of students, so that the teacher can be closer and evoke active involvement of students in learning.

At the second meeting, students and teacher began to get used to learn with REACT strategy. Where the teacher evaluates learning at the first meeting. The teacher's role in managing the class is so important that learning is carried out maximally. The third meeting until the tenth meeting received more attention from observers. It is because of the increased activity of students who are seen while studying in group discussions. In addition, increasing the confidence of students is also seen in classroom learning, for example most students want to come to the front of the class to present answers on the board, ask the teacher without feeling awkward, and express their opinions both in group discussions and class discussions. The solving problems in the worksheet actively involves students. It can be seen from the high curiosity of the students to solve the problem. If students experience difficulties, they will not hesitate to ask friends and teacher. It is in accordance with the teacher's task as a facilitator and motivator.

The implementation of REACT strategy uses instruments and learning tools that have been validated by experts. Hand out that are part of learning tools are developed and used to construct students' knowledge. Hand out for the first to the fifth meeting are given by the teacher to students to learn at home. However, some students do not learn it, even the hand out given by the teacher is not carried out during the learning process. It is seen when students feel confused working on the worksheet and always ask the teacher. Based on this results, the teacher changed the strategy at the sixth to tenth meetings. The teacher shares hand outs and worksheets simultaneously during learning. It has a positive impact because students construct their thinking by learning hand out and worksheet at once (Darling-Hammond, Flook, Cook-Harvey, Barron, & Osher, 2019).

CONCLUSION

REACT strategy is an implementation of a contextual learning approach. REACT strategy can be applied to develop the mathematical representation, reasoning, and disposition ability that involve students actively through the stages. Implementation of the REACT strategy runs smoothly and gets enthusiastic responses from students. It can be seen from the activity of students in every learning activity in the classroom. The activeness of students in the class can be seen from high motivation in learning, activeness of students in discussions in groups, asking the teacher. Students are more excited when learning, especially when the teacher asks students to present the results of their group work in front of the class. The implementation of REACT strategy should be carried out continuously so that the learning objectives can be achieved by integrating various integrated mathematical abilities.

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THE LEARNING TRAJECTORY OF NUMBER PATTERN LEARNING USING *BARATHAYUDHA* WAR STORIES AND UNO STACKO

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Abstract

In recent years, several researchers have tried to use stories and games as a starting point for learning mathematics. This is allegedly able to increase students' mathematical abilities and make learning mathematics more enjoyable. Therefore, this research is aimed to design a mathematics learning trajectory in pattern number using *Barathayudha* War Stories and Uno Stacko games as a starting point or context in the learning process with the Indonesian Realistic Mathematics Education (IRME) approach. The research method used is a design research that contains three stages, preliminary design, teaching experiment, and retrospective analysis. The result of this research is the learning trajectory design of number pattern learning using *Barathayudha* war stories and Uno Stacko. The design consists of four activities, which is a detective of *Barathayudha* war; rebuilt *Abimayu* fortress at the battlefield of *Kurusetra*; find the unique secret number code of *Abimayu* fortress; and built another fort using number pattern. The results showed *Barathayudha* war stories and Uno Stacko can stimulate students to understand their knowledge of pattern number concept which is the stages in the learning trajectory of student have an essential role in understanding the concept.

Keywords: Learning Trajectory, Number Pattern, *Barathayudha* War Stories, Uno Stacko, Design Research

Abstrak

Dalam beberapa tahun terakhir, sejumlah peneliti mencoba untuk menggunakan cerita dan permainan sebagai titik awal pembelajaran matematika. Hal ini disinyalir dapat menumbuhkan kemampuan matematis siswa dan membuat pembelajaran matematika menjadi lebih menyenangkan. Oleh karena itu, penelitian ini bertujuan untuk mendesain lintasan belajar matematika pada materi pola bilangan menggunakan cerita peperangan *Barathayudha* dan permainan Uno Stacko sebagai titik awal atau konteks dalam proses pembelajaran menggunakan pendekatan Pendidikan Matematika Realistik Indonesia (PMRI). Metode yang digunakan dalam penelitian ini adalah penelitian desain yang terdiri dari 3 tahapan, yaitu desain pendahuluan, percobaan pengajaran, dan analisis retrospektif. Hasil dari penelitian ini merupakan desain lintasan belajar pada pembelajaran pola bilangan menggunakan cerita peperangan *Barathayudha* dan permainan Uno Stacko. Desain ini terdiri dari 4 aktivitas, yaitu seorang detektif dari perang *Barathayudha*, membangun kembali benteng *Abimayu* di medan perang Kurusetra; menemukan kode nomor rahasia unik dari benteng *Abimayu*; membangun benteng lain menggunakan pola angka. Hasil penelitian menunjukkan bahwa kisah peperangan *Barathayudha* dan Uno Stacko dapat merangsang siswa untuk menumbuhkan pemahaman siswa tentang konsep pola bilangan, yang mana seluruh tahapan dalam lintasan belajar yang dilalui siswa memiliki peran penting dalam penanaman konsep tersebut.

