



INDONESIAN
MATHEMATICAL SOCIETY

JOURNAL ON
MATHEMATICS
EDUCATION

(IndoMS-JME)

Journal on MATHEMATICS Education	Vol.10	No. 3	Page 315-470	Palembang September, 2019	ISSN 2087-8885
---	--------	-------	--------------	------------------------------	-------------------

ditorial Team

Editor in Chief

1. [Prof. Dr. Zulkardi Zulkardi](#), (SCOPUS ID: 55808334900), Universitas Sriwijaya, Palembang, Indonesia

Advisory International Editorial Boards

1. [Prof. Dr. Kaye Stacey](#), (SCOPUS ID: 7003572015), University of Melbourne, Australia
2. [Prof. Dr. Cyril Julie](#), (SCOPUS ID: 9434368700), University of the Western Cape, School of Science and Mathematics Education, Bellville, South Africa
3. [Prof. Dr. Berinderjeet Kaur](#), (SCOPUS ID: 36466505700), National Institute of Education, Nanyang Technological University, Singapore
4. [Prof. Dr. Maarten Dolk](#), (SCOPUS ID: 8541841100), Freudenthal Institute, Utrecht, Netherlands
5. [Prof. Dr. David C Webb](#), (SCOPUS ID: 56109603400), University of Colorado at Boulder, Boulder, United States
6. [Prof. Dr. Christa Kaune](#), (SCOPUS ID: 55386996300), Institute for Cognitive Mathematics, University of Osnabrueck, Germany
7. [Prof. Dr. Lee Peng Yee](#), (SCOPUS ID: 8967077700), National Institute of Education, Nanyang Technological University, Singapore
8. [Prof. Dr. Tom Lowrie](#), (SCOPUS ID: 55955596500), University of Canberra, Australia
9. [Prof. Dr. Zaleha Ismail](#), (SCOPUS ID: 36760992000), Universiti Teknologi Malaysia, Malaysia
10. [Prof. Dr. Minoru Ohtani](#), (SCOPUS ID: 57193851081), Kanazawa University, Kanazawa, Japan
11. [Prof. Dr. Ratu Ilma Indra Putri](#), (SCOPUS ID: 55874038900), Universitas Sriwijaya, Palembang, Indonesia
12. [Prof. Dr. Dolly van Eerde](#), (SCOPUS ID: 35070570900), Freudenthal Institute (FI), Utrecht University, Netherlands
13. [Assoc. Prof. Dr. Hartono Tjoe](#), (SCOPUS ID: 55571607700), The Pennsylvania State University, Pennsylvania, United States
14. [Assoc. Prof. Drs. F.H.J. \(Frans\) van Galen](#), (SCOPUS ID: 55975430300), Freudenthal Institute (FI), Utrecht University, Netherlands
15. [Assoc. Prof. Dr. Michiel Doorman](#), (SCOPUS ID: 6504396378), Utrecht University, Freudenthal Institute for Science and Mathematics Education (FISME), Utrecht, Netherlands
16. [Assoc. Prof. Dr. Wanty Widjaja](#), (SCOPUS ID: 24177678200), School of Education, Deakin University, Geelong, Australia
17. [Assoc. Prof. Dr. Al Jupri](#), (SCOPUS ID: 56378251200), Universitas Pendidikan Indonesia, Bandung, Indonesia
18. [Assoc. Prof. Dr. Maman Fathurrohman](#), (SCOPUS ID: 57205744828), Universitas Sultan Ageng Tirtayasa (Untirta), Serang, Banten, Indonesia
19. [Dr. Masitah Shahrill](#), (SCOPUS ID: 55873798300), Universiti Brunei Darussalam, Brunei Darussalam

Editorial Assistant

1. [Wahyu Hidayat](#), (SCOPUS ID: 57200720567), IKIP Siliwangi, Cimahi, Indonesia
2. [Ahmad Wachidul Wachidul Kohar](#), (SCOPUS ID: 57188993923), Universitas Negeri Surabaya, Surabaya, Indonesia
3. [Bustang Bustang](#), (SCOPUS ID: 55808584400), Universitas Negeri Makassar, Makassar, Indonesia
4. [Ranni Permatasari](#), (SCOPUS ID: 57204473251), Universitas Sriwijaya, Palembang, Indonesia

Journal on Mathematics Education

Doctoral Program on Mathematics Education
Faculty of Teacher Training and Education, Universitas Sriwijaya
Kampus FKIP Bukit Besar
Jl. Srijaya Negara, Bukit Besar
Palembang - 30139
email: jme@unsri.ac.id

p-ISSN: [2087-8885](#) | e-ISSN: [2407-0610](#)

Table of Contents

Articles

<u>A COMPARATIVE STUDY OF QUADRILATERALS TOPIC CONTENT IN MATHEMATICS TEXTBOOKS BETWEEN MALAYSIA AND SOUTH KOREA</u> <i>Abdul Halim Abdullah, Bomi Shin</i>	315-340
<u>MATH AND MATE IN JAVANESE PRIMBON: ETHNOMATHEMATICS STUDY</u> <i>Niken Wahyu Utami, Suminto A Sayuti, Jailani Jailani</i>	341-356
<u>DIAGNOSING STUDENTS' LEARNING DIFFICULTIES IN THE EYES OF INDONESIAN MATHEMATICS TEACHERS</u> <i>Ariyadi Wijaya, Heri Retnawati, Wahyu Setyaningrum, Kazuhiro Aoyama, Sugiman Sugiman</i>	357-364
<u>EXPLORING MATHEMATICAL REPRESENTATIONS IN SOLVING ILL-STRUCTURED PROBLEMS: THE CASE OF QUADRATIC FUNCTION</u> <i>Ika Santia, Purwanto Purwanto, Akbar Sutawidjadja, Sudirman Sudirman, Subanji Subanji</i>	365-378
<u>FLAWS IN PROOF CONSTRUCTIONS OF POSTGRADUATE MATHEMATICS EDUCATION STUDENT TEACHERS</u> <i>Zakaria Ndemo</i>	379-396
<u>THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA</u> <i>Heris Hendriana, Rully Charitas Indra Prahmana, Wahyu Hidayat</i>	397-408
<u>TEACHERS EXPECTATION OF STUDENTS' THINKING PROCESSES IN WRITTEN WORKS: A SURVEY OF TEACHERS' READINESS IN MAKING THINKING VISIBLE</u> <i>Abdur Rahman As'ari, Dian Kurniati, Subanji Subanji</i>	409-424
<u>SENIOR HIGH SCHOOL MATHEMATICS LEARNING THROUGH MATHEMATICS MODELING APPROACH</u> <i>Bambang Riyanto, Zulkardi Zulkardi, Ratu Ilma Indra Putri, Darmawijoyo Darmawijoyo</i>	425-444
<u>REFLECTIVE THINKING SKILLS OF ENGINEERING STUDENTS IN LEARNING STATISTICS</u> <i>Rindu Alriavindra Funny, Muhammad Abdul Ghofur, Wuli Oktiningrum, Ni Luh Sakinah Nuraini</i>	445-458
<u>AQUATIC IN ASIAN GAMES: CONTEXT OF PISA-LIKE MATHEMATICS PROBLEM</u> <i>Levana Maharani, Ratu Ilma Indra Putri, Yusuf Hartono</i>	459-470

A COMPARATIVE STUDY OF QUADRILATERALS TOPIC CONTENT IN MATHEMATICS TEXTBOOKS BETWEEN MALAYSIA AND SOUTH KOREA

Abdul Halim Abdullah¹, Bomi Shin²

¹Faculty of Social Sciences and Humanities, Universiti Teknologi Malaysia (UTM), Johor Bahru, Malaysia

²College of Education, Chonnam National University, Gwangju, South Korea

Email: p-halim@utm.my

Abstract

This study compares Malaysian and Korean geometry content in mathematics textbooks to help explain the differences that have been found consistently between the achievement levels of Malaysian and South Korean students in the Trends in International Mathematics and Science Study (TIMSS). Studies have shown that the use of textbooks can affect students' mathematics achievements, especially in the field of geometry. Furthermore, to date, there has been no comparison of geometry content in Malaysian and Korean textbooks. Two textbooks used in the lower secondary education system in Malaysia and South Korea were referred. The topic of quadrilaterals was chosen for comparison, and the topic's chapter in the South Korean textbook has been translated into English. The findings show four main aspects that distinguish how quadrilaterals are taught between the two countries. These aspects include the composition of quadrilaterals topics, the depth of concept exploration activities, the integration of deductive reasoning in the learning content and the difficulty level of mathematics problems given at the end of the chapter. In this regard, we recommend the Division of Curriculum Development of the Malaysian Ministry of Education reviews the geometry content of mathematics textbook used today to suit the curriculum proven to produce students who excel in international assessments.

Keywords: Geometry, Quadrilaterals, Textbook, Malaysia, South Korea

Abstrak

Penelitian ini membandingkan konten geometri pada buku matematika di Malaysia dan Korea Selatan, untuk membantu menjelaskan perbedaan yang telah ditemukan secara terus menerus antara tingkat pencapaian siswa Malaysia dan Korea dalam hasil *Trends in International Mathematics and Science Study* (TIMSS). Sejumlah penelitian sebelumnya menunjukkan bahwa penggunaan buku teks dapat mempengaruhi prestasi matematika siswa, terutama pada bidang geometri. Selain itu, sampai sekarang, belum ada perbandingan konten geometris di buku teks Malaysia dan Korea Selatan. Dua buku teks yang digunakan dalam sistem pendidikan menengah bawah di Malaysia dan Korea Selatan dirujuk dan salah satu topik geometri yang diajarkan diidentifikasi. Materi Segiempat telah dipilih sebagai topik perbandingan dan bab topik dalam buku teks Korea Selatan telah diterjemahkan ke dalam bahasa Inggris. Hasil penelitian menunjukkan bahwa ada empat aspek utama yang membedakan bagaimana materi segiempat diajarkan pada kedua negara. Aspek-aspek ini termasuk komposisi topik Segiempat, kedalaman kegiatan eksplorasi konsep, integrasi penalaran deduktif dalam konten pembelajaran dan tingkat kesulitan masalah matematika yang diberikan pada akhir bab ini. Dalam hal ini, direkomendasikan kepada Divisi Pengembangan Kurikulum dari Kementerian Pendidikan Malaysia untuk merevisi isi geometri dari buku teks matematika yang digunakan hari ini untuk menyesuaikan dengan kurikulum yang telah terbukti menghasilkan siswa yang unggul dalam penilaian internasional.

Kata kunci: Geometri, Segi Empat, Buku Teks, Malaysia, Korea Selatan

How to Cite: Abdullah, A.H., & Shin, B. (2019). A Comparative Study of Quadrilaterals Topic Content in Mathematics Textbooks between Malaysia and South Korea. *Journal on Mathematics Education*, 10(3), 315-340. <http://doi.org/10.22342/jme.10.3.7572.315-340>

Classrooms use multiple educational resources. Perhaps the primary source used most often as reference material is textbooks. Textbooks refer to books designed and developed to translate the desired curriculum objectives. They are an important component of the education system and

curriculum such that learning at school is synonymous with textbooks. However, according to Koedel, Li, Polikoff, Hardaway, and Wrabel (2017), though textbooks are the most widely used sources of education, studies on their impact on student learning are very limited. Therefore, Fan, Mailizar, Alafaleq, and Wang (2018) state that school textbooks have become subjects of international research. Rezat (2009) found that students use mathematics textbooks not only when they are told by the teacher, but also for self-learning. Also, students make their mathematics textbooks a medium for solving problems, consolidating, acquiring mathematical knowledge, and activities related to the interest in mathematics.

While according to Ceretkova, Sedivy, Molnar, and Petr (2008), textbooks also have several functions: (1) motivational function, i.e. well-written textbooks can stimulate the interest of students reading them, (2) communication functions, i.e. textbooks that can expand their vocabulary including technical terms, (3) regulatory functions, i.e. the curriculum divided into parts that can elaborate sequence logically, (4) application function, i.e. it consists of ideas using things that practice and express examples of real life, (5) integrated function, i.e. textbooks that are not tied to their subjects but contain links to other disciplines that lead to more complex cognitive processes, (6) innovative functions that introduce new knowledge, and (7) control and corrective functions which students use text, exercises and problems to test themselves, students discover what they understand or do not understand, and they are reviewing the matter. According to Lepik, Grevholm, and Viholainen (2015), textbooks are equally important resources for both students and teachers. Students use them to learn mathematics and teachers use them for planning and teaching mathematics lessons. Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) stressed that the structure of mathematical textbooks might have an impact on teaching in the classroom. They state that the shape and structure of textbooks affect different pedagogical models in mathematical classes.

COMPARISON OF GEOMETRY CONTENT IN TEXTBOOKS

Many studies compared mathematics textbooks as a whole, with some studies focusing on geometry. Among the earliest was the study by Kim (1993) comparing the geometry content of South Korean and US textbooks. His study found that geometry content in American textbooks was spirally arranged as the same topic emerged and extended to many grade levels. On the contrary, in Korean textbooks, geometry content was structured so that a concept or skill dominated each grade level. In Kelley (2013), eight American textbooks were studied from the 1980s and 2000s with the aim of identifying the differences between the two textbook groups in terms of approaches to teaching proof and writing of geometric proofing. All exercises in each text were encoded using parameters such as proof, type of proof, and reasoning task. It was found that new textbooks incorporated conjecture-based learning for the theorem and paid more attention to the evidence in the context of geometric reasoning. Hsu and Ko (2014) compared the geometry content of teaching materials in the mathematics textbooks of Taiwan, Finland, and Singapore. Content analysis was used, and

mathematics problems were analysed. Problems were classified based on their cognitive type, representational form, and context. The study showed that most problems were classified as 'procedure without connection' with only a few problems under the 'doing mathematics' category. Most of the contexts of problems and representations in the three countries are non-contextual and visual forms. The obvious differences between the three countries are the presentation of problem examples and the ratios between examples of concepts and mathematics problems. From the aspect of problem delivery, Taiwan and Singapore provide a more detailed and focused process to help students solve the problems, but brief explanations and demonstrations are found in textbooks. The ratio between examples and mathematics problems is around 1:3 in Taiwan and Singapore and 1:25 in Finland.

Usiskin et al. (2008) conducted a micro curriculum analysis using a variety of textbooks in the United States on the concept of quadrilaterals. It discussed the issue of how a particular quadrilateral can be mathematically defined in the same way, and that definition can be inclusive or exclusive. Furthermore, the geometry thinking level in the van Hiele model has been used as the basis of analysis of primary and secondary school textbooks (grades K to 8) used in 42 States of the United States. Newton (2010) reported that learning objectives were in line with the general principles of the van Hiele theory, especially the principle that the level of geometry thinking is sequential. Fujita (2012) proposed an arrangement of plans to engage students with complex quadrilaterals definition hierarchical issues, which involve nurturing students' understanding of quadrilaterals concepts, encouraging them to seek inclusive relationships between definitions and properties, and critical and non-critical discussion based on what they already know.

Mironychev (2016) compared the sequence of topics in geometry courses in high school curricula in Texas, USA and Russia. Four textbooks used in Texas and Russia were selected for this comparison. The objectives of this research were to compare the sequence of topics in the course, determine how the sequence of topics corresponds to the objectives of the geometry topic, and understand why the course topics are structured in such ways. Mironychev (2016) used two approaches in developing geometry courses, namely (1) Topic approach - when parts of the book are arranged accordingly with object/ terms difficulties after consideration, and (2) Proof of evidence - when parts of the book were arranged according to theoretical evidence or form properties. His study found that in the Texas book (HISD), topics were arranged by object, without proof. In the books used in Moscow, the content was compiled in the order of proof. These books were used in geometry courses for different periods. For HISD, students learn this subject for only one year. Therefore, there were not many opportunities to explore the properties of geometry objects in sequence. The main focus was on applying the formula and the nature of the different calculation steps. In conclusion, the geometry textbook used in Texas is easy to use in short courses. They do not need in-depth analysis of geometry properties and are easily understood by students. Many calculating problems help to develop practical skills for applying the nature of learning in life. Russian geometry textbooks were

more suited for advanced courses. They pay more attention to the subject's basics and logical relationships. They are suitable for high-level courses such as pre-university.

Wang and Yang (2016) compared the differences in the use of geometry in primary school textbooks between Finland, China, Singapore, Taiwan, and the United States. The results showed significant differences in representation, problem types, and question formats between mathematical textbooks of the five countries. In Singapore, mathematical textbooks focus primarily on visual forms combined with other forms of representation. There were significant differences between contextual and non-contextual geometry questions between the five mathematical textbooks. In particular, Chinese textbooks have the highest percentage of contextual problems. Mathematical textbooks from China and the US have more open-ended geometry questions.

The main objective of Silalahi and Chang (2017) was to identify geometry equations and differences by analysing the contents of junior high school mathematics textbooks (grades 7-9) in California, Singapore and Indonesia. They found that problem-solving questions were provided at the end of subtopics in geometry textbooks in California and Singapore but not in Indonesia. In contrast, the similarities from California, Singapore and Indonesia were that all three textbooks provided more non-application problems than applying questions. Yang, Tseng, and Wang (2017) analysed geometry problems in four series of high school mathematics textbooks from Taiwan, Singapore, Finland, and the United States. The analytical framework developed for the analysis of mathematics text problems has three dimensions: representational form, contextual feature, and type of feedback. The findings showed that Taiwan and Singapore textbooks contain more problems in combination, while Finnish and American textbooks contain more problems in both oral and visual forms. The problem distribution across various forms of representation is more balanced in Finnish and Singaporean textbooks than in Taiwanese and American textbooks. Most problems are non-application and close-ended compared to the application and open-ended problems. The Taiwanese textbook contains the lowest actual situation problems, rather than the American textbook that has the highest open-type problem. Wong (2017) discussed the opportunity for students to study the proving and reasoning of the geometry topics of the school's mathematical textbook in Hong Kong. The results showed that the Hong Kong Education Ministry took a traditional approach where the proof was taught mainly in geometry, and two-column proofing was emphasised. Overall, the results show that proofing plays a marginal role in mathematics schooling in Hong Kong.

Cao (2018) compared 3-D geometry content in American and Chinese textbooks. His study showed that the main topic of 3-D geometry in the US curriculum is the volume and surface area of a prism, pyramid, sphere, and real-world objects. The US curriculum emphasises connecting 3-D geometry to the real life of students through mathematical modelling. In China, the main topics required in the curriculum are abstract reasoning in spatial positional relationships, parallel relationships, perpendicular relationships and angles, and combining algebraic methods with spatial vectors. Volume and surfaces of three types of polyhedrons (prisms, pyramids, and pyramid

frustums), and four types of solid revolutionary (cylinders, cones, circular frustums, and spheres) are required, but few are found in the Chinese curriculum as opposed to abstract reasoning. Both countries have very different topics in 3-D geometry texts. In the United States, the 3-D main geometry topics taught at school are volume, surface area, and categorisation of objects like a prism and real-world or composite solids. On the contrary, in China, volume and surface area are not the main focus. On the other hand, spatial position relationships, parallel relationships, perpendicular relationships and angles based on abstract graphs, as well as real-world or composite solids and prisms become the main 3-D geometry topic. The findings revealed that the topics found in Chinese texts are quite complex and have a broad spectrum. Also, the content load and cognitive demand are higher than the US text.

LACK OF COMPARATIVE MATHEMATICS EDUCATION STUDIES BETWEEN MALAYSIA AND SOUTH KOREA

Many countries have made South Korea the basis of comparison in mathematics education, particularly in the areas of curriculum, pedagogy and assessment due to the excellent performance of South Korean students in the subject. The many studies on mathematical curriculum include Kuang, Yao, Cai, and Song (2015) concerning the difficulty level of primary school mathematics textbooks in South Korea, and other countries such as France, Russia, Japan and China; Cao, Wu, and Dong's (2017) study on the difficulty level of mathematical textbooks in junior high schools in China, USA, South Korea, Singapore and Japan; Son and Senk's (2010) analysis of the development of the concept of multiplication and division of fraction in two curricula: Everyday Mathematics (EM) from the United States and Korean mathematics curriculum; and Kim's (2012) study of non-textual elements in South Korean and US mathematical textbooks using a conceptual framework that includes accuracy, connectivity, contextuality and conciseness; Shin and Lee's (2018) study on how mathematical textbooks in Korea and the United States helped in the development of student learning from the aspects of recursive partitioning, common partitioning, and distributive partitioning. Studies were also conducted for algebraic learning. Hwang (2004) concluded several elements are distinguishing the South Korean mathematics curriculum with that of the British. He identified that in South Korea, algebraic content is exposed only once to the students, while in England, the algebraic content tends to be repeated or evolving at every level. The algebraic curriculum in England emphasises approximation, mental calculations, and the use of calculators. Consequently, the English mathematics curriculum is less concerned with writing methods and introduces the written approach slightly later than the Korean curriculum. The English curriculum uses a more flexible approach through rounding, mental methods, calculator usage, ratio, and proportion, while the South Korean curriculum emphasises formal and abstract mathematical knowledge and the understanding of certain mathematical concepts. All mathematical content implemented in England and South Korea is provided in the national mathematics curriculum.

Choi and Park (2013) compared the curriculum standards, textbook structure, and textbook items for geometrics topics between the U.S. and Korea. The study found that the Korean curriculum standards do not focus on real-life situations and the textbooks used in the study only included a few real-life application problems. The study also found that the American CMP textbooks begin each section with real-life examples and activities that can familiarise students with abstract ideas, while Korean textbooks introduce real-world situations related to the lessons without any activity or examples that promote student engagement in actual real-world problem-solving situations. Only a small number of life-related problems are found at the end of each part of the textbook. On the topic of probability, Han, Rosli, Capraro, and Capraro (2011) found that Malaysian, South Korean and American textbooks are routine, open-ended, and non-contextual.

From the pedagogical perspective, several comparative studies compared pedagogical practices and implementation in South Korean mathematics classes despite the study of Mustafa, Evrim, and Serkan (2016) indicating that pedagogical practices are not related to South Korean students' achievement in mathematics. In Siraj-Blatchford and Nah (2013), the pedagogical practices in mathematics classes in England and South Korea were compared in the areas of cultures, classroom activity observations and document analysis. Teachers in both countries use integrated activities to teach mathematics. In England, mathematics classes are more structured, more dominated by teachers and less holistic, while the classes are more structured and didactically independent in South Korea. Mathematics education in the UK is more systematic, using more individualised approaches and incorporates a wide range of hands-on activities and comprehensive outdoor activities, while in Korea, mathematical activities are more group-oriented, use limited material and less outdoor activities. Leung and Hew (2013) examined the use of counterexamples, which play a role in encouraging deductive reasoning skills in mathematics learning process among South Korean and Hong Kong students. O'Dwyer, Wang, and Shields (2015) examined eighth-grade teaching practices in the United States, South Korea, Japan and Singapore that support students' conceptual understanding as well as studied the relationship between practice and mathematical tests.

However, in the context of mathematics education, not many comparative studies exist on the differences between Malaysian and South Korean in the perspective of curriculum, pedagogy and assessment. Studies have shown that for mathematics education, most comparative studies were carried out between Malaysia and Singapore. Ibrahim and Othman (2010) and Ahmad (2016) were among the studies which compared the Malaysian curricula with its Singaporean counterpart. Ibrahim and Othman (2010) concluded that there was a need for the Malaysian mathematics curriculum to be revised to enable mathematical literacy among students and for them to be able to apply mathematics into other disciplines at higher educational levels. Han, Rosli, Capraro, and Capraro (2011) examined the analysis of Malaysian, South Korean and US textbooks on the topic of probability. Ismail and Awang (2008) and Thien and Ong (2015), on the other hand, studied the factors that contribute to the success of Singaporean students in the field of mathematics.

Based on the literature review, most comparative studies on components of mathematics curriculum as well as pedagogy and assessment in mathematical classes were conducted on South Korea and other countries but not Malaysia. The only comparative study in the curriculum perspective in which Malaysia was compared to South Korea was Han, Rosli, Capraro, and Capraro (2011). Most comparative studies in mathematics curriculum components were carried out between Malaysia and Singapore.

A COMPARISON OF GEOMETRY CONTENT DOMAIN ACHIEVEMENTS IN TIMSS BETWEEN MALAYSIA AND KOREA

Malaysia joined the Trends in International Mathematics and Science Study (TIMSS) assessment for the first time during the second cycle in TIMSS 1999. To date, Malaysia has participated in six TIMSS cycles in TIMSS 1999, 2003, 2007, 2011 and 2015 and 2019. Malaysia's participation is to foster effective science and mathematics learning among students compared to their peers in other countries. Since Malaysian participation in TIMSS, the best achievement of Malaysia was during the first participation in TIMSS 1999 at 519 points and above the TIMSS average score of 500 points. However, there were declines in performance after TIMSS 1999 in the next three cycles, whereby Malaysian students scored 508 points in TIMSS 2003 which is still above the TIMSS average score. In TIMSS 2007 and 2011, the score of the mathematics achievement was 474 and 440 respectively. However, an increase of 25 points to reach 465 points in TIMSS 2015 renders the fifth round of the assessment the fourth highest ever since TIMSS 1999. Even though there was an increase in points, Malaysia's performance is still on the Low-Level Benchmarking and is below the TIMSS average score.

South Korea has been involved in the TIMSS since its establishment. South Korea has achieved remarkable achievements throughout the involvement of TIMSS. Throughout the participation in TIMSS, South Korea is one of the top three countries with an average score of achievement for grade 4 and grade 8 students in mathematics subjects compared to other countries. In TIMSS 1999, assessment is only done for grade 8. Based on the findings, if we compare with the minimum score set by TIMSS of 500, the average score of South Korean Mathematics is at a very satisfactory level. When referring to the measurement level in TIMSS, the average score of South Korean Mathematics is in the category of high international benchmarking. Thus, we can conclude that South Korean students can apply basic mathematics knowledge in difficult questions and non-routine problems. TIMSS has organised into two domains namely, (1) content domain which refers to the subjects to be evaluated in mathematics, and (2) cognitive domain which focuses on the thinking processes expected from students as they engage in mathematical content. Figure 1 shows that from TIMSS 1999 until TIMSS 2015, 8th-grade South Korean students outperformed Malaysian 8th-grade students in geometry domain.

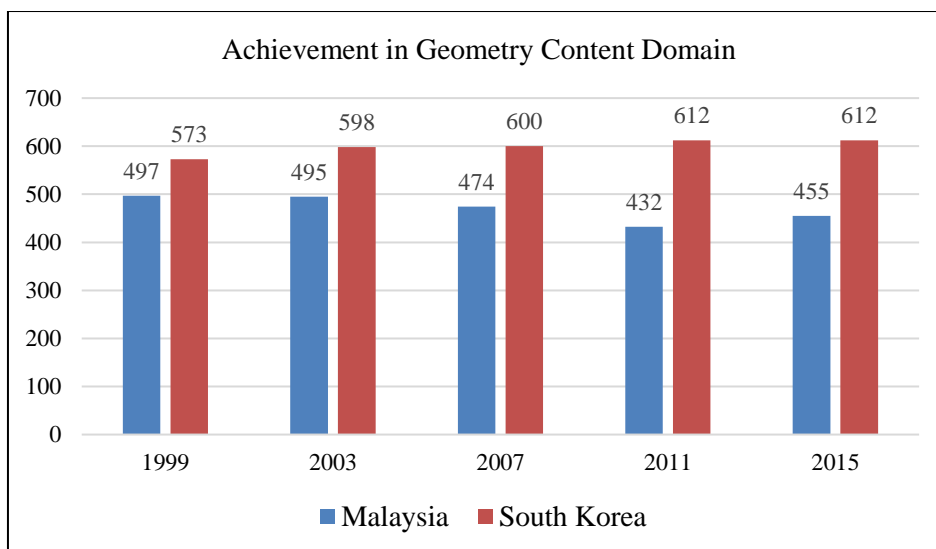


Figure 1. Achievement in Geometry Content Domain Between Malaysia and South Korea

Figure 2 shows a question in geometry domain in which 87% of South Korean students answered the question successfully as opposed to only 32% of Malaysian students. The percentage of Malaysians is not just below the international average, but also among the lowest countries. In this regard, the study compares one of the geometry topics in a textbook at the lower secondary level in Malaysia and South Korea. In the TIMSS 2011 report (Mullis et al., 2012), mathematics teachers of both countries reported that they use mathematical textbooks as a major source in mathematical classes.

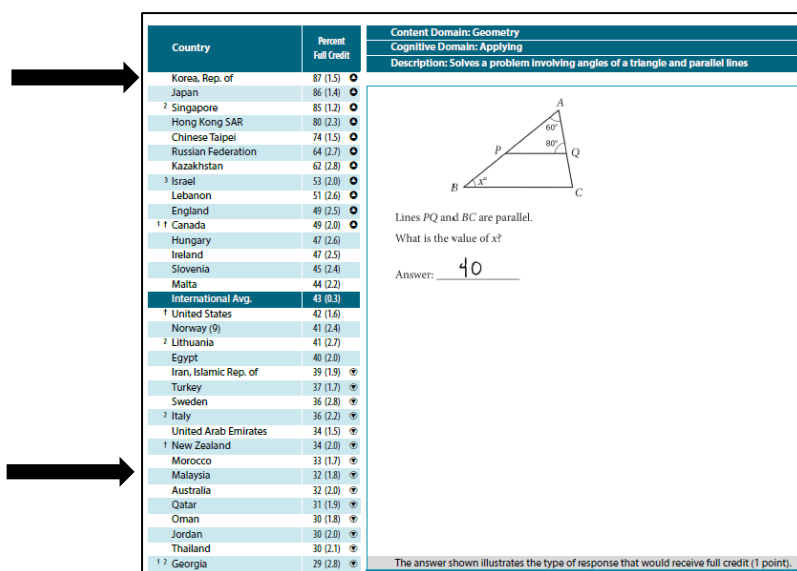


Figure 2. A Sample of Domain Geometry Question in TIMSS 2015

Source: Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016)

Previous studies have found a significant relationship between the textbook used and students' achievement in mathematics (Tornroos, 2005). The Hadar Study (2017) discussed the correlation

between learning opportunities provided in mathematical textbooks and student achievement in national examinations. The findings show that students who use different textbooks have different scores in the national examination. If a textbook gives the students the opportunity to engage in a task that requires a higher level of understanding, students who use this book will obtain a higher score. Xin (2007) examined the potential impact of learning opportunities provided in a USA mathematical textbook and a Chinese textbook on the achievement of student problem-solving. Additionally, Xin studied the learning opportunities provided in textbooks by analysing the problematic distribution of problems across a wide range of problems, as well as the potential impact of learning opportunities on students' ability to solve arithmetical problems. The research shows a positive correlation between the presentation of problem task in textbook and students' mathematical problem-solving skills.

Furthermore, no previous study compared geometry content in mathematical textbooks between Malaysia and South Korea. Choi and Park (2013) analysed the comparison of geometry education related to curriculum standards, textbook structure and items in textbooks between the United States and Korea. While Hong and Choi (2018) analysed and compared the opportunities of reasoning and proofing activities in geometry lessons from American and Korean textbooks to understand how the textbook provides students with the opportunity to engage in reasoning and proving activities. Therefore, this study compares Malaysian and Korean geometry specifically for quadrilaterals topic content in mathematics textbooks to help explain the differences that have been found consistently between the achievement levels of Malaysia and Korean students in the TIMSS especially in geometry content domain.

METHOD

The textbook used as a comparison is the main mathematics textbook used in the education system in both countries. Quadrilaterals topic was selected because education systems in both countries teach the same topic, as well as comparable content. As shown in Figure 3, for South Korea, the content of the selected topic was then translated into English. For Malaysia, the English version of the mathematics textbook was used in this study. This study adapted the framework by Morgan (2004), which looks at content and structure, while also referring to Gracin (2018) who looked at content, cognitive demands, question types and mathematical activities. In this regard, this study examines the composition of quadrilaterals topics, the depth of concept exploration activities, the integration of deductive reasoning in the learning content and the difficulty level of mathematical problems given at the end of the chapter. The van Hiele Model and the Revised Bloom Taxonomy are the basis of comparison in this study. The van Hiele model has been a subject of continuous academic research in geometry and has been applied in various geometry studies (Battista, 2002; Bruni & Seidenstein, 1990; Clement & Battista, 1992; Halat, 2008; Noraini, 2005). Many researchers have recognised the geometry model of van Hiele (Fuys & Liebov, 1997; Usiskin, 1982). Battista (2002) also noted that students' thinking patterns on two-dimensional geometry is clear and best described by

van Hiele's geometry thinking model. Researchers argued that lower secondary students are usually able to achieve up to three levels of van Hiele's geometry thinking of informal deduction (Husnaeni, 2006; Saifulnizan, 2007; Usiskin, 1982). NCTM (1989, 2000) emphasised that the van Hiele model can be applied in to effectively teach geometry. NCTM also emphasised the importance of structured learning as proposed in the van Hiele model. The revised Bloom Taxonomy (Anderson & Krathwohl, 2001) is often used as a framework in differentiating the difficulty of questions, especially in mathematics subject. There are six levels in the taxonomy which are knowing, understanding, applying, analysing, evaluating and creating.

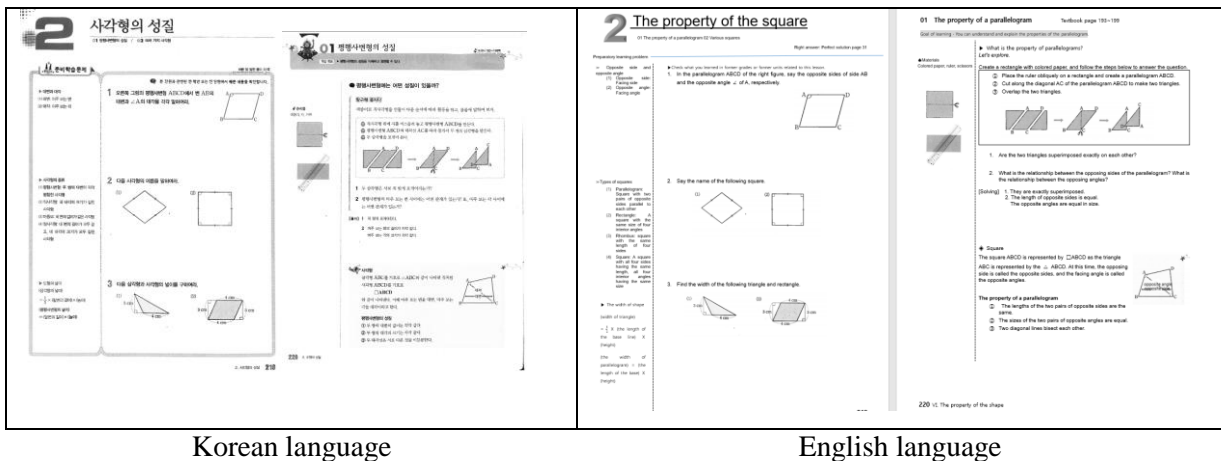


Figure 3. Translation of Quadrilaterals Content Topic in South Korean Textbook

RESULT AND DISCUSSION

The Content Arrangement of Quadrilateral Topics

The content of quadrilaterals in the South Korean mathematics curriculum is based on the van Hiele model. Based on Figure 4, the content of this topic is collected by asking students to recognise the names of the quadrilaterals in South Korean textbook. This is in line with the first level in the van Hiele model which is known as visualisation. At this point, students recognise geometrical shapes. For example, students can identify the names of the quadrilateral group such as rectangle, square, parallelogram, trapezium and so on. However, no such activity is found in Malaysian mathematics textbooks.

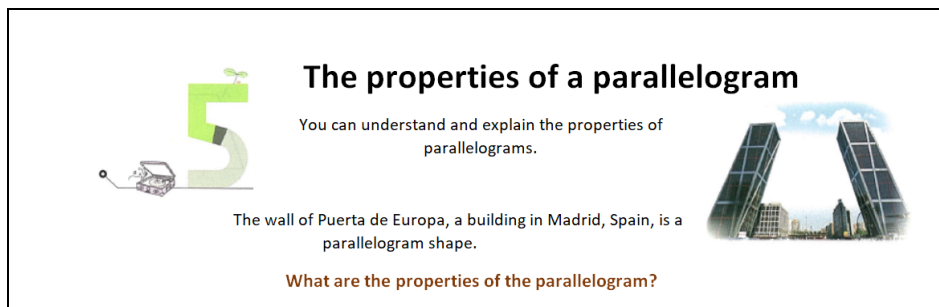


Figure 4. Students are introduced with the shape (Ministry of Education Korea, 2018, p. 165)

The subsequent activities provide students with the opportunity to explore a variety of quadrilaterals. Many of the activities provided in South Korean textbooks are hands-on by using manipulative materials. This is in line with the second level of the van Hiele model which is analysis. At this level, students will be able to recognise the characteristics of shape through observational and experimental activities. Figure 5, for example, shows the activity in which students are to look for properties of parallelogram. According to the National Academies of Sciences, Engineering, and Medicine (2015), the current curriculum focuses on manipulative activities in which students can achieve intuitive ideas about the topic they are currently learning as well as enhance their creativity. Such activities provide more time for creativity and foster a positive attitude towards mathematics. This is important as the TIMSS 2015 results have shown that even though South Korean students displayed encouraging results in mathematics, they scored among the lowest attitudes towards the subject.

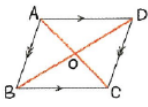
The properties of diagonal in the parallelogram

Let's see if two diagonal lines of a parallelogram are bisecting each other.

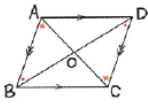
When the intersection of two diagonal lines in the $\square ABCD$ is O, let's explain $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$ and $\overline{OA} = \overline{OC}$, $\overline{OB} = \overline{OD}$

Opening your mind

We can show that the $\triangle ABO$ and the $\triangle CDO$ made by two diagonals are congruent.



Using the properties of the alternate angle in the parallel line can explain they are congruent.



Explaining Step 1. Drawing two diagonals

Let O be the intersection of two diagonals AC and DB in the parallelogram ABCD.

Step 2. Show that both triangles are congruent.

On $\triangle ABO$ and $\triangle CDO$, $\overline{AB} \parallel \overline{DC}$

Therefore,

$\angle OAB = \angle OCD$ (alternate angle).....①

$\angle OBA = \angle ODC$ (alternate angle)....②

Since the lengths of the opposite sides in the parallelogram are the same. Therefore,

$\overline{AB} = \overline{DC}$ ③

Since the corresponding length of one side is the same and the sizes of both end angles are the same in ①, ②, and ③, $\triangle ABO = \triangle CDO$.

Step 3. Find the properties of diagonal in parallelogram

Therefore,

$\overline{OA} = \overline{OC}$, $\overline{OB} = \overline{OD}$

That is, two diagonal lines of a parallelogram bisect each other.

Figure 5. Exploring the characteristics of a quadrilateral (Ministry of Education Korea, 2018, p. 168)

Once students have understood the characteristics of quadrilaterals, they would be able to make connections between them. Figure 6 illustrates the relationship between quadrilaterals. According to the van Hiele model, the third level is an informal deduction. In this level, students can see or prove the relationship between shapes and create a relationship.