Kata kunci: lintasan belajar, pola bilangan, Cerita Peperangan *Barathayudha*, Uno Stacko, Penelitian Desain

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The development and application of the mathematics concept daily problems are part of a learning process (Tanujaya, Prahmana, & Mumu, 2017). Freudhental (1991) explained that mathematics is a human activity and must be related to daily life. However, in reality, the mathematics in schools tend to be taught using practical formulas and most often, not seamlessly associated with everyday life and culture, as should be experienced by the students (Stacey, 2011; Arisetyawan, Suryadi, Herman, & Rahmat, 2014; Nurhasanah, Kusumah, & Sabandar, 2017). The society, including the teachers, generally does not regard mathematics to be related to culture, and the learning of mathematics in the classroom can also be regarded with almost having no relation to culture. In fact, culture is part of a student's life that may guide the way a student learns and regard mathematics (Stacey, 2011; Revina, 2018; Revina & Leung, 2019). It may significantly influence the student's ability to solve mathematics projects that relate to daily life. The results of the Programme for International Student Assessment (PISA) for Indonesia showed that the students' abilities to solve and interpret problems in various situations are still considered at a level, which is low (Kamaliyah, Zulkardi, & Darmawijoyo, 2013).

Subsequently, Irawan and Kencanawaty (2017) and Sembiring, Hoogland, and Dolk (2010) suggested that appropriate strategies and learning methods are needed to develop students' thinking ability that orientate towards technical skills and the reformation of mathematics education based on problem-solving in the daily life. The learning method suggested is the *Pendidikan Matematika Realistik Indonesia* (PMRI), which is an adaptation of the Realistic Mathematics Education (RME). The PMRI is aligned with the Indonesian culture, geography and the ability of Indonesian society in general (Soedjadi, 2007; Sembiring, Hadi, & Dolk, 2008; Prahmana, Zulkardi, & Hartono, 2012, Arsaythamby & Zubainur, 2014).

Furthermore, Wahyudi, Zulkardi, and Darmawijoyo (2016) and Subijanto (2015) explained that one of the contexts that can be used in PMRI is a culture that is applied to realistic mathematics learning and modified according to the local context where the school is located. Consequently, it may result in engaging, contextual knowledge if it is to be taught in schools as it may increase the students' ability to solve a problem that has a relation to their daily life. Also, the cultural context can be a solution to the lost aesthetic value and character of a student due to the influence of modernization (Prahmana, 2017; Grigoryan, Lebedeva, & Breugelmans, 2018; Uge, Neolaka, & Yasin, 2019). Other researchers also use cultural contexts such as folklore as a starting point for learning mathematics, including the use of *Legend Putri Dayang Merindu* story as folklore in understanding Least Common Multiple (Triyani, Putri, & Darmawijoyo, 2012) and the legend of *Kemaro* Island story for supporting students in learning average (Lestariningsih, Putri, & Darmawijoyo, 2012). These results show that the cultural context can support students to develop their mathematics knowledge.

On the other hands, professional teachers, as the product of reform in education, must have higher education qualifications and be able to innovate in teaching and learning (Prahmana, Zulkardi, & Hartono, 2012; Risdiyanti & Prahmana, 2018). So, every prospective teacher should be prepared to become a professional teacher to equip himself through higher education and knowledge of the learning and teaching process.

In Yogyakarta, there is a study club called the Yogyakarta Mathematics Study Club (YMSC) consisting of several mathematics education graduates who are engaged in innovating mathematics learning as a way to improve the qualifications and innovative learning abilities for graduates of mathematics education in Yogyakarta. In this research, some of YMSC members act as research subjects (students) who are given treatment in the form of mathematics learning activities using cultural context and games, namely *Barathayudha* war stories and Uno Stacko game.

The culture context product should be fun and contained concepts of mathematics for learning purposes and aspects of moral values (Radovic, Black, Williams, & Salas, 2018; Risdiyanti, Prahmana, & Shahrill, 2019). The design learning activities of the number pattern using *Barathayudha* war stories and Uno Stacko game are expected to be innovative in terms of learning mathematics so that the concept will be easy for students to understand thus enabling them to solve any daily-related problems. This context was chosen mainly because of its familiarity with the participants from the perspectives of culture as well as their daily life. Furthermore, this design is expected to cultivate and develop the cultural values that may influence a student's character.