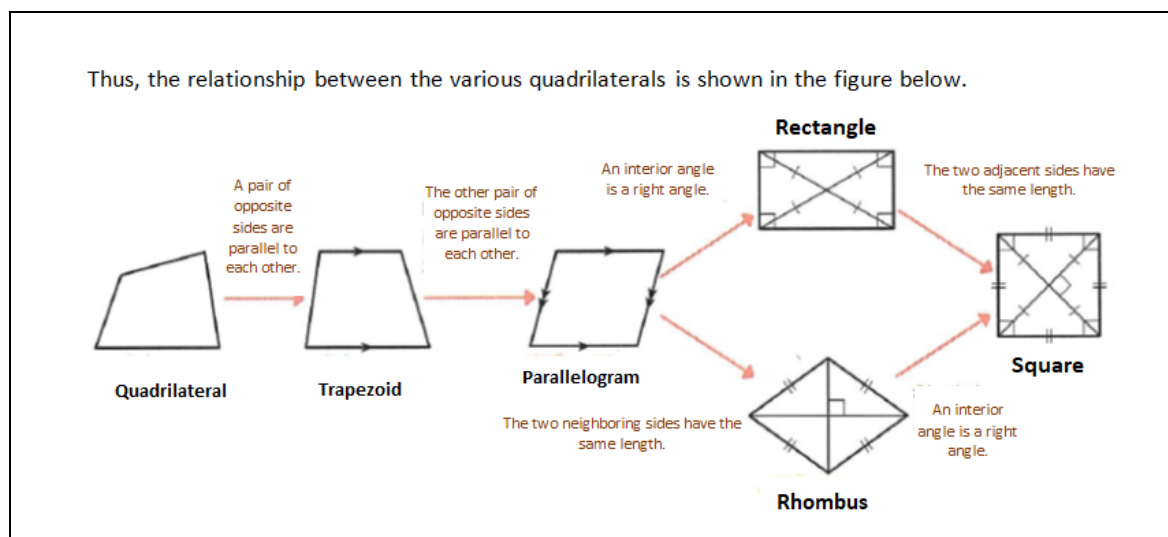


Figure 6. Relationship between the quadrilaterals (Ministry of Education Korea, 2018, p. 182)

Judging from the Malaysian textbooks, quadrilaterals are not compiled based on the van Hiele model. No quadrilateral identification activities were noted in the beginning of the chapter. Students are exposed to the activities of finding quadrilaterals' properties through dynamic geometry software as shown in Figure 7.

1. Open the file *Quadrilaterals geometric properties.ggb* using *GeoGebra*.

GeoGebra interface showing a quadrilateral with vertices A, B, C, D and its diagonals AC and BD. The left sidebar lists properties for various quadrilaterals: Square, Rectangle, Rhombus, Parallelogram, Trapezium, Kite. The right sidebar shows numerical values for side lengths and diagonals.

GeoGebra interface showing a quadrilateral with vertices J, K, L, M and its diagonals JL and KM. The left sidebar lists properties for various quadrilaterals: Square, Rectangle, Rhombus, Parallelogram, Trapezium, Kite. The right sidebar shows numerical values for side lengths and diagonals.
2. Click the checkbox for the first type of quadrilateral. Click and drag the vertices of the quadrilateral to change the dimensions of the quadrilateral. Explain the properties of the quadrilateral.
3. Repeat the exploration in Step 2 for all the other types of quadrilaterals.
4. Discuss with your friends the geometric properties of the various types of quadrilaterals.
5. Open and print the file *Quadrilaterals axes of symmetry.pdf*. Cut out the quadrilaterals in the printout.
6. By folding each of the quadrilaterals, or otherwise, explain how you can find the number of axes of symmetry for each type of quadrilateral.

Figure 7. Activity of identifying characteristics of quadrilaterals using the GeoGebra application (Ministry of Education Malaysia, 2016, p.212)

Figure 8 describes the properties of quadrilaterals. Information presented in such a way could encourage facts memorisation among students. According to Boyraz (2008), Brahier (2005), and Faucett (2007), for geometry content, most textbooks in encouraging memorisation and discouraging

effective learning. Active activities involving students are limited (Nik Azis, 1992). No activity in the form of a conjecture investigation is included in textbooks and theorems are only delivered by the teacher through textbooks (Gillis, 2005).

The following table shows the types of quadrilaterals and their geometric properties.

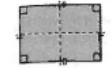
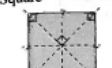




Type of quadrilateral	Number of axes of symmetry	Geometric properties
 Rectangle	2	<ul style="list-style-type: none"> The opposite sides are parallel and of equal length. All of its interior angles are 90°. The diagonals are of equal length and are bisectors of each other.
 Square	4	<ul style="list-style-type: none"> All the sides are of equal length. The opposite sides are parallel. All of its interior angles are 90°. The diagonals are of equal length and are perpendicular bisectors of each other.
 Parallelogram	None	<ul style="list-style-type: none"> The opposite sides are parallel and of equal length. The opposite angles are equal. The diagonals are bisectors of each other.
 Rhombus	2	<ul style="list-style-type: none"> All the sides are of equal length. The opposite sides are parallel. The opposite angles are equal. The diagonals are perpendicular bisectors of each other.
 Trapezium	None	<ul style="list-style-type: none"> Only one pair of opposite sides is parallel.
 Kite	1	<ul style="list-style-type: none"> Two pairs of adjacent sides are of equal length. One pair of opposite angles is equal. One of the diagonals is the perpendicular bisector of the other. One of the diagonals is the angle bisector of the angles at the vertices.

Figure 8. Various characteristics of quadrilaterals presented in a table (Ministry of Education Malaysia, 2016, p.213)

The content of this topic is then formulated in the form of classification as shown in Figure 9 without specifying the relationship between one quadrilateral and another.

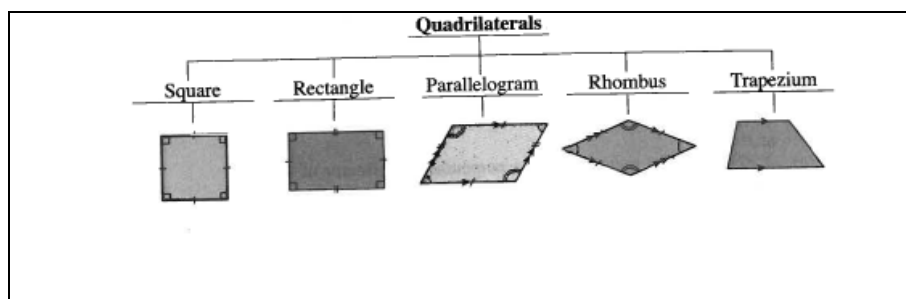



Figure 9. Classification of Quadrilaterals (Ministry of Education Malaysia, 2016, p.219)

Depth Concept Exploration Activities

The second aspect that distinguishes the content of this topic is the depth of the activities of a given concept. Compared to Malaysia which only provides the look of quadrilaterals properties using dynamic geometry software, as shown in Figure 10, the curriculum in South Korea provides immersive exploration activities with questions that test their thinking skills in each quadrilaterals

concept. Ozlem and Jale (2011) showed that the learning process enriched with hands-on activities could improve students' achievement compared to traditional methods. Many studies show that hands-on learning, if often integrated into the learning process, can enhance students' cognitive achievements (Scharfenberg & Bogner, 2010; Thompson, & Soyibo, 2002; Revina, et al. 2011). In the South Korean example, hands-on activities are provided for each type of quadrilaterals. The learning method of discovery is a learning practice that involves students actively, is process-oriented and more focused on self-learning (Agus, Dian, & Ajat, 2017). Based on the results of the study by Sinambela, Napitupulu, Mulyono, and Sinambela (2018), there is a positive impact on learning methods through the discovery of the students' understanding of the mathematical concept, while Yunita, Wahyudin & Sispiyati (2017) concluded that the discovery method would enhance mathematical problem-solving skills. The findings of Balim (2009) study using the findings of learning discoveries showed that there is a significant difference in academic achievement among the students in the experimental group compared to the students in the control group concerning academic achievement, learning retention score, and the perception score on the study skills either at the cognitive or effective levels.

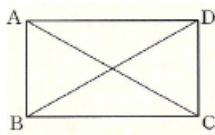

Research learning

What is the property of a rectangle?


Draw two diagonal lines from the rectangle ABCD and compare the lengths.

Draw two diagonal lines on the rectangle ABCD and it is like at the figure on the right. Compare the lengths of the two diagonal lines with the compass, and it is AC and BD.

Are the lengths of the two diagonals the same in all rectangles?



In research learning, the lengths of two diagonals of a rectangle are the same to each other.

 A rectangle is a square whose four interior angles are all the same size.

What you learned previously

On the other hand, the rectangles are equal in size of two pair of opposite angles to each other because the four interior angles are the same size. Thus, the rectangle is a parallelogram and satisfies all the properties of a parallelogram. That is, two diagonal lines of a rectangle bisect each other.

Figure 10. Discovery activities in South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 177)

When students discovered and understood the properties of the rectangle, they will then be given low-level questions and questions that can improve their high-level thinking skills. Students are given a simple question (see Figure 11) followed by difficult questions (see Figure 12).

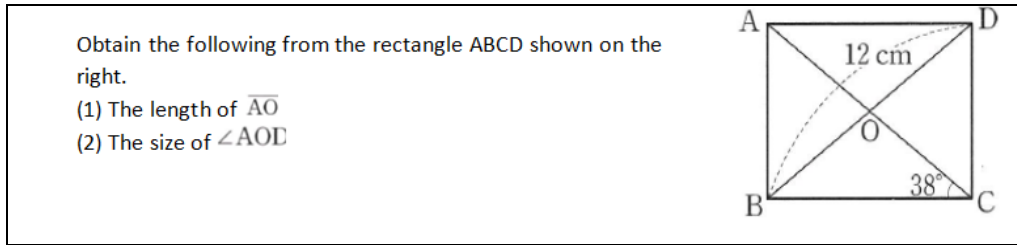


Figure 11. Low-level question for rectangle concept (Ministry of Education Korea, 2018, p. 178)

However, no such activity is found for this topic in Malaysian textbooks. After the introduction of the quadrilaterals concept, Malaysian textbooks introduced interior and exterior angles of the quadrilaterals while in the South Korean context, the concept is introduced together with each type of quadrilateral.

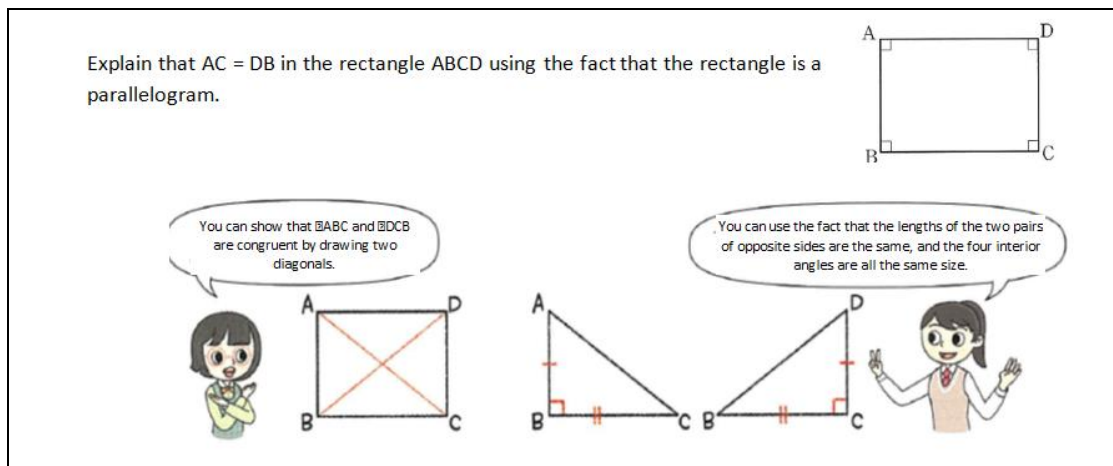
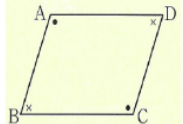


Figure 12. High-level question for rectangle concept (Ministry of Education Korea, 2018, p.177)

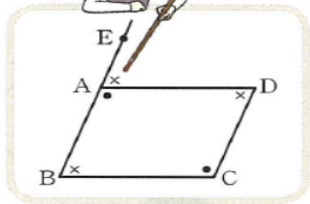
Integration of Deductive Reasoning in Learning Content

Although according to Husnaeni (2006), Saifulnizan (2007) and Usiskin (1982), lower secondary students are usually able to reach up to three levels of geometry thinking based on van Hiele's model of informal deduction, the South Korean textbooks extends to the fourth level of formal deductions. The fourth level of the van Hiele model is the deduction level. Students at this level understand the meaning and importance of deduction and the role of postulate, theorem and evidence. They are able to prove themselves of their understanding. They also understand that the proving process can be done in more than one way and the proof is not obtained by memorisation (Crowley, 1987). The most fundamental reasoning is logical reasoning that consists of inductive reasoning and proven reasoning. Inductive reasoning is one of the reasoning processes which requires students to engage in gathering, interpreting and generalising information. Whereas for deductive reasoning, students can analyse, describe the relationship between forms and prove the theorem deductively. Students also understand that the process of verification can be done in more than one way and the proof is not obtained by memorisation (Crowley, 1987; Prahmana & Suwasti, 2014). Figure 13 shows the reasoning method provided to prove that the given shape is a parallelogram.

Explain that $\square ABCD$ with $\angle A = \angle C$, $\angle B = \angle D$ is a parallelogram as shown in the figure at right.



You can find the corresponding angle of and show that their sizes are the same.



Solving
 If you take point E on the extension line of \overline{BA} ,
 $\angle DAB + \angle DAE = 180^\circ$ ①
 In $\square ABCD$, $\angle A + \angle B + \angle C + \angle D = 360^\circ$
 And $\angle A = \angle C$, $\angle B = \angle D$
 Therefore, $\angle A + \angle B = 180^\circ$ ②
 In ① and ②, $\angle DAE = \angle B$.
 Since the sizes of the corresponding angles are the same, $\overline{AD} \parallel \overline{BC}$. Likewise, $\overline{AB} \parallel \overline{DC}$.
 Therefore, ABCD is a parallelogram because the two pairs of opposite sides are parallel to each other.

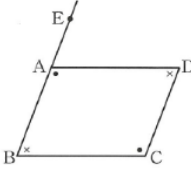
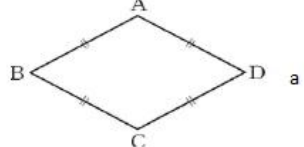


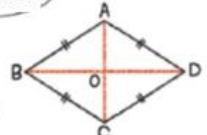
Figure 13. Proof of parallelogram (Ministry of Education Korea, 2018, p. 173)

Figure 14 helps the students perform deductive reasoning to prove that the diagonals are perpendicular in a rhombus.

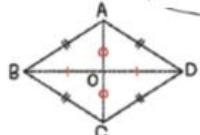
Explain $\overline{AC} \perp \overline{BD}$ in rhombus ABCD using that the rhombus is parallelogram.



You can show $\triangle ABO$ and $\triangle ADO$ are congruent by drawing two diagonals.

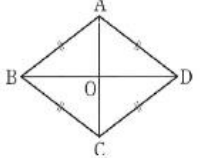


You can use the fact that two diagonals bisect each other to explain they are congruent.



Step 1.
Draw two diagonals
 Let O be the intersection of AC and BD which are two diagonals in the rhombus ABCD.

Step 2. Show two triangles are congruent.
 In $\triangle ABO$ and $\triangle ADO$,
 $\overline{AB} = \overline{AD}$... ①
 A rhombus is a parallelogram. Therefore,
 $\overline{BO} = \overline{DO}$... ②
 \overline{AO} is common ③
 The length of three corresponding sides in ①, ②, ③ are the same respectively.
 Therefore, $\triangle ABO \cong \triangle ADO$



Step 3. Show that the two diagonals are perpendicular to each other
 Therefore, $\angle AOB = \angle AOD$ And $\angle AOB + \angle AOD = 180^\circ$.
 So $\angle AOB = \angle AOD = 90^\circ$ In other words, $\overline{AC} \perp \overline{BD}$.
 The two diagonal lines of the rhombus are perpendicular to each other.

Figure 14. Proof of rhombus (Ministry of Education Korea, 2018, p. 179)

The Difficulty Level of Mathematics Problems Provided at Chapter End

For this, both Malaysian and South Korean curriculum end the learning with mathematical problems related to the topic. Based on the analysis, the problems provided in the Malaysian textbook for quadrilaterals are directed at asking students to look for values of angles in a particular form. For example, as shown in Figure 15, students are asked to find the internal angle of the rhombus and the parallelogram.

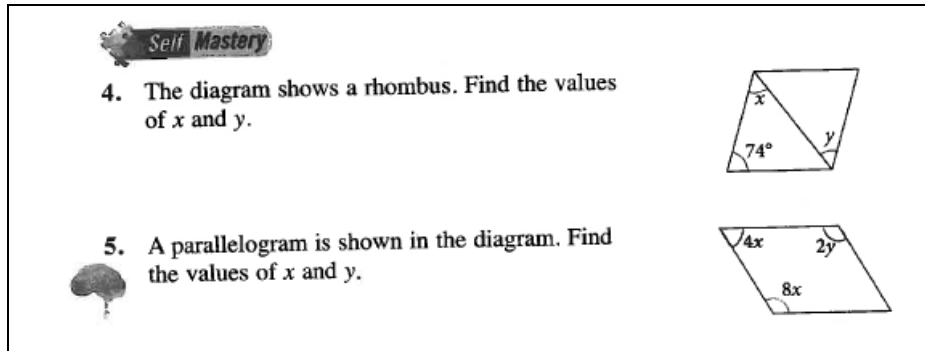


Figure 15. Problems at “applying” level in the Malaysian mathematics textbook (Ministry of Education Malaysia, 2016, p.221)

The six levels in the Revised Bloom Taxonomy introduced by Anderson and Krathwohl (2001) are remembering, understanding, applying, analysing, evaluating and creating. If the student runs or uses a specific procedure to get an answer, then the problems are only at the level of application. There are also questions of understanding level provided in Malaysia textbook as in Figure 16. According to Anderson and Krathwohl (2001), constructing the meaning of various types of functions in writing, graphics or activity such as interpreting, proving, classifying, formulating, concluding, comparing, or explaining is problems at the understanding level.

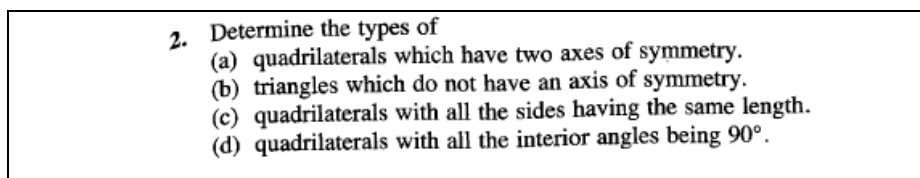


Figure 16. Problems at “understanding” level in the Malaysian Mathematics textbook (Ministry of Education Malaysia, 2016, p.221)

The problems presented for this topic in South Korean textbooks are more diverse and challenging. Undeniably, problems that require students to look for the angle, length or width of a shape which is the problem for application level as shown in Figure 17 were also present.

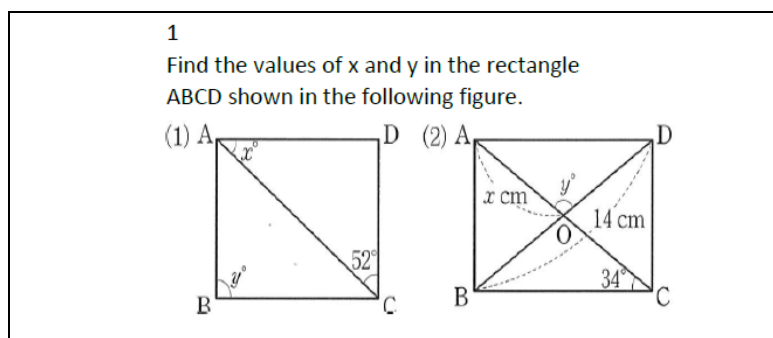


Figure 17. Problem at “applying” level in the South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 183)

Problems for analysing and evaluating levels are also provided in South Korean mathematics textbooks.

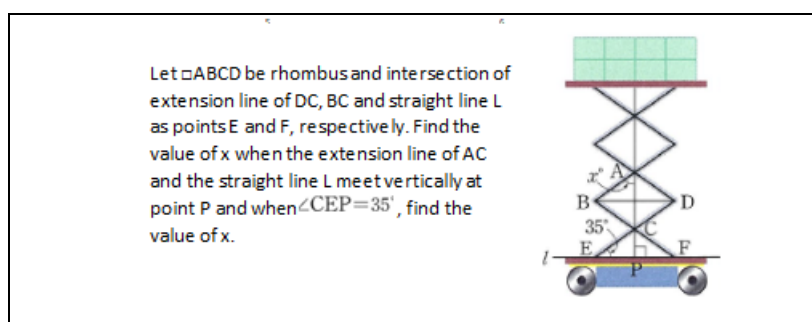


Figure 18. Problem at “analysing” level in the South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 183)

Figure 18 shows one of the samples of problem at “analysing” level. According to Anderson and Krathwohl (2001), mathematics problems at the analysing level involve convincing concepts to their sections, determining how they are related to each other or how they are interrelated and how they complement the overall concept. Thinking skills at this level include comparing and distinguishing between components or parts. The mathematics problem shown in Figure 19 is a problem for evaluating level which is the second highest level based on the Revised Bloom Taxonomy. According to Anderson and Krathwohl (2001), at the evaluating level, students make decisions based on criteria and standards through checks and criticisms. Criticisms, suggestions, and reports are some of the methods that can be done to indicate that evaluation process.

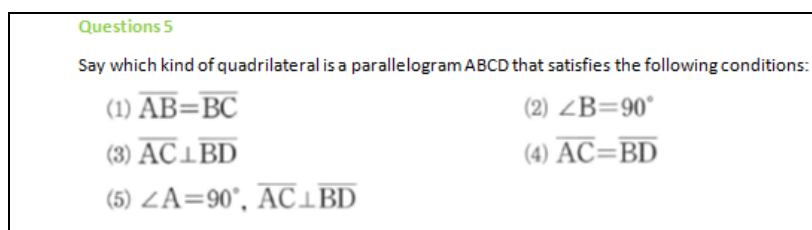


Figure 19. Problem at “evaluating” level in the South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 183)

CONCLUSION

There is a difference in the arrangement of the content for quadrilaterals between the Malaysian and South Korean curriculum. Geometric content, especially in the topic of Quadrilaterals in South Korean mathematics textbooks, is based on the van Hiele model which begins with visualisation, analysis and informal deductions. This means the content of Quadrilaterals in South Korean textbooks is based on the van Hiele model, which is the best model in the arrangement of geometric content. Students should first recognise shapes before they can analyse and find relationships between shapes. According to Noraini (2005), van Hiele states that the geometric thinking of secondary school students is below the expected levels of student thinking for this age cohort. Accordingly, the arrangement of geometric content should begin with visualisation. This model was also proposed by NCTM (1989, 2000) as the best model for effective geometry learning. NCTM also emphasised the importance of structured learning as proposed in the van Hiele model. Many studies show that the geometric content arrangement based on the van Hiele model supports increases in the level of geometric thinking of students. While many computer-assisted learning based on van Hiele models (Chew, 2009; Abdulah & Zakaria, 2013; Muhtadi, et al. 2018; Sukirwan, et al. 2018; Ahamad, et al. 2018) have a positive impact on students' geometry thinking, activities in South Korean textbooks emphasise hands-on manipulative-based activities. Whereas the composition of the content of the topics in the Malaysian curriculum is structured starting with technology-based activities to look for the characteristics of the quadrilaterals followed by the description of the properties in the table and then emphasises the concept of interior angles and the exterior angles of the quadrilaterals. Additionally, every quadrilateral concept in the South Korean curriculum is presented in the form of immersive and engaging activities. According to Nik Azis (2008), the ultimate goal of mathematical learning based on constructivist support based on pragmatism philosophy is the construction of mathematical strength by all students which involves some special abilities that each student needs to develop such as the ability to explore and reason, solve problems, relate ideas mathematics, communicating mathematics and developing self-beliefs about mathematics.

Maheshwari and Thomas (2017) showed that students had high motivation levels and achieved higher mean scores when they were taught using a constructivist teaching approach compared to non-constructivist teaching approaches. South Korean textbooks, especially in quadrilaterals, include deductive aspects of mathematical reasoning. According to Thompson, Senk & Johnson (2012), if the opportunity to address and prove that it is not available in textbooks, it is impossible for reasoning and proven activities to be implemented in mathematics classes. In this regard, secondary mathematics curriculum developers with the Malaysian Ministry of Education (MOE) should consider this in future curriculum reviews. In South Korea's curriculum, although many reasoning and proving activities can be found in the textbooks, according to Hong and Choi (2018), the reasoning and proving opportunities in South Korean textbooks are slightly different as they provide some problems to be proven reasonably while more statements are proven deductively. Many general statements are proven

deductively to give students the opportunity to read and familiarise themselves with deductive proving. From the aspect of the difficulty of mathematical problems provided especially for this topic, the problems presented in South Korean textbooks are seen as more diverse and challenging. Mathematical problems in Malaysian textbooks are still at the level of applying and below based on the Revised Bloom Taxonomy (Anderson & Krathwohl, 2001). Therefore, we recommend the Division of Curriculum Development of the Malaysian Ministry of Education reviews the geometry content of mathematical textbooks used today to suit the curriculum proven to produce students who excel in international assessments.

ACKNOWLEDGEMENT

This work is supported by the Korea Foundation for Advanced Studies 'Chey Institute for Advanced Studies' International Scholar Exchange Fellowship for the academic year of 2018- 2019.

REFERENCES

- Abdullah, A. H. & Zakaria, E. (2013). Enhancing students' level of geometric thinking through van Hiele's phase-based learning. *Indian Journal of Science and Technology*, 6(5), 4432-4446.
- Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, 943(1), 012008. <https://doi.org/10.1088/1742-6596/943/1/012008>.
- Ahmad, M.N.N. (2016). A Discourse Analysis of Malaysian and Singaporean Final Secondary Level Mathematics Textbooks. *Unpublished Master Dissertation*. University of Malaya, Kuala Lumpur, Malaysia.
- Anderson, L.W., & Krathwohl, D.R. (2001). *A Taxonomy for Learning, Teaching and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives: Complete Edition*. New York: Longman.
- Balim, A.G. (2009). The Effects of Discovery Learning on Students' Success and Inquiry Learning Skills. *Egitim Arastirmalari-Eurasian Journal of Educational Research*, 35, 1-20.
- Battista, M.T. (2002). Learning geometry in a dynamic computer environment. *Teaching Children Mathematics*, 8(6), 333-340.
- Boyraz, S. (2008). The effects of computer based instruction on seventh grade students' spatial ability, attitudes toward geometry, mathematics and technology. *Unpublished Master Dissertation*. Middle East Technical University, Turkey.
- Brahier, D.J. (2005). *Teaching Secondary and Middle School Mathematics (2nd ed.)*. Boston, MA: Pearson Education, Inc.
- Bruni, J.V., & Seidenstein, R.B. (1990). *Geometric concepts and spatial sense. Mathematics for the Young Child*. Reston, Va.: National Council of Teachers of Mathematics.
- Cao, M. (2018). An Examination of Three-dimensional Geometry in High School Curricula in the US and China. *Unpublished Doctoral Dissertation*. Columbia University, New York, USA.
- Cao, Y., Wu, L., & Dong, L. (2017) Comparing the Difficulty Level of Junior Secondary School

- Mathematics Textbooks in Five Nations. In: Son, J.W., Watanabe, T., Lo, J.J. (Eds.) *What Matters? Research Trends in International Comparative Studies in Mathematics Education. Research in Mathematics Education*. Cham : Springer.
- Ceretkova, S., Sedivy, O., Molnar, J., & Petr, D. (2008). The Role and Assessment of Textbooks in Mathematics Education. *Problems of Education in the 21st Century*, 6, 27-37.
- Chew, C.M. (2009). Enhancing students' geometric thinking through phase-based instruction using Geometer's Sketchpad : A case study. *Jurnal Pendidik dan Pendidikan*, 24, 89-107.
- Choi, K.M., & Park, H-J. (2013). A Comparative Analysis of Geometry Education on Curriculum Standards, Textbook Structure, and Textbook Items between the U.S. and Korea. *Eurasia Journal of Mathematics, Science & Technology Education*, 9(4), 379-391. <https://doi.org/10.12973/eurasia.2013.947a>.
- Clements, D.H., & Battista, M.T. (1992). Geometry and spatial reasoning. In D.A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning*. New York: Macmillan.
- Crowley, M. (1987). The van Hiele model of development of geometric thought. In M. M. Lindquist, (Ed.). *Learning and teaching geometry K-12*, pp.1-16. Reston, VA: NCTM.
- Fan, L., Mailizar, M., Alafaleq, M., & Wang, Y. (2018). A Comparative Study on the Presentation of Geometric Proof in Secondary Mathematics Textbooks in China, Indonesia, and Saudi Arabia . In L. Fan, L. Trouche, C. Qi, S. Rezat, J. Visnovska (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues*, Springer, Cham.
- Faucett, C.W. (2007). Relationship between type of instruction and student learning in geometry. *Unpublished Doctoral Dissertation*. Walden University, Minnesota, USA.
- Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon. *Journal of Mathematical Behavior*, 31(1), 60-72. <https://doi.org/10.1016/j.jmathb.2011.08.003>.
- Fuys, D.J., & Liebov, A.K. (1997). Concept learning in geometry. *Teaching Children Mathematics*, 3, 248–251.
- Gillis, J.M. (2005). An investigation of student conjectures in static and dynamic geometry environments. *Unpublished Doctoral Dissertation*. Auburn University, Alabama, USA.
- Gracin, D.G. (2018). Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples, *International Journal of Mathematical Education in Science and Technology*, 49(7), 1003-1024. <https://doi.org/10.1080/0020739X.2018.1431849>.
- Hadar, L.L. (2017). Opportunities to learn: Mathematics textbooks and students' achievements. *Studies in Educational Evaluation*, 55, 153–166. <https://doi.org/10.1016/j.stueduc.2017.10.002>.
- Halat, E. (2008). In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender. *The Mathematics Educator*, 18(1), 8–14.
- Han, S., Rosli, R., Capraro, R.M., & Capraro, M.M. (2011). The Textbook Analysis on Probability: The Case of Korea, Malaysia and U.S. Textbooks. *Journal of the Korean Society of Mathematical Education Series D: Research in Mathematical Education*, 15(2), 127-140. <https://doi.org/10.7468/jksmed.2011.15.2.127>.
- Hong, D.S., & Choi, K.M. (2018). Reasoning and proving opportunities in textbooks: A comparative analysis. *International Journal of Research in Education and Science (IJRES)*, 4(1), 82-97.

- Hsu, W-M., & Ko, F-Y. (2014). A Comparison of Geometry Content in Instructional Materials of Elementary School Mathematics Textbooks in Taiwan, Finland, and Singapore. *Journal of Textbook Research*, 7(3), 101-141.
- Husnaeni. (2006). Penerapan model pembelajaran Van Hiele dalam membantu siswa kelas IV SD membangun konsep segi tiga. *Jurnal Pendidikan*, 7(2), 67-78.
- Hwang, H.J. (2004). A comparative analysis of mathematics curricula in Korea and England focusing on the content of the algebra domain. *International Journal for Mathematics Teaching and Learning*. Available online at: <http://www.ex.ac.uk/cimt/ijmtl/hwang.pdf>
- Ibrahim, Z.B., & Othman, K.I. (2010). Comparative Study of Secondary Mathematics Curriculum between Malaysia and Singapore. *Procedia - Social and Behavioral Sciences*, 8, 351-355. <https://doi.org/10.1016/j.sbspro.2010.12.049>.
- Ismail, N.A., & Awang, H. (2008). Mathematics Achievement among Malaysian Students: What Can They Learn from Singapore. Paper presented at the 3rd IEA International Research Conference held on 18-20 September 2008 in Taipei, Taiwan.
- Kelley, G.D. (2013). Approaches to Proof in Geometry Textbooks: Comparing Texts from The 1980s and 2000s. *Unpublished Master Thesis*. University of Maryland, College Park, USA.
- Kim, H. (1993). A Comparative Study Between an American and a Republic of Korean Textbook Series' Coverage of Measurement and Geometry Content in First Through Eighth Grades. *Measurement & Geometry*, 93(3), 123-126.
- Kim, R.Y. (2012). The quality of non-textual elements in mathematics textbooks: an exploratory comparison between South Korea and the United States. *ZDM*, 44(2), 175-187. <https://doi.org/10.1007/s11858-012-0399-9>.
- Kistian, A., Armanto, D., & Sudrajat, A. (2017). The Effect of Discovery Learning Method on The Math Learning of The V SDN 18 Students of Banda Aceh, Indonesia. *British Journal of Education*, 5(11), 1-11.
- Koedel, C., Li, D., Polikoff, M.S., Hardaway, T., & Wrabel, S.L. (2017). Mathematics Curriculum Effects on Student Achievement in California. *AERA Open*, 3(1), 1-22.
- Kuang, K., Yao, C., Cai, Q., & Song, N. (2015). An International Comparison on the Degrees of Difficulty of Primary Mathematics Textbooks and Its Enlightenments. *Comparative Education Research*, 9, 75-80.
- Lepik, M., Grevholm, B., & Viholainen, A. (2015). Using textbooks in the mathematics classroom – the teachers' view. *Nordic Studies in Mathematics Education*, 20(3-4), 129-156.
- Leung, I.K.C., & Lew, H.-c. (2013). The ability of students and teachers to use counter-examples to justify mathematical propositions: A pilot study in South Korea and Hong Kong. *ZDM: The International Journal on Mathematics Education*, 45(1), 91-105. <https://doi.org/10.1007/s11858-012-0450-x>.
- Maheshwari, G., & Thomas, S. (2017). An analysis of the effectiveness of the constructivist approach in teaching business statistics. *Informing Science: The International Journal of an Emerging Transdiscipline*, 20, 83-97.
- Ministry of Education Korea. (2018). *Grade 8 Middle School Mathematics Textbook*. Seoul: www.ktbook.com

- Ministry of Education Malaysia. (2016). *Mathematics Textbook Form 1*. Johor Bahru: Penerbitan Pelangi Sdn. Bhd.
- Mironychev, A.F. (2016). Logical Arrangement of Topics in the High School Geometry Curriculum: An International Comparison. *Journal of Universality of Global Education Issues*, 3, 1-16.
- Morgan, C. (2004). *Writing Mathematically: The Discourse of Investigation*. London: Falmer Press.
- Muhtadi, D., Wahyudin, Kartasasmita, B.G., & Prahmana, R.C.I. (2018). The Integration of technology in teaching mathematics. *Journal of Physics: Conference Series*, 943(1), 012020. <https://doi.org/10.1088/1742-6596/943/1/012020>.
- Mullis, I.V.S., Martin, M.O., Foy, P., & Arora, A. (2012). TIMSS 2011 international results in mathematics. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College. <http://timssandpirls.bc.edu/timss2011/reports/international-results-mathematics.html>
- Mustafa, S.T., Evrim, E., & Serkan, A. (2016). Factors Predicting Turkish and Korean Students' Science and Mathematics Achievement in TIMSS 2011. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(7), 1711-1737. <https://doi.org/10.12973/eurasia.2016.1530a>.
- National Academies of Sciences, Engineering, and Medicine. (2015). *Mathematics Curriculum, Teacher Professionalism, and Supporting Policies in Korea and the United States: Summary of a Workshop*. Washington, DC: The National Academies Press. <https://doi.org/10.17226/21753>
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston: VA.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: VA.
- Newton, J. (2010). An examination of K-8 geometry state standards through the lens of van Hiele levels of geometric thinking. In J. P. Smith (Ed.), *Variability is the rule: a companion analysis of K-8 state mathematics standards* (pp. 71–94). Charlotte: InfoAge Publishing.
- Nik Azis Nik Pa. (1992). *Agenda Tindakan: Penghayatan Matematik KBSR dan KBSM*. Kuala Lumpur: Dewan Bahasa dan Pustaka.
- Nik Azis Nik Pa. (2008). *Isu-isu kritikal dalam pendidikan matematik*. Kuala Lumpur: Dewan Bahasa dan Pustaka.
- Noraini Idris. (2005). *Pedagogy in Mathematics Education*. Second Edition. Kuala Lumpur: Utusan Publication Sdn. Bhd.
- O'Dwyer, L., Wang, Y., & Shields, K. (2015). Teaching for conceptual understanding: A cross-national comparison of the relationship between teachers' instructional practices and student achievement in mathematics. *Large Scale Assessments in Education*, 3(1), 1-30. <https://doi.org/10.1186/s40536-014-0011-6>.
- Ozlem Sadi & Jale Cakiroglu. (2011). Effects of hands-on activity enriched instruction on students' achievement and attitudes towards science. *Journal of Baltic Science Education*, 10(2), 87-97.
- Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, 5(1), 17-26. <https://doi.org/10.22342/jme.5.1.1445.17-26>.
- Revina, S., Zulkardi, Darmawijoyo, & Galen, F.V. (2014). Spatial visualization tasks to support

- students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, 2(2), 127-146. <http://dx.doi.org/10.22342/jme.2.2.745.127-146>.
- Rezat, S. (2009). The utilization of mathematics textbooks as instruments for learning. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of CERME6*, Lyon France. <http://www.inrp.fr/editions/cerme6>. 15 July 2011.
- Saifulnizan Che Ismail. (2007). *Pembinaan modul pembelajaran menggunakan perisian geometri interaktif*, *Unpublished Master Dissertation*. Universiti Teknologi Malaysia, Johor Bahru, Malaysia.
- Scharfenberg, F. J., & Bogner, F. (2010). Instructional efficiency of changing cognitive load in an out-of-school laboratory. *International Journal of Science Education*, 32, 829-844. <https://doi.org/10.1080/09500690902948862>.
- Shin, J., & Lee, S.J. (2018). The alignment of student fraction learning with textbooks in Korea and the United States. *The Journal of Mathematical Behavior*, 51, 129-149. <https://doi.org/10.1016/j.jmathb.2017.11.005>.
- Silalahi, S.M., & Chang, C.C. (2017). A Comparative Study of Geometry Problems in Junior Secondary Mathematics Textbooks From US, Singapore, and Indonesia. *Proceedings of 76th IASTEM International Conference*, Seoul, South Korea, 18th-19th September 2017.
- Sinambela, J.H., Napitupulu, E.E., Mulyono, & Sinambela, L. (2018). The Effect of Discovery Learning Model on Students Mathematical Understanding Concepts Ability of Junior High School. *American Journal of Educational Research*, 6(12), 1673-1677.
- Siraj-Blatchford, I., & Nah, K. (2013). A comparison of the pedagogical practices of mathematics education for young children in England and South Korea. *International Journal of Science and Mathematics Education*, 12, 145-165. <https://doi.org/10.1007/s10763-013-9412-1>.
- Son, J., & Senk, S. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics*, 74(2), 117-142. <https://doi.org/10.1007/s10649-010-9229-6>.
- Sukirwan, Darhim, Herman, T., & Prahmana, R.C.I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series*, 943(1), 012026. <https://doi.org/10.1088/1742-6596/943/1/012026>.
- Thien, L.M., & Ong, M.Y. (2015). *Malaysian and Singaporean students' affective characteristics and mathematics performance: evidence from PISA 2012*. Springer Plus, 4, 563.
- Thompson, D.R., Senk, S.L., & Johnson, G.J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education*, 43, 253-295. <https://doi.org/10.5951/jresmetheduc.43.3.0253>.
- Thompson, J., & Soyibo, K. (2002). Effects of lecture, teacher demonstrations, discussions and practical work on 10th graders' attitudes to chemistry and understanding of electrolysis. *Research in Science & Technological Education*, 20, 25-37. <https://doi.org/10.1080/02635140220130902>.
- Tornroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies In Educational Evaluation*, 31(4), 315-327. <https://doi.org/10.1016/j.stueduc.2005.11.005>.

- Usiskin, Z. (1982). *Van Hiele levels of achievement in secondary school geometry. Final report of the Cognitive Development and Achievement in Secondary School Geometry Project*. Chicago, University of Chicago
- Usiskin, Z., Griffin, J., Witonsky, D., & Willmore, E. (2008). *The Classification of quadrilaterals: a study of definition*. Charlotte: Information Age Publishing.
- Valverde, G.A., Bianchi, L.J., Wolfe, R.G., Schmidt, W.H., & Houang, R.T. (2002). *According to the Book - Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer.
- Wang, T.L., & Yang, D.C. (2016). A Comparative Study of Geometry in Elementary School Mathematics Textbooks from Five Countries. *European Journal of STEM Education*, 1(3), 58. <http://dx.doi.org/10.20897/lectito.201658>.
- Wong, K-c. (2017). Reasoning-and-proving in geometry in school mathematics textbooks in Hong Kong. CERME 10, Feb 2017, Dublin, Ireland.
- Xin, Y.P. (2007). Word Problem Solving Tasks in Textbooks and Their Relation to Student Performance. *The Journal of Educational Research*, 100(6), 347-360. <https://doi.org/10.3200/JOER.100.6.347-360>.
- Yang, D-c, Tseng, Y-k & Wang, T-l. (2017). A Comparison of Geometry Problems in Middle-Grade Mathematics Textbooks from Taiwan, Singapore, Finland, and the United States. *EURASIA Journal of Mathematics Science and Technology Education*, 13(7), 2841-2857. <https://doi.org/10.12973/eurasia.2017.00721a>.
- Yunita, H., Wahyudin, & Sispiyati, R. (2017). Effectiveness of discovery learning model on mathematical problem solving. *AIP Conference Proceedings*, 1868(1), 050028. <https://doi.org/10.1063/1.4995155>.



MATH AND MATE IN JAVANESE *PRIMBON*: ETHNOMATHEMATICS STUDY

Niken Wahyu Utami^{1,2}, Suminto A Sayuti², Jailani²

¹Universitas PGRI Yogyakarta, Jl. IKIP PGRI I Sonosewu No.117, Yogyakarta 55182, Indonesia
²Yogyakarta State University, Jl. Colombo No.1, Karang Malang, Caturtunggal, Yogyakarta 55281, Indonesia
Email: niken@upy.ac.id

Abstract

Marriage is an essential part of life for most people. In the Javanese tradition, great attention is paid to the *weton* of the couple through Javanese *Primbon*. It predicts the fate of the couple in the marriage. This prediction of the Javanese outcome after the wedding has some numerical values. Therefore, this study aims to uncover these numerical values using ethnomathematics. This research uses a qualitative method. Analyzing the Javanese *Primbon* documents is meant to explore the numerical value of Javanese people. Also, analyzing the data using the Javanese *Primbon* documents was not only based on the interpretation of the researcher but also the result of discussions with cultural and mathematical experts. This study proposes numerical values such as number bases, remainder theorem, modulo, and modulus of the congruence in formal mathematics, which is associated with matchmaking using Javanese *Primbon*.

Keywords: Ethnomathematics, number bases, remainder theorems, modulo, the modulus of congruence

Abstrak

Pernikahan adalah bagian penting dari kehidupan bagi kebanyakan orang. Dalam tradisi Jawa, perhatian terbesar diberikan kepada *weton* pasangan melalui *Primbon* Jawa. Hal ini dapat memprediksi nasib pasangan dalam pernikahan. Prediksi setelah pernikahan ini memiliki beberapa nilai numerik. Oleh karena itu, penelitian ini bertujuan untuk mengungkap nilai-nilai numerik tersebut menggunakan ethnomathematics. Penelitian ini menggunakan metode kualitatif. Menganalisis dokumen *Primbon* Jawa dimaksudkan untuk mengeksplorasi nilai numerik orang Jawa. Selain itu, peneliti juga menganalisis data menggunakan dokumen *Primbon* Jawa, tidak hanya didasarkan pada interpretasi peneliti tetapi juga hasil diskusi dengan para ahli budaya dan matematika. Penelitian ini mengusulkan nilai-nilai numerik seperti bilangan dasar, teorema sisa, modulo, dan modulus kongruensi dalam matematika formal, yang dikaitkan dengan perjodohan menggunakan *Primbon* Jawa.

Kata kunci: Etnomatematika, bilangan basis, teorema sisa, modulo, kekongruenan modulo

How to Cite: Utami, N.W., Sayuti, S.A., & Jailani. (2019). Math and Mate in Javanese *Primbon*: Ethnomathematics Study. *Journal on Mathematics Education*, 10(3), 341-356. <https://doi.org/10.22342/jme.10.3.7611.341-356>

Indonesia is a maritime nation with land and sea territories stretching from Sabang to Merauke. The geographical location of the country spans across a group of small islands making Indonesia a land with various ethnic and cultural characteristics (Sutarto, 2006). Culture is a phenomenon which is common in any society. Javanese has one of the most popular customs in Indonesia known for its noble character, as it is also the tradition of other Eastern religions (Ember & Ember, 2001; Hatley, 2008; Sutarto, 2006).

The Javanese culture is a well-known custom filled with mystical issues, with no scientific basis. Java's mystic beliefs came into the open since the early nineteenth century and a poem, *Serat Centhini*, composed in 1815 in the court of Surakarta, was an aspect of Java's mystic belief (Ricklefs, 2007). Based on this mystical atmosphere, the Javanese culture is less explored scientifically. However, it contains various numerical values including ancestral civilization in terms of Mathematics (Maryati & Prahmana, 2019a; Maryati & Prahmana, 2019b; Supiyati, et al. 2019; Risdiyanti & Prahmana, 2018;

Muhtadi, et al. 2017). The development of this subject started a long time in Java, although not yet to the level of formal mathematics or the Western ones.

The objects in Javanese culture contain numerical values in various forms of literature. Ambrosio and Rosa (2017) stated that the different types of culture appear in the form of the artifact, venti-fact, or socio-fact. One of the socio-fact in Javanese culture is the foreknowledge matchmaking in marriage. Moreover, it is a general fact that marriage is something special in the lives of almost every individual, and it must be well planned, including with whom to marry. Before the wedding takes place, the Javanese usually find out about their next lives after the wedding. They carry out foreknowledge matchmaking in marriage before planning the married life. The traditional Javanese elders are the ones who usually start the whole process, guided by the Javanese *Primbon*, which is a life guide used by the people in their daily activities. The Javanese *Primbon* contains the human life guidelines from the womb, birth, adulthood, and even to the day of death.

The guide namely Javanese *Primbon* contains numerical values or mathematical knowledge from Javanese people. Although, the emergence of the *Primbon* did not have any connection with the formal mathematics taught in schools. However, there is a mathematical requirement when it comes to foreknowledge matchmaking in marriage as contained in this *Primbon* and Javanese marriage customs pay attention to the counting *weton* of the couples before they get married. It is usually used to predict the fate of the couple. The exploration of these numerical values in foreknowledge matchmaking in marriage as contained in the Javanese *Primbon* is an area of ethnomathematics studies.

Ethnomathematics Studies

Tutak, Bondy, and Adams (2011) stated that the term of ethnomathematics was first introduced by D'Ambrosio in 1978 at the annual meeting of the American Association for the Advancement of Science (AAAS). Subsequently in 1985, D'Ambrosio, Gloria Gilmer and Rick Scott formed a group known as The International Study Group on Ethnomathematics (ISGEm), whose aim was to increase the understanding of mathematical practices on cultural diversity and apply the knowledge for the purpose of education and development (Tutak, et al. 2011).

D'Ambrosio (1985) stated that ethnomathematics is the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on. Furthermore, it defines ethnomathematics as the intersection of mathematics with the historical, cultural, and social roots of mathematic, and also that the ethnomathematics comes from the combination of two words - ethno and mathematics, where ethno denotes the socio-cultural context, and mathematics talks about mathematical knowledge such as counting, weighing, measuring, comparing, sorting, classifying, designing, and playing (Katsap, 2018). However, Rosa and Orey (2016) explained that ethnomathematics includes ideas, procedures, processes, methods, and practices that are rooted in different cultural environments. Therefore, ethnomathematics is so related to mathematics and culture in the nation.

There are several ethnomathematics related to studies carried out in other countries. One of such is the study conducted by Mohamad, Adam, and Embong (2010), which discusses an ethnomathematical study on weaving among the Malay tudung Saji or food cover weavers, focussing on investigating the mathematical ideas that are relevant to the weaving techniques, framework construction, and pattern formation. On the other hands, Fonseca (2010) discusses how youth and adult educators found ethnomathematics to be a resource useful in knowing their students better and producing an improved result in mathematics, as well as in intercultural studies related to ethnomathematics. This research offers a new perspective on conceiving mathematical knowledge and practices in youth and adult education. We also discuss the dimensions of the approach of teaching and learning practices to present a reflection about the relations among youth and adult education, ethnomathematics and the search for better understanding the teaching and learning of mathematics at schools, as well as making it an inclusion project. A related study was also conducted by Pais (2011), which clarifies the positions of ethnomathematics and its implications in the recent pedagogical research in the field. Meanwhile, François and Stathopoulou (2012) present the development of the political dimension of ethnomathematics and the researchers of critical mathematics education, also explore their similarities.

In other ethnomathematics related studies, D'Ambrosio (2013) explained about the position of ethnomathematics as a theoretical framework capable of guiding practice and serving as a curriculum for different educational projects. We also explained the fact that ethnomathematics is positioned as one which centers the children in a world of social equity and justice. Also, Matang and Owens (2014) discuss the role of indigenous traditional counting systems in children's development of numerical cognition in Papua New Guinea, and then Rosa and Orey (2015) discuss the curriculum for mathematics based on literacy, matheracy, and technoracy from ethnomathematics perspective.

Considering of all these existing articles, no author has discussed foreknowledge matchmaking in marriage guided by Javanese *Primbon* (the special book from Java) which is related to mathematics learning. Therefore, the research focus on to explore the ethnomathematics value of foreknowledge matchmaking in marriage found in Javanese *Primbon*, the body of knowledge from the ethnomathematics value of foreknowledge matchmaking in marriage found in Javanese *Primbon*, and the developed learning mathematics that can be used from the Javanese *Primbon*.

METHOD

The method used in this research is a qualitative method. The purpose of research method is to reveal ethnomathematics in Javanese *Primbon*. The design of this research follows the framework of ethnomathematics study from Alangui (2010) to conduct a qualitative method through ethnomathematics in Javanese *Primbon*. The framework is presented in Table 1.

Table 1. The Framework of Ethnomathematics Study

Generic Question	Initial Answer	Critical Construct	Specific Activity
Where is it to look?	Cultural practices in a cultural context, namely the use of the Javanese <i>Primbon</i> in determining matchmaking of the couple.	Culture	Analysis of Document Javanese <i>Primbon</i> . Describe how were the rules of Javanese <i>Primbon</i> that were used in determining the matchmaking of the couple.
How is it to look?	Investigating QRS (Qualitative, Relational, and Spatial) aspects of the determining the matchmaking of the couple.	Alternative thinking	Determine what QRS ideas are contained in the determining the matchmaking of the couple.
What is it?	Proof of alternative concept	Philosophical mathematics	Identify criteria to justify the rules of external customs in determining the matchmaking of the couple.
What does it mean?	The significant value of culture, and mathematics	Anthropology	Describe the relationship between the two forms of knowledge (mathematics and culture). Write a new mathematical concepts found in the determining the matchmaking of the couple.

Gathering Data

To answer the research question, we analyzed the literature of Javanese *Primbon*. Analyzing the document involves activities like skimming (cursory examination), through reading, and interpretation (Bowen, 2009). The analyzed document produced data in quotations, themes or categories, and the examination, as well as the interpretation of the Javanese *Primbon* document, was carried out to obtain the scientific values in the foreknowledge matchmaking marriage and the experience after the wedding.

Interviewing informants were also carried out as supplementary data in the study. The informants are the traditional Javanese elders, who knew deeply about the Javanese tradition. During the interview, we made notes in the form of field notes and tape recorders. This interviews conducted was aimed at verifying the validity of data from the interpretation of the researcher.

The Data Analysis

This research is limited to the rules the determining the matchmaking of the couple. The data analysis technique used in this study is taxonomic analysis. The taxonomic analysis is conducted to analyze the determining of the matchmaking of the couple, and the values of school mathematics that can be taught.

RESULT AND DISCUSSION

The foreknowledge practice through Javanese *Primbon* is still a common act among the Javanese people, and it is carried out before a wedding could take place. Foreknowledge is known in Javanese as

Petung, and the real meaning of this word in English is called computation. However, *petung* in the Javanese language does not always mean computation. It means consideration in determining something.

The *weton* of the couple is critically considered in matchmaking using the Javanese *Primbon*. *Weton* is a calculation of birthdays based on days in the Javanese Calendar. *Weton* is a composite calculation of *neptu dina* and *neptu pasaran* from the two mates. *Neptu* is certain calculation numbers. Based on interviews conducted with *Mbah Cip*, one of the traditional Javanese elders:

“*Yen cara niki lho niki kula mboten ngguroni, yen ten mriki sing abot niku ana tembung geing. Geing niku netune sing siji Wage sing siji Pahing*” (The interview, 18 May 2018).

It means that consideration in matchmaking in Javanese society is not permissible for a prospective couple who have a *neptu wage* and *pahing* for the wedding.

The major consideration in matchmaking through Javanese *primbon* uses computation, and there are two types of matchmaking computations. The first one is the computation with the couple *weton*, and the other is computation with the names of the couple. This study focuses on the computation of matchmaking based on *weton* from the couple. In Javanese culture, the *weton* of the couple needs to be paid attention to before the wedding, and when it is computed, it helps to predict the fate of the couple during the wedding.

Computation in Predicting the Future of the Bride based on Weton of the Two Brides

There are many computations in predicting the future of the couple based on their *weton*. Also, this *weton* is a compilation of *dina* and *pasaran*. *Dina* (Days) is *Akad* (Sunday), *Senen* (Monday), *Selasa* (Tuesday), *Rebo* (Wednesday), *Kemis* (Thursday), *Jumuah* (Friday), and *Setu* (Saturday). Table 2 present the correlation between days (*Dina*) and *Neptu* namely *Neptu Dina*.

Table 2. *Neptu Dina*

Days	<i>Neptu</i>
Sunday	5
Monday	4
Tuesday	3
Wednesday	7
Thursday	8
Friday	6
Saturday	9

Pasaran is *Kliwon*, *Legi*, *Pahing*, *Pon*, and *Wage*. However, determine the match matching to predict the future of the couple after the wedding must be attention to the *neptu dina* and *neptu pasaran* of the bride and groom. Hence, attention must be paid to Table 3, for a proper discussion on computation in predicting the couple’s future based on their *weton* (Tjakraningrat & Soemodidjojo, 2017).

Table 3. *Neptu Pasaran*

<i>Pasaran</i>	<i>Neptu</i>
<i>Kliwon</i>	8
<i>Legi</i>	5
<i>Pahing</i>	9
<i>Pon</i>	7
<i>Wage</i>	4

There are four types of the computations in predicting the future based on *weton* of couples, such as.

1. Type 1

The computation is performed by summing up of each *neptu dina* and *neptu pasaran* of both the bride and groom and then divided by 9. Then, the foreknowledge decision is made using the remaining value after the division. Therefore, the conclusion is based on the meaning shown on Appendix: The Meaning of the Remain Type 1, and the part of it shown in Table 4.

Table 4. The Part of the Meaning of the Remainder in Type 1

Remainder	and	Remainder	Meaning
4		4	<i>Kerep lara</i> (often sick)
4		5	<i>Akeh rencanane</i> (many plans)
4		6	<i>Sugih rejeki</i> (lots of luck)
4		7	<i>Mlarat</i> (poor)
4		8	<i>Akeh pengkalane</i> (many obstacles)
4		9	<i>Kalah siji</i> (One loses)
5		5	<i>Tulus begjane</i> (have continuous luck)
5		6	<i>Cepak rijekine</i> (easy of fortune)
5		7	<i>Tulus sandhang pangane</i> (have continuous prosperous)
5		8	<i>Akeh sambekalane</i> (many obstacles)
5		9	<i>Cepak sandhang pangane</i> (prosperous)

Then, after the computation, if the remainder has a good meaning, according to the Javanese *Primbon*, then the wedding takes place. However, the wedding is canceled if the result is bad.

The first example of the computation is a situation where the bridegroom is born on *Kliwon* Friday, and the bride born on *Pahing* Friday. Then, the foreknowledge of the two brides is determined by calculating the *weton* of each bride:

- a. *Weton* of the bridegroom born on *Kliwon* Friday means: Friday = 6, and *kliwon* = 8

Hence, the computation is $\frac{(6+8)}{9} = 1$ and the remainder is 5.

- b. *Weton* of the bride born on *Pahing* Friday, means: Friday = 6, and *pahing* = 9

Hence, the computation is $\frac{(6+9)}{9} = 1$ and the remainder is 6.

Considering the computations above with the remainders of 5 and 6, the conclusion based is based in Table 4, which is easy of fortune. In other words, if they get married, they probably have a better life, easy of fortune.

Another example is a situation where the bridegroom is born on *Kliwon* Friday, and the bride born on *Pahing* Thursday. Then, the foreknowledge of the two brides is determined as follows:

- a. *Weton* of the bridegroom born on *Kliwon* Friday means: Friday = 6, and *kliwon* = 8

Hence, the computation is $\frac{(6+8)}{9} = 1$ and the remainder is 5.

- b. *Weton* of the bride born on *Pahing* Thursday means: Thursday = 8, and *pahing* = 9

Hence, the computation is $\frac{(8+9)}{9} = 1$ and the remainder is 8.

From the computations, the remainders are 5 and 8. Checking these with the meanings in Table 4, we have many obstacles. In other words, if they get married, probably their life will be many obstacles.

2. Type 2

This computation in Type 2 is carried out by summing up all the *neptu dina* and *neptu pasaran* of bride and groom, then divided by 4. The foreknowledge decision in this is also made using the remaining value after the division. The conclusion of the computation based on the meaning from the Javanese *Primbon* in shown in Table 5.

Table 5. The Meaning of the Remainder in Type 2

Remainder	Meaning
1	<i>Gentho, larang anak</i> (rarely have children)
2	<i>Gembili, sugih anak</i> (have many children)
3	<i>Sri, sugih rejeki</i> (lots of fortune)
4	<i>Punggel, mati siji</i> (one died)

The first example here is a situation in which the bridegroom is born on *Kliwon* Friday, and the bride is born on *Pahing* Friday. Then, the foreknowledge of the couple is determined as follows: *Weton* of the bridegroom born on *Kliwon* Friday means Friday = 6, and *kliwon* = 8, and then *weton* bride *Pahing* Friday, means Friday = 6, and *pahing* = 9. Hence, the computation is $\frac{(6+8)+(6+9)}{4} = 7$, while the remainder is 1.

Since the remainder is 1, the conclusion of the computation as shown from the meaning in Table 5 is rarely have children. Therefore, in a situation where the bridegroom is born on *Kliwon* Friday and the bride born on *Pahing* Friday, there is no good future for them.

The second example is a situation where the bridegroom is born on *Kliwon* Friday, and the bride born on *Pahing* Thursday. Then, the foreknowledge of the couple is determined as follows: *Weton* of the bridegroom born on *Kliwon* Friday, means Friday = 6, and *kliwon* = 8, and the *weton* of the bride born on *Pahing* Thursday, means Thursday = 8, and *pahing* = 9. Hence, the computation is $\frac{(6+8)+(8+9)}{4} = 7$ and the remainder is 3.

The conclusion with the remainder of 3 as shown from the meanings in Table 5 is lots of fortune. Based on this, the bridegroom born on *Kliwon* Friday with the bride born on *Pahing* Thursday has a good future, lots of fortune.

3. Type 3

Predicting the future with Type 3 is similar to Type 2, which involves summing up of all *neptu dina* and *neptu pasaran* of bride and groom. However, the divider is 10 or 7, and the remainder of the division starting from 1 to 7. Then, the conclusion is drawn from the meanings of the Javanese *Primbon* shown in Table 6.

Table 6. The Meaning of the Remainder in Type 3

Remainder	Meaning
1	<i>Wasesasegara, kamot, jembar budine, sugih pangapura gedhe prabawane</i> (easy to forgive, accept suggestions)
2	<i>Tunggaksemi, cepak rijekine</i> (lots of luck, lots of friends)
3	<i>Satriya wibawa, oleh kamulyan lan laluhuran</i> (authoritative, noble)
4	<i>Sumursinaba, dadi pangungsening kapinteran</i> (smart, as a questioned person, a source of knowledge)
5	<i>Satriyaning, nandhang dukacita, kawirangan, isarat panulake ngetokake getih, umpamane mbleh ayam</i> (hit by disaster, affected, miserable)
6	<i>Bumikapetak, petengan aten, nanging taberi ing gawe, kuat nandhang lara lapa, resikan, isarat panulake mendhem lemah</i> (diligent work but miserable)
7	<i>Lebu katiup angin, nandang papa cintraka, kabeh karepe ora dadi, kerep ngalih omah, isarat panulake gabul-abul lemah</i> (the desires do not come true, often move home, miserable)

Like the first example in Type 2, the bridegroom is born on *Kliwon* Friday, and the bride born on *Pahing* Friday. *Kliwon* Friday means Friday = 6, and *kliwon* = 8, and *Pahing* Friday means Friday = 6, and *pahing* = 9. Hence, the computation is $\frac{(6+8)+(6+9)}{10} = 2$ and the remainder is 10 (should not). But when 7 is used, we have $\frac{(6+8)+(6+9)}{7} = 4$ and the remainder is 1.

Based on this, the conclusion is drawn from the meanings in Table 6. Since the remainder is 1, it is easy to forgive, accept suggestions. In other words, a bridegroom born on *Kliwon* Friday will have a good future with a bride born on *Pahing* Friday.

Also, just like the second example in Type 2, the bridegroom is born on *Kliwon* Friday, and the bride born on *Pahing* Thursday. *Kliwon* Friday means Friday = 6, and *kliwon* = 8, and *Pahing* Thursday means Thursday = 8, and *pahing* = 9. Hence, the computation is $\frac{(6+8)+(8+9)}{10} = 3$, while the remainder is 1. Checking for this remainder with the meaning shown in Table 6 gives easy to forgive, accept suggestions. Therefore, it is concluded that the bridegroom born on *Kliwon* Friday will have a good future with a bride born on *Pahing* Thursday.

4. Type 4

The computation of Type 4 is the same with type 2 and 3, with the summing up of all *neptu dina* and *neptu pasaran* of bride and groom. However, the divider is 5, and the remainder of the division starts from 1 to 5. Then, the conclusion is drawn from the meanings shown by Javanese *Primbon* in Table 7.

Table 7. The Meaning of the Remainder in Type 4

Remain	Meaning
1	<i>Sri</i> (noble)
2	<i>Dana</i> (charity)
3	<i>Lara</i> (sick)
4	<i>Pati</i> (die)
5	<i>Lungguh</i> (possess a position)

Like the first example in Type 2 and Type 3, the bridegroom is born on *Kliwon* Friday, and the bride is born on *Pahing* Friday. *Kliwon* Friday means Friday = 6, and *kliwon* = 8, and *Pahing* Friday means Friday = 6, and *pahing* = 9. Hence, the computation is $\frac{(6+8)+(6+9)}{5} = 5$ and the remainder is 4. Based on the remainder, which is 4, and checking it with reference to the meanings shown in Table 7, it is *pati*, meaning died. Therefore, it is concluded that the bridegroom born on *Kliwon* Friday will have bad marriage experience with the bride born on *Pahing* Friday.

Also, like the second example in Type 2 and Type 3, the bridegroom is born on *Kliwon* Friday, and the bride is born on *Pahing* Thursday. *Kliwon* Friday means Friday = 6, and *kliwon* = 8, and *Pahing* Thursday means Thursday = 8, and *pahing* = 9. Hence, the computation is $\frac{(6+8)+(8+9)}{5} = 6$ and the remainder is 1.

Considering the remainder, which is 1, the conclusion is drawn from the meanings shown in Table 7, and this is noble. Therefore, there is a good future between the bridegroom born on *Kliwon* Friday and the bride born on *Pahing* Thursday. If they get married, probably they will be very rich.

Considering this foreknowledge presented above, we could deduce that the results are inconsistent. However, the focus is not to discuss these inconsistencies; it is an indication that foreknowledge is someone's estimate, which is not necessarily true — nevertheless, it showed that numbers have a mystical aura for the Javanese and this does not only occur in Java. There was a pair of friendly numbers in the ancient Greeks, which attained a mystical aura, and superstition. Then, it later maintained that two talismans bearing these numbers would have a perfect friendship. According to Eves (1969), these numbers came to play an essential role in magic, sorcery, astrology, and the casting of horoscopes.

Relating Mate and Mathematics

The act of getting a mate and the prediction of future based on the *weton* of the couples through Javanese *Primbon* has a numerical value within the Javanese community. These computations in predicting the future have similarities in the case of repeating numbers. Ascher (1991) stated that numbers are only names given for the series formed, and the capacity to calculate is universally related to human language. Furthermore, when writing or naming numbers for each culture, it may be different from what it means in the other culture (Van Maanen, 2011; Abdullah, 2017). However, numbers usually have patterns and have implicit relationships in arithmetic.

In addition, Eves (1969) states that the calculation process must be systematized in a number base. Human fingers are more comfortable to use. Hence, it was not surprising that base 10 was chosen. Besides base 10, Eves (1969) suggests the use of 2, 3, and 4 as number bases. In line with this, certain tribes of Tierra del Fuego have several first number names based on 3, and some South American tribes also use 4. Eves (1969) also suggests that base 5 could be used extensively. There are some South American tribes that currently count by hands, such as "one, two, three, four, hands, hands and one,." Yukaghir Siberia uses a mixed scale by counting "one, two, three, three and one, five, two three, one more, two four, ten with one missing, ten."

Like calculations from other cultures that use bases, computation in predicting the future of couples based on *weton* also uses number bases. If we look at Type 1, foreknowledge used base number 9, Type 2 used base number 4, foreknowledge in Type 3 used base number 7, and Type 4 used base number 5. This shows that the computation in predicting the future of couple based on their *weton*, use different number bases for different types. In other words, Javanese *Primbon* recognizes more than one base.

Moreover, calculating through Javanese *Primbon* could be performed using other mathematical elements, as we could symbolize the calculation on Type 1 in the formal mathematical sentence. If *neptu dina* of the bride is symbolized as a , and *neptu pasaran* of the bride is symbolized as b , and divisor 9 is symbolized in q , and the remainder of this calculation is symbolized by r , then it can be written as:

$$\frac{a+b}{q} \text{ and remainder } r, \text{ where } a = \{3,4,5,6,7,8,9\} \text{ and } b = \{4,5,7,8,9\}.$$

Also, we also could symbolize the calculation in Type 2, 3, and 4 in the formal mathematical sentence. If *neptu dina* of the bride is symbolized as a , *neptu dina* of the groom is symbolized as b , *neptu pasaran* of the bride is symbolized as c , *neptu pasaran* of the groom is symbolized as d , and the divisor is symbolized in q , and the remainder of the calculation is symbolized as r , it can be written as:

$$\frac{a+b+c+d}{q} \text{ and remainder } r, \text{ where } a = c = \{3,4,5,6,7,8,9\} \text{ and } b = d = \{4,5,7,8,9\}.$$

In the general form, the result of the sum is symbolized as a then divided by b and remainder r or written as $a = bq + r$. This is referred to as the remainder theorem in formal mathematics. In other words, one of the mathematical elements in the foreknowledge is a remainder theorem.

According to Nagell (1952), in the remainder theorem, if a and b are integers and $b \neq 0$, a unique integer q exists such that

$$a = bq + r, \text{ where } 0 \leq r < |b|.$$

This theorem is also known as the division algorithm (Fine & Rosenberger, 2007). We take the following examples of the division algorithm:

- (i) 22 divided by 5 is 4, remainder 2, so we can write $22 = 5 \cdot 4 + 2$

- (ii) 26 divided by 4 is 6, remainder 2, so we can write $26 = 4 \cdot 5 + 2$
- (iii) 28 divided by 3 is 9, remainder 1, so we can write $28 = 3 \cdot 9 + 1$

Another commonly used notation is $a \text{ mod } m = r$ (read "a modulo m"), which denotes that r is the remainder obtained when a is divided by m . Suppose that a is an integer and m is an integer more than 0, hence:

$$a \text{ mod } m = r, \text{ so that } a = mq + r, \text{ where } 0 \leq r < m$$

We can take the following examples of the modulo arithmetic:

- (i) $22 = 5 \cdot 4 + 2$ is $22 \text{ mod } 5 = 2$
- (ii) $26 = 4 \cdot 5 + 2$ is $26 \text{ mod } 4 = 2$
- (iii) $28 = 3 \cdot 9 + 1$ is $28 \text{ mod } 3 = 1$

Furthermore, the mathematical element in the foreknowledge is a **congruent modulo** n . In the definitions and fundamental properties, let n be an integer $\neq 0$. The integers a and b are said to be congruent modulo n , or for the modulus n , when their difference $a - b$ is divisible by n . To express this mathematical element:

$$a \equiv b(\text{mod } n),$$

Where the symbol \equiv is to be read "is congruent to", and the number n is the modulus of the congruence (Nagell, 1952). We take the following examples of the congruent modulo:

- (i) $22 \text{ mod } 5 = 2$ and $17 \text{ mod } 5 = 2$, so $22 \equiv 17(\text{mod } 5)$
- (ii) $26 \text{ mod } 4 = 2$ and $30 \text{ mod } 4 = 2$, so $26 \equiv 30(\text{mod } 4)$
- (iii) $28 \text{ mod } 3 = 1$ and $25 \text{ mod } 3 = 1$, so $28 \equiv 25(\text{mod } 3)$

Therefore, we could conclude that foreknowledge computations contain mathematical elements including number bases, remainder theorem, modulo, and modulus of the congruence.

Proposed a Mate in Javanese Primbon as a Contextual for Teaching Mathematics

A mate in Javanese *Primbon* is a contextual problem for Javanese students. Learning uses a context that can provide comprehensive understanding and links to students to provide direct experience with hands-on experience in real life. Cultural context issues can be used in learning through ethnomathematical problems. Hence, mate or foreknowledge matching in Javanese *Primbon* can be used as an ethnomathematical problem. Ethnomathematics problems are mathematical problems in which verbal texts use narration to describe mathematical practices that exist in daily habits, traditions, and experiences of various socio-cultural groups, and solution to the problem must be examined in its social context (Katsap & Silverman, 2016; Risdiyanti & Prahmana, 2018).

Problem's text using ethnomathematics can be built by integrating problems related to a culture which contain mathematical content. Katsap and Silverman (2016) argue that the problem contains two parts; the first part contains a prelude, which is a segment of information related to the culture, tradition, or habits of society. The second part of the problem's text contains mathematical questions (the solution

is discussing mathematical objects and structures requires investigation or evidence) and non-mathematical questions (referring to social problems or facilitating mathematical practices in the community). Here is an example.

Problem: Prediction of fate marriage through base and modulo

Read the prelude before you answer the following questions.

Prelude

In the tradition of marriage in Javanese, various things need to prepare, including determination about the compatibility of the two brides. Traditional Javanese elders usually do match matching between the two brides. The determination of the compatibility of the two brides contains predictions about life later after marriage based on the *weton*. If the foreknowledge results are good, then the determination continues on the search for a good wedding day for the bride and groom. However, if the foreknowledge results are not good, the marriage of the two brides will not take place.

The task

1. Describe in your own words what you've learned about the rural setting of the prediction of fate marriage from the prelude information.
2. A girl of sufficient marriage (a mature woman), has Tuesday *Legi weton*. The girl's parents looking for a partner to her daughter. If the forecasting used is to use the modulo base 4, the computation is carried out by summing up all the *neptu dina* and *neptu pasaran* of bride and groom, then divided by 4. Then, the foreknowledge decision is use Table 8. Furthermore, find *weton* of a man, who has a good prediction if he is married to the girl.

Table 8. The Meaning of the Remainder in Modulo 4

Remainder	Meaning
1	<i>Gentho, larang anak</i> (rarely have children)
2	<i>Gembili, sugih anak</i> (have many children)
3	<i>Sri, sugih rejeki</i> (lots of fortune)
4	<i>Punggel, mati siji</i> (one died)

CONCLUSION

The use of Javanese *Primbon* in marriage is something unique in predicting the future. We have shown that matchmaking of mates using foreknowledge through Javanese *Primbon* is attributed to number bases, remainder theorems, modulo, and modulus of congruence in formal mathematics. Hence, the matchmaking of mates using foreknowledge through Javanese *Primbon* has potential material as a context in mathematics learning, such as the context in making an ethno-mathematical problem. The limitation of this study is needed to be continued for further research on its empirical use in mathematics learning by the topic.

ACKNOWLEDGMENTS

I express my profound gratitude to LPDP (BUDI DN) for funding this research. Also, I sincerely appreciate my colleagues and lecturers for their valuable suggestions and discussions.

REFERENCES

- Abdullah, A.S. (2017). Ethnomathematics in Perspective of Sundanese Culture. *Journal on Mathematics Education*, 8(1), 1-16. <http://dx.doi.org/10.22342/jme.8.1.3877.1-15>.
- Alangui, W.V. (2010). *Stone walls and water flows: Interrogating cultural practice and mathematics*. (Doctoral dissertation, ResearchSpace@ Auckland).
- Ambrosio, U.D., & Rosa, M. (2017). Ethnomathematics and Its Pedagogical Action in Mathematics Education. In *Ethnomathematics and its Diverse Approaches for Mathematics Education* (pp. 285–305). Springer, Cham. <https://doi.org/10.1007/978-3-319-59220-6>.
- Ascher, M. (1991). *Ethnomathematics: A Multicultural View of Mathematical Ideas*. United States of America: Wadsworth, Inc.
- Bowen, G.A. (2009). Document Analysis as a Qualitative Research Method. *Qualitative Research Journal*, 9(2), 27–40. <https://doi.org/10.3316/QRJ0902027>.
- D'Ambrosio, U. (1985). Ethnomathematics and its Place in the History and Pedagogy of Mathematics. *For the Learning of Mathematics*, 5(1), 44–48. Retrieved from <https://www.jstor.org/stable/40247876>.
- D'Ambrosio, U., & D'Ambrosio, B.S. (2013). The Role of Ethnomathematics in Curricular Leadership in Mathematics Education. *Journal of Mathematics Education at Teachers College*, 4, 19–25.
- Ember, M., & Ember, C.R. (2001). *Cultures Countries and Their Cultures Volume 2*. United States of America Printing: Macmilan Reference USA.
- Eves, H. (1969). *An Introduction to The History of Mathematics*. United States of America: Holt, Rinehart and Winston, Inc.
- Fine, B., & Rosenberger, G. (2007). *Number theory*. Birkhuser Boston.
- Fonseca, M.D.C.F.R. (2010). Adult Education and Ethnomathematics: appropriating results, methods, and principles. *ZDM - International Journal on Mathematics Education*, 42(3), 361–369. <https://doi.org/10.1007/s11858-010-0257-6>.
- François, K., & Stathopoulou, C. (2012). In-Between Critical Mathematics Education and Ethnomathematics. A Philosophical Reflection and an Empirical Case of a Romany Students' Group Mathematics Education. *Journal for Critical Education Policy Studies*, 10(1), 234–247.
- Hatley, B. (2008). *Javanese Performances on an Indonesian Stage: Celebrating Culture, Embracing Change*. Singapore: NUS Press.
- Katsap, A. (2018). Opening the Door to Ethnomathematics in Israel. In *K-12 Mathematics Education In Israel: Issues And Innovations* (pp. 377–384). <https://doi.org/10.1142/9789813231191>.
- Katsap, A., & Silverman, F.L. (2016). Ethnomathematics in Mathematics Curriculum via Ethnomathematical Word Problems. In *Ethnomathematics of Negev Bedouins' Existence in Forms, Symbols and Geometric Patterns* (pp. 289–304). Rotterdam: SensePublishers.
- Maryati & Prahmana, R.C.I. (2019a). Ethnomathematics: Exploring the activities of culture festival. *Journal of Physics: Conference Series*, 1188(1), 012024. <https://doi.org/10.1088/1742-6596/1188/1/012024>.
- Maryati & Prahmana, R.C.I. (2019b). Ethnomathematics: Exploration of the Muntuk Community. *International Journal of Scientific and Technology Research*, 8(6), 47-49.

- Matang, R.A.S., & Owens, K. (2014). The role of indigenous traditional counting systems in children's development of numerical cognition: Results from a study in Papua New Guinea. *Mathematics Education Research Journal*, 26(3), 531–553. <https://doi.org/10.1007/s13394-013-0115-2>.
- Mohamad, D., Adam, N.A., & Embong, R. (2010). An ethnomathematical study of triaxial weave in tudung saji weaving. In *CSSR 2010 - 2010 International Conference on Science and Social Research* (pp. 64–67). <https://doi.org/10.1109/CSSR.2010.5773861>.
- Muhtadi, D., Sukirwan, Warsito, & Prahmana, R.C.I. (2017). Sundanese ethnomathematics: mathematical activities in estimating, measuring, and making patterns. *Journal on Mathematics Education*, 8(2), 185-198. <https://doi.org/10.22342/jme.8.2.4055.185-198>.
- Nagell, T. (1952). *Introduction to Number Theory*. Stockholm: Almqvist & Wiksell.
- Pais, A. (2011). Criticisms and contradictions of ethnomathematics. *Educational Studies in Mathematics*, 76(2), 209–230. <https://doi.org/10.1007/s10649-010-9289-7>.
- Ricklefs, M.C. (2007). *Polarising Javanese society: Islamic and other visions, c. 1830-1930*. Singapore: NUS Press.
- Risdiyanti, I., & Prahmana, R.C.I. (2018). Ethnomathematics: Exploration in Javanese culture. *Journal of Physics: Conference Series*, 943(1), 012032. <https://doi.org/10.1088/1742-6596/943/1/012032>.
- Rosa, M., & Orey, D.C. (2015). A trivium curriculum for mathematics based on literacy , matheracy , and technoracy : an ethnomathematics perspective. *ZDM - International Journal on Mathematics Education*, 47(4), 587–598. <https://doi.org/10.1007/s11858-015-0688-1>.
- Rosa, M., & Orey, D.C. (2016). State of the Art in Ethnomathematics. In *Current and Future Perspectives of Ethnomathematics as a Program* (pp. 11–37). Springer, Cham. <https://doi.org/10.1007/978-3-319-30120-4>.
- Tjakraningrat, K.P.H., & Soemodidjojo, R. (2017). *Kitab Primbon Betaljemur Adammakna*. Yogyakarta: Soemididjojo Mahadewa.
- Tutak, F.A., Bondy, E., & Adams, T.L. (2011). Critical pedagogy for critical mathematics education. *International Journal of Mathematical Education in Science and Technology*, 42(1), 65–74. <https://doi.org/10.1080/0020739X.2010.510221>.
- Supiyati, S., Hanum, F., & Jailani. (2019). Ethnomathematics in Sasaknese Architecture. *Journal on Mathematics Education*, 10(1), 47-58. <https://doi.org/10.22342/jme.10.1.5383.47-58>.
- Sutarto, A. (2006). Becoming a true Javanese: A Javanese view of attempts at Javanisation. *Indonesia and the Malay world*, 34(98), 39-53.
- Van Maanen, J. (2011). *Tales of the field: On writing ethnography*. Chicago: University of Chicago Press.

Appendix: The Meaning of the Remain Type 1(Source: Document of Javanese *Primbon*)

Wetoné pangantèn lanang lan wadon, neptuné dina lan pasaran digunggung, banjur kabagé 9, lanang turah pira, wadon turah pira :

Yèn turah :

1 lan 1. becik kinasihan	3 lan 5. gelis pegat
1 " 2. becik	3 " 6. olèh nugraha
1 " 3. kuat, adoh rijekiné	3 " 7. akèh bilahiné
1 " 4. akèh bihahiné	3 " 8. gelis mati siji
1 " 5. pegat	3 " 9. sugih rejeki
1 " 6. adoh sandhang pangané	4 " 4. kerep lara
1 " 7. sugih satru	4 " 5. akèh rencanane
1 " 8. kasurang - surang	4 " 6. sugih rejeki
1 " 9. dadi pangauban	4 " 7. mlarat
2 " 2. slamet, akèh rijekiné	4 " 8. akèh pangkalane
2 " 3. gelis mati siji	4 " 9. kalah siji
2 " 4. akèh godané	5 " 5. tulus begjané
2 " 5. akèh bilaine	5 " 6. cepak rijekiné
2 " 6. gelis sugih	5 " 7. tulus sandhang pangané
2 " 7. anaké akèh mati	5 " 8. akèh sambékalané
2 " 8. cepak rijekiné	5 " 9. cepak sandhang pangané
2 " 9. akèh rijekiné	6 " 6. gedhé bilahiné
3 " 3. mlarat	
3 " 4. akèh bilaine	
6 lan 7. rukun	7 lan 8. nemu bilahi saka awake dhéwé
6 " 8. sugih satru	7 " 9. tulus palakramané
6 " 9. kasurang - surang	8 " 8. kinasihan déning wong
7 " 7. ingikum maring rabiné	8 " 9. akèh bilahiné
	9 " 9. giras rijekiné

Katrangan : Saupama wetoné pangantèn lanang Jumuah Kliwon neptuné $6 + 8 = 14$ kabagé 9, turah 5. Wetoné pangantèn wadon Jumah Paing, neptuné $6 + 9 = 15$, kabagé 9, turah 6. Dadi $5 + 6$ tiba cepak rijekiné, iku becik.



DIAGNOSING STUDENTS' LEARNING DIFFICULTIES IN THE EYES OF INDONESIAN MATHEMATICS TEACHERS

Ariyadi Wijaya¹, Heri Retnawati¹, Wahyu Setyaningrum¹, Kazuhiro Aoyama², Sugiman¹

¹Universitas Negeri Yogyakarta, Jl. Colombo No.1, Karang Malang, Caturtunggal, Yogyakarta 55281, Indonesia

²Aichi University of Education, 〒448-0001 Aichi, Kariya, Igayacho, Hirosawa- 1, Japan

E-mail: a.wijaya@uny.ac.id

Abstract

Teachers' diagnostic practice on students-difficulties is one of the important steps in designing and managing classroom lessons. The purpose of this study was to explore teachers' perception and practices regarding diagnosing students' learning difficulties. The participants of the study were 28 Indonesian mathematics teachers of Junior High School. The data was collected through a Focus Group Discussion and a teacher questionnaire. The data were analyzed qualitatively to describe how the teachers perceive learning difficulties and how the teachers diagnosed students' learning difficulties. The results of the analysis reveal that the teachers do not yet perform an in-depth diagnosis of students' difficulties in learning mathematics. The teachers only focus on the mathematics topics and non-mathematical issues, instead of on students' thinking process. The teachers also do not differentiate the diagnosis, evaluation, and prediction test. With regard to the strategies used by the teachers to diagnose students' difficulties, analyzing students' responses to tests was the majority. In this respect, observing students' learning process during classroom activities is rarely done by the teachers in the purpose of diagnosing students' learning difficulties. The results of the diagnosis are mainly used as the basis for remedial and drill and practices. The results imply that more support is needed for teachers to improve their competences particularly in diagnosing students' thinking process difficulties when learning mathematics.

Keywords: diagnosis, learning difficulties, teachers' perception, teachers' practices

Abstrak

Praktik diagnosis terkait kesulitan siswa dalam belajar merupakan salah satu langkah penting ketika mendesain dan mengelola proses pembelajaran di kelas. Tujuan penelitian ini adalah mengeksplorasi persepsi dan praktik yang dilakukan guru terkait mendiagnosa kesulitan belajar siswa. Penelitian ini melibatkan 28 guru matematika Sekolah Menengah Pertama (SMP) dari Indonesia. Data dikumpulkan melalui *Focus Group Discussion* (FGD) dan angket guru. Data dianalisis secara kualitatif untuk mendeskripsikan bagaimana persepsi guru tentang kesulitan belajar dan bagaimana cara yang dilakukan guru dalam mendiagnosa kesulitan belajar siswa. Hasil analisis menunjukkan bahwa guru belum melakukan diagnosa kesulitan siswa dalam belajar matematika secara mendalam. Guru hanya fokus pada aspek matematika itu sendiri tanpa memperhatikan proses berpikir siswa. Selain itu, guru juga kurang mampu membedakan antara tes diagnosa, evaluasi, dan prediksi. Strategi yang paling sering digunakan guru untuk mendiagnosa kesulitan belajar siswa adalah dengan menganalisis hasil tes siswa. Dalam hal ini pengamatan proses belajar siswa selama kegiatan belajar mengajar jarang dilakukan guru untuk mendiagnosa kesulitan belajar siswa. Hasil penelitian ini menunjukkan bahwa kemampuan diagnostik guru masih perlu dukungan terutama untuk mendiagnosis kesulitan proses berpikir matematika siswa.

Kata kunci: diagnosa, kesulitan belajar, persepsi guru, praktik mengajar guru

How to Cite: Wijaya, A., Retnawati, H., Setyaningrum, W., Aoyama, K., & Sugiman. (2019). Diagnosing Students' Learning Difficulties in the Eyes of Indonesian Mathematics Teachers. *Journal on Mathematics Education, 10*(3), 357-364. <https://doi.org/10.22342/jme.10.3.7798.357-364>.

A number of studies have shown that mathematics is experienced as a difficult subject by many students in various levels of education. In elementary school level, Wijaya (2017), for example, revealed students' difficulties with fractions. Students in his study understood fractions as parts of a whole, but more than 90% of these students were unable to solve problems involving fractions as parts of a collection of objects. Mathematics is also difficult for secondary school students. Retnawati,

Kartowagiran, Arlinwibowo, and Sulistyarningsih (2017) revealed that only 5% of junior high school students in their study who could deal with fractional exponents such as $4^{2/3}$. The concept of subset and parallel lines are also difficult for these students as indicated by the low percentages of students' correct answer, i.e. only about 30%. In upper secondary school, Coşkun (2008) revealed that students experience difficulties with: (1) division algorithm that forms the basis of modular arithmetic, (2) symbolic representation of the division algorithm with modular arithmetic notation, and (3) equivalence class with the concept of mod. Difficulties with mathematics are also experienced by university students. A study of Klymchuk, Zverkova, Gruenwald, and Sauerbier (2010) revealed that many university students could not construct a simple function that representing a familiar context. A general perspective on students' difficulties in mathematics is given by Russell, O'Dwyer, and Miranda (2009) who found that students' difficulties in mastering concept occur are caused by students' inability to link between the knowledge that they are studying and the prior knowledge they have.

Awareness of the difficulties experienced by students in their learning process is an important first step for teachers to design and manage mathematics lessons (Çiltas & Tatar, 2011; Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014; Wijaya, 2016; Saleh, Prahmana, Isa, & Murni, 2018). In this respect, analyzing students' learning difficulties is often seen as a crucial step to access students' reasoning (Brodie, 2014; Muttaqin, Putri, & Somakim, 2017; Mutohir, Lowrie, & Patahuddin, 2018). Analyzing students' difficulties can be a preliminary step in the process of improving student performance because it sheds light on key aspects of students' learning process that need to be developed. After diagnosing students' difficulties in learning mathematics, Tall and Razali (1993) recommend that less able students cannot be simply helped by providing them with specific strategies to overcome their specific errors. These students also need overall powerful mathematics strategies. Furthermore, on the basis of their analysis Tall and Razali also highlight that developing the confidence of less able students is also an important step to help them gain a better result in mathematics. Another example of instructional recommendation on the basis of analyzing students' difficulties can be found in the study of Wijaya, et al. (2014). The results of an error analysis conducted by Wijaya, et al. (2014) imply that improving the task comprehension of students requires a focus not only on students' language competence, but also on the ability to select relevant information. Furthermore, the ability to identify the required procedure or concept was found to be another key competence that needs to be improved.

To identify and overcome students' difficulties during their learning process are not only a requirement of modern education, but also parts of teachers' responsibilities (Çiltas & Tatar, 2011; Nor, Ismail, & Yusof, 2016). This is in agreement with one of the principles for school mathematics proposed by the National Council of Teachers of Mathematics (2000) that an effective mathematics teaching requires teachers to understand what students know and need to learn. Such understanding forms a basis for teachers to support their students to learn mathematics well. Therefore, it is crucial for teachers to be aware of their students' difficulties in learning mathematics in order to develop and perform learning activities effectively (Yetkin, 2003; Aris, Putri, & Susanti, 2017; Nuari, Prahmana, & Fatmawati, 2019).

This expectation implies that teachers need to have a competence to identify students' learning difficulties. With respect to the teaching of foreign language, Edelenbos and Kubanek-German (2004) defined a so called 'teacher's diagnostic competence' as "the ability to interpret students' foreign language growth, to skillfully deal with assessment material and to provide students with appropriate help in response to this diagnosis" (p. 260). Transforming this idea into the teaching of mathematics, teacher's diagnostic competence could be defined as teachers' ability to interpret students' thinking and reasoning process, to monitor students' progress and difficulties, and to provide appropriate responses to the results of the diagnosis. With respect to diagnostic competence, students have different preconditions therefore teachers need to recognize each student (Tolsdorf & Markic, 2017) and must be able to describe and interpret the individual student's abilities and difficulties.

The present study was aimed to investigate diagnostic practices reported by teachers particularly mathematics teachers. This investigation includes not only how teachers perform the diagnosis, but also teacher's perspective on students' learning difficulties.

METHOD

The present study was a qualitative research employing Focus Group Discussion (FGD) and survey through questionnaire. FGD was chosen because of the following reasons it is a powerful exploratory tool that could provide information about how people think, feel, and act regarding a particular topic (Freitas, Oliveira, Jenkins, & Popjoy, 1998). A total of 28 Indonesian mathematics teachers of Junior High School participated in the FGD. The participants were selected using convenience sampling. The FGD was aimed to uncover diagnostic practices that have been done by teachers in relation to students' learning difficulties. It was conducted in about 4 hours and recorded yet remain anonymously.

In addition to the FGDs, the teacher's participants also filled in an open questionnaire. The FGD and the questionnaire covered the following issues:

1. teachers' perception on students' learning difficulties,
 - Do you notice if your students have difficulties in learning mathematics?
 - How do you know about diagnosing students' learning difficulties? Have you read particular books about diagnosing students' learning difficulties?
2. teachers' practices with regard to diagnosing students' learning difficulties,
 - How do you diagnose your students' learning difficulties?
 - When do you usually diagnose students' learning difficulties?
 - Do you encounter any difficulty in developing proper instrument to diagnose students' learning difficulties?
 - When using multiple-choices items, do you use proper distractor as a tool to diagnose students' learning difficulties?

3. teachers' actions as the responses to the results of diagnosis,
 - In general, what kind of learning difficulties do your students have?
 - What kind of difficulties do your students have when particularly learning mathematics?
 - How do you classify students' learning difficulties?
 - How do you respond or follow up the results of diagnosis? What kind of actions do you usually take?
 - How do you report students' learning difficulties? To whom do you give the report?
 - In your opinion, what parties who should be responsible to take an action in response to students' learning difficulties?
4. the parties who are involved or informed regarding students' learning difficulties
 - Are the other parties that also take an action in response to students' learning difficulties? If so, who are they?
5. needs or tools to diagnose students' learning difficulties
 - Do you use particular software(s) as a tool to diagnose students' learning difficulties? If so, what are the advantages and disadvantages of using the software(s)?
 - In your opinion, do you need a software that is particularly designed to diagnose students' difficulties in learning mathematics? If so, what kinds of specification?

In the first step of analysis, the questionnaire data was reported quantitatively to show the occurrence of particular practices (in percentage) with regard to diagnosing students' learning difficulties. In the next process, this data were described qualitatively and supported by the results from the FGDs.

RESULT AND DISCUSSION

The results of this study are organized into four categories, i.e. (1) teachers' perception on students' learning difficulties, (2) teachers' practices with regard to diagnosing students' learning difficulties, (3) teachers' actions as the responses to the results of diagnosis, and (4) the parties who are involved regarding students' learning difficulties.

Teachers' Perception on Students' Learning Difficulties

The teachers were asked to mention the difficulties experienced by their students in learning mathematics. The teachers reported various difficulties, around 61% of the difficulties indicated by the teachers were related to mathematics. These difficulties include a lack of calculation skills, low understanding of algebra, and also inability to make a mathematical model representing a contextual problem. Some other Indonesian teachers only listed mathematics domains or topics, such as algebra, geometry, and function without detailed indication about the students'. These Indonesian teachers seemed to focus on the mathematics itself, not on their students' thinking process. The remaining 39%

of the difficulties referred to non-mathematical difficulties. These difficulties include a lack of motivation, careless, and ignorance. Interestingly, such non-mathematical difficulties are still mentioned when the Indonesian teachers were asked specifically about their students' difficulties in learning mathematics, i.e. about 23% of teachers' responses.

Teachers' Practices with Regard to Diagnosing Students' Learning Difficulties

With regard to teachers' practice in diagnosing students' learning difficulties, there are two main concerns, i.e. the period when teachers diagnose students' difficulties and the instruments which are used by teachers. The questionnaire shows that 53% of the Indonesian teachers diagnosed their students' learning difficulties during the mathematics lessons. About 42% of the Indonesian teachers reported that they performed the diagnosis after the completion of teaching a particular topic. The remaining 5% of the Indonesian teachers said that they diagnosed students' difficulties at the beginning of the semester. Based on these results, it seems that the Indonesian teachers did not distinguish diagnosis (or formative assessment), evaluate (or summative assessment), and prediction test. The fact that the teachers 'diagnose' students' difficulties after the completion of a particular topic indicates that the teachers perform an evaluation, instead of a diagnosis. During the FGD many teachers reported that they perform the diagnosis after a series of lesson addressing a particular topic in order to see whether their students accomplished the learning objectives. Few teachers reported that their schools administer a kind of bridging test or a so called of matriculation test at the beginning of semester in order to get information about students' prior knowledge. Such practice also does not fit the purpose of diagnosis. Despite these inappropriate practices of diagnosis, more than a half of the teachers showed a good understanding of diagnosis process. These teachers reported that they investigate students' difficulties during the teaching and learning process.

The second concern regarding teachers' practice in diagnosing students' difficulties is the instrument or ways to diagnose. The questionnaire data show that most Indonesian teachers used more than one ways or instruments. The most frequently used ways to diagnose students' learning difficulties was test, i.e. 56%. The Indonesian teachers reported that they diagnosed students' learning difficulties by administering a test and analyzing its results. The Indonesian teachers also reported that they also diagnose students' difficulties through observation during the learning process, i.e. 28%. The teachers explained that they could identify students who experienced difficulties by observing students' gesture and the kinds of questions posed by students. The last way that used by the Indonesian teachers to identify students' difficulties was interview or conversation, i.e. 17%. The teachers explained that they could diagnose students' difficulties through conversation. However, the FGD data indicates that some teachers did not entirely diagnose students' difficulties during the conversation because what they asked mainly about whether or not their students like mathematics and what make students like or dislike mathematics.

Teachers' Actions as the Responses to the Results of Diagnosis

The third issue that was explored in the present study is investigating what the teachers do after diagnosing students' learning difficulties. The finding about this issue could provide information whether the teachers diagnose students' difficulties as a part of their attempt to conduct an effective teaching or not. Based on the data from questionnaire and FGD, there are four types of action that were performed by the Indonesian teachers after diagnosing students' learning difficulties. The most frequently performed action was remedial, i.e. 48%. The teachers reported that they conducted a remedial when many students in their class had difficulties in learning mathematics. This remedial was mainly in the form of re-teaching the difficult topic. For the remedial, the teachers still used their regular teaching strategies. The teachers also often gave drill and practices, i.e. 31%, to overcome students' learning difficulties. They gave students similar tasks to solve. Only 17% of teachers' actions were in the category of developing or planning new teaching strategies. Some teachers explained that students' learning difficulties might be influenced by the teachers' teaching strategies. Therefore, these teachers tried to find new teaching strategies to overcome students' learning difficulties. In this respect, these teachers explicitly mentioned joyful learning. It indicates that the teachers consider students' uncomfortableness during the learning process is a cause for students' learning difficulties. The least frequently performed action was developing teaching media, i.e. 5%. Some teachers argued that a lack of media might contribute to students' learning difficulties; therefore, designing appropriate media is an important action to take after diagnosing students' difficulties.

The Parties Who are Responsible to Overcome Students' Learning Difficulties

The teachers were asked about the parties who should take action regarding the results of the diagnosis. In general, the Indonesian teachers mentioned four parties, i.e. the school principal, teachers, school counselor, and parents. According to the teacher, school principal need to take action at school level to overcome and prevent students' learning difficulties. It might be in the form of school program such as extra study hours at school. The teachers themselves are the main actor who should take action on the basis of their students' learning difficulties. In an addition to principal and teacher, school counselor is also an important party who should participate in overcoming students' learning difficulties. The involvement of school counselor seems to correlate to teachers' perception that students' difficulties in learning mathematics include non-mathematical aspects, such as students' motivation and careless. Lastly, the Indonesian teachers emphasize that overcoming students' learning difficulties is not only the responsibility of teachers and schools, but also the parents' responsibility.

The present study was aimed to explore teachers' perception on students' learning difficulties and teachers' practice in diagnosing students' learning difficulties. With regard to the teachers' perception, the results of the study show that the teachers do not perform in-depth diagnosis of students' difficulties in learning mathematics. The teachers do not thoroughly identify students' thinking process in relation to the mathematics concepts they are learning. Paying more attention to such specific aspect

could help teachers perform an appropriate follow-up action other than re-teaching. As recommended by Ciltas and Tatar (2011), teachers could identify appropriate teaching methods when they have enough information about students' thinking process. Similarly, NCTM (2000) also emphasize a need for understanding what students know and need to learn in order to provide support for students. Another important finding of the present study is the fact that the teachers also do not differentiate diagnosis, evaluation, and prediction test. Regarding the strategies used by the teachers to diagnose students, analyzing students' responses to tests was the majority followed by observation and interview.

CONCLUSION

The teachers perform various actions on the basis of their finding in diagnosing students' difficulties in learning mathematics. The major action taken by the teachers is remedial in the form of re-teaching. Many teachers also perform drill and practices to overcome students' learning difficulties. Developing teaching strategies and media that fit students' learning difficulties is a minor action taken by the teachers.

ACKNOWLEDGMENTS

We would like to express our gratitude to the Graduate Program of Yogyakarta State University for providing a research grant. We would also like to thank our Japanese colleague and the Indonesian teachers who participated in the study for their valuable information and experiences.

REFERENCES

- Aris, R.M., Putri, R.I.I., & Susanti, E. (2017). Design study: Integer subtraction operation teaching learning using multimedia in primary school. *Journal on Mathematics Education*, 8(1), 95-102. <https://doi.org/10.22342/jme.8.1.3233.95-102>.
- Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, 85(2), 221–239. <https://doi.org/10.1007/s10649-013-9507-1>.
- Çiltas, A., & Tatar, E. (2011). Diagnosing learning difficulties related to the equation and inequality that contain terms with absolute value. *International Online Journal of Educational Sciences*, 3(2), 461–473.
- Coşkun, O. (2008). Diagnosing learning difficulties in modular arithmetic. *Unpublished Thesis*. Yakutiye: Atatürk University.
- Edelenbos, P., & Kubanek-German, A. (2004). Teacher assessment: the concept of “diagnostic competence.” *Language Testing*, 21(3), 259–283. <https://doi.org/10.1191/0265532204lt284oa>.
- Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2010). University students' difficulties in solving application problems in calculus: Student perspectives. *Mathematics Education Research Journal*, 22(1), 81–91. <https://doi.org/10.1007/BF03217567>.
- Mutohir, T.C., Lowrie, T., & Patahuddin, S.M. (2018). The development of a student survey on attitudes towards mathematics teaching-learning processes. *Journal on Mathematics Education*, 9(1), 1-14. <https://doi.org/10.22342/jme.9.1.4193.1-14>.

- Muttaqin, H., Putri, R.I.I., & Somakim. (2017). Design research on ratio and proportion learning by using ratio table and graph with OKU Timur context at the 7th grade. *Journal on Mathematics Education, 8*(2), 211-222. <https://doi.org/10.22342/jme.8.2.3969.211-222>.
- National Council of Teachers of Mathematics. (2000). *Principles and Standard for School Mathematics*. Reston: The National Council of Teachers of Mathematics, Inc.
- Nor, N.A.K.M., Ismail, Z., & Yusof, Y.M. (2016). The relationship between emotional intelligence and mathematical competency among secondary school students. *Journal on Mathematics Education, 7*(2), 91-100. <https://doi.org/10.22342/jme.7.2.3534.91-100>.
- Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. *Journal on Mathematics Education, 10*(1), 127-142. <https://doi.org/10.22342/jme.10.1.6913.127-142>.
- Retnawati, H., Kartowagiran, B., Arlinwibowo, J., & Sulistyaningsih, E. (2017). Why are the mathematics national examination items difficult and what is teachers' strategy to overcome it? *International Journal of Instruction, 10*(103), 257–276. <https://doi.org/10.12973/iji.2017.10317a>.
- Russell, M., O'Dwyer, L. M., & Miranda, H. (2009). Diagnosing students' misconceptions in algebra: Results from an experimental pilot study. *Behavior Research Methods, 41*(2), 414–424. <https://doi.org/10.3758/BRM.41.2.414>.
- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education, 9*(1), 41-54. <https://doi.org/10.22342/jme.9.1.5049.41-54>.
- Tall, D., & Razali, M. R. (1993). Diagnosing students' difficulties in learning mathematics. *International Journal of Mathematical Education in Science and Technology, 24*(2), 209-222. <https://doi.org/10.1080/0020739930240206>.
- Tolsdorf, Y., & Markic, S. (2017). Exploring Chemistry student teachers' diagnostic competence – A qualitative cross-level study. *Education Sciences, 7*(4), 86. <https://doi.org/10.3390/educsci7040086>.
- Wijaya, A. (2016). Students' information literacy: A perspective from mathematical literacy. *Journal on Mathematics Education, 7*(2), 73-82. <https://doi.org/10.22342/jme.7.2.3532.73-82>.
- Wijaya, A. (2017). The relationships between Indonesian fourth graders' difficulties in fractions and the opportunity to learn fractions: A snapshot of TIMSS results. *International Journal of Instruction, 10*(4). <https://doi.org/10.12973/iji.2017.10413a>.
- Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *Mathematics Enthusiast, 11*(3), 555-584.
- Yetkin, E. (2003). *Student difficulties in learning elementary mathematics*. *Eric Clearinghouse for Science, Mathematics and Environmental Education*. Available at (Retrieved 05 September 2009). <http://www.ericdigests.org/2004-3/learning.html>.



EXPLORING MATHEMATICAL REPRESENTATIONS IN SOLVING ILL-STRUCTURED PROBLEMS: THE CASE OF QUADRATIC FUNCTION

Ika Santia^{1,2}, Purwanto², Akbar Sutawidjadja², Sudirman², Subanji²

¹Universitas Nusantara PGRI Kediri, Jl. KH. Ahmad Dahlan 76, Kediri, Jawa Timur 64112, Indonesia

²Universitas Negeri Malang, Jl. Semarang 5, Malang, Jawa Timur 65145, Indonesia

E-mail: ika.santia.1703119@students.um.ac.id

Abstract

Mathematical representation has an essential role in solving mathematical problems. However, there are still many mathematics education students who have difficulty in representing ill-structured problems. Even though the ill-structured-problem-solving tasks designed to help mathematics education students understand the relevance and meaningfulness of what they learn, they also are connected with their prior knowledge. The focus of this research is exploring the used of mathematical representations in solving ill-structured problems involving quadratic functions. The topic of quadratic functions is considered necessary in mathematics teaching and learning in higher education. It's because many mathematics education students have difficulty in understanding these matters, and they also didn't appreciate their advantage and application in daily life. The researchers' explored mathematical representation as used by two subjects from fifty-four mathematics education students at the University of Nusantara PGRI Kediri by using a qualitative approach. We were selected due to their completed all steps for solving the ill-structured problem, and there have different ways of solving these problems. Mathematical representation explored through an analytical framework of solving ill-structured issues such as representing problems, developing alternative solutions, creating solution justifications, monitoring, and evaluating. The data analysis used technique triangulation. The results show that verbal and symbolic representations used both subjects to calculate, detect, correct errors, and justify their answers. However, the visual representation used only by the first subject to detect and correct errors.

Keywords: mathematical representation, problem-solving, ill-structured problem, quadratic function

Abstrak

Representasi matematika memiliki peran penting dalam memecahkan masalah matematika. Namun, masih banyak siswa pendidikan matematika yang mengalami kesulitan dalam merepresentasikan masalah yang tidak terstruktur. Meskipun tugas penyelesaian masalah yang tidak terstruktur dirancang untuk membantu siswa pendidikan matematika memahami relevansi dan kebermaknaan dari apa yang mereka pelajari, mereka juga terhubung dengan pengetahuan mereka sebelumnya. Fokus dari penelitian ini adalah mengeksplorasi penggunaan representasi matematis dalam memecahkan masalah yang tidak terstruktur yang melibatkan fungsi kuadrat. Topik fungsi kuadrat dianggap perlu dalam pengajaran dan pembelajaran matematika di pendidikan tinggi. Itu karena banyak siswa pendidikan matematika mengalami kesulitan dalam memahami hal-hal ini, dan mereka juga tidak menghargai keunggulan dan aplikasi mereka dalam kehidupan sehari-hari. Para peneliti mengeksplorasi representasi matematika yang digunakan oleh dua subjek dari lima puluh empat mahasiswa pendidikan matematika di Universitas PGRI Kediri dengan menggunakan pendekatan kualitatif. Kami dipilih karena telah menyelesaikan semua langkah untuk menyelesaikan masalah yang tidak terstruktur, dan ada berbagai cara untuk menyelesaikan masalah ini. Representasi matematis dieksplorasi melalui kerangka kerja analitis untuk memecahkan masalah yang tidak terstruktur seperti merepresentasikan masalah, mengembangkan solusi alternatif, membuat justifikasi solusi, memantau, dan mengevaluasi. Analisis data menggunakan teknik triangulasi. Hasilnya menunjukkan bahwa representasi verbal dan simbolik menggunakan kedua subjek untuk menghitung, mendeteksi, memperbaiki kesalahan, dan membenarkan jawaban mereka. Di sisi lain, representasi visual hanya digunakan oleh subjek pertama untuk mendeteksi dan memperbaiki kesalahan.

Kata kunci: Representasi matematis, pemecahan masalah, masalah tidak terstruktur, fungsi kuadrat

How to Cite: Santia, I., Purwanto, Sutawidjadja, A., Sudirman, & Subanji. (2019). Exploring Mathematical Representations in Solving Ill-Structured Problems: The Case of Quadratic Function. *Journal on Mathematics Education*, 10(3), 365-378. <https://doi.org/10.22342/jme.10.3.7600.365-378>.

Mathematical representation means expressing a mathematical concept in various ways, such as languages, symbols, pictures, diagrams, models, graphs, or physical objects (Caglayan & Olive, 2010; Goldin & Beilock, 2010; Villegas, Castro, & Gutiérrez, 2009; Putri & Zulkardi, 2018; Saleh, et al. 2018). National Council of Teachers of Mathematics explains that the use of mathematical representation forms, such as charts, graphs, tables, and symbols, as well as the transition between the fore-mentioned forms, is an important aspect in mathematics learning (NCTM, 2000). Meanwhile, Nizarudin (2014) stated that mathematical representation is an internal generalization of mathematical ideas that are constructed as internal mental networks. Based on all the definitions mentioned, mathematical representations can be seen as external products and internal processes.

Mathematical representation as an external product means the expression of mathematical ideas; as an internal process, it is a technique of expressing mathematical ideas (Minarni, et al. 2016; Nizar, et al. 2018). Mathematical representations can be observed as they occur in the mind when performing mathematical activities. Mathematical representation must be emphasized in the mathematics learning process because students' ability to represent problems can help them solve mathematics problems.

Studies about the importance of mathematical representation in problem-solving were carried out by Bal (2014), Caglayan & Olive (2010), and Villegas, Castro, & Gutiérrez (2009). These studies found that students' representation ability is the main aspect for success in solving mathematical problems and understanding mathematical concepts. Therefore, one of the problems that support multiple representations is ill-structured problems.

Jonassen (2011) defined ill-structured problems like types of problems with conflicting goals, many methods of solutions, unexpected problems, scattered knowledge, and various representations. Furthermore, solving ill-structured problems will support students' mathematical representation ability (Byun, Kwon, & Lee, 2014; Ge & Land, 2003).

Several studies on ill-structured problems state that authenticity, complexity, and openness are properties of ill-structured problems (Hong & Kim, 2011; Jonassen 1997; Palm, 2008; Shin, Jonassen, & McGee, 2003). Authenticity means that the problem is by following everyday life, with problems that describe real-life outside of school (Palm, 2008; Muhtadi, et al. 2017; Saleh, et al. 2018). A problem can be said to be authentic if it covers the context of everyday life and is relevant enough to deduce an integral part of the actual situation. In terms of complexity, Jonassen (1997) considers it to mean the uncertainty of concepts, rules, and principles needed to solve problems, or how problems are managed, and also that the relationship between concepts and rules and principles is not established. In terms of openness, Jonassen (1997) states that first, several evaluation criteria must be in place to solve the problem; second, the clarity of the problem is not presented; third, students must express their opinions and personal beliefs about their interpretations of the problem.

In this study, the researchers focused on ill-structured problems involving quadratic functions because these are important components of mathematics education and many students find it difficult

to solve because of the lack of accurate representation used to solve ill-structured problems. They didn't successes when to build problem representations in problem-solving situations.

Quadratic functions area subject that students have difficulty in understanding (Akgün, 2011; López, Robles, & Martínez, 2016; Samo, 2009), and they also do not understand their advantage and application in daily life (Akgün, 2011). Quadratic-function problems have been used in many studies on representation and problem-solving (Bannister, 2014; Samo, 2009; Schoenfeld, 1992). Likewise, they are used in secondary education and early years of university in the fields of science and mathematics education. Therefore this material needs to be discussed further related to the reasons why many students fail to solve ill-structured problems with this topic. Related to this topic, Bosse, et al. (2014), in their study found that there are four activities done by the students in doing translation from the function graph to the symbolic. The activities are unpacking the source, preliminary coordination, constructing the targets, and determining equivalence. Bosse, et al. (2014) suggests that further studies, about mathematical representation translation besides the graph to the symbolic, are also needed in completing the research to examine in more detail in the translation process.

Therefore, the purpose of this study is to explore the types of representations and translation processes between representations, which are used by mathematics education students in solving ill-structured problems involving quadratic functions. The specific objectives in this study are to describe the types of representations used by mathematics education students in solving ill-structured quadratic-function problems and to explain mathematics education students' translation between representations when solving ill-structured quadratic-function problems.

A lot of research has been done regarding students' use of mathematical representations and their usefulness as a tool in problem-solving (Goldin, 1993; Moreno, Hegedus, & Kaput, 2008; Villegas, et al. 2009). Also besides, some mathematical representations can be used to develop an understanding of the processing of mathematical concepts at a deeper level (Anwar, Yuwono, Asari, Sisworo, & Dwi, 2016; Hiebert & Carpenter, 2000; Pape & Tchoshanov, 2001). While it is important for students to have various representations of mathematical concepts and to allow for flexible use of mathematical concepts in problem-solving, to successfully manage the information used in problem-solving, mathematical representations must be correct and related to each other (Villegas, et al. 2009).

Given the importance of problem-solving in mathematics education (NCTM, 2000), the study of mathematical representations created by students when they solve problems is an interesting topic in both education and research in the field of mathematics. NCTM (2000) recognizes that a suitable use of multiple representations contributes to building a set of flexible tools for problem-solving. One of the goals of learning in middle-level and high-level mathematics education is that students acquire a good understanding of concepts with different representations and that they use precise and accurate representations during problem-solving because this is important for successful problem-solving (Schloeglmann, 2004).

Zhang (1997) said that the success of problem-solvers is based on their ability to build problem representations in problem-solving situations. The process of selecting problem representations allows students to practice balancing the advantages and disadvantages of various forms of representations (Chapman, 2010), and use them as tools in problem-solving. Lesh, Post, & Behr (1987) describe the role of mathematical representation, and the translation between representations, in mathematics learning, especially in problem-solving. They use the term 'representation' in a limited sense, as an external expression of students' mathematical conceptualization. For this study, different forms of representation will be shown and the translation between representations and transformations will be explored in problem-solving performance.

Rahmawati, Purwanto, Subanji, Hidayanto, & Anwar (2017) state the importance of translating activities from one representation system to another, starting with problem representations. Therefore, researchers need to explore mathematical representations and translations between the mathematical representations in solving quadratic-function problems. Kieran (2004) considers the concept of quadratic function as a form of understanding algebra with multiple representations. Therefore, in this study, the researchers explore three types of representations. Firstly, verbal representations consist of representations expressed both in writing and verbally. Secondly, visual representations consist of images, diagrams, or graphs. Finally, symbolic representations consist of numbers, operations, relationship signs, and algebraic symbols.

Some experts explain ill-structured problems as having definitions, goals, and boundaries that are not clearly stated (Voss, 1988); having several solutions, solution paths, or no solutions at all (Kitchner, 1983); having a relationship between concepts, rules, and principles that is not appropriate (Jonassen, 1997). These problems contain unique human interpersonal activities. Ill-structured problems are a type of problem faced in everyday life, so they usually create a dilemma (Jonassen, 1997). Because an ill-structured problem is not limited by the content learned in class, the solution is unpredictable.

Several studies on structured problems define the characteristics of ill-structured problems such as authenticity, complexity, and openness (Byun, Kwon & Lee, 2014; Ge & Land, 2003; Hong & Kim, 2016; Jonassen 1997; Palm, 2009). A problem can be said to be authentic if it covers the context of everyday life and is quite relevant to deducing an integral part of the real-life situation. Jonassen (1997) said that complexity includes the uncertainty of concepts, rules, and principles needed to solve problems, or how the problem is regulated, and the fact that the relationship between concepts and rules or principles is not specified. In terms of openness, Jonassen (1997) states that: first, several evaluation criteria must be in place to solve the problem; second, the clarity of the purpose of the problem is not presented; and third, students must express their opinions and personal beliefs about the problem.

Several researchers propose an ill-structured-problem solving process that is different from structured-problem solving (Ge & Land, 2003; Jonassen, 1997; Sinnott, 1989). Sinnott (1989)

proposes the following problem-solving model: the problem space design process, making choices and creating solutions, monitoring and storing, and non-cognitive factors. Ge and Land (2003) outline the four steps of the solving process, namely: representation of problems, developing solutions, developing justification, and monitoring and evaluation. Jonassen (1997) proposes the same process: representation of problems, generating solutions, justification, and monitoring, and evaluation. In several studies such as Palm (2008) also Ge and Land (2003), students were given a problem to solve. In doing so, they took into account the real-life situation when developing practical solving strategies and organizing the information. Thus, their minds were led to a new understanding of the problem, by evaluating and examining various alternatives to find the most appropriate solution.

Therefore, in this study, ill-structured problems are problems that are complex and open and involve quadratic functions. The matters of quadratic functions application in solving mathematical and real-life problems are widely used in mathematics education. Visual representations of real-world problems with quadratic-function concepts provide more understanding of the application of mathematics in life (Janakievska, Stopanska, & Bogatinosca, 2012).

METHOD

This exploratory study with a qualitative approach because an analysis based on the data obtained then developed a certain relationship pattern or become a hypothesis then the researcher does triangulation for analyzed data. Data was undertaken by fifty-four mathematics education students at the University of Nusantara PGRI Kediri attending the algebra course. Data was obtained through the completion of an ill-structured problem task about the quadratic function that it's graph through one point by thinking aloud method. Some notes were taken by researchers during the task-solving session. The researchers reviewed the participants' answer sheets and field notes to assess the mathematical representations obtained. Based on the results of the worksheet analysis, two subjects were selected due to their completed all steps for solving the ill-structured problem, there have different ways of solving these problems, and their good response when interviewed.

The ill-structured problem in this study is called Ill-Structured Problem Mathematics Test (TPMI) and it was given to the students as follows: "*Determine the quadratic function that it's graph through point $K(2, 3)$.*" These problems include ill-structured problems because it had complexity from the limited information and have an openness method also solution. These problems can also explore multiple representations because the topic of function allows being presented in various forms of representations (López, Robles, & Martínez, 2016).

The research procedure conducted to obtain the data of mathematical representations used in solving an ill-structured problem. First, the participants were asked to solve the problem as outlined on the worksheet by different ways of solving the problem. At this stage, only two participants have different ways of solving the problem. Second, the researchers interviewed the subjects to verify that the test data is written based on the analysis framework, which is shown in Table 1.

Table 1. Framework for Mathematical Representation Analysis

Type of Mathematical Representation	Description	Indicators for Mathematical Representation Analysis	Codes
Type 1: Verbal representation in solving ill-structured problems	Stating problems, problem-solving strategies, and monitoring results and discussions verbally.	Expressed problems similar to the text problem.	Vb11
		Expressed problems, change a few words with their own style of speech.	Vb12
		State the problem by writing the same text as the problem.	Vb13
		State the problem by writing a paraphrase text from the question.	Vb14
		Describe the strategies developed to get solutions.	Vb21
		Describe the strategies developed to get solutions and there are no solutions to paper problems	Vb22
		Describe the arguments and facts that support the solution chosen orally.	Vb31
		Describe the arguments and facts that support the chosen solution by writing text.	Vb32
		Describe the results of evaluation and effectiveness of solutions that are appropriate orally.	Vb41
		Type 2: Visual representation in solving ill-structured problems	Stating problems, problem solving strategies, and monitoring results and discussion using embodiments of images, graphs, diagrams, number lines, and other mathematical drawings.
Describe the strategies developed to get solutions and alternative solution to the problem by pictures, graphics, diagrams, number lines and other mathematical drawings on the worksheet.	Vs21		
Describe the arguments and facts that support the chosen solution by pictures, graphs, diagrams, number lines, and other mathematical drawings on the worksheet.	Vb31		
Describe the results of the evaluation and effectiveness of solution expressed by pictures, graphs, diagrams, number lines, and other mathematical drawings on the worksheet.	Vb41		
Type 3: Symbolic representation in solving ill-structured problems	Stating problems, problem solving strategies, and monitoring results and discussion using numbers, variables and other mathematical expressions.	Express problems by using numbers, variables and other mathematical expressions on the worksheet	Sb11
		Describe the strategies developed to get solutions and alternative solutions to problems by using numbers, variables and other mathematical expressions on the worksheet.	Sb21
		Describe the arguments and facts that support the chosen solution using numbers, variables and other mathematical expressions on the worksheet.	Sb31
		Describe the results of the evaluation and the	Sb41

Type of Mathematical Representation	Description	Indicators for Mathematical Representation Analysis	Codes
		effectiveness of solution expressed by using numbers, variables and other mathematical expressions on the worksheet.	

In this stage, interviews were conducted to match the data with the written data from the students' work. Data validity checks were carried out using the triangulation technique, which is checking the consistency between the written result and the interview result. Data were considered valid if it showed consistency between the written and the interview data. If the data was invalid, then data collection was repeated until the data was valid.

RESULT AND DISCUSSION

Each subject's paper results and their transcript of think aloud were analyzed, described based on the framework in Table 1. The first subject, CML, used a more procedural method in solving the problem. CML realized that there was a lack of information in the problem, so she assumed that there were many correct answers. CML's reasoning showed in Figure 1.

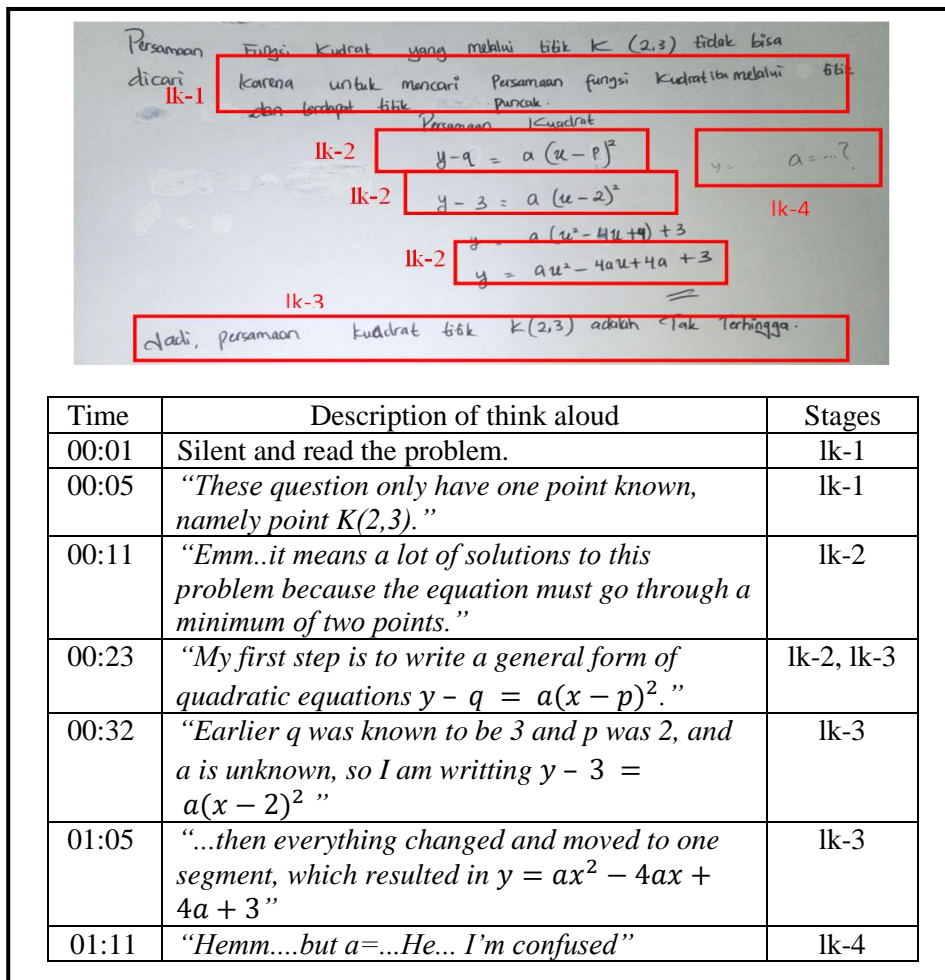


Figure 1. The Work Result of Subject-1 (CML)

Figure 1 stated that at the problem-representation stage (lk-1), CML's verbal representation expressed that the problem known was point K (2,3), and the quadratic function found passed through that point K. CML stated that the equation of the quadratic function that passes through point K cannot be found because the quadratic equation must go through one point and is known to be the peak point. According to CML, there must be a minimum of two known points to form a quadratic function. At the developing-solution stage (lk-2), CML used an algebraic equation $y - q = a(x - p)^2$ to represent symbolically the quadratic function in the problem. Then she reverted to verbal representation by stating that, "Earlier q was known to be 3 and p was 2, and a was unknown, then everything changed and moved to one segment, which resulted in $y = ax^2 - 4ax + 4a + 3$." At the argumentation stage (lk-3), CML chose a correct solution by representing symbolically and verbally that the number of quadratic function at point K (2,3) is infinite. However, CML could not provide an argument for her decisions, the monitoring and evaluation stage (lk-4), she reflected and used a symbolic representation by writing $a=?$

Similar to CML, the second subject, AZZ, also realized that there was a lack of information in the problem, thus supposing that there are many correct answers. AZZ chose a trial and error method by substituting points into predetermined equations. AZZ's work showed in Figure 2.

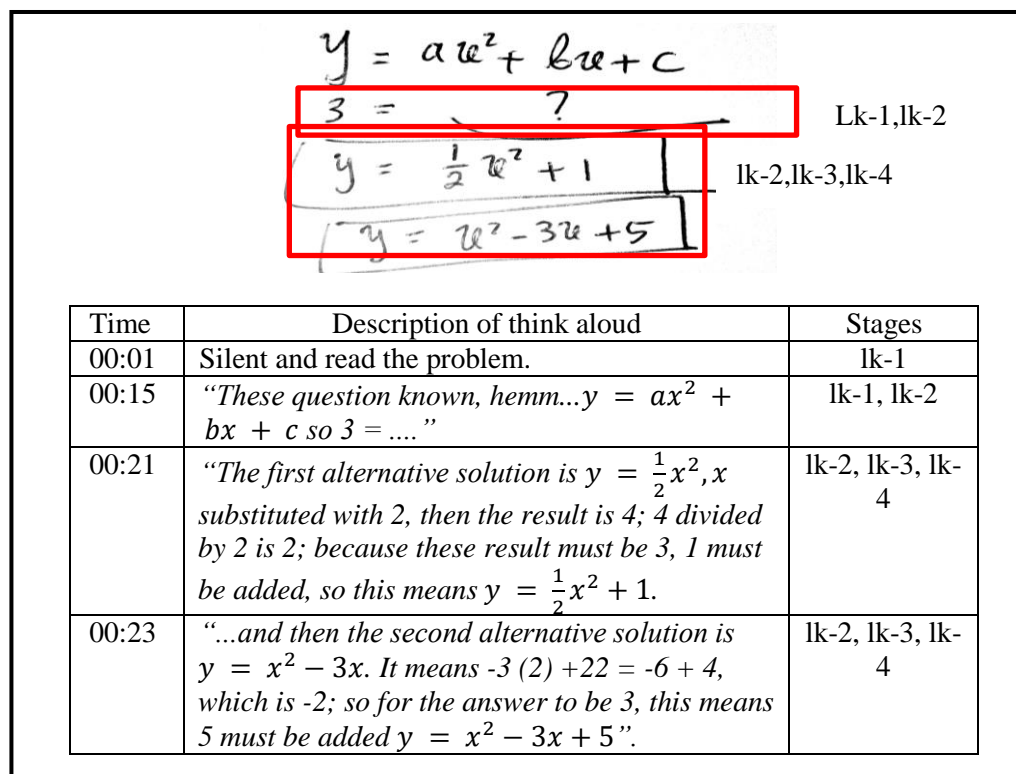


Figure 2. The Work Result of Subject-2 (AZZ)

Figure 2 shows that at the problem-representation stage (lk-1), AZZ represented symbolically the quadratic function in question as $y = ax^2 + bx + c$, and then used another symbolical representation by writing ' $3 = ?$ '. AZZ's verbal representation consisted of pointing the K point on the

problem and declaring the y value to be 3 and x value to be 2. Next, at the developing-solution stage (Ik-2), AZZ wrote $y = \frac{1}{2}x^2$ (symbolic representation) and then stated (verbal representation) that if x is substituted with 2, then the result is 4; 4 divided by 2 is 2; because these result must be 3, 1 must be added, so this means $y = \frac{1}{2}x^2 + 1$ (symbolic representation). AZZ carried out the justification (Ik-3) and monitoring and evaluation (Ik-4) stages simultaneously; for example, $y = x^2 - 3x$ means $-3(2) + 2^2 = -6 + 4$, which is -2 ; so for the answer to be 3, this means 5 must be added $y = x^2 - 3x + 5$.

In the results above, CML (first subject)'s stages to solve the ill-structured problem can be described as follows: 1) The subject used verbal representation to identify the problem; 2) the subject used translation from verbal representation to symbolic representation to describe the data; 3) the subject used translation between symbolic representation and verbal representation to compute the answer; and 4) the subject failed to uncover the necessary information about the scale to justify her answers.

AZZ (the second subject)'s solution stages to solve the ill-structured problem can be described as follows: 1) the subject used verbal representation to identify the problem, 2) the subject used translation from verbal representation to symbolic representation to describe the data, 3) the subject used symbolic representation to compute the answer, and 4) the subject used verbal representation to justify his answers. Both of subjects' translation-of-representation processes are shown in Figure 3.

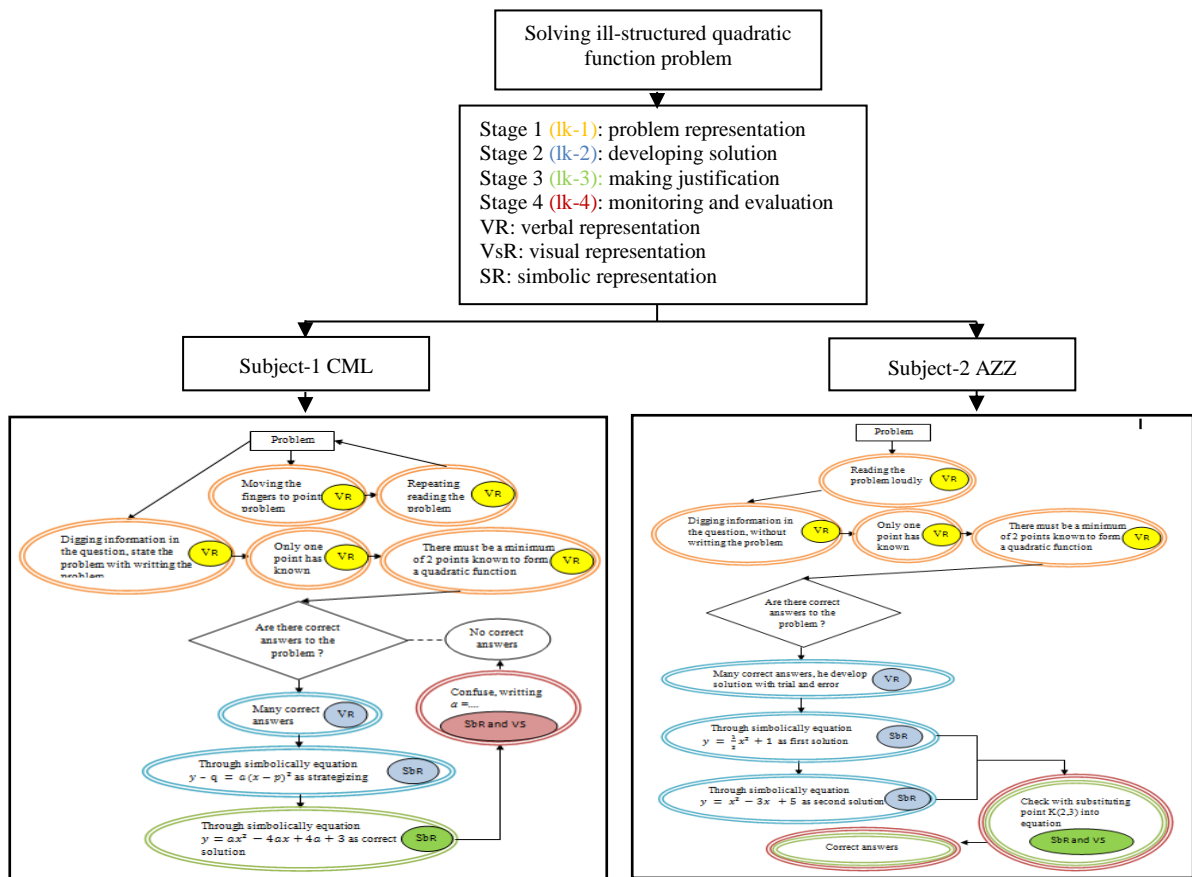


Figure 3. The Schema both of Subjects' Representation in Solving TPMI

The empirical data shows the mathematical representations created by the students when solving the ill-structured problem. They made their notations regarding concept understanding, but often had difficulty in transitioning their symbolic representations. There was a lack of understanding of symbol notations, which could be seen in CML's confusion when monitoring and evaluating to develop the final solution, and in the fact that AZZ rarely made verbal representations in writing but used the morally to describe the analysis and evaluation, he made in developing the final solution. Without using many types of representations and tending to use verbal representations, AZZ was able to give two solutions.

Based on the theoretical framework and results, we found that the two subjects had different methods of using mathematical representations to solve an ill-structured problem. It showed that the success of problem-solvers is based on their ability to build problem representations in a problem-solving situation (Zhang, 1997). The process of selecting problem representations causes differently characteristics when solving ill-structured problems, which are verbal-symbolic and symbolic-verbal problem solvers. These characteristics show that there is a strong relationship between success in solving ill-structured problems and translation of representation skills. The findings are by following Villegas et al. (2009), who states that there was a strong relationship between success in solving problems and skills in construction, use, and articulation of representation.

Another finding from this research is that in addition to the diversity of representations, the accuracy of the translation process between types of representations also determines the success of problem-solving. Pape & Tchoshanov (2001) said that translation representation reducing the level of abstraction of a representation. It is in line with Bosse et al., (2014), Rahmawati et al., (2017) and Swastika et al., (2018) said that the accuracy in determining the representation of sources and targets, prior calling of knowledge, and the mapping process between source representation and target representation are important aspects in the translation process.

CONCLUSION

This research analysed about the process of mathematical-representation translation and identified mathematical representations by applying ill-structured-problem solving activities to problem-solving learning approaches for mathematics education students. The finding of the present research that students' mathematical representation forms affect ill-structured-problem solving competencies. The schemas of mathematics education students' representations in solving ill-structured problems shows that they to tend to use verbal-symbolic and symbolic-verbal representations. Verbal and symbolic representations are used mathematics education students to calculate, detect, correct errors, and justify their answers, but visual representations are used mathematics education students to detect and correct errors. Furthermore, the studies will be carried out by increasing the number of subjects and extending the study period so that the pattern of translation of mathematical representation in solving ill-structured problems is obtained.

ACKNOWLEDGMENTS

The researchers would like to express their gratitude to the Graduate Program of Mathematics Education at The State University of Malang and the Department of Mathematics Education at The Faculty of Teacher Training and Education of University of Nusantara PGRI Kediri.

REFERENCES

- Akgün, L. (2011). Experiences of undergraduate students with literal symbols. *Scientific Research and Essays*, 6(7), 1489-1497. <https://doi.org/10.5897/SRE10.075>.
- Anwar, R.B., Yuwono, I., Ashari, A.R., Sisworo, & Rahmawati, D. (2016). Mathematical representation by students in building relational understanding on concepts of area and perimeter of rectangle. *Educational Research and Reviews*, 11(21), 2002–2008. <https://doi.org/10.5897/ERR2016.2813>.
- Bal, A.P. (2014). The examination of representations used by classroom teacher candidates in solving mathematical problems. *Educational sciences: Theory and practice*, 14(6), 2349-2365. <https://doi.org/10.12738/estp.2014.6.2189>.
- Bossé, M.J., Adu-Gyamfi, K., & Chandler, K. (2014). Students' differentiated translation processes. *International Journal for Mathematics Teaching and Learning*, 1–28.
- Bannister, V.R.P. (2014). Flexible conceptions of perspectives and representations: an examination of pre-service mathematics teachers' knowledge. *International Journal of Education in Mathematics, Science and Technology*, 2(3), 223–233. <https://doi.org/10.18404/ijemst.23592>.
- Byun, J.N., Kwon, D.Y., & Lee, W.G. (2014). Development of ill-structured problems for elementary learners to learn by computer-based modeling tools. *International Journal of Computer Theory and Engineering*, 6(4), 292-296. <https://doi.org/10.7763/IJCTE.2014.V6.877>.
- Caglayan, G., & Olive, J. (2010). Eighth grade students' representations of linear equations based on a cups and tiles model. *Educational Studies in Mathematics*, 74(2), 143–162. <https://doi.org/10.1007/s10649-010-9231-z>.
- Chapman, O. (2010). Teachers' self-representations in teaching mathematics. *Journal of Mathematics Teacher Education*, 13(4), 289–294. <https://doi.org/10.1007/s10857-010-9153-9>.
- Ge, X., & Land, S.M. (2003). Scaffolding students' problem-solving processes in an Ill-structured task using question prompts and peer interactions. *Educational Technology Research and Development*, 51(1), 21–38. <https://doi.org/10.1007/BF02504515>.
- Goldin-Meadow, S., & Beilock, S.L. (2010). Action's influence on thought: The case of gesture. *Perspectives on Psychological Science*, 5(6), 664–674. <https://doi.org/10.1177/1745691610388764>.
- Hong, J.Y., & Kim, M.K. (2016). Mathematical abstraction in the solving of ill-structured problems by elementary school students in Korea. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(2), 267–281. <https://doi.org/10.12973/eurasia.2016.1204a>.
- Janakiewska, B., Stopanska, L.F., & Bogatinoska, D.J. (2012). Enhanced study of quadratic functions with GeoGebra and preparation for Calculus. *Proceedings of the 8th International Symposium on Geometric Function Theory and Applications* (pp. 1-6). Ohrid, R. Macedonia.

- Jonassen, D.H. (1997). Instructional design model for well-structured and ill-structured problem-solving learning outcomes. *Educational Technology Research and Development*, 45(1), 65-95.
- Jonassen, D.H. (2011). *Learning to Solve Problems a Handbook for Designing Problem-Solving Learning Environments*. London: Pfeiffer Publisher.
- Kieran, C. (2004). Algebraic Thinking in the Early Grades: What Is It? *Mathematics Educator*, 8(1), 139–151. <https://doi.org/10.1080/13670050.2017.1323445>.
- Kitchner, K.S. (1983). Cognition, metacognition, and epistemic cognition. *Human Development*, 26(4), 222–232. <https://doi.org/10.1159/000272885>.
- López, J., Robles, I., & Martínez, P.R. (2016). Students' understanding of quadratic equations. *International Journal of Mathematical Education in Science and Technology*, 47(4), 552–572. <https://doi.org/10.1080/0020739X.2015.1119895>.
- Minarni, A., Napitupulu, E., & Husein, R. (2016). Mathematical understanding and representation ability of public junior high school in north sumatra. *Journal on Mathematics Education*, 7(1), 43-56. <https://doi.org/10.22342/jme.7.1.2816.43-56>.
- Moreno, A.L., Hegedus, S.J., & Kaput, J.J. (2008). From static to dynamic mathematics: Historical and representational perspectives. *Educational Studies in Mathematics*, 68(2), 99–111. <https://doi.org/10.1007/s10649-008-9116-6>.
- Muhtadi, D., Sukirwan, Warsito, & Prahmana, R.C.I. (2017). Sundanese ethnomathematics: mathematical activities in estimating, measuring, and making patterns. *Journal on Mathematics Education*, 8(2), 185-198. <https://doi.org/10.22342/jme.8.2.4055.185-198>.
- NCTM. (2000). *National Council of Teacher of Mathematics, Principles and Standards for School Mathematics*. Retrieved at December 11th, 2014, from <http://standards.nctm.org/document/appendix/numb.htm>.
- Nizar, H., Putri, R.I.I., & Zulkardi. (2018). Developing PISA-Like Mathematics Problem Using the 2018 Asian Games Football and Table Tennis Context. *Journal on Mathematics Education*, 9(2), 183-194. <https://doi.org/10.22342/jme.9.2.5246.183-194>.
- Nizarrudin. (2014). *Role of Multiple Representations in Mathematical Problem Solving*. Conference on Mathematics and Natural Sciences Education Faculty, PGRI, Semarang University Indonesia, pp 163-168.
- Palm, T. (2009). Gender Aspects of Sense Making in Word Problem Solving. *Mathematical Modelling*, 1(1), 59–76.
- Pape, S.J., & Tchoshanov, M.A. (2001). The role of representation (s) in developing mathematical understanding. *Theory into Practice*, 40(2), 118–127. <https://doi.org/10.1207/s15430421tip4002>.
- Putri, R.I.I., & Zulkardi. (2018). Higher-order thinking skill problem on data representation in primary school: A case study. *Journal of Physics: Conference Series*, 948(1), 012056. <https://doi.org/10.1088/1742-6596/948/1/012056>.
- Rahmawati, D., Purwanto, Subanji, Hidayanto, E., & Anwar, R.B. (2017). Process of mathematical representation translation from verbal into graphic. *International Electronic Journal of Mathematics Education*, 12(4), 367–381.

- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education*, 9(1), 41-54. <https://doi.org/10.22342/jme.9.1.5049.41-54>.
- Samo, M.A. (2009). Students' perceptions about the symbols, letters and signs in Algebra and how do these affect their learning of Algebra: A case research in a government girls secondary school Karachi. *International Journal for Mathematics Teaching and Learning*, 35.
- Schloeglmann, W. (2004). Routines in non-routine problem solving processes. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 4, (pp 161-168).
- Schoenfeld, A.H. (1992). *Learning To Think Mathematically: Sense-Making in Mathematics*. In D. Grouws (Ed.), (pp. 334-370). New York: MacMillan.
- Sinnott, J.D. (1989). A model for solution of ill-structured problems: Implications for everyday and abstract problem solving. In J. D. Sinnott (Ed.), *Everyday problem solving: Theory and applications* (pp. 72-99). New York: Praeger.
- Swastika, G., As'ari, A., Irawan, E.B., Nusantara, T., Subanji, & Irawati, S. (2018). Representation translation analysis of junior high school student in mathematics problem solving. *International Journal of Insights for Mathematics Teaching*, 1(2), 115-129.
- Villegas, J.L., Castro, E., & Gutiérrez, J. (2009). Representations in problem solving: a case research with optimization problems. *Electronic Journal of Research in Educational Psychology*, 7(1), 279–308.
- Voss, J.F., & Post, T.A. (1988). *On the solving of ill structured problems*. In MTA.Chi, Glaser&M.J.Farr (Eds). *The Nature of Expertise*. Hilldale, NJ: Lawrence Erlbaum.
- Wood, P.K. (1993). Inquiry systems and problem structured: Implication for cognitive. *Development. Human Development*, 26, 249-265.
- Zhang, J. (1997). The nature problem of external in solving representations. *Cognitive Science*, 21(2), 179–217. https://doi.org/10.1207/s15516709cog2102_3.

FLAWS IN PROOF CONSTRUCTIONS OF POSTGRADUATE MATHEMATICS EDUCATION STUDENT TEACHERS

Zakaria Ndemo

Bindura University of Science Education, 741 Chimurenga road, Bindura, Zimbabwe
E-mail: zndemo@gmail.com

Abstract

Intending to improve the teaching and learning of the notion of mathematical proof this study seeks to uncover the kinds of flaws in postgraduate mathematics education student teachers. Twenty-three student teachers responded to a proof task involving the concepts of transposition and multiplication of matrices. Analytic induction strategy that drew ideas from the literature on evaluating students' proof understanding and Yang and Lin's model of proof comprehension applied to informants' written responses to detect the kinds of flaws in postgraduates' proof attempts. The study revealed that the use of empirical verifications was dominant and in situations. Whereby participants attempted to argue using arbitrary mathematical objects, the cases considered did not represent the most general case. Flawed conceptualizations uncovered by this study can contribute to efforts directed towards fostering strong subject content command among school mathematics teachers.

Keywords: mathematical proof, transpose and multiplication of matrices, flawed conceptualisations, levels of proof comprehension

Abstrak

Berniat untuk meningkatkan pengajaran dan pembelajaran gagasan bukti matematika penelitian ini berusaha untuk mengungkap jenis-jenis kelemahan dalam pendidikan pascasarjana guru matematika siswa. Dua puluh tiga guru siswa menanggapi tugas pembuktian yang melibatkan konsep transposisi dan penggandaan matriks. Strategi induksi analitik yang menarik ide-ide dari literatur tentang mengevaluasi pemahaman bukti siswa dan model pemahaman bukti Yang dan Lin diterapkan pada tanggapan tertulis informan untuk mendeteksi jenis kesalahan dalam upaya bukti pascasarjana. Studi ini mengungkapkan bahwa penggunaan verifikasi empiris dominan dan dalam situasi. Dimana peserta berusaha untuk berdebat menggunakan objek matematika yang arbitrer, kasus-kasus yang dipertimbangkan tidak mewakili kasus yang paling umum. Konseptualisasi yang cacat yang ditemukan oleh penelitian ini dapat berkontribusi pada upaya yang diarahkan untuk menumbuhkan perintah konten pelajaran yang kuat di antara guru matematika sekolah.

Kata kunci: bukti matematis, transformasi dan perkalian matriks, konseptualisasi yang salah, tingkat pemahaman bukti

How to Cite: Ndemo, Z. (2019). Flaws in Proof Constructions of Postgraduate Mathematics Education Student Teachers. *Journal on Mathematics Education*, 10(3), 379-396. <https://doi.org/10.22342/jme.10.3.7864.379-396>.

Over the past decades research on mathematical proof has gained increased attention and most research studies have revealed that student teachers have a fragile understanding of mathematical proof (Bieda, 2010; Lesseig, 2016; Mejia-Ramos, & Inglis, 2009; Maya & Sumarmo, 2011; Noto, et al. 2019). However, a huge number of those studies were based on conviction issues. In other words, studies sought to determine how convincing a given argument would be to the participant (e.g., Bleiler, Thompson, & Krajčevski, 2014; Martin & Harel, 1989). Hence, there is scarcity of research that examines how students construct proofs of mathematical statements, particularly on students' competencies in resolving proof tasks.

Precisely, the argument is that little is known about students' reasoning as they immerse themselves with proof tasks as instructional and assessment strategies have tended to promote

memorization and regurgitation of lecture notes (Ndemo, Zindi, & Mtetwa, 2017; Stylianou, Blanton, & Rotou, 2015). Hence, if we conceive proving as a problem solving process then arguments generated by students when they engage with proof tasks should illuminate the kinds of students' thoughts about mathematical proof (Lee & Smith, 2009). In stark contrast, many school and university students and even teachers of mathematics have only a superficial grasp of the idea of a mathematical argument (Jahnke, 2007; Brown & Stillman, 2009; Saleh, et al. 2018). Yet prospective secondary mathematics teachers need to exit teacher preparation with a firm grasp of the concepts of the concepts of transposition and multiplication of matrices (Brown & Stillman, 2009).

The notion of mathematical proof has persistently caused severe discomforts among teachers and learners at many scholastic levels. Hence, it is necessary that teachers engage in substantial learning of the concept of mathematical proof. Furthermore, mathematics education that encourages student teachers to engage in autonomous proof constructions is crucial for their learning in order to build their capacity to explain the concept in a persuasive manner to their future students (Jones, 1997).

To bring the research problem in proper perspective the researcher refers to Harel, Selden and Selden's (2006) comment about students' struggles with the notion of mathematical proof. Harel, et al. (2006) wrote:

We know where the students are, we know where the mathematicians are, but we just don't know how to get mathematics students from where they are to where we want them to be (p. 148).

Harel et al.'s quote points to an undesirable gap between students' and experts' understandings of mathematical proof. Hence, one of the primary goals of mathematics instruction at tertiary level is to promote among student teachers conceptions of mathematical proof held by expert mathematicians. Therefore, mathematics education instruction should aim to enhance expert conceptions of the concept of mathematical proof among student teachers. In this regard, the researcher was impelled to explore student teachers' thinking as reflected in in-class mathematics problem solving tasks as student teachers engage with concepts deemed to be within their conceptual reach. The research problem is explained in the next section.

Statement of the Problem

In-service student teachers do not have a firm grasp and appreciation of the idea that proofs that explain can be more elucidating and help to foster justification. Doing proofs at school level and even at undergraduate level has been characterised by students and instructors resorting to rote memorisation and regurgitation of instructor notes. The researcher argues that if comprehension tests only ask students to regurgitate memorised facts then such students are likely to develop a superficial understanding of those mathematical facts (Mejia-Ramos, et al. 2012). Further, such students are likely to emphasise form over substance, that is, the ritual proof scheme becomes dominant (Harel & Sowder, 2007). Yet, teachers need a flexible and firm understanding of mathematics content they are supposed to teach (Shahrill, et al. 2018; Prahmana & Suwasti, 2014). The researcher reiterates that the notion of mathematical proof has been

reported to develop reasoning, that is, the ability to think rationally and logically among learners. However, the ability to reason can be impeded by flaws in students' mental representations of the concept of mathematical proof (Garret, 2013).

Promoting argumentation skills can illumine the kinds of flaws in students' thoughts about the concepts of transposition and multiplication of matrices. It is in an argument that we likely to find the most significant way in which higher order thinking can manifest during mathematics learning (Jonassen & Kim, 2010; Putri & Zulkardi, 2018; Ahmad, et al. 2018). The goal of the current study is to gain insights into the kinds of limitations in postgraduate students' argumentation schemes—chunks of reasoning with respect to the notions of transpose and multiplication of matrices. Pertinent questions that therefore, come to mind in this respect are: how can we develop an understanding of the flaws in student teachers' mental representations of the idea of proof? What is the nature of these limitations among in-service mathematics teachers?

Generating answers to these questions can contribute to useful ideas for teacher education in Zimbabwe. While many studies on students' discomforts with the concept of a mathematical proof have been carried out most of such studies have been based on arguments participants find convincing from those availed by researchers (e.g., Bleiler, et al. 2014; Matin & Harel, 1989). Therefore there is scarcity of empirical studies based on students' own proof constructions, that is, their own actual voices which in turn could illuminate their thinking processes as they engage with proof and proving (Duval, 2006; Mejia-Ramos & Inglis, 2009; Ndemo, Zindi, & Mtetwa, 2017; Mumu, et al. 2018). This study intends to respond to this dearth in studies grounded in students' own proof constructions by addressing the research question, such as what kinds of flaws characterise postgraduate students' conceptions of the concepts of transposition and multiplication of matrices?

By addressing this question the study may provide mathematics teachers with a clearer picture of what is needed to help students to develop a good command of the concept of mathematical proof in order to teach it effectively to their future students. Furthermore, it was anticipated that evaluating postgraduate students' understandings of the notions of transposition and multiplication of matrices could in turn inform teachers and mathematics educators of what specific aspects they understand and what aspects they do not understand.

Furthermore, proof serves as a vehicle for discovering new mathematical ideas and if learners properly grasp the notion of proof then they learn from it (Mejia-Ramos, et al. 2012). Mathematical proof also serves the purpose of promoting reasoning skills (Weber & Mejia-Ramos, 2015), which in turn would contribute to the student's cognitive development about the concept of mathematical proof. In this regard, the study aims to inject new ideas into the growing body of theoretical frameworks and methodologies for understanding mathematical concepts.

Matrix Theory

If A is a $m \times n$ matrix of a field K , of scalars then the transpose of matrix A is the $n \times m$ matrix whose rows are the columns of A in same order (Goodaire, 2014). The transpose of matrix A is denoted A^t . Hence, if $A = (a_{ij})$ then $A^t = (a_{ji})$ (Lipschutz, 1991). In other words, if

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \text{ then } A^t = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{pmatrix}.$$

First, we observe that the product AB of two matrices is somewhat complicated and hence there is need to describe prerequisite ideas for the definition of the product of matrices. The product $A \cdot B$ of a row matrix $A = (a_i)$ and a column matrix $B = (b_i)$ is defined as:

$$A \cdot B = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n = \sum_{k=1}^n a_k b_k. \text{ Second, we observe that the row}$$

matrix A and the column B should have the same number of elements for the product $A \cdot B$ to be defined. Finally, the product $A \cdot B$ is a scalar or a 1×1 matrix (Lipschutz, 1991).

The researcher now describes the definition of a product of two matrices. Suppose $A = (a_{ij})$ and $B = (b_{ij})$ are matrices over the field of scalars K , the product AB is defined if the number of columns of matrix A is equal to the number of rows of matrix B . If the number of columns of matrix A is equal to the number of rows of B we say the two matrices A and B are of compatible sizes (Goodaire, 2014). Thus, if matrices A and B are compatible say A is an $m \times p$ and B is a $p \times n$ matrix then the product AB is an $m \times n$ matrix whose ij -th entry is obtained by multiplying the i -th row A_i of the matrix A by the j -th column, B^j , of the matrix B . That is: $AB =$

$$\begin{pmatrix} A_1 \cdot B^1 & A_1 \cdot B^2 & \dots & A_1 \cdot B^n \\ A_2 \cdot B^1 & A_2 \cdot B^2 & \dots & A_2 \cdot B^n \\ \dots & \dots & \dots & \dots \\ A_m \cdot B^1 & A_m \cdot B^2 & \dots & A_m \cdot B^n \end{pmatrix}. \text{ Alternatively, we can write the product as:}$$

$$AB = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1p} \\ a_{21} & a_{22} & \dots & a_{2p} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mp} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{p1} & a_{p2} & \dots & a_{pn} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix}, \text{ where } c_{ij} =$$

$a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{ip}b_{pj}$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. The researcher emphasizes that if A and B are not compatible then the product AB is not defined. So if A is an $m \times p$ matrix and B is a $q \times n$ matrix then the product AB exists only if $p = q$. This study sought to explore how postgraduate students could apply the notions of transpose and multiplication of matrices to determine whether the product $A^t A$ exists.

Yang and Lin's (2008) Model

The goal of the study was to uncover the sorts of flaws in postgraduate students' proof attempts. To assess in-service teachers' understandings of mathematical proof the researcher drew ideas from a model for assessing comprehension of mathematical proof by Yang and Lin (2008). Yang and Lin introduced what has come to be known as a model of reading comprehension of

geometric proof (RCGP). The model consists of four levels which represent increasing levels of cognition. Next, the researcher briefly describes these levels.

First, there is the *surface level* of the RCGP model whereby a prover acquires basic knowledge regarding the meaning of the statements and symbols in the proof. For example, for the theorem: *A real sequence converges to a real number L if given $\varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all but infinitely terms of the sequence (a_n)* , surface level understanding of this theorem can include the conception of ε as a small radius, an understanding of a finite set and the basic idea that a sequence is a mapping with domain the set of natural numbers, (\mathbb{N}) and range in real numbers, (\mathbb{R}). The second level has been called *recognising elements*. At this level a prover should be capable of identifying the logical status of statements that are explicitly or implicitly involved in the proof construction exercise. The researcher now uses the example given at the surface level that concerns a theorem on the characterisation of converging sequences in \mathbb{R} to describe proof understanding anticipated at this level. At this second level of Yang and Lin’s YCGP model, a prover should recognise that for every $\varepsilon > 0$, there is a natural number, $N(\varepsilon)$, dependent on ε such that if $n > N(\varepsilon)$ then $|a_n - L| < \varepsilon$. A prover’s chunk of reasoning at the second level should show also awareness that the interval $(L - \varepsilon, L + \varepsilon)$ contains infinitely many terms of the sequence (a_n) .

At the third level there is what Yang and Lin refer to as *chaining of elements*. Central at this level is the fact a prover demonstrates his/her understanding of the way in which different statements are connected in the proof by identifying the logical relations between them (Mejia-Ramos et al., 2012). Following up on the example given to illustrate the YCGP model, the researcher describes the logical relations between statements a prover should depict in his/her argumentation. A prover should make the connection that since $\varepsilon > 0$ then there is a natural number, $N(\varepsilon)$, such that if $n > N(\varepsilon)$ then

$$|a_n - L| < \varepsilon$$

$$\Rightarrow -\varepsilon < a_n - L < \varepsilon$$

$$\Rightarrow L - \varepsilon < a_n < L + \varepsilon$$

$$\Rightarrow a_n \in (L - \varepsilon, L + \varepsilon) \text{ for } n > N(\varepsilon).$$

Hence, a prover who would have attained the third level of Yang and Lin’s model of reading of comprehension of geometric proof (RCGP) should make a series of logical inferences illustrated. The third level of Yang and Lin’s model is similar to Azarello’s (2007) conceptualisation of proof as a sequence of inferences. Azarello (2007) in Mejia-Ramos and Weber (2014) view proof as a series of claims of the form $P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow \dots \rightarrow P_n$, where P_n is the consequent statement or conclusion while, P_1, P_2, \dots, P_{n-1} constitute the premises of the proof construction. In this conceptualisation of composing proofs the focus is on how each new inference is derived from the previous inference, for instance, how does claim P_3 lead to claim P_4 (Mejia-Ramos & Weber, 2014).

Finally, Yang and Lin’s RCGP model has a fourth level called encapsulation whereby a prover is anticipated to interiorize a proof holistically and develop an appreciation and understanding of its

application in other contexts (Mejia-Ramos & Weber, 2014). The researcher illustrates the encapsulation phase using the example of real sequence already given. A prover at the fourth level of proof understanding can draw ideas from this theorem and use them together with ideas from the completeness property of \mathbb{R} to prove our first criterion for convergence that: *A bounded monotone sequence converges*. In their study, Yang and Lin focused on the first three levels of their model. The current study that involves postgraduate students seeks to explore those students' understanding of ideas drawn from Elementary Linear Algebra of Matrices. The concepts had been covered during their studies at undergraduate level. Hence, the study seeks to explore elements of the encapsulation level of Yang and Lin's model in postgraduates' solution attempts to the task.

Other Useful Ideas on Evaluating Students' Understanding of Mathematical Proof

Mejia-Ramos and Weber (2014) and Mamon-Downs and Downs (2013) concur that one way to evaluate whether a student understands a proof is by determining the student's proof behaviour or problem solving behaviour. In addition to scrutinising one's proof behaviour, Hanna and Barbeau (2008) suggest that a student's proof construction competence can be ascertained by determining the extent to which the student applies pertinent ideas to the proof task or theorem in other situations. It can be, therefore, noted that Hanna and Barbeau's suggestion on how to evaluate proof understanding is similar to the encapsulation level of proof conceptualisation proposed by Yang and Lin (2008).

Conradie and Frith (2000) stated that students often fail to grasp the meaning of terms when trying to comprehend a proof thereby hindering their ability to understand other aspects of the proof. To get a sense of students' grasp of key terms to a theorem, Conradie and Frith (2000) suggest that a researcher can ask students to define the key terms or sentences. This technique of determining understanding features at the surface level of Yang and Lin's (2008) model. Alternatively, students can be asked to identify examples that illustrate a given theorem or a term in a theorem. Generating examples that illustrate a theorem or ideas embedded in that theorem is similar to Yang and Lin's second level of their RCGP model. Finally, a prover should develop a firm grasp of the logical relationships of the statement being proven and the major assumptions and conclusions of the proof. In other words, proof understanding involves grasping the proof framework (Selden & Selden, 2009).

Morselli (2006) writes that generating examples has several benefits in proving. The benefits of generating examples to illustrate key concepts involved in constructing proof of a given theorem include illuminating the defining property of pertinent mathematical ideas to the proof being constructed, revealing logical connections in ideas embedded in the mathematical statement that can form the crux of the proof, and providing a pictorial representation of the mathematical ideas especially in proof situations where graphical instantiations can be found. However, despite these benefits, empirical verifications have a severe limitation that the statement can be true for particular examples a prover could have considered but can be false for just one instance not considered by the prover. This is so called notion of a counter example (Stylianides, 2011). Hence, empirical

explorations do not provide complete and conclusive evidence about the truth-value of a statement a prover may seek to establish.

Finally, another useful idea in evaluating an individual's ability to compose proof relates to the structure of a mathematical argument. A mathematical argument consists of a connected sequence of assertions in which the consequent statement is called the conclusion and the rest of which is called the premises (Curd, 1992). The premises provide valid reasons for inferring that the conclusion is true. Furthermore, a mathematical proof is said to be valid if it is deductive, contains no errors and provides complete and conclusive evidence about the truth-value of a mathematical statement (Weber & Mejia-Ramos, 2015). The ideas presented in this section were important in evaluating students' responses to the task. For instance, arguments generated by mathematics education post graduate students were checked for logical consistency between premises and conclusions drawn.

METHOD

Research Design

A cross-sectional survey research design with a qualitative bent was used in this study (Flick, 2011). The intent of the study was to capture the in-service teachers' state of knowledge structures about the notions of multiplication of matrices and transpose of a matrix. Further, the study sought to investigate mathematical connections built by the postgraduate students which would in turn allow them to determine whether the product $A^t A$ was defined. Hence, the study was designed to get what Flick (2011) calls "a picture of the moment" (p. 67). Such picturing was based on the assumption that concepts examined were considered to be within the conceptual reach of the participants since they teach these at secondary school level and at teacher training colleges.

Study Informants

The study involved in-service mathematics education students who were studying towards a master degree in mathematics education. The cohort consisted of 23 members: 15 males and 8 females. Of the 15 male participants, 4 were lecturers from primary teacher training colleges while the rest were high school mathematics teachers with more than 10 years teaching experience. One out of 8 female in-service teachers was a lecturer at a primary teacher training college while the other 7 female student teachers were high school mathematics teachers with more than 8 years of teaching experience. Both college and high school mathematics curricula cover subject content on Elementary Linear Algebra of Matrices. Hence, the researcher had assumed on reasonable grounds that the concepts were within the conceptual reach of the participants.

The postgraduate programme in mathematics education offered by the university that served as the study site is now described. The master degree programme has duration of 2 years during which the in-service teachers major in mathematics content and professional courses. Professional courses deal with 3 mathematics education modules and a module covering foundations in science education.

In addition professional studies also include Research Methods and Statistics module that is designed to prepare postgraduate students for research projects during the second and final year of their studies. There are 5 modules under the Professional component of the postgraduate programme just described. Mathematics content courses include: Metric Spaces and Topology, Functional Analysis and Non-linear Differential Equations. Subject content courses drawn from the learning area of Statistics are: Multivariate Statistics, Survey Sampling Methods, and Operations Research. Mathematics content and Professional modules are covered during the first two semesters of year one.

During the third and fourth semesters, that is, during year two, student teachers engage in research projects. The content of the postgraduate programme articulated here supports this researcher's assumption that theory of matrix multiplication and the notion of transpose of a matrix were within students' conceptual reach. The study involved all the 23 students who had enrolled for the master degree studies. Data collection took place during week 10 of semester 2 of year one and this period was deemed strategic for data collection because lectures had been completed and students were doing individual studies and hence pressure had eased.

Research Instrument and Data Collection Procedure

A proof task with an open instruction: *If A is a $m \times n$ matrix, determine whether the product $A^t A$ is defined. Justify your answer*, was posed. Although the task involved elementary concepts in Matrices, it was anticipated that the answer generated would not be a result of applying standard procedure involving a known mathematical result or a known theorem. In other words, an attempt was made to avoid assessing proven results in literature on Linear Algebra such as: (i) $(kA)^t = kA^t$ where k is a scalar (ii) $(AB)^t = B^t A^t$.

The argument here is that assessing students' competences at composing such proofs could possibly lead to regurgitation of proofs from textbooks and so would serve very little purpose with respect to the research goal of establishing the kinds of flaws in postgraduate mathematics education students' conceptualisations of mathematical proof. Instead of reproducing memorised theorems such as those stated here, the postgraduate students were expected to tap from their knowledge of matrix multiplication and the notion of a transpose of a matrix to decide whether the product $A^t A$ is defined. It was also considered that these are well known mathematical ideas for the postgraduates. Hence, students could face no difficult in establishing the connections between these ideas. Thus, the task was intended to measure students' competence at autonomous proof writing.

Following Dahlberg and Housman (1997) task-based interviews were used to gather data. To collect data a task sheet with space for writing the answer was provided. Students responded to the task individually in the mathematics lecture room. No time restrictions were imposed. The participants took about 15 minutes working on the task. All task sheets distributed were returned with some text scribbled on by the students. During data collection process, participants were encouraged to document their thinking as much as possible and request for extra answer sheets when necessary.

Data Analysis

The data analysis procedure employed the analytic induction strategy (Punch, 2005). Analytic induction comprises a series of alternating inductive and deductive steps whereby data driven inductive codes are followed by deductive examination as described in the following steps. First, a marking guide was devised to evaluate postgraduate students' proof efforts. Second, the researcher then performed content analysis (Berg, 2009) of the students' written efforts. Content analysis was facilitated by a refinement of a classification originally developed by Stylianides and Stylianides (2009). The refinement was driven by the desire not only to identify correct proofs but to also explicate the different thinking styles displayed by the postgraduate student teachers. Following Stylianides and Stylianides' (2009) classification as well as a scrutiny of the students' written efforts, a data matrix with the following format was then constructed. Column 1 entries consist of categories identified from students' written responses, in column 2 each category is described, column 3 entries are frequencies of the categories. Steps 1 and 2 so far described led to a data matrix. The creation of the data matrix constituted the induction analysis part of the analytic induction strategy employed in this study.

Finally, the emerging categories from content analysis of postgraduates' proof attempts were mapped to levels of proof understanding in Yang and Lin's (2008) model and other ideas about proof understanding explained in Section 2.3 of this paper in order to ascertain postgraduate students' level of grasp of the concept of mathematical proof. The mapping of results of inductive content analysis constituted the deductive analysis part of the analytic induction strategy. Furthermore, in-vivo codes (Corbin & Strauss, 2008; Varghese, 2009) were used to support inferences made about postgraduate students' proof construction competences.

Ethical Considerations

Flick (2011) suggests that research should involve participants who have been informed about the aim of the study and that participation should be voluntary. Hence, with regards to informed consent the researcher explained to the postgraduate mathematics education students why research into the nature of students' flawed conceptualisations of mathematical proof was necessary as it could promote conceptual teaching of the mathematical ideas. The students were asked to complete informed consent forms. Further, the researcher emphasised that consent was to be given voluntarily (Flick, 2011).

Another ethical concern was about anonymization of data. Anonymizing data involved removing any identifiers from the students' responses (Punch, 2005). Furthermore, during research reporting pseudonyms were used to describe postgraduates' flaws in their conceptualisations of mathematical proof.

RESULT AND DISCUSSION

Inductive content analysis of postgraduate students' written responses revealed the categories summarised in Table 1.

Table 1. Emerging categories from content analysis of postgraduate students' proving attempts ($n = 23$)

Category	Description	Frequency
C1	Correct proof	4
C2	Empirical argument used	8
C3	Argument adduced does not represent the most general case	10
C4	Consequent statement missing or wrongly formulated	7
C5	Argument has algebraic slips	1
Total		30

Table 1 shows that the dominant scheme of argumentation was one in which student teachers produced justifications that failed to offer complete and conclusive evidence about the fact that the product $A^t A$ is defined for any matrix $A = (a_{ij})$ over a field of scalars K . From the same Table 1 it can be seen that postgraduate students' efforts were also dominated by use of specific examples, denoted by C2 with 8 out of 30 responses and a very significant number of responses (7 out of 30) were in the category in which the conclusion did not follow logically from the premises— represented by code C4. Finally, the same Table 1 illustrates that very few (4 out of 30) student teachers managed to justify the existence of the product $A^t A$ – a worrisome result at postgraduate level, more so in light of the fact that elementary concepts of transpose and multiplication of matrices were explored. Presented next is a discussion of thinking styles displayed in each category summarised in Table 1 for the purpose of illuminating flaws detected in postgraduate mathematics education student teacher informants.

C1: Correct proof constructed

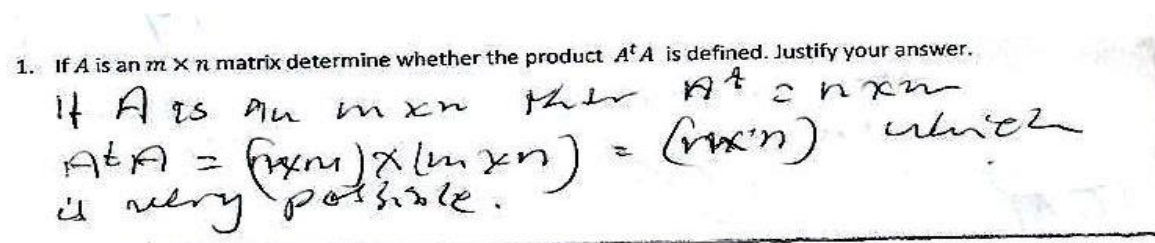
**Figure 1:** Trevor's written response to the task on multiplication of matrices

Figure 1 illustrates that Trevor could state the order of the transpose matrix A^t correctly as $n \times m$. Trevor then tested for compatibility of the two matrices by determining whether the number of columns of A^t was equal to the number of rows of matrix A and then reached the conclusion that the product $A^t A$ is defined. Hence, Trevor's written response shows that he had a firm grasp of the concepts of transposition and multiplication of matrices. Further, there was proper chaining of these knowledge elements which then led to the valid conclusion that the product $A^t A$ exists (Yang & Lin, 2008).

C2: Empirical arguments used

Typical examples in this category were produced by Munya and Tauya. Munya and Tauya's efforts are shown in Figures 2 and 3 respectively.

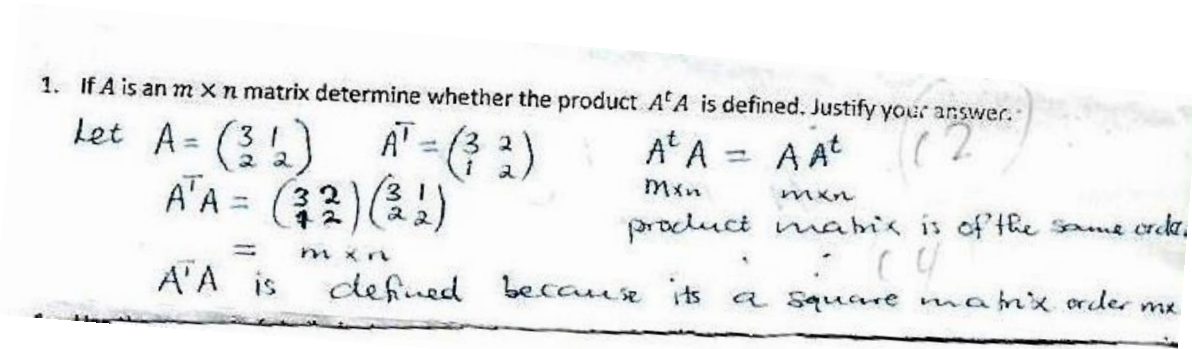


Figure 2. Munya’ written response to the task

Figure 2 illustrates that Munya chose a specific example of a 2 –square matrix A and proceeded to write the transpose of matrix A correctly. The product $A^t A$ is then equated to mxn , presumptively referring to the order. Munya went on to write $A^t A = AA^t$, which is a false assertion because a matrix A and its transpose are not commutative over the binary operation multiplication over the field K . It is a serious flaw in reasoning displayed by Munya. Further, Munya represented the order of the product $A^t A$ to be mxn and the order of AA^t was written also as mxn . It was yet another flawed reasoning as Munya could not identify that the operation of transposing entails interchanging the rows and columns of a matrix.

Finally, Munya concluded that the “product matrix is of the same order.” This is a flawed argument because for m by n matrix A the product $A^t A$ is a n –square matrix while the product AA^t is a m –square matrix. The discussion of Munya’s proof effort reveals that he had not even attained the second level of Yang and Lin’s (2008) model of comprehension of geometric proof. Munya was operating at surface level of Yang and Lin’s model as he could state the transpose of the 2 –square matrix he had written. Furthermore, Munya concluded on the basis of the specific example of the 2 –square matrix that “ AA^t is defined because it a square matrix order m .” The claim shows that Munya had not grasped the fundamental limitation that empirical verifications cannot be elevated to the status of a proof (Ndemo, Zindi, & Mtetwa, 2017; Stylianides, 2011). In other words, Munya exhibited a weak command of the notion of counter-argumentation in mathematics.

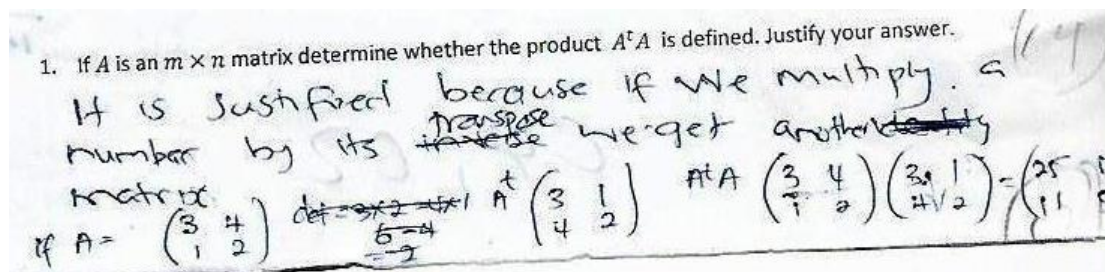


Figure 3. Tauya’s written response to the task

From Tauya’s solution attempt the word *number* was used to refer to a matrix when he wrote “when we multiply a number by its transpose we get another matrix.” This claim by Tauya reveals

lack of precision in the manner he used the word *number* which he used interchangeably with the word *matrix*. Further, Tauya's effort also reveals lack of deep grasp of the basic limitation that specific examples cannot be used to represent general matrix multiplication. In other words, although matrix multiplication held in the single instantiation considered, the specific example picked by Tauya should not be regarded as a proof. In terms of Yang and Lin's (2008) model of geometric proof comprehension, Tauya's effort reveals that he had not interiorised matrix multiplication. According to Stylianides and Stylianides (2009) an argument is deemed to be valid if it is deductive and the premises logically imply the consequent statement. A true deductive argument once constructed offers complete and conclusive evidence about the truth of a mathematical statement. Hence, it becomes superfluous to look for further evidence about the truth-value of the mathematical statement (Weber & Mejia-Ramos, 2014). It can, therefore be, inferred from Tauya's use of a single example to resolve the proof task that he lacked a good grasp of these fundamental ideas about proving and proof in the area of Elementary Algebra.

C3: Argument produced does not represent the most general case

Table 1 shows that this category that emerged from inductive content analysis of students' was the most dominant among mathematics education postgraduates with 10 out of 30 responses. Next, the researcher now presents typical students' responses in this category and discusses these results within the perspective of Yang and Lin's model and other ideas about students' conceptions of mathematical proof. First Ticha's response is considered.

1. If A is an $m \times n$ matrix determine whether the product $A^t A$ is defined. Justify your answer.

The product of $A^t A$ is defined if A is $m \times n$ matrix.

Let $A = \begin{pmatrix} a \\ b \end{pmatrix}$; $A^t = (a, b) \Rightarrow (a, b) \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a^2 + b^2 \end{pmatrix}$

$A = (a, b)$; $A^t = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} a, b \end{pmatrix} = \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}$.

Figure 4. Ticha's written attempt to the task

Figure 3 shows that whilst Ticha produced his argument in terms of arbitrary mathematical objects by using the column matrix $A = \begin{pmatrix} a \\ b \end{pmatrix}$ whose transpose was correctly written as $A^t = (a \ b)$, the argument cannot be considered to represent the general $m \times n$ matrix A . Similarly, a row matrix $A = (a \ b)$ whose transpose $A^t = \begin{pmatrix} a \\ b \end{pmatrix}$ was also considered and the product $A^t A = \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}$ was then determined.

Hence, although in both cases Ticha's efforts involved manipulating arbitrary mathematical objects, the two cases cannot be deemed to represent the most general case of the product AB of two matrices A and B since Ticha's arguments involved column and row matrices. However, there was

proper chaining of the elements (Azarello, 2007; Mejia-Ramos et al., 2012) as Ticha could carry out matrix multiplication correctly that led to the conclusion that “ A^tA is defined if A is $m \times n$ matrix.” It can be argued that traces of the encapsulation phase of proof comprehension were missing because Ticha could not conceive matrix multiplication in terms of the broader and more general case. Another example in this category by Mutaka is now presented and discussed.

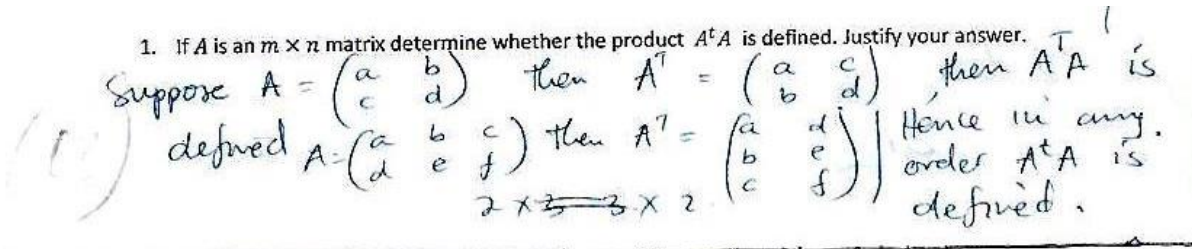


Figure 5. Mutaka’s written effort to resolve the task

First, Mutaka considered the special case of a 2 –square matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and wrote the transpose of the matrix A correctly as $A^t = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$. Mutaka’s argument up to this stage indicated that the surface level of Yang and Lin’s (2008) model of proof comprehension had been attained as shown by correct use of the definition of the idea of a transpose. However, chaining of the elements of A^t and A was not evident because after transposing the matrix A , Mutaka just wrote the conclusion that “then A^tA is defined.” Second, Mutaka considered another special case of a 2×3 matrix A with arbitrary entries and as before he managed to interchange rows and columns of A to get the transpose A^t . Similar to his proof behaviour in the first example discussed, there was no chaining of elements observed here that led to the conclusion stated as “Hence in any order A^tA is defined.” In a similar fashion to Ticha’s example the 2 –square and 2×3 matrices used by Mutaka to support his conclusion that the product A^tA is defined for any $m \times n$ matrix A over a field of scalars K cannot be considered to be representative of the most general case of matrix multiplication. Hence, the encapsulation level (Yang & Lin, 2008) was not reached by Mutaka and Ticha. Next, the researcher focuses on informants’ written responses in which the conclusion was either missing or wrongly formulated.

C4: Consequent statement missing or wrongly formulated

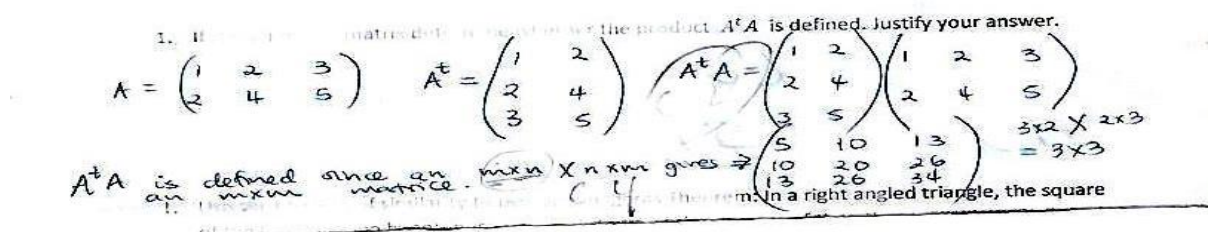


Figure 6. Mushai’s written response to the task on matrices

Figure 6 shows that the categories formed from the analytic induction of data were not mutually exclusive because similar to efforts by Munya and Tauya, Mushai also used specific examples to

validate the statement that $A^t A$ is defined. In addition to employing empirical verifications Mushai's arguments also revealed the following limitation. Mushai did not recognise that matrix multiplication is not commutative as he referred to A^t as a $m \times n$ matrix yet matrix A has been given as a $m \times n$ matrix. Furthermore, he identified A as $n \times m$ in stark contrast to the assertion that A was given as a $m \times n$ matrix. This chaotic proof behaviour led to the conclusion that $A^t A$ is "a $m \times m$ matrix," instead of an n -square matrix. Hence, whilst Mushai later on chained the elements correctly his failure earlier on to correctly identify the order of the matrix A led to the wrong conclusion. Another example in this category is now examined.

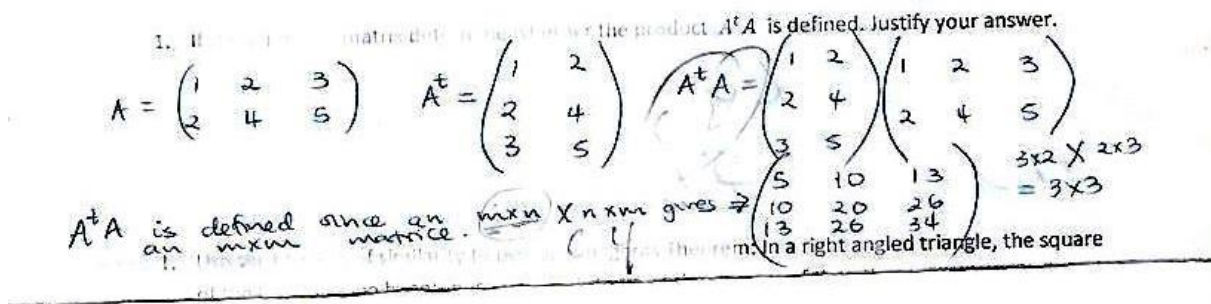


Figure 7. Maunja's written response to task on matrix multiplication

The product $A^t A$ denotes that the matrix A is post-multiplying the transpose matrix A^t . Hence, for compatibility of the multiplication operation the number of columns of A^t should be equal to the number of rows of A . Figure 7 illustrates that Maunja's argument contradicts to the assertion just stated concerning matrix multiplication. He wrote that " $A^t A$ will be defined ... the no of columns in matrix A will always be equal to the no of rows in the 2^{nd} matrix." The second matrix mentioned by Maunja presumptively referred to the post-multiplying matrix which in this case should be the matrix A . Maunja's conclusion is a typical example of many such conclusions (7 out of 30) drawn by postgraduate students that did not logically follow from the premises. Finally, the researcher focuses on a typical example of a flawed argument caused by an algebraic slip made by Mujuru.

C5: Argument has algebraic slips

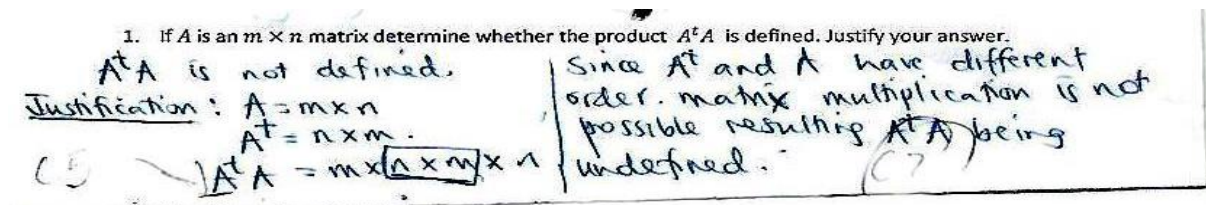


Figure 8. Mujuru's proof attempt to the task on matrices

Figure 8 shows that Mujuru started very well as she was able to write the transpose matrix A^t correctly as $n \times m$ matrix. However, her woes with the proof task manifested at the chaining stage (Yang & Lin, 2008) whereby her representation of matrix A^t as $m \times n$ led to the assertion that " $A^t A = m \times n \times m \times n$." This statement made it impossible for her to determine the product $A^t A$ and led to the

conclusion that “ $A^t A$ is not defined.” It can be observed also from the assertion “ $A^t A = mxn \times mxn$,” that the two matrices A^t and A have the same order, which is a false assertion. Hence, the wrong conclusion drawn can be attributed to the algebraic slip made by Mujuru.

CONCLUSION

Postgraduate had not grasped the fundamental limitation that empirical verifications do not count as proofs. A case in point was the use of a single instantiation by Tanya. Furthermore, students produced arguments which were not typical of the most general. Although student teachers’ efforts to justify that the product is defined were terms of arbitrary mathematical objects, those objects did not represent the most general case. For instance, a column matrix $A = \begin{pmatrix} a \\ b \end{pmatrix}$ was used to prove that the product $A^t A$ exists. The cases used by postgraduate were not representative of the general matrix multiplication discussed in Section on multiplication of matrices. However, the use arbitrary objects was a huge step forward in current efforts to promote deductive argumentation among students in mathematics education. Lastly, postgraduate students’ written responses revealed that the premises and the conclusion drawn were not logically connected. Further, in other cases the definition of the transpose of a matrix was not properly grasped as shown by proof behaviour such as referring to A^t as a mxn matrix when the matrix was stated as a mxn matrix. In addition, in some cases for the product, $A^t A$, the students focused on the number of columns of the matrix A instead of the number of rows since A was the post-multiplying matrix.

As concluding remarks, the researcher emphasizes that postgraduate students’ flawed conceptions uncovered by this study have important implications for teacher preparation in Zimbabwe. Subject content mastery by students was fragile and hence the need for continuing professional development of in-service mathematics on subject content knowledge. The need to promote good grasp of concepts of Linear Algebra implies that mathematics educators and researchers need to find ways of ameliorating flawed conceptions of the concept of a mathematical proof. Furthermore, teacher preparation needs to include content and instructional strategies that foster and enhance prospective secondary mathematics teachers’ explanatory role. Such content and strategies should also develop an appreciation of the function of a mathematical proof in justifying why a given mathematical assertion is true or false.

REFERENCES

- Ahmad, S., Prahmana, R.C.I., Kenedi, A.K., Helsa, Y., Arianil, Y., & Zainil, M. (2018). The instruments of higher order thinking skills. *Journal of Physics: Conference Series*, 943(1), 012053. <https://doi.org/10.1088/1742-6596/943/1/012053>.
- Azarello, F. (2007). The proof in the 20th century: From Hilbert to automatic theorem proving. In P. Boero (Ed.), *Theorems in schools: From history, epistemology, and cognition to classroom practice*. Rotterdam: Sense Publishers.

- Berg, B.L. (2009). *Qualitative Research Methods for the Social Sciences Topics*. Boston: Allyn Brown.
- Bieda, K.N. (2010). Enacting proof-related in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education*, 41(4), 351-382.
- Bleiler, S.K., Thompson, D.R., & Krajčevski, M. (2014). Providing written feedback on students' mathematical arguments: proof validations of prospective secondary mathematics teachers. *Journal of Mathematics Teacher Education*, 17(2), 105-127. <https://doi.org/10.1007/s10857-013-9248-1>.
- Brown, J., & Stillman, G. (2009). Preservice teachers' competencies in proof. In F.-L. Lin, F.-J., Hsieh, G. Hanna, M. de Villiers (Eds), *Proceedings of the ICMI Study 19 Conference: Proof and Proving in Mathematics Education: Vol. 2.* (pp.196-201). Taipei, Taiwan: National Taiwan University.
- Conradie, J., & Frith, J. (2000). Comprehension tests in mathematics. *Educational Studies in Mathematics*, 42(3), 225-235. <https://doi.org/10.1023/A:1017502919000>.
- Corbin, J., & Strauss, A. (2008). *Basics of Qualitative Research*. Thousand Oaks: Sage.
- Curd, M. (1992). *Arguments and Analysis: An Introduction to philosophy*. London: West Publishing.
- Dahlberg, R.P., & Housman, D.L. (1997). Facilitating learning events through example generation. *Educational Studies in Mathematics*, 33(3), 283-299. <https://doi.org/10.1023/A:1002999415887>.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1), 103-131. <https://doi.org/10.1007/s10649-006-0400-z>.
- Flick, W. (2011). *Introducing Research Methodology: A Beginner's Guide to Doing a Research Project*. London: Sage Publications.
- Garret, L. (2013). Flawed mathematical conceptualizations: Marlon's dilemma. *Journal of Developmental Education*, 37(2), 2-4, 6-8.
- Goodaire, E.D. (2014). *Linear Algebra: Pure and Applied*. Toh Tuck: World Scientific Publishing.
- Hanna, G., & Barbeau, E. (2008). Proofs as bearers of mathematical knowledge. *ZDM*, 40(3), 345-353. <https://doi.org/10.1007/s11858-008-0080-5>.
- Harel, G., and Sowder, L. (2007). Toward a comprehensive perspective on proof. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp 805-842). Charlotte, NC: Information Publishing.
- Harel, G., Selden, A., & Selden, J. (2006). Advanced mathematical thinking: Some PME perspectives. In A. Gutierrez and P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Post, Present and Future*. Rotterdam: Sense, pp. 147-172.
- Jahnke, H.N. (2007). Proofs and hypotheses. *ZDM*, 39(1-2), 79-86. <https://doi.org/10.1007/s11858-006-0006-z>.
- Jonassen, D.H., & Kim, B. (2010). Arguing to learn and learning to argue: Design justifications and guidelines. *Educational Technology Research and Development*, 58(4), 439-457. <https://doi.org/10.1007/s11423-009-9143-8>.
- Jones, K. (1997). Student teachers' conceptions of mathematical proof. *Mathematics Education Review*, 9, 21-32.

- Lee, K., & Smith, P. (2009). Cognitive and linguistic challenges in understanding proving. In F-L. Lin, F-J. Hsieh, G. Hanna, & M. de Villiers (Eds.), *Proceedings of the ICMI Study 19 Conference on Proof and Proving in Mathematics Education: Vol. 2.* (pp. 15-21). Taipei, Taiwan: National Taiwan Normal University.
- Lesseig, K. (2016). Investigating mathematical knowledge for teaching proof in professional development. *International Journal of Research in Education and Science*, 2(2), 253-270.
- Lipschutz, S. (1991). *Schaum's Outline of Theory and Problems of Linear Algebra*. New York: McGraw-Hall.
- Mamona-Downs, J., & Downs, M. (2013). Problem solving and its elements in forming proof. *The Mathematics Enthusiast*, 10(1), 137-162.
- Martin, G., & Harel, G. (1989). Proof frames of pre-service elementary teachers. *Journal for Research in Mathematics Education*, 20(1), 41-51.
- Maya, R., & Sumarmo, U. (2011). Mathematical understanding and proving abilities: experiment with undergraduate student by using modified Moore learning approach. *Journal on Mathematics Education*, 2(2), 231-250. <http://doi.org/10.22342/jme.2.2.751.231-250>.
- Mejia-Ramos, J.P., & Inglis, M. (2009). Argumentative and proving activities in mathematics education research. In F-L. Lin, F-J. Hsieh, G. Hanna, & M. de Villiers (Eds), *Proceedings of the ICMI Study 19 Conference on Proof and Proving in Mathematics Education: Vol. 1.* (pp. 88-93). Taipei, Taiwan: National Taiwan Normal University.
- Mejia-Ramos, J.P., & Weber, K. (2014). Why and how mathematicians read proofs: Further evidence from a survey study. *Educational Studies in Mathematics*, 85(2), 161-173. <https://doi.org/10.1007/s10649-013-9514-2>.
- Mejia-Ramos, J.P., Fuller, E., Weber, K., Rhoads, K., & Samkoff, A. (2012). An assessment model for proof comprehension in undergraduate mathematics. *Educational Studies in Mathematics*, 79(1), 3-18. <https://doi.org/10.1007/s10649-011-9349-7>.
- Morselli, F. (2006). Use of examples in conjecturing and proving: An exploratory study. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Ed), *Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education: Vol. 4.* (pp. 185-192). Prague: PME.
- Mumu, J., Prahmana, R.C.I., & Tanujaya, B. (2018). Construction and reconstruction concept in mathematics instruction. *Journal of Physics: Conference Series*, 943(1), 012011. <https://doi.org/10.1088/1742-6596/943/1/012011>.
- Ndemo, Z., Zindi, F., & Mtetwa, D. (2017). Mathematics Undergraduate Student Teachers' Conceptions of Guided Inductive and Deductive Teaching Approaches. *Journal of Curriculum and Teaching*, 6(2), 75-83.
- Noto, M.S., Priatna, N., & Dahlan, J.A. (2019). Mathematical proof: The learning obstacles of preservice mathematics teachers on transformation geometry. *Journal on Mathematics Education*, 10(1), 117-126. <https://doi.org/10.22342/jme.10.1.5379.117-126>.
- Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, 5(1), 17-26. <https://doi.org/10.22342/jme.5.1.1445.17-26>.

- Punch, K.F. (2005). *Introduction to Social Science Research: Quantitative and Qualitative Approaches*. London: Sage.
- Putri, R.I.I., & Zulkardi. (2018). Higher-order thinking skill problem on data representation in primary school: A case study. *Journal of Physics: Conference Series*, 948(1), 012056. <https://doi.org/10.1088/1742-6596/948/1/012056>.
- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education*, 9(1), 41-54. <https://doi.org/10.22342/jme.9.1.5049.41-54>.
- Selden, J., & Selden, A. (2009). Understanding the proof construction process. In F.-L. Lin, F.-J. Hsieh, G. Hanna, M. de Villiers (Eds), *Proceedings of the ICMI Study 19 Conference: Proof and Proving in Mathematics Education: Vol. 2*. (pp.196-201). Taipei, Taiwan: National Taiwan University.
- Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings*, 1952(1), 020019. <https://doi.org/10.1063/1.5031981>.
- Stylianides, A.J. (2011). Towards a comprehensive knowledge package for teaching proof: A focus on the misconception that empirical arguments are proofs. *Pythagoras*, 32(1), 1-10.
- Stylianides, A.J., & Stylianides, G.J. (2009). Proof constructions and evaluations. *Educational Studies in Mathematics*, 72(3), 237-253. <https://doi.org/10.1007/s10649-009-9191-3>.
- Stylianou, D., Blanton, M.L., & Rotou, O. (2015). Undergraduate students' understanding of proof: Relationships between proof conceptions, beliefs, and classroom experiences. *International Journal of Research in Mathematics Education*, 1(1), 91-134. <https://doi.org/10.1007/s40753-015-0003-0>.
- Varghese, T. (2009). Secondary-level teachers' conceptions of mathematical proof. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, 1, 1-13.
- Weber, K., & Mejia-Ramos, J.P. (2014). Mathematics majors' beliefs about proof reading. *International Journal of Mathematics Education in Science and Technology*, 45(1), 89-103. <https://doi.org/10.1080/0020739X.2013.790514>.
- Weber, K., & Mejia-Ramos, J.P. (2015). On relative and absolute conviction in mathematics. *For the Learning of Mathematics*, 35(2), 3-18.
- Yang, K., & Lin, F.L. (2008). A model of reading comprehension of geometry proof. *Educational Studies in Mathematics*, 67(1), 59-76. <https://doi.org/10.1007/s10649-007-9080-6>.



THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA

Heris Hendriana¹, Rully Charitas Indra Prahmana², Wahyu Hidayat¹

¹IKIP Siliwangi, Jalan Terusan Jenderal Sudirman, Cimahi 40526 Cimahi, Indonesia

²Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo 55161, Yogyakarta, Indonesia
Email: rully.indra@mpmat.uad.ac.id

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used as a basic concept of multiplication. Finally, the students were able to understand the concept of multiplication more easily, and they showed interest in using this learning trajectory.

Keywords: multiplication, learning trajectory, design research, rural area

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research, daerah pedesaan

How to Cite: Hendriana, H., Prahmana, R.C.I., & Hidayat, W. (2019). The Innovation of Learning Trajectory on Multiplication Operations for Rural Area Students in Indonesia. *Journal on Mathematics Education*, 10(3), 397-408. <https://doi.org/10.22342/jme.10.3.9257.397-408>.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al. 2012). It is because learning number operations involves an understanding of symbols, notation, and reference number (or other forms to represent) (NCTM, 2000), and it also plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operations would be one of the prior knowledge that students must have to learn other topics in

mathematics.

The concept of number operations, especially in multiplication, is one of the students' difficulties in understanding mathematics concepts (Ahmad, 2010; bin Syed Ismail, 2010; Drews, et al. 2005; Kilian, et al. 1980; Tanujaya, et al. 2017; Unlu & Ertekin, 2012). Teachers usually teach multiplication operations using symbolic form or something abstract (Unlu & Ertekin, 2012). As a result, students learn multiplication operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010). They also have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010). On the other hands, they also have a poor understanding of the place value (tens and ones) concept concerning multiplication (Drews, et al. 2005; Kilian, et al. 1980).

The result of the previous research explored about number operations is in line with the preliminary classroom observation results of the rural area's students came from, namely Serui, Ambon, and Sorong Selatan. Teachers introduced the concept of division using the formula without involving the concept itself (Prahmana & Suwasti, 2014). Therefore, this research focuses on multiplication operation as one of the concept of number operation that students must be mastered to support their knowledge in learning another mathematics subject.

Several studies indicated that constructivism approach could improve students' understanding of learning multiplication (Ahmad, 2010; Prahmana, et al. 2012; Chang, et al. 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana & Suwasti, 2014; Prahmana, 2015; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This method has been applied to students from rural areas in Indonesia starting from the introduction of integer number and number operations (Prahmana & Suwasti, 2014; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This situation underlies the researchers of this present study to try designing learning trajectory on number operations especially for multiplication operation in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Indonesia. Therefore, the focus of this study is to describe the learning activities on students' performance to do multiplication in Math GASING. It is also because several researcher stated that Math GASING is the suitable method to use in teaching mathematics, especially number operation, more easy, fun, and meaningful.

Finally, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to a formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for the students.

In this research, the literature on Math GASING and number operations are studied as basic knowledge to design sequential activities that will be passed by students ranging from concrete situations to abstract levels. All literature will be explained further in the next section.

Number Operations

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al. 1998). The following four relations operation that has a relationship with each other, and students must understand the relationships. Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING, such as:

1. Multiplication and division are inverse operations

$$4 \times 6 = 24 \text{ ----- } 24 : 4 = 6$$

2. Multiplication can be seen as a repeated addition

$$4 \times 6 \text{ ----- } 6 + 6 + 6 + 6$$

3. Division can be seen as a repeated subtraction

$$24 : 6 \text{ ----- } 24 - 6 - 6 - 6 - 6$$

Mathematics GASING

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns, and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic math facts is easy if it is based on conceptual learning and visual representations.

The learning process makes students’ learning easy (*GAmpang*), fun (*ASyIk*), and enjoyable (*menyenaNGkan*) in Math GASING (Shanty & Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember — exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). On the other hand, Prahmana (2013) stated that Math GASING shows how to change a concrete sample into an abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING, as one of the innovations in learning mathematics, offers critical point in its learning process. The critical point of GASING means the condition that students must pass during the learning process and studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya & Moss, 2012). The critical point in learning multiplication is that students must master the multiplication concept of 1×1 to 10×10 . Students could learn various problems of multiplication operations more easily after passing a critical point.

This research uses Math GASING to describe the learning outcomes of rural area's student in learning multiplication as a repeated addition and see student responses. Researchers conducted research on rural area's students because students have experience difficulties in multiplication operations based on the pre-evaluation results. In addition, students are less focused, less accurate in counting, and easy to forget. Therefore, this study could be solved the students' mathematical problem by using Math GASING.

Hypothetical Learning Trajectory

Hypothetical Learning Trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina, et al. 2011). Furthermore, Prahmana (2017) stated that HLT is a hypothesis or prediction of how students' thinking and understanding develop in a learning activity. The HLT in this study had several learning goals expected to be reached by the students during one phase.

METHOD

Design research is used as the research method of this study. Design research consists of five characteristics, such as interventionist nature, process-oriented, reflective component, cyclic character, and theory-oriented (Akker, et al. 2006; Gravemeijer, 2004; Prahmana, 2017). There are two important aspects related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning trajectory taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). The first component is the purpose of mathematics teaching for the students. Secondly, it is the sequence activity that students must do during the learning process. Lastly, the conjecture is the various answers, strategies, and models that researcher expected from student understanding that emerge and develop when learning activities are carried out in class. Furthermore, there are three phases of design research, such as preliminary design, teaching experiment, and retrospective analysis that can be seen in Figure 1.

The research data came from various data sources. All data sources used aim is to get a visualization of mastery of the basic concepts of student multiplication operations. There are documentation (photos), video, student worksheets, and observation sheets. Furthermore, the data were analyzed retrospectively with HLT as a guide. This research was conducted and completed in 2 days. The research subjects are 11 matriculation teacher candidates at one of the College of Teacher Training and Education in Tangerang. All research subjects came from rural areas in Indonesia, such as Yapen, Ambon, South Sorong, Serui, and also a teacher model.

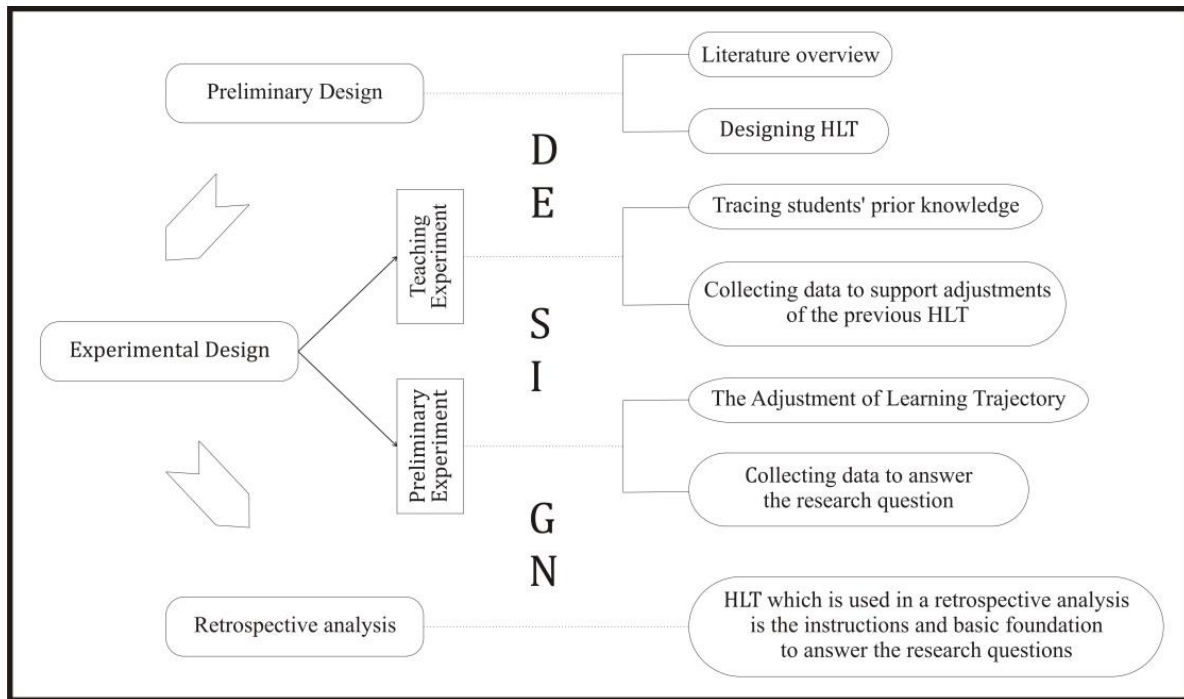


Figure 1. The phase of the design research (Prahmana, et al. 2012).

RESULTS AND DISCUSSIONS

The learning activities start from making the same perceptions of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, the students were trained to memorize the multiplication for 1 to 10 using several methods. Lastly, the teacher provides an evaluation to study students' understanding of multiplication by using mental arithmetic activities namely *mencongak* as one of the evaluation processes in these learning activities and exercises using student worksheet and also evaluation sheets. The results show that students master the multiplication operations based on the final evaluation results. On the other hands, the important results is student would like to leave the old way in learning mathematics and change to the Math GASING way. Furthermore, another results indicate that the design of multiplication learning operations in Math GASING has a crucial role as a starting point and increases student motivation in learning. The details would be discussed in the further section.

Preliminary Design

The researchers start to do literature review, conduct observation, and design the learning trajectory as a sequence of instructional learning for the learning of multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). The activities are designed by HLT consisting of six activities for two meetings through several easy, fun, and enjoyable activities. Students should be interested and engaged during the learning process. The last activity is evaluation process by using student worksheet and also evaluation sheet to measure the understanding of student in learning multiplication.

Table 1. Overview of the learning trajectory of multiplication (adapted from Surya (2011)).

Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box," for example 2×3 means 2 boxes are containing three things in that box, and so on.
Using some method to memorize this part easier	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students can master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as $1 \times 1, 2 \times 2, \dots, 10 \times 10$	Students learn about the same numbers of multiplication, such as $1 \times 1, 2 \times 2, 3 \times 3, \dots, 10 \times 10$.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. The teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the student ability in learning multiplication	Teacher evaluates the student about the multiplication problem in the formal and informal form.

Teaching Experiment

Teaching experiment phase consists of several activities that already design in the preliminary stage. In these phases, researchers implement the learning activities using HLT as a teacher guide for the teacher model. The various educational games provided are to make teaching and learning activities more fun and enjoyable for students. This activity is one of the characteristics of learning Math GASING.

The five activities conduct using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box," for example 2×3 means 2 boxes are containing three things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students can master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different from the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5.

Furthermore, students learn about the same numbers of multiplication, such as 1×1 , 2×2 , 3×3 , ... , 10×10 . Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.

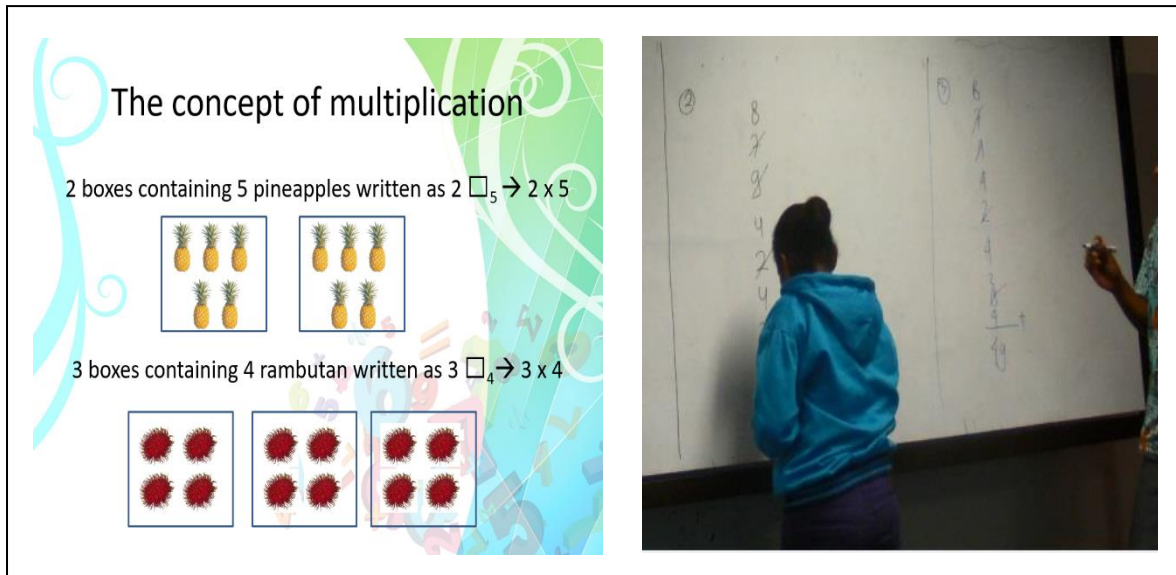


Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

There are some differences between the multiplication process in Math GASING and the multiplication process in general. These differences are the answer for the research question in this research. The difference is manifested in the learning trajectory to be analyzed retrospectively.

The designing learning trajectory seen in Table 1 is the student-guided activities to mastering the multiplication operations. Therefore, the researcher designed an activity using Math GASING aids. The goal is that students can understand the concrete form of multiplication using the understanding of boxes and something in there. The student must understand that multiplication in the form of repeated addition. The teacher used combination learning tools such as presentation and whiteboard to make learning process effective and efficiency that can be seen in Figure 2. Next, the teacher guides students lead the concept of multiplication as a form of repeated addition during this activity. The teacher uses several methods to remember doubling for one to ten more easily and meaningfully.

On the other hands, teacher makes the order of memorizing the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1×1 , $2 \times$

2, 3×3 , ... , 10×10 . After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

The researchers used multiplication learning phases in the Math GASING (Table 1). The introduction activity in learning the basic concept of multiplication have several good discussion. During the discussion, students look like easy, fun, and enjoyable in learning multiplication in Math GASING. Therefore, the learning trajectory guides students to understand the concept of multiplication.

All activities describe the process of students understanding from informal to a formal level according to the multiplication concept. Their experience supported by the Math GASING learning aids can make students pass the critical point of multiplication so that students can master multiplication as a whole. Surya and Moss (2012) stated that student would be able to master the mathematics subject regarding in Math GASING after their pass the critical point of the subject.

The results show that the students can apply the multiplication in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or other words, the design of this study can be used as the starting point of learning multiplication. In the last activities, teacher gives evaluation to measure the students' understanding in multiplication that can be seen in Figure 3.



Figure 3. Student evaluation process using student worksheet.

The retrospective analysis has shown that one of the ways making student understanding in learning multiplication makes the learning process can be imaging for students. This result is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hand, learning environment also can support the result of the

learning process (Putri, et al. 2015; Nuari, et al. 2019). Finally, all students can solve several problems and exercises regarding multiplication operation.

CONCLUSION

The learning of multiplication operation in Math GASING have a significant role as the starting point and improve students' motivation in learning multiplication. Also, the designed students' activities find the multiplication concept. The activities are starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10 , which is the critical point in learning multiplication in Math GASING. The students solve several multiplication problems more easily after passing the critical point. Lastly, students can do mental arithmetic for any given multiplication problem and answer many multiplication questions very quickly and precisely. Both of evaluation is the characteristics of the assessment in Math GASING.

ACKNOWLEDGMENTS

Firstly, we thank to STKIP Surya for providing the opportunity to do this research and give facilitated until this research is completed. Then, we also thank to Petra Suwasti as a teacher model and all of students as a research subject in this research.

REFERENCES

- Ahmad, N.S.B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences*, 8, 72-78. <https://doi.org/10.1016/j.sbspro.2010.12.010>.
- Akker, J.V.D., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.
- bin Syed Ismail, S.A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences*, 8, 129-133. <https://doi.org/10.1016/j.sbspro.2010.12.018>.
- Caron, T.A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 80(6), 278-282. <https://doi.org/10.3200/TCHS.80.6.278-282>.
- Chang, K. E., Sung, Y.T., Chen, Y.L., & Huang, L.H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in Human Behavior*, 24(6), 2904-2916. <https://doi.org/10.1016/j.chb.2008.04.015>.
- Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, 125(2), 271-279.
- Drews, D., Dudgeon, J., Hansen, A., Lawton, F., & Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.
- Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Kluwer Academic Publishers.
- Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, 6(2), 105-128.

https://doi.org/10.1207/s15327833mtl0602_3.

- Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, 30(4), 1365-1375. <https://doi.org/10.1016/j.neuroimage.2005.11.016>.
- Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25. Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 27(5), 22-25.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. *Journal on Mathematics Education*, 10(1), 127-142. <https://doi.org/10.22342/jme.10.1.6913.127-142>.
- Prahmana, R.C.I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.
- Prahmana, R.C.I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, 5(1), 49-61.
- Prahmana, R.C.I. (2017). *Design Research (Teori dan Implementasinya: Suatu Pengantar) [Design Research (Theory and Its Implementation: An Introduction)]*. Depok: Rajawali Pers.
- Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, 5(1), 17-26. <https://doi.org/10.22342/jme.5.1.1445.17-26>.
- Prahmana, R. C. I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, 3(2), 115-132. <https://doi.org/10.22342/jme.3.2.1931.115-132>.
- Putri, R.I.I., Dolk, M., & Zulkardi. (2015). Professional development of PMRI teachers for introducing social norms. *Journal on Mathematics Education*, 6(1), 11-19. <https://doi.org/10.22342/jme.6.1.1900.11-19>.
- Revina, S., Zulkardi, Darmawijoyo, & Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, 2(2), 127-146. <http://dx.doi.org/10.22342/jme.2.2.745.127-146>.
- Reys, R.E., Suydam, M.N., Lindquist, M.M., & Smith, N.L. (1998). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.
- Shanty, N.O., & Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, 3(2), 175-186. <http://dx.doi.org/10.22342/jme.3.2.603.175-186>.
- Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING [Teachers' Guide: Smart Basics of Counting's GASING]*. Tangerang: PT. Kandel.
- Surya, Y., & Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National

University of Education, Seoul, South Korea Selatan, 6223-6229.

- Tanjaya, B., Prahmana, R. C. I., & Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, 15(3), 287-291.
- Unlu, M., & Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions?. *Procedia-Social and Behavioral Sciences*, 46, 490-494. <https://doi.org/10.1016/j.sbspro.2012.05.148>.

TEACHERS EXPECTATION OF STUDENTS' THINKING PROCESSES IN WRITTEN WORKS: A SURVEY OF TEACHERS' READINESS IN MAKING THINKING VISIBLE

Abdur Rahman As'ari¹, Dian Kurniati², Subanji¹

¹Universitas Negeri Malang, Jl. Semarang No. 5, Malang, Indonesia

²Universitas Jember, Jl. Kalimantan No. 37, Jember, Indonesia

Email: abdur.rahman.fmipa@um.ac.id

Abstract

The trends of teaching mathematical thinking and the existence of two thinking skills (critical dan creative thinking) the required by 21st-century skills have created needs for teachers to know their students' thinking processes. This study is intended to portray how mathematics teachers expect their students showing their thinking processes in students' written work. The authors surveyed Whatsapp and Telegram group of mathematics teachers. First, the authors shared the result of the literature review and the governmental regulations about the need to develop thinking skills. Second, the authors stated that the potentials of students' written works as a tool for knowing students' thinking processes. Third, the authors sent a simple mathematical problem with the topic of algebra and asked the mathematics teachers how should their students answer that problem such that they can easily monitor and assess their students' thinking processes. A total of 25 teachers participated voluntarily in this survey. Results of the survey were triangulated with direct trial data in lecture classes at both undergraduate and postgraduate levels. The result indicates that participating mathematics teachers do not expect too much for their students to show their thinking processes in written work. Teacher's focus is mostly on the accuracy and the correctness of their students' mathematics answer.

Keywords: mathematics teachers' expectation, making thinking visible, thinking processes, written works

Abstrak

Kecenderungan tentang pembelajaran berpikir matematis, dan keberadaan dua keterampilan berpikir (kritis dan kreatif) sebagai bagian dari keterampilan abad ke-21, telah menjadikan guru perlu mengetahui proses berpikir para siswanya. Sebagai salah satu alat yang dapat digunakan untuk melihat proses berpikir siswa, yaitu pekerjaan tertulis siswa, penelitian ini dimaksudkan untuk memotret bagaimana guru matematika mengharapkan para siswanya menunjukkan proses berpikirnya pada tugas tertulis. Peneliti mengadakan survey terhadap kelompok guru yang membentuk grup guru dalam Whatsapp dan Telegram. Pertama, peneliti membagikan hasil kajian pustaka dan peraturan menteri tentang perlunya mengembangkan keterampilan berpikir. Kedua, peneliti menyampaikan perlunya memanfaatkan tugas tertulis siswa sebagai wahana untuk mengetahui proses berpikir siswa. Ketiga, peneliti mengirimkan satu soal matematika sederhana dengan topik aljabar, dan meminta guru menuliskan jawaban yang seharusnya ditulis oleh siswa sehingga para guru dengan mudah mengetahui proses berpikir siswanya. Sebanyak 25 orang guru yang bergabung pada kedua grup aplikasi berpartisipasi dalam penelitian ini tanpa paksaan. Hasil dari survey dilakukan triangulasi dengan data ujicoba langsung di dalam kelas perkuliahan baik di tingkat sarjana ataupun pascasarjana. Hasil menunjukkan bahwa guru-guru matematika tidak berkeinginan untuk mengetahui proses berpikir siswanya dari pekerjaan tertulisnya. Guru matematika lebih terfokus kepada ketepatan dan kebenaran konten matematika pada jawaban tertulis siswanya.

Kata kunci: harapan guru matematika, menjadikan berpikir terlihat, proses berpikir, pekerjaan tertulis

How to Cite: As'ari, A.R., Kurniati, D., & Subanji. (2019). Teachers Expectation of Students' Thinking Processes in Written Works: A Survey of Teachers' Readiness in Making Thinking Visible. *Journal on Mathematics Education*, 10(3), 409-424. <https://doi.org/10.22342/jme.10.3.7978.409-424>.

Mathematical thinking now becomes the focus of mathematics instruction (Hollebrands & Dick, 2009; Stacey, 2006). Mathematical thinking is useful for students and also for teachers (Stacey, 2006). For students, mathematical thinking is a way of learning mathematics. Through specializing,

generalizing, conjecturing, and convincing, which are the fundamental processes of mathematical thinking, students can learn mathematics effectively. They can understand the mathematical concepts, principles, procedures deeply, and even better solve mathematical problems. For the teachers, Stacey (2006) claimed that mathematical thinking could help mathematics teachers provide suitable lessons for their students. The Royal Society (2014) stated that the ability to think mathematically is useful for decision making either in the classroom context or in real life situations.

Considering the importance of teaching mathematical thinking, The National Research Center (Burns et al., 2006) has created a publication on a comprehensive approach for teaching thinking skills. Even a book illustrating the use of tasks and questions to strengthen the thinking practices and processes is just currently published (Small, 2017). Chinese education, even, evolutionarily has set mathematical thinking as the focus of mathematics instruction (Li, Mok, & Cao, 2019). In addition, several other countries, namely Japan, Singapore, and Australia, also focus on mathematical thinking in every implementation of mathematics learning in class (Katagiri, 2004). So, teaching mathematical thinking now become international trends in mathematics teaching. Consequently, mathematics teaching in Indonesia should also change the focus into teaching mathematical thinking rather than mathematical content acquisition.

Furthermore, in this 21st century, two types of thinking skills (critical thinking and creative thinking skills) are considered as 21st-century skills. Several scholars have set the 4Cs (critical thinking, creative thinking, collaboration, and communication) as the essential skills needed a human being to live in the 21st century (Kay, 2009; National Education Association, 2014; Pacific Policy Research Center, 2010; Vockley & Lang, 2008). In the year 2016, the government of Indonesia through the Ministry of Education and Culture has set these thinking skills as the goals of educational practices in primary and secondary school level (Menteri Pendidikan dan Kebudayaan Republik Indonesia, 2016). Teachers should implement teaching and learning activities, which could lead their students to have critical and creative thinking skills.

Therefore, Indonesian mathematics teachers should also shift their teaching focus from content acquisition into developing thinking skills. This call is truly appropriate since facts indicated that Indonesian students have the lowest score on Programme for International Student Assessment (Argina, Mitra, Ijabah, & Setiawan, 2017) who measured students' reasoning and thinking skills (OECD, 2017; Prenzel, Blum, & Klieme, 2015; Stacey, 2011). Even prospective mathematics teachers do not have excellent thinking skills. They are not critical thinkers yet (As'ari, Mahmudi, & Nuerlaelah, 2017).

To make the teaching thinking skills is carried out optimally, teachers need to continuously know the level and process of their students' thinking skills. They need to know how their students use their thinking skills in understanding the mathematical concepts, principles, procedures, and in solving the problems. Without understanding students' thinking, teachers will not be able to provide

suitable teaching treatments. Therefore, teachers should encourage their students to show their thinking process on every occasion. Teachers need to make their students' thinking is visible.

Many ways can be used by teachers to make students' thinking visible. Teachers may ask their students to think aloud, write reflective journals, involve in focus group discussion, and produce written answers of mathematical tasks. Compare to the other types of tasks; the students' written works is auspicious. The students' written work has some advantages. First, it can be done regularly every day. Second, it can be integrated with the teaching of mathematical content. Third, it is very easy to record and analyze the students' works. Fourth, teachers and students are familiar with this type of assignment.

From the review of literature, many scholars have tried to study students work that mostly related to feedback provision (Gjerde, Padgett, & Skinner, 2017; Fu & Nassaji, 2016; Cohen-Sayag, 2016; Chaqmaqchee, 2015; Fonseca et al., 2015; Osterbur, Hammer, & Hammer, 2016; Costello & Crane, 2013). None of these existing studies are talking about teachers expectation of their students' written work. The only research result available related to teachers' expectation says that having several times involvement in workshops about working on challenging tasks, teachers finally expect students to persist when working on challenging task (Clark, Roche, Cheeseman, & Sullivan, 2014). There is no information available about the type of mathematical thinking processes that should be incorporated into students' work.

The existence of teachers' examples of how to develop written works where thinking processes are shown in it is very important. Examples given by mathematics teachers represent their expectation on students' mathematical thinking processes. From the examples, teachers could guide the students what and how their mathematical thinking processes should be described in their written works. Examples are references for students. If the students follow the examples, teachers will have a chance to understand better their students' mathematical thinking skills. Zhu, Yu, & Cai (2018) stated that understanding students' thinking is helpful for their teaching practice. On the other hands, Neumann (2014) explains that teachers should listen, probe, interpret and responds to students' thinking.

This article is a report of the authors' investigation about teachers expectation in incorporating their students' mathematical thinking through written works. From the study, it can be seen whether the expected thinking processes by the teachers are useful or not. Therefore, the result of this study is significant for the development of teaching thinking. This study provides a portray of the current mathematics education practices which can be used to provide a recommendation of better practices in the future. Mathematics teacher educators can use the result of this study as insights on how to create the best tasks for their pre-service mathematics teacher students to enable the students to have many ideas on how to monitor and evaluate students' thinking processes. In-service mathematics teacher trainers may also use the result of this study to train and up-grade existing mathematics teachers' competencies on how to encourage their students to produce better students works that express their thinking skills.

METHOD

General Background of Research

This research was a qualitative descriptive study, as it was aimed at knowing mathematics teachers expectation in incorporating their students' mathematical thinking through written works.. Stages of this research referred to the stage of a qualitative content analysis method. Written works referred to in this reserach were students' written answers expected by mathematics teachers to determine their mathematical thinking processes.

Participants

Participants in this study were 25 mathematics teachers who joined both WhatsApp and Telegram groups. The conditions for determining the research participants were (1) mathematics teacher who have teaching experience between 10-15 years, (2) mathematics teachers who respond to problems given by writing down the written works, and (3) the mathematics teacher who are willing to be the research participants.

Instruments

To best way to know teachers' expectation about the mathematical thinking processes that their students should incorporate into their written works is through asking the teachers to provide examples of how students should express their thinking processes in their written works. The examples have the potential as the reference for the students to write their responses. The examples could function as the standards criteria for the students to develop their written responses. Therefore, in this study, the authors send a simple mathematics problem to the participating mathematics teachers and they are then expected to pretend as students, They have to write the solution to the problem such that all of the thinking processes required to solve the problem are made visible in it. The simple mathematical problem sent to the group was displayed in Figure 1.

Original Problem	Translation
Harga dua topi dan dua kaos adalah Rp100.000. Harga 3 topi dan 1 kaos adalah Rp80.000. Berapa harga 1 topi?	The price of two hats and two shirts is Rp100.000. The price of three hats and one shirt is Rp80.000. What is the price of one hat?

Figure 1. The Simple Mathematical Problem

This simple problem was used in this study for several reasons. First, the authors wanted the participating teachers focused mostly on their thinking processes. The authors did not want these teachers to reject to response caused by the difficulty of the problem. Second, there are several thinking processes required to solve this problem; (1) the mathematics teachers should use apply horizontal mathematizing to develop mathematical representation suitable to the problem, (2) the mathematics teachers should also employ mathematical things to simplify the mathematical expression, and (3) the mathematics teachers should use reasoning skills to interpret the result correctly and evaluate the appropriateness of the answer.

Data Collection Methods

In a digital era now, Whatsapp and Telegram are two among many mobile phone application that facilitates teachers to communicate and share ideas, research finding, and other things. These applications have been utilized by most teachers, including mathematics teachers, to communicate with each other. Even, 2985 mathematics teachers across the nation have organized themselves as a group in Telegram application where they routinely discuss and share many aspects of mathematics teaching (see Figure 2). Groups of a smaller number of mathematics teachers may use Whatsapp application to communicate and share their ideas. Therefore, the authors decided to use these two mobile phone applications as the way for data collection.

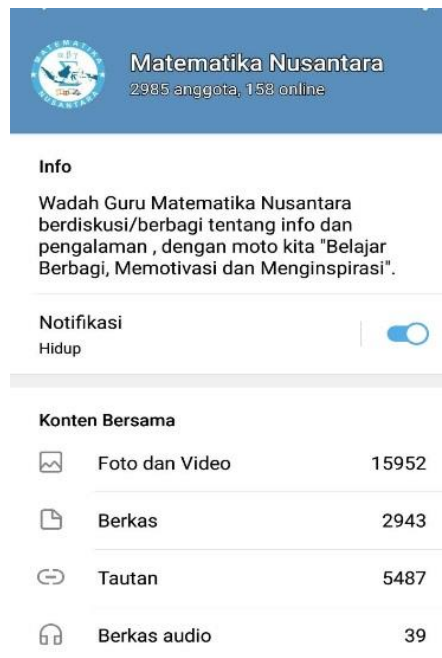


Figure 2. The Screenshot of Telegram Group

The teachers were asked to write the best-expected of their students' written work so that they can see their students' mathematical thinking processes. The authors assumed that teachers' answer expressed in the written answer is the one that they are expecting from their students.

Data Analysis

Answers from participated respondents were then analyzed qualitatively using content analysis. The stages were (1) providing math problems in both groups namely Whatsapp and Telegram, (2) asking to mathematics teachers who were in the group with 10-15 years of teaching experience to respond to the problems given by writing down the answers and sending photos of the answers in the group, (3) leveling research participants in 3 levels of mathematics knowledge based on teaching experience, namely advanced level, good level, and medium or lover level, (4) analyzing written works that given a mathematics teacher as a research participant, (5) triangulating the data with the answers of prospective mathematics teacher at the undergraduate or postgraduate levels, and (6)

summarizing the expectations of the mathematics teacher to their students by showing their thought processes on written works.

RESULTS AND DISCUSSIONS

A number of 25 mathematics teachers responded to the survey who join both the WhatsApp and Telegram groups. By analyzing their responses, it can be seen that all are related to its mathematical content. The focus of their answer is showing the procedure that should students used to solve the problem. Based on the data analysis, there are no indications that they are expecting their students to show and communicate their thinking processes. Students' thinking processes seem to be implicit and invisible. Students' thinking processes were assumed to be automatic, self-contained, and there is no expectation from the teacher side that their students need to explain the reasons for every step of their answer. It seems that the teachers do not want to know and understand the thinking processes happening in their students' mind when the students are working to solve the problem. Teachers did not take care of students' thinking processes in solving the problem. Teachers seem to assume that students' mathematical thinking is good if they can provide the correct answer to the problems.

To make it clear, following are 25 answers provided by 25 mathematics teachers, consists of 5 teachers with advanced level mathematical knowledge, 15 teachers with good mathematical knowledge, and 5 teachers with medium or lower mathematical knowledge. The only reason for showing teachers' answers by their mathematical competencies is that although they have different mathematical competencies, they have the same expectation. They expect that the correct answer to the mathematical aspect is the most important in their students' written work.

One of the participants for the advanced level mathematics teachers is Participant #1 replied authors' request explained in Figure 3.

<p>Harga dua topi dan dua kaos adalah 100.000 rupiah. Harga tiga topi dan satu kaos adalah 80.000 rupiah. Berapa harga satu topi?</p> <p>Alternatif jawaban: Misal, x = harga satu topi y = harga satu kaos</p> <p>Dari soal diperoleh $2x + 2y = 100.000 \Rightarrow x + y = 50.000 \dots (1)$ $3x + y = 80.000 \dots (2)$</p> <p>Eliminasi y pada persamaan (1) dan (2) $x + y = 50.000$ $3x + y = 80.000$ $\underline{-2x = -30.000 \Rightarrow x = 15.000}$</p> <p>Jadi, harga satu topi adalah 15.000</p>	<p>Translation: The price of two hats and two t-shirts are IDR100.000. The price of three hats and one t-shirt is IDR80.000. What is the price of one hat?</p> <p>Alternative solution Let $x = \text{the price of one hat}$ $y = \text{the price of one t-shirt}$ From the problem, therefore $2x + 2y = 100.000 \rightarrow x + y = 50.000 \dots (1)$ $3x + y = 80.000 \dots (2)$ Eliminating y in equation (1) dan (2) $x + y = 50.000$ $3x + y = 80.000$ $\underline{-2x = -30.000 \rightarrow x = 15.000}$ So, the price of one hat is IDR15.000</p>
--	---

Figure 3. One of Five Advanced Mathematics Teacher's Expectation

He is an outstanding and very famous mathematics teacher in the group, even in Indonesia. He is a very experienced mathematics teacher. He has also written books for national exam preparation, and he also manages his blog for mathematics education. He is known as a very strategic mathematics tutor in helping students and other teachers to solve mathematics exams smartly, efficiently, and effectively.

From his written response (Figure 3), it can be seen that he is very good at mathematics. He knows that x and y are symbols of variable, and therefore, he let x and y correctly as the price of each thing, not the price of things. However, he did not mention the origin of the equation $2x + 2y = 100.000$. He just mentioned "from the given information" as its premis for writing the equation. He did not expect his students to write a *bridge* statement, namely: "Since 100.000 is the price of two hats and two shirts, then $2x + 2y = 100$ " which will make him better understand his students' thinking processes. It seems that writing this bridge statement, which telling teachers about their students thinking process, is not important for him.

In another chance, he writes a procedure to solve the problem, namely: elimination procedure. He did not expect their students to show the reasons why he/she choose that procedure. There are no indicators that he wants their students to express their thinking process during comparing and contrasting all the procedures that can be used to solve the problem, process of deciding the most appropriate procedure to solve the problem. He did not pay attention to students thinking processes. He cares more on the correctness of its mathematical answer. The mathematical thinking process is allowed to be implicit and invisible.

One of the participants for the good level mathematics teachers is Participant #2 replied authors' request described in Figure 4.

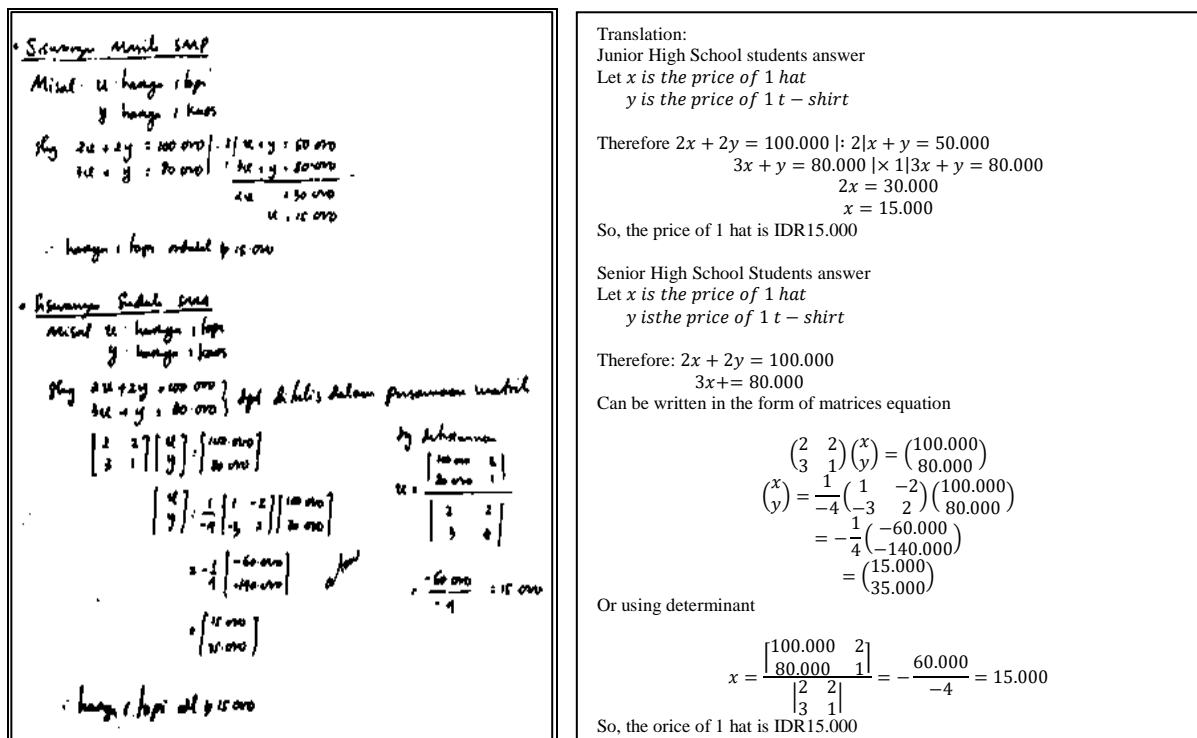


Figure 4. One of Fifteen Good Mathematics Teacher's Expectation

He is also an excellent mathematics teacher. However, he is not as productive as participant 1. He just regular good mathematics teacher who teaches mathematics at a big city in East Java. He is good in mathematics. His mathematical concept about the variable is good. His ability to let x as the price of 1 hat (not the price of hats or the hat itself) shows that he understand the concept variable correctly. He shows that he is also mastery in employing mathematical procedures to solve the problem. Even, he shows that he has good pedagogical content knowledge for the topic. He showed two different approaches that can be used by students from different level to solve the problems.

Unfortunately, his response also indicated that he did not pay attention to the mathematical thinking processes that should be expressed by his students. He also focused on the mathematical aspects of the solution. He did not show the origin of coefficient 2 from expression $2x + 2y = 100.000$. He just writes it down with no explanation.

When he used $\begin{cases} 2x + 2y = 100.000 & | \div 1 | & x + y = 50.000 \\ 3x + y = 80.000 & | \div 2 | & 3x + y = 80.000 \end{cases}$, which resulting in $2x = 30.000$,

there is no information at all about the thinking processes involved in it. The reader will never know the reasons why he chose to use division rather than multiplication in the problem-solving process. The thinking process is invisible and implicit. The students' work did not tell automatically all of their mathematical thinking processes. Students' written works do not function well in inspiring the teachers about the thinking processes done by students. Teachers need additional efforts to know and understand students mathematical thinking processes.

The tendency of the response given by participant 2 has the same tendency as the Graduate Mathematics Education student namely her mathematical concept about the variable is good (see Figure 5). Her ability to let x as the price of 1 hat (not the price of hats or the hat itself) and let y as the price of 1 t-shirt shows that she understand the concept variable correctly. Even, she shows that she has good pedagogical content knowledge for the topic especially the process of elimination. But, her response also indicated that she did not pay attention to the mathematical thinking processes that should be expressed by her students. According to the authors, students only focus on the truth of the solution of the problem given.

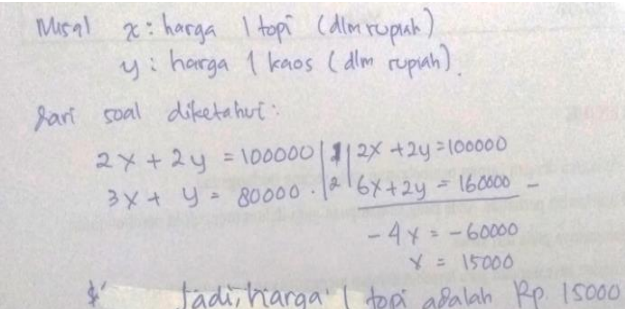
 <p>Misal x: harga 1 topi (dlm rupiah) y: harga 1 kaos (dlm rupiah). dari soal diketahui: $2x + 2y = 100000$ 1 $2x + 2y = 100000$ $3x + y = 80000$ 2 $6x + 2y = 160000$ - $-4x = -60000$ $x = 15000$ Jadi, harga 1 topi adalah Rp. 15000</p>	<p>Translation</p> <p>Let: x = price of 1 hat (in IDR) y = price of 1 t-shirt (in IDR)</p> <p>Form the given: $2x + 2y = 100000$: 1 : $2x + 2y = 100000$ $3x + y = 80000$: 2 : $6x + 2y = 160000$ $-4x = -60000$ $x = 15000$</p> <p>So the price of one hat is IDR. 15.000.00</p>
--	--

Figure 5. Response of Graduate Student

Another the women participant for the good level mathematics teachers is Participant #3 replied authors' request as seen in Figure 6.

Misal $t = \text{harga topi}$	Translation Let $t =$ the price of hats $K =$ the price of t-shirt Therefore, from the information given (1) $2t + 2k = 100.000$ $t + k = 50.000$ (2) $3t + k = 80.000$ $2t + t + k = 80.000$ $2t + 50.000 = 80.000$ $2t = 30.000$ $t = 15.000$
$k = \text{harga kaos}$	
Berarti dari soal diketahui :	
(i) $2t + 2k = 100.000$ ($\div 2$)	
$t + k = 50.000$	
(ii) $3t + k = 80.000$	
$2t + t + k = 80.000$	
$2t + 50.000 = 80.000$	
$2t = 30.000$	
$t = 15.000$	

Figure 6. One of Fifteen Good Mathematics Teacher's Expectation

She is a junior mathematics teacher, and she has not many experiences in mathematics teaching. However, right now, she is taking her first semester of master degree in mathematics education. Therefore, in this study, the authors classify her as a good mathematics teacher too.

From her written response, it seems that her mathematical concepts could be incorrect. Instead of writing $t = \text{the price of ONE hat}$, she wrote $t = \text{the price of hats}$. She did not differentiate between "harga 1 topi" which can be translated into *the price of one hat* and "harga topi" which can be translated in, to the *prince of hats*. It seems that she did not realize that what she did may lead to different and wrong mathematical representation. She also relied on *what is known* or *what is given* as to come up with an equation $2t + 2k = 100.000$. There is no information about the thinking processes used to decide that its mathematical representation is $2t + 2k = 100.000$.

Having let $t = \text{the price of hats}$, she wrote an expression $2t + 2k = 100.000$ and followed by a sign " $(\div 2)$ ". There was no information on what the meaning of this sign and why she used it. For those who are having difficulty on solving this kind of problem, they will never know why they have to divide the each side of the equation by 2. These students will be having difficulty in answering the question, "Is dividing by 2 applicable to every problem?"

She also did not communicate the origin of $2t + t + k = 80.000$ in the (ii) part of the response. The thinking process that "since $3t = 2t + t$, and $3t + k = 80.000$, then $2t + t + k = 80.000$ was implicit. No explicit explanation was provided. According to the authors, the good mathematics teacher also did not pay attention to the thinking processes.

One of the participant for the medium or lower level mathematics teachers is Participant #4 replied authors' request as seen in Figure 7.

<p>Alternatif</p> <p>Misal topi = t kaos = k</p> $\begin{array}{r l} 2t + 2k = 100.000 & \times 1 \\ 3t + k = 80.000 & \times 2 \end{array}$ $\begin{array}{r} 2t + 2k = 100.000 \\ 6t + 2k = 160.000 \\ \hline -4t = -60.000 \\ t = \frac{-60.000}{-4} \\ t = 15.000 \end{array}$ <p>harga 1 topi : Rp 15.000,00</p>	<p>Translation Alternative Let</p> $\begin{array}{l} \text{topi} = t \\ t - \text{shirt} = k \end{array}$ $\begin{array}{l} 2t + 2k = 100.000 \quad \times 1 2t + 2k = 100.000 \\ 3t + k = 80.000 \quad \times 2 6t + 2k = 160.000 \\ 4t = -60.000 \\ t = -\frac{60.000}{-4} \\ t = 15.000 \end{array}$ <p>The price of 1 hat is IDR15.000</p>
---	--

Figure 7. One of Five Medium or Lower Mathematics Teacher's Expectation

Figure 7 shows that he is not good at mathematics. He did not show us that he has a sense of understanding about the concept of variable in mathematics. He did not realize that the equal sign in his answer was wrongly used in this expression. He did not realize that his mathematical expression is meaningless, that is: the left side (the x) represents a quantity, and the right side represents the thing that not a quantity, and those two things are related using the equation sign. However, he showed his capability to execute analytical procedures fluently. He used the procedure clearly and came up to the correct solution.

The tendency of the response given by participant 4 has the same tendency as the Postgraduate Mathematics Education student who is not able to distinguish the given variables, namely for the variable x = price of 1 hat with the variable x = hat (see Figure 8). It seems that he did not realize that what he did might lead to different and wrong mathematical representation. He also relied on what is known or what is given as to come up with an equation $2x + 2y = 100,000$. There is no information about the thinking processes used to decide that its mathematical representation is $2x + 2y = 100,000$.

<p>Jawaban :</p> <ul style="list-style-type: none"> ❖ Diketahui: harga 2 buah topi dan 2 kaos adalah Rp.100.000,00 harga 3 buah topi dan 1 kaos adalah Rp.80.000,00 ❖ Ditanya : Berapa harga 1 buah topi ? ❖ Penyelesaian : Misalkan Topi = x Kaos = y Sehingga diperoleh : $2x + 2y = 100.000,00$ $3x + y = 80.000,00$ Dengan melakukan proses eliminasi diperoleh nilai $x = 15.000,00$ Karena yang ditanya hanya topi dimana topi merupakan pemisalan dari variable x sehingga harga 1 buah topi adalah Rp.15.000,00 	<p>Translation</p> <p>Given: Price of 2 hats and 2 t-shirts are IDR. 100.000 Price of 3 hats and 1 t-shirt are IDR. 80.000 Ask: What is the price of 1 hat? Solution: Let: Hat = x T-shirt = y So: $2x + 2y = 100.000,00$ $3x + y = 80.000,00$ With the elimination process, we get the value of $x = 15.000,00$ Because what is asked is only the hat where the hat is a variable x so the price of one hat is IDR. 15.000.00</p>
---	--

Figure 8. Response of Postgraduate Student

Furthermore, the student writes "with a process of elimination" without writing down the process of elimination like what so as to get the value of variable x . At the end of the completion process, the student also wrote "Because what is asked is only the hat where the hat is an example of the variable x so the price of 1 hat" and indicates that the student has not been able to distinguish the "price of hat" variable from the "hat". According to the authors, students do not focus on the thought process and only focus on the truth of the solution of the problem given.

So, from the above of mathematics teachers responds, teachers' expectation on students thinking processes in students written work is inexistent. Teachers still assume that thinking skills do not need to be made explicit. Thinking skills are still implicit and informally developed. The researchers then followed up this activity by asking their reasons for focusing on its mathematical content only. Two types of responses were identified.

First, teachers feel that the development of 4Cs is not essential. They refer to the type of national examination test which assesses the mathematical content only. They argue that if the development of 4Cs is essential, the government should develop an assessment approach that measures students 4Cs, and up to now, there is no such examination.

Second, teachers feel that the content to be taught in the curriculum is already very tight, and students readiness are not right. The coverage of mathematical content to be taught in a semester is very much with a limited time allocated. Furthermore, teachers have to spend much time to review and teach the prerequisite materials every time they have to facilitate their students to learn new mathematical materials.

According to the authors, many things can be used to discuss the result of the study. According to the authors, many factors influence the practices of teaching conducted by teachers. Among all of the factors, some of them are teachers understanding about the mathematical thinking itself, their perceptions about changing teaching practices, their perception about teaching mathematical thinking.

First, the discussion about the importance of developing mathematical thinking is a new topic in mathematics education (Royal Society, 2014). Used to be, the focus of discussion in mathematics education is mostly about mathematical content. During their pre-service mathematics teachers education program, especially in Indonesia, there was a minimal discussion related to the development of mathematical thinking. Therefore, not every teacher understand the complete figure mathematical thinking. Some teachers even consider students' mathematical thinking just as their prior knowledge (Celik & Guzel, 2017). They perceived mathematical thinking just as knowledge, not as skills. If their students have shown their prior knowledge correctly, students mathematical thinking are assumed to be already excellent. How the students use their logic to process their existing or prior knowledge into another knowledge is not considered as an essential issue.

Second, teachers' perceptions of changing teaching practices. According to Cho (2014), many factors may hinder teachers from changing their teaching practices. Factors such as curriculum, teacher culture, school environment, the educational policy could lead to teachers' reluctant in

implementing innovation. Based on this Cho's claim, it is clear why these participating mathematics teachers do not give the best response related to how their students should show their mathematical thinking through their written works. In the curriculum documents, the government does not encourage teachers to request their students to show their thinking process. The national exam form, which uses multiple choice for most of the years, also do not encourage mathematics teachers to encourage their students to show their mathematical thinking.

Ketelaar (2012) identified three crucial factors that influence teachers' readiness in implementing innovation. The factors are sense-making, ownership, and agency. In the sense-making stage, the teachers relate the innovation to their existing knowledge, experiences, and expectations. If the teachers feel that innovation makes sense to them, there is a big chance for the teachers to develop ownership. Unfortunately, encouraging mathematical thinking, visible innovation seems not to make sense for these participating teachers. Their experiences during the pre-service mathematics teachers program which focused more on content acquisition did not give a chance for them to make-sense this innovation. The multiple-choice types of national examination that should be taken by students could also be the reasons why teachers feel that encouraging students to show their thinking processes through their written work is not a make sense activity.

Call for encouraging students' mathematical thinking through their students' written work could be new for the participating teachers. This call is not coming from themselves. There is also no formal training given to these participating teachers. Ownerships from these participating teachers could be very low. Therefore, it makes sense if their responses are not encouraging mathematical thinking.

In term of agency, although government, through Ministry of National Education and Culture (MOEC) has trained almost every mathematics teacher in Indonesia (Puri & Ilma, 2014), there are no specific agencies that were designed specially to ensure the encouragement of mathematical thinking processes through students' written work. There were no agencies who train teachers on how to encourage students' mathematical thinking through students' written work, monitor and supervise its implementation, assess and evaluate its achievement. Leadership who can direct and lead the implementation of teaching mathematical thinking is not available. Therefore, what is Cook (2014) said as the most important is absent for this encouraging mathematical thinking into written works.

Based on the above discussion, the authors recommend others to follow up this research by comparing teachers' expectation about thinking processes to be incorporated into students works based on teachers' familiarity with mathematical thinking, and their involvement in educational innovation. Finally, the national government could also conduct researches to evaluate the effectiveness of having a national examination system which students thinking can be made visible, toward students' thinking development, and teachers' tendencies in implementing teaching and learning activities in the classroom.

CONCLUSION

Students' thinking processes are not expected to be included in students' written work yet. Teachers' focus on students' written work is still on its mathematical content only. Teachers do not see the importance of making their students' thinking visible. Lack understanding of mathematical thinking, teachers' perception on the innovation especially things that are related to change their practices are among factors that could make the teachers do not include students' mathematical thinking in written works. Sense-making about encouraging students to show their mathematical thinking processes in their written works should be improved. Teachers' ownership of these innovative practices and the existence of agents who can monitor and supervise the practices are also needed to make this innovation happens.

REFERENCES

- Argina, A.W., Mitra, D., Ijabah, N., & Setiawan, R. (2017). Indonesia Pisa Result : What Faktors and What Should be Fixed? *The 1st Education and Language International Conference Proceedings Center for International Development of Unissula*, 69–79.
- As'ari, A.R., Mahmudi, A., & Nuerlaelah, E. (2017). Our Prospective Mathematic Teachers Are Not Critical Thinkers Yet. *Journal on Mathematics Education*, 8(2), 145–156. <https://doi.org/10.22342/jme.8.2.3961.145-156>.
- Burns, D.E., Leppien, J., Stuart, O., Gubbins, E.J., Muller, L., & Vahidi, S. (2006). *Teachers' Guide for the Explicit Teaching of Thinking Skills*. Retrieved from <https://nrcgt.uconn.edu/wp-content/uploads/sites/953/2015/04/rm06218.pdf>.
- Celik, A.O., & Guzel, B.E. (2017). Mathematics Teachers' Knowledge of Student. *Journal on Mathematics Education*, 8(2), 199–210. <https://doi.org/10.22342/jme.8.2.4144.199-210>
- Chaqmaqchee, Z.A. (2015). Teacher's Attitude into Different Approach to Providing Feedback to Students in Higher Education. *Journal of Education and Practice*, 6(2), 150–162. Retrieved from <http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1083811&site=ehost-live>.
- Cho, J.H. (2014). *Teachers' perceptions of changes to teaching practice and the influence of professional development: Experienced EFL teachers in South Korea*. Victoria University of Wellington.
- Clark, D., Roche, A., Cheeseman, J., & Sullivan, P. (2014). Encouraging students to persist when working on challenging tasks Some insights from tea. *Australian Mathematics Teacher*, 70(1), 1–11.
- Cohen-Sayag, E. (2016). Student-teachers Across the Curriculum Learn to Write Feedback Does it reflect on their writing? *The Journal of Effective Teaching*, 16(2), 5–19. Retrieved from http://www.uncw.edu/jet/articles/Vol16_2/Cohen-sayag.pdf.
- Cook, J.W. (2014). Sustainable school leadership: The teachers' perspective. *NCPEA International*

- Journal of Educational Leadership Preparation*, 9(1), 1–17.
- Costello, J., & Crane, D. (2013). Technologies for Learner-Centered Feedback. *Open Praxis*, 5(3), 217–225.
- Fonseca, J., Carvalho, C., Conboy, J., Valente, M., Gama, A., Salema, M., & Fiúza, E. (2015). Changing Teachers' Feedback Practices: A Workshop Challenge. *Australian Journal of Teacher Education*, 40(40), 59-- 82. <https://doi.org/10.14221/ajte.2015v40n8.4>.
- Fu, T., & Nassaji, H. (2016). Corrective feedback, learner uptake, and feedback perception in a Chinese as a foreign language classroom. *Studies in Second Language Learning and Teaching*, 6(1), 159–181. <https://doi.org/10.14746/ssllt.2016.6.1.8>.
- Gjerde, K.P., Padgett, M.Y., & Skinner, D. (2017). The impact of process vs. outcome feedback on student performance and perceptions. *International Journal of the Academic Business World*, 13(1), 73–82.
- Hollebrands, K.F., & Dick, T.P. (2009). *Focus in High School Mathematics: Technology to Support Reasoning and Sense Making*. National Council of Teachers of Mathematics (NCTM).
- Katagiri, S. (2004). *Mathematical Thinking and How to Teach it*. Tokyo: Meijitosyo Publishers.
- Kay, K. (2009). 21st Century Skills: Why They Matter, What They Are, and How We Get There. In J. Bellanca & R. Brandt (Eds.), *21st Century Skills: Rethinking How Students Learn* (1st ed., pp. xiii–xxxi). Retrieved from https://faculty.uoit.ca/kay/courses/educ5303g/lessons/lesson5/21Century_Skills.pdf.
- Ketelaar, E. (2012). *Teachers and Innovations: on the role of ownership, sense-making, and agency*. <https://doi.org/http://dx.doi.org/10.6100/IR732586>.
- Li, N., Mok, I., & Cao, Y. (2019). The Evolution of Mathematical Thinking in Chinese Mathematics Education. *Mathematics*, 7(3), 1–18. <https://doi.org/10.3390/math7030297>.
- Menteri Pendidikan dan Kebudayaan Republik Indonesia. (2016). *Peraturan Menteri Pendidikan dan Kebudayaan Pemerintah Indonesia Nomor 21 Tahun 2016 Tentang Standar Isi Pendidikan Dasar dan Menengah* [Regulation of the Minister of Education and Culture of the Government of Indonesia Number 21 of 2016 concerning Basic and Secondary Education Content Standards]. Jakarta, 6 June 2016.
- National Education Association. (2014). *Preparing 21st Century Students for a Global Society: An Educator ' s Guide to the " Four Cs ."* 1–38.
- Neumann, M.D. (2014). Mathematics Teaching: Interpreting and Responding to Children ' s Thinking. *Investigations in Mathematics Learning*, 6(3), 1–28.
- OECD. (2017). How does PISA for Development measure mathematical literacy? *PISA for Development Brief*, 1(February 2014), 0–1. <https://doi.org/10.1787/9789264208780-en>.
- Osterbur, M.E., Hammer, E., & Hammer, E. (2016). Does Mechanism Matter? Student Recall of Electronic versus Handwritten Feedback. *Georgia Educational Researcher*, 9(1), 1–15. <https://doi.org/10.20429/ijst.2015.090107>.

- Pacific Policy Research Center. (2010). 21 st Century Skills for Students and Teachers. *Research & Evaluation*, (August), 1–25. Retrieved from www.ksbe.edu/spi.
- Prenzel, M., Blum, W., & Klieme, E. (2015). Assessing Mathematical Literacy. In *Assessing Mathematical Literacy: The PISA Experience*. <https://doi.org/10.1007/978-3-319-10121-7>.
- Puri, I., & Ilma, R. (2014, March). Evaluasi program pelatihan pendidikan matematika realistik indonesia (PMRI) bagi guru matematika Sumatera Selatan [Evaluation of Indonesia's realistic mathematics education training program (PMRI) for mathematics teachers in South Sumatra]. In *Prosiding Seminar Nasional Evaluasi Pendidikan* (Vol. 1, No. 1, pp. 522-527). Program Studi PEP Program Pascasarjana UNJ.
- Small, M. (2017). *Teaching Mathematical Thinking: Tasks and Questions for Strengthen Practices and Processes*. New York, NY: Teachers College Press.
- Stacey, K. (2006). What Is Mathematical Thinking and Why Is It Important? *Progress Report of the APEC Project: Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II)—Lesson Study Focusing on Mathematical Thinking*. Retrieved from [Http://E-Archives.Criced.Tsukuba.Ac.Jp/Data/Doc/Pdf/2](http://E-Archives.Criced.Tsukuba.Ac.Jp/Data/Doc/Pdf/2).
<https://doi.org/10.3102/0034654312457429>.
- Stacey, K. (2011). The PISA view of mathematical literacy in Indonesia. *Indonesian Mathematical Society Journal on Mathematics Education*, 2(2), 95–126.
<https://doi.org/10.22342/jme.2.2.746.95-126>.
- The Royal Society. (2014). *Vision for science and mathematics education*. Retrieved from <https://royalsociety.org/~media/education/policy/vision/reports/vision-full-report-20140625.pdf>.
- Vockley, M., & Lang, V. (2008). *21st Century Skills, Education & Competitiveness*. <https://doi.org/6th August 2016>.
- Zhu, Y., Yu, W., & Cai, J. (2018). Understanding students' mathematical thinking for effective teaching: A comparison between expert and nonexpert chinese elementary mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(1), 213–224.
<https://doi.org/10.12973/ejmste/78241>.

SENIOR HIGH SCHOOL MATHEMATICS LEARNING THROUGH MATHEMATICS MODELING APPROACH

Bambang Riyanto, Zulkardi, Ratu Ilma Indra Putri, Darmawijoyo

Universitas Sriwijaya, Jalan Padang Selasa 522, Palembang, Indonesia
Email: zulkardi@unsri.ac.id

Abstract

By modeling learning students enjoy learning and doing mathematics in new ways. This study aimed firstly to produce senior high school mathematics modeling tasks, lesson plan, and student worksheet for valid mathematical learning; secondly, to produce senior high school mathematics modeling, lesson plan, and student worksheet for practical mathematics learning; lastly, to produce senior high school mathematics modeling tasks, lesson plan, and student worksheet for potentially effective mathematics learning. This study used method of development research that consisting of 3 steps, i.e., analysis, design, and evaluation. In the analysis stage, researcher did student analysis, curriculum, and mathematical modeling. Second stage are to design and product. Finally, researchers applied a design of formative evaluation consists of self-evaluation, one-to-one, experts review, small group, and field test. Based on experts review, one-to-one, small groups, and field test were obtained valid, practical, and potentially effective, i.e. mathematical modeling tasks, lesson plan, student worksheet to teach mathematical modeling in senior high school and Mathematical modeling tasks and student worksheets to learn mathematical modeling in senior high school.

Keywords: learning, senior high school mathematics, modeling approach

Abstrak

Dengan pembelajaran pemodelan siswa menyenangi belajar dan *doing mathematics* di dalam cara baru. Tujuan Penelitian ini adalah, pertama, menghasilkan tugas pemodelan matematika sekolah menengah atas, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika yang valid, kedua, menghasilkan tugas pemodelan matematika sekolah menengah atas,), lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika yang praktis, terakhir, menghasilkan tugas pemodelan matematika sekolah menengah atas, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP) untuk pembelajaran matematika mempunyai efek potensial. Studi ini menggunakan metode penelitian pengembangan yang memiliki 3 langkah, yaitu analisis, desain kemudian evaluasi. Langkah pertama dilakukan analisis siswa, kurikulum, dan pemodelan matematika. Langkah kedua adalah desain dan *product*. Kemudian langkah terakhir, peneliti menggunakan desain evaluasi formatif, yaitu dilakukan *self-evaluation*, *one-to-one*, *review* ahli, *small group*, dan *field test*. Berdasarkan *experts review*, *one-to-one*, *small group*, dan *field test* diperoleh, yaitu soal pemodelan matematika, lembar kerja peserta didik (LKPD), dan rencana pelaksanaan pembelajaran (RPP), dan yang valid, praktis dan memiliki efek potensial untuk mengajar pemodelan matematika di sekolah menengah atas dan tugas pemodelan dan lembar kerja peserta didik untuk belajar pemodelan matematika di sekolah menengah atas.

Kata kunci: pembelajaran, matematika sekolah menengah atas, pendekatan pemodelan

How to Cite: Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2019). Senior High School Mathematics Learning through Mathematics Modeling Approach. *Journal on Mathematics Education*, 10(3), 425-444. <https://doi.org/10.22342/jme.10.3.8746.425-444>.

Modeling learning makes Mathematics more meaningful and interest for students in this approach (Arseven, 2015). Traditional teaching and separate teaching from problem solving are not enough to prepare students for the 21st century. Bliss et al. (2016) stated that the preparing of 21st century students can be achieved by mathematical modeling because modeling and competency modeling have important contributions to make whether learns able learn and how they must learn in mathematics education. In traditional learning, students are conditioned by "results orientation"

(Kantowski, 1981). The traditional approach also aimed that students produce correct answers validated by the teacher. Students are considered achievers if their actions are the same as the teacher's expectations. The teacher prioritizes the purpose of the following specific procedure instructions to be effective (Cobb et al., 1992). It is very contrary to the modeling learning.

Kilbane and Milman (2014) state that learning in the 21st century is characterized by the use of information and thinking to solve problems. In Problem-Based Learning models, students do both of these, resulting in their development of 21st century skills. It is very compatible with mathematical modeling learning that modeling learning is learning matching the characteristics of 21st century learning, and modeling can access 21st century students' skills (Bliss et al., 2016). Research conducted by Maaß et al. (2018) show that with learning modeling the teachers explained the positive changes in general motivation for mathematics, and stated that their students decided to study mathematics or science, or technology because they enjoyed learning and doing mathematics in new ways. This shows the teacher is very happy to teach mathematics in new ways and students enjoy learning mathematics in new ways.

Yanagimoto (2005) states that traditional mathematics currently taught in schools does not contain subject areas under development (everything is already complete and finished). There is a little examples of modeling in the practice of learning mathematics in many countries. The cause for the abyss between educational programs and facts is that mathematical modeling is not easy for teachers (Arseven, 2015). The looking for of student's problems when modeling is the aerly step before giving teacher intervention or a feedback (Ferri, 2018). This shows that mathematical modeling aiming to diagnose student weaknesses in mathematics and teachers having to have content and pedagogical knowledge to do the mathematical modeling can be done by focusing on modeling tasks. Mathematical modeling taught in school mathematics must be introduced together with strengthening new content (Kawasaki, 2012).

Mathematical modeling methods make learners understand better the relationship between real-world problem and mathematics (Bonotto, 2007; Blum, 2002). Then, studies have shown that mathematical modeling is a approach that teachers do not know well (Frejd, 2012). Quite similarly, introducing modeling into curriculum of mathematics resulted on teachers creating their strategy of learning, causing their learning of mathematics more related to real-world problems (Martinez-Luacles, 2005). Consequently, models have an essential influence in creating mathematics real to learns (Anderson, 2010).

There were three research questions in this Study. Firstly, how senior high school mathematics modeling tasks, student worksheet, and lesson plan were designed valid. Secondly, how senior high school mathematics modeling tasks, student worksheet, and lesson plan were designed practical. Lastly, how senior high school mathematics modeling tasks, student worksheet, lesson plan were designed potentially effective.

METHOD

This study applied method of development research. This method consists of 3 steps, i.e., Analysis, design, then evaluation (Akker, et. al., 2006). On the analysis stage, researcher did student analysis, curriculum, and mathematical modeling. The second stage, researcher did design and produce mathematical modeling tasks, LKPD and RPP. Finally, this research applied a design of formative evaluation (Figure 1). On this stage, researcher did self-evaluation, one-to-one, experts review, small group, and field tests (Tessmer, 1993, Zulkardi, 2006).

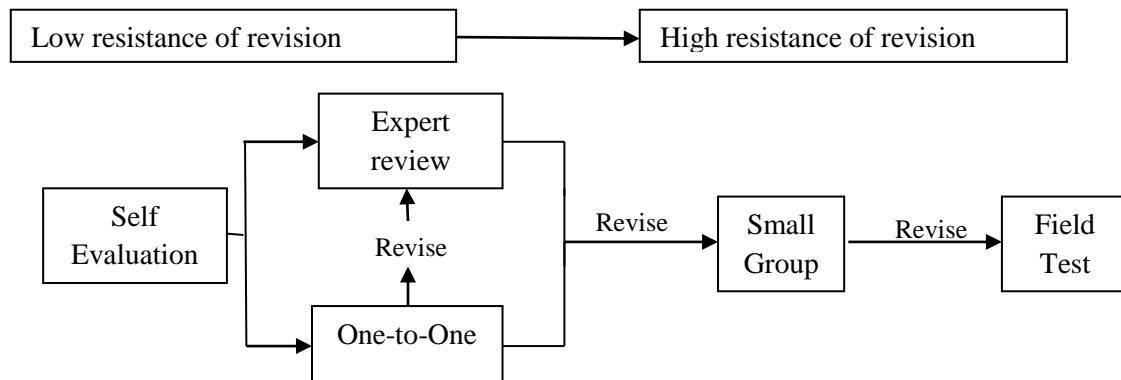


Figure 1. Design of Formative Evaluation.

The criteria of success of this research used the form of mathematical modeling asks, student worksheet , and lesson plan in Senior High School that was valid, practical and potentially effective. The validity was got from the comments of experts of mathematics, RME, mathematics education and mathematical modeling. The practicality was got from the students' comments since working mathematical modeling tasks via observations of the one-to-one, small group by video and interview, and the effective was obtained from the field test. Practicality implies simple to implement. It can be interpreted, and is not double meaning.

The collecting of data techniques were, firstly, walkthrough. It based on the comments of experts to produce a valid mathematical modeling tasks, student worksheet, lesson plan on language, content, construct and contexts aspects, secondly, interview and student's solution, it produced from on-to-one, small group and field test to get the practicality and potentially effective of the mathematics modeling tasks, student worksheet, lesson plan. The obtained data were analyzed by implementing method of descriptive analysis, firstly, walkthrough, walkthrough by sheet analysis based on the comments of validation of experts to produce valid mathematical modeling tasks, student worksheet, and lesson plan, secondly, interview and student's solution, it analyze the outcomes of one-to-one and small group to produce practicality and potentially effective of the mathematics modeling tasks, student worksheet, and lesson plan.

RESULT AND DISCUSSION

Preliminary research, i.e. the first cycle of research was done at SMP Negeri 6 Kayuagung with 3 students Grade VIII.2 students as research subjects, namely K. F. P., M. R. A. P. and O. A. In this study the valid and practical mathematical modeling tasks were produced using UBER contexts. Figure 2 shows that the students' commented that the task of mathematical modeling with UBER context was very good and made them think, and reason, and need revisions to terms in the process of modeling that they were not yet familiar with it. It is because the mathematical modeling was new for them.

Saran:
Menurut saya, ~~itu~~ ^{materi} sudah sangat bagus, karena bisa membuat anak berfikir menalar, tapi bahasa yang digunakan terlalu tinggi untuk anak SMP, sehingga anak akan kurang paham dengan soal yang diberikan

Suggestion:

In my opinion, the material is very good, because it can make children to think reasoning, but the language that used too high for junior high school, therefore, children do not understand to the tasks that is given.

Figure 2. Students' comments on the modeling task

The first cycle of the study also indicated that learners could create identification of the problems, make assumptions, work mathematically to get results, and provide recommendations even though they were not able to generalize. Figure 3 is a solution for students of UBER context modeling tasks.

Melakukan Matematika: Untuk mendapatkan solusi

$$x = \frac{200 \text{ siswa}}{8 \text{ orang}} = 25 \text{ mobil}$$

$$y = 50.000 \times 25 \text{ mobil}$$

$$= 1.250.000.$$

Analisislah dan Evaluasilah Model dan Solusi yang Anda peroleh

Model nya sudah cocok / sesuai

Apakah Anda ingin memperbaiki dan memperbaiki model Anda?

tidak.

Model yang Anda Rekomendasikan untuk diimplementasikan dan laporkan hasilnya.

Kesimpulannya: \rightarrow
Bahwa jika yang mengikuti tamanya berjumlah 200 orang maka akan disewa 25 mobil dengan tarif 1.250.000.

Do the math: to get solution

$$x = \frac{200 \text{ people}}{8} = 250 \text{ car}$$

$$x = 50.000 \times 25 = 1.250.000$$

Analyze and assess the model and the solutions

The model is suitable

Iterate as needed to refine and extend the model

No

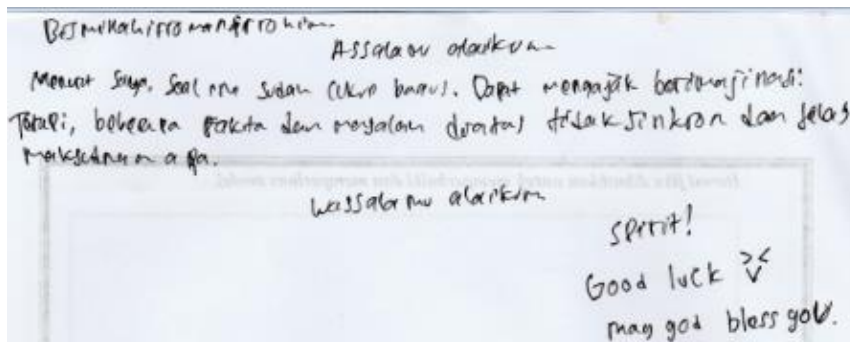
Implement the model and report the results

Conclusion

If there are 250 people on the trip, 25 car will be rented at cost 1.250.000 rupiah

Figure 3. Students Solution

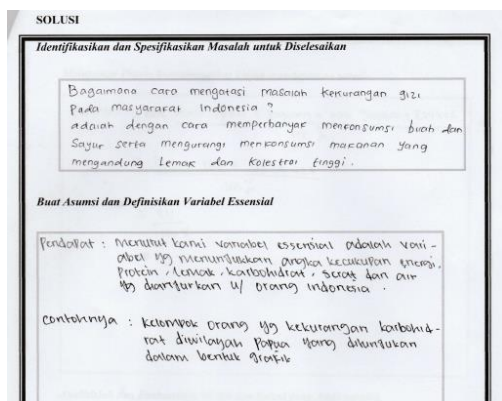
The students' commented that the tasks of mathematical modeling with nutritional context was very good and made them imaginative, but it was quite difficult. This shows that the modeling task is good and needs revision. The comments of student can be seen in Figure 4.



In my opinion, this task is good. It can invite to imagine. But, some fact and problem above out of sync and not clear what's the meaning.

Figure 4. Student comment's on mathematical tasks

The second cycle of the study indicates that students could only identify and simplify the tasks but could not to complete the modeling tasks according to the modeling process. This resulted the fact that mathematical modeling was new to them. Figure 5 shows the solution for students in the task of modeling nutrition contexts.



Identify and specify the problem to be solved
 How to solve the problem of malnutrition in Indonesian society? whether by increasing consumption of fruits and vegetables and reducing the consumption of foods that contain high fat and cholesterol

Melakukan Proses bermatematika: Untuk mendapatkan solusi

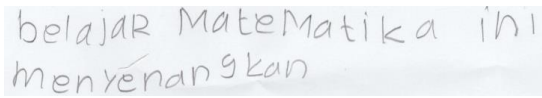
Dalam ukuran porsi : 100 gram

	kal.	Lemak	karb.	protein
Nasi putih	129	0,285	27,94	2,664
Jagung	132	1,62	29,29	4,966
Pati Gandum	67	1,07	12,269	2,376
Burial barley	123	0,44	28,23	2,263

Do the math: get a solution
 In one serving size
 White rice: 129 0,285 27,94 2,664
 Corn : 132 1,62 29,29 4,966
 Wheat bread: 67 1,07 12,269 2,376
 Pearled barley: 123 0,44 26,229 2,263

Figure 5. Student's Solution

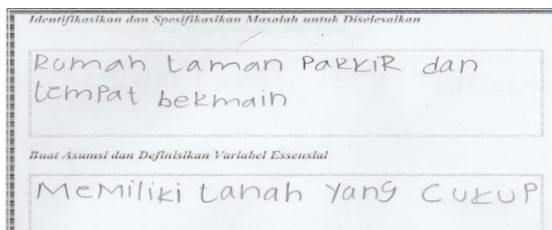
The Third Cycle of the study took 3 subjects as the subjects of SDIT Bina Insani Kayuagung, namely C. A. M., A., and F. The students' commented that they learn interesting mathematics with mathematical modeling learning. Figure 6 is a student's comment on a mathematical modeling task.



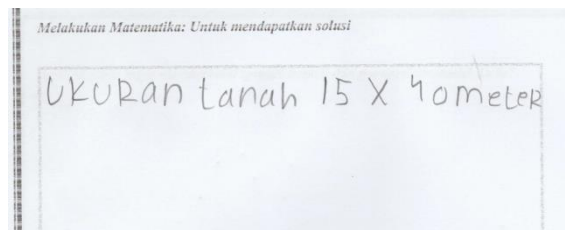
This learning mathematics is interesting

Figure 6. Students' comments on modeling tasks

In this third cycle of the study, students could only identify and specify problems and make assumptions. They were not able to do a mathematical process to get a solution and cannot evaluate and generalize mathematical models (Figure 7).



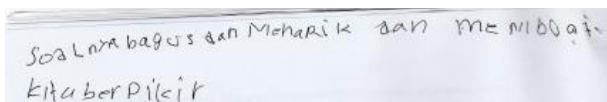
Identify and specify the problem to be solved
home, parking lot and playground



Do the math: get a solution
Land size 15 x 40 meter

Figure 7. Student Solution to the Modeling Task

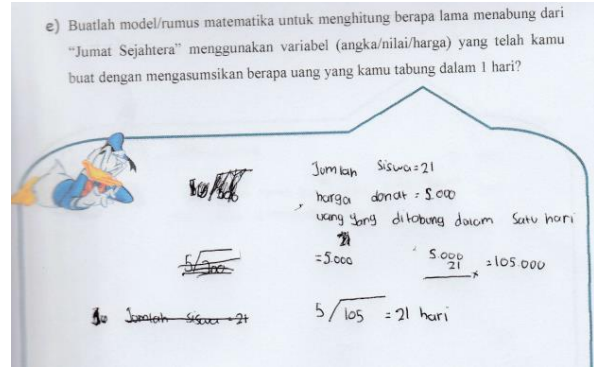
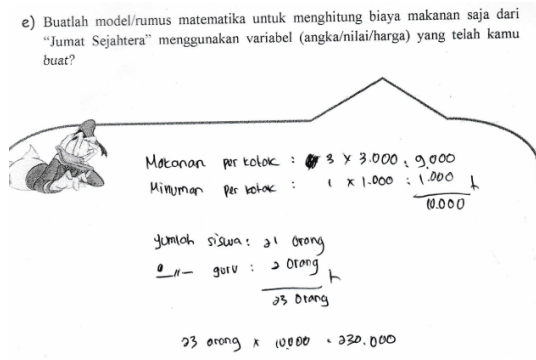
The Fourth Cycle took 6 students as research subjects of SDIT Bina Insani Kayuagung, namely S. A., K. N. I., F. A., M. S. A., M. F. A. H. and D. R. F. The students commented that the mathematics learning became interesting and made them think with mathematical modeling learning. Figure 8 explains a student's comment on a mathematical modeling task. This fourth cycle study used the context of "Jumat Sejahtera".



The problem is good and interesting and makes us think

Figure 8. Students' comments on modeling tasks

In this fourth cycle, students could only identify and specify problems and make assumptions, determine important variables, perform mathematical processes to get solutions, produce mathematical models, but they were not able to generalize and do iterations that can be seen in Figure 9.

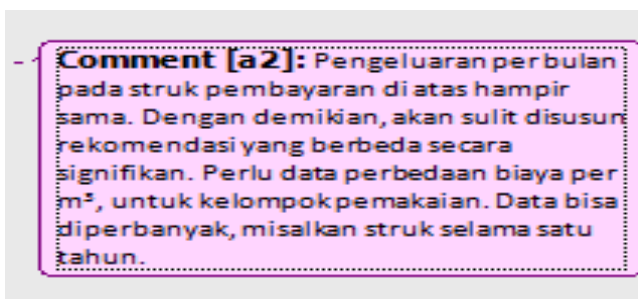


Create a mathematical model/formula to calculate only the cost of food from "Prosperous Friday" using the variables (numbers / values / prices) that you have created!
 Food per box = $3 \times 3000 = 9000$
 Drink per box = $1 \times 1000 = 1000$
 = 10.000 rupiah
 Total of students = 21 people
 Total of teacher = 2 teachers
 23 orang
 $23 \text{ orang} \times 10.000 = 230.000 \text{ rupiah}$

Create a mathematical model/formula to calculate how long to save from "Prosperous Friday" using the variables (numbers / values / prices) that you have created by assuming how much money you save per day!
 Total of students = 21
 The price of donut = 5000
 The money that save in one day = 5000
 $5000 \times 21 = 105.000 \text{ rupiah}$
 $105 / 5 = 21 \text{ hari}$

Figure 9. Student Solution to the Modeling Task

This study was conducted at SMA Negeri 1 Palembang. In this study, modeling learning used the context of Musi 2 Bridge, Toll Fee, Online and Conventional taxi, Car Speed on Toll Road, and Water subscription fee of PDAM. These mathematical modeling tasks and students worksheet was conducted experts review, i.e. Prof. Hendra Gunawan, Prof. Edi Cahyono, Al Jupri, S.Pd., M.Sc., Ph.D., and Dr. Rusdy A. Siroj. Figure 10 show the comment of expert of mathematical modeling, i.e. Edi Cahyono (Figure 10).

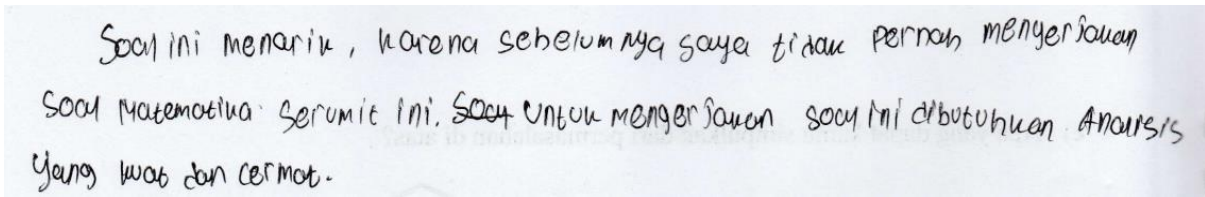


Expenditure per month on the payment receipt above is almost the same. Thus, it would be difficult to formulate significantly different recommendations. Data cost per m^2 difference is needed, for usage groups. Data can be reproduced, for example, receipt for 1 year

Figure 10. Expert' comment to the Modeling Tasks on PDAM contexts

After experts review, this study conducted one-to-one. The subjects of one-to-one were the learners of Grade XI IPA 1 and 3 of SMA Negeri 1 Palembang as many as 15 students, namely A. K., A. A., A. K. A. I., A. A. R., A. Y. N., D. T. T., H. K., J. A, M. A. A., M. B. H., M. D. S. N., M. A., N. K., N. S. A, and R. R. A.

One of the comments of students when one-to-one students, named A. K. state that this modeling task was interesting and he never studied modeling tasks like this pervously and this task needed a strong and careful analysis. A. K.'s comments is shown in the Figure 11.



This problem is interesting, because before I never solved a math problem this complicated. To do this problem requires a strong and careful analysis.

Figure 11. Student comment's on one-to-one

Judging from the solution of students, the task of modeling with the context of the Toll Road Fee and Musi 2 Bridge, the students were not able to bring up mathematical models even though they did the mathematical processes. But for the other three contexts, the students were able to make mathematical models even though they could not validate and evaluate and do iterations to improve the mathematical models obtained. This is due to new mathematical modeling for them. Figure 12 is a solution for students who work mathematically.

b) Berdasarkan data yang telah diketahui di atas, lengkapilah tabel di bawah ini:

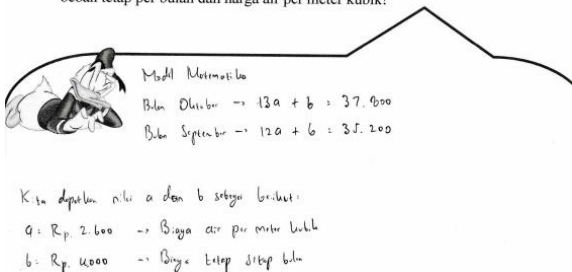
Pengisian ke-n uang pada kartu tol (e-money)	Jumlah total uang yang dikeluarkan untuk pengisian kartu tol (Rp)	Jumlah total biaya administrasi (Rp)	Jumlah uang (e-money) pada kartu tol	Jumlah uang yang dibayar untuk perjalanan ke-n (Rp)	Jumlah uang pada kartu tol pada perjalanan ke-n	Uang pada kartu tol yang terisi untuk berapa kali jalan?	Sisa uang (e-money) pada kartu tol pada perjalanan ke-n
1	Tidak ada Pengisian	Rp 1.500	Rp 48.500	Rp 20.000	Rp 45.200	2	Rp 25.200
2	Tidak ada Pengisian	Rp 1.500	Rp 48.500	Rp 20.000	Rp 53.200	2	Rp 33.200
3	Rp 50.000	Rp 1.500	Rp 48.500	Rp 20.000	Rp 33.200	1	Rp 13.200
4	Tidak ada Pengisian	Rp 1.500	Rp 48.500	Rp 20.000	Rp 61.700	3	Rp 41.700
5	Tidak ada Pengisian	Rp 1.500	Rp 48.500	Rp 20.000	Rp 41.700	2	Rp 21.700
6	Tidak ada Pengisian	Rp 1.500	Rp 48.500	Rp 20.000	Rp 58.700	2	Rp 38.700
7	Rp 50.000	Rp 1.500	Rp 48.500	Rp 20.000	Rp 38.700	1	Rp 18.700

charging nth money on the toll card (e-money)	The total amount of money spent on toll card filling (Rp)	The total amount of administrative costs (Rp)	The amount of money on the toll card (Rp)	the amount of money paid for the nth trip (Rp)	The amount of money on the toll card on the nth trip (Rp)	Money on the card filled in for the number of trips?	Remaining money (e-money) on the toll card on the nth trip (Rp)
1	Not filling	1.500	48.500	20.000	45.200	2	25.200
2	Not filling	1.500	48.500	20.000	53.200	2	33.200
3	50.000	1.500	48.500	20.000	33.200	1	13.200
4	Not filling	1.500	48.500	20.000	61.700	3	41.700
5	Not filling	1.500	48.500	20.000	41.700	3	31.700
6	Not filling	1.500	48.500	20.000	58.700	2	38.700
7	50.000	1.500	48.500	20.000	38.700	1	18.700

Figure 12. The results of doing mathematics students.

After conducting one-to-one, the small group was also conducted using the context of "Musi 2 Bridge", "Toll Road Fee," "Online and Conventional Taxis", "Car Speed on Palindra Toll Road ", and "Water subscription fee of PDAM". The result of small group was that students could solve mathematical modeling problems in the context of "PDAM". The solution to the mathematical model is shown in Figure 13.

d) Buatlah model matematika untuk menghitung biaya beban dan harga air bersih per meter kubik menggunakan variabel yang telah kamu buat? Tentukan biaya beban tetap