METHOD

This research uses design research as a research method. Design research was chosen in this research because this method is a systematic and flexible method to improve the quality of learning in the classroom by collaborating between researchers and teachers to develop a learning design (Gravemeijer, 1994). The development of learning design is carried out in three phases, which are preliminary design, design experiment, and analysis retrospective (Bakker, 2004; Gravemeijer & Cobb, 2006; Simonson, 2006; Prahmana, 2017).

The preliminary design aims to design the Hypothetical Learning Trajectory (HLT), which is then refined in the design experiment stage (Prahmana, 2017). The activities carried out in this stage are collaborating with the teacher to conduct a literature review of the concept of number patterns, realistic mathematics education, and contexts that can be used in learning number patterns namely *Barathayudha* war stories and Uno Stacko game. Also, researchers analyzed the concept of number patterns in the mathematics education curriculum in Indonesia. Furthermore, the results of the literature study and curriculum analysis were used as a basis for designing learning trajectories and developing conjectures to become HLT. In this case, theory aims as guidelines that will improve in each learning activity, so it is flexible and can be revised during the experimental design stage.

In the design experiment stage, the learning trajectory that has been designed at the preliminary design stage is then implemented in the learning process (Prahmana, 2017). The purpose of this implementation is to explore and observe the strategies and thoughts of students. There are two cycles in this stage; the first cycle is a pilot experiment that aims to evaluate and improve the learning trajectory that has been designed. The second cycle is a teaching experiment that seeks to implement a learning trajectory that is evaluated and revised in the pilot experiment of the design experiment stage. The implementation of number pattern learning activity using *Barathayudha* war stories and Uno Stacko game consists of four activities.

The last stage is retrospective analysis. All data collected in the design experiment stage are analyzed by comparing conjecture and HLT with the results of the application of the learning trajectory that has been carried out in the design experiment stage (Gravemeijer & Cobb, 2006). From the results of the analysis will obtain a learning trajectory description of number pattern learning using

Barathayudha war stories and Uno Stacko game.

RESULTS AND DISCUSSION

The results of this study obtained a trajectory description of the number pattern learning using the *Barathayudha* war story and Uno Stacko. The learning activities consist of four activities. The first activity is to be a detective of *Barathayudha* war. Activity two is to rebuild *Abimayu* Fortress at Battlefield of *Kurusetra*. Furthermore, the third activity is to find the unique secret number code of *Abimayu* fortress. Lastly, the fourth activity is to build another fortress using the number pattern. Students can understand the concept of number patterns using the *Barathayudha* war story and Uno Stacko. It viewed from the results of the final evaluation and the positive responses of students.

Regarding this learning can be seen from the comments, students feel more comfortable understanding the number pattern using this context. The results of this study indicated that the learning design of number patterns using *Barathayudha* War Story and Uno Stacko has very important to be the starting point and can increase student motivation in the learning process. In detail, the researchers discusses the results of this study as follows.

Activity 1: Be a detective of Barathayudha War

The learning activities begin with the teacher describing the *Barathayudha* war, which is a civil war between *Kurawa* and *Pandhawa* in *Pewayangan* Stories. The student will be told the original story, for, in the end, it will be slightly modified to fit the material to be learned. *Barathayudha*'s story was chosen as the starting point because this learning was carried out in Java, which was very thick with the *Pewayangan* stories culture. So, it would create a new stigma for students who had felt that actually, mathematics was far from their lives to think that mathematics existed and became part of their culture.

In this activity, it told that at once upon a time. There was a civil war between *Kurawa* and *Pandhawa*. In the *Pewayangan* story, *Kurawa* and *Pandhawa* have the same father named *Prabu Pandhu*, but from different mother. Before dying, *Prabu Pandhu* handed over the authority of the state to purify *Pandhawa*, because he was considered capable of managing and leading wisely. *Kurawa* did not accept his father's decision and always tried to seize the power of *Pandhawa* (Susetya, 2007). Finally, one day, a civil war took place on a battlefield, namely *Kurusetra*. The Hindus believe that *Kurusetra* existed on this earth precisely in India, but no one had succeeded in proving the truth.

At the time of the war, *Abimayu*, one of the members of *Pandhawa* made a triangular fortress of rock arranged in a unique arrangement of numbers, consisting of the results of repetitive number operations. According to archeologists, there are several ways to prove whether the war really happened and took place in *Kurusetra*, India are by breaking the secret code used *Abimayu* to compile the fort (Susetya, 2007; Suparjo, 2011; Priyatni, 2016). Based on historical records, it is known that the fortress was composed of 30 pieces of stone, consisting of 8 levels, the most basic of which had eight bricks, and the top is one brick (Hatley, 2005; Susetya, 2007). Until now, no one has been able to crack the secret code.

After the story is complete, the teacher provokes the students' interest in breaking the secret code. The teacher invites students to be a detective looking for truth. They seem to be in *Kurusetra*, India and found the ruins of the fort there, but did not know whether the debris was a fortress built by *Abimayu* as in the *Pewayangan* story. Therefore, students formed a group of 4 to 5 people who acted as a detective team. The team did research by collecting debris that was suspected of being *Abimayu*'s fortress and then rebuilt and solved its secret code. In this study, the fortress debris is illustrated using Uno Stacko sticks. As a result, at this stage, the students were enthusiastic about listening to the *Pewayangan* story and were interested in deciphering the secret code of *Abimayu* fortress which was actually a pattern number.

Activity 2: Rebuilt Abimayu fortress at battlefield of Kurusetra

In this activity students who have collected fortress debris that is suspected to be the fortress of *Abimayu*, then they try to compile the fort with the arrangement as recorded in history that the *Abimayu* fortress is triangular in shape, arranged uniquely, consisting of the results of repetitive number operations, organized of 30 pieces of stone that from 8 levels, the bottom is composed of 8 pieces of stone. The top is arranged for one stone. At this stage, students need creativity and critical thinking because students must expect a form of the fort that they have never seen, and they rebuild that fort only based on the clues given. As a result, students managed to make a fortress with an arrangement that formed a triangle with eight levels and the provision of each level, creating a number pattern. The students' activities of rebuilt the *Abimayu*'s fortress can be seen in Figure 1.

Figure 1. Students rebuilt *Abimayu* fortrees at battlefield *Kurusetra*

Activity 3: Find the unique secret number code of Abimayu fortress

In the third activity, students are given a student worksheet, which will serve to help students find the secret code of the *Abimayu* fortress arrangement. The student worksheet consists of columns that will be filled with the number of stones arranged in each level. Then students look for the relationship of the number operation of the number of rocks arranged on each level. It is done to test whether the structure that students make has been the same as the one recorded in history, and to prove the truth of the ruins of the fortress in *Kurusetra,* India is the landscape of *Abimayu* in the puppet story. At this stage, students then look for formulas from the number pattern that has been found. The activity in the third stage is essential to do as a bridge to understanding the concept of number patterns from the formal level to the informal level.

Figure 2. Students find the unique secret number code of *Abimayu* fortress

Activity 4: Built another fortress using number pattern

In this fourth activity, students make a fortress with a pattern according to the number pattern they want as this stage, students' understanding has been on understanding the informal level. Students have understood the concept of number patterns and can make these patterns without using the help of Uno Stacko sticks and can visualize the number pattern in the form of a fort made using Uno stick.

Figure 3. Students built another fortress using number pattern

All activities could change the stigma of students and society that mathematics that is felt far from daily life exists and becomes part of the culture of the community. This study was able to take on the role of developing the learning trajectory of number pattern learning using *Barathayudha* war stories and Uno Staco as the local context of education. In addition, a few of researchers have documented the results of their research related to the implementation of daily activities of students in the learning process of mathematics, such as using *Tepuk Bergambar* Indonesian traditional game in learning number operations (Prahmana, Zulkardi, & Hartono, 2012), playing one house in learning number operations (Nasrullah & Zulkardi, 2011), *Patok Lele* stakes in learning measurements (Wijaya, 2008), *Kubuk Manuk* Indonesian traditional game as stimulated starting point to understand the knowledge of the social arithmetic concept (Risdayanti, Prahmana, & Shahrill, 2019), and *Gasing* game in measuring time learning (Jaelani, Putri, & Hartono, 2013), and several mathematical activities in estimating, measuring, and making patterns using Sundanese culture (Muhtadi, Sukirwan, Warsito, & Prahmana, 2017). Therefore this study takes a role to add to the study of contexts that can be used as a starting point for learning mathematics.

CONCLUSION

The learning trajectory can be practiced using local contexts such as culture or other things easily found in the daily activities of the students. The students were able to understand the concept of number pattern more easily since it is fun for them and importantly after doing all learning activities. Lastly, the game is also relatable to activities in their daily life.

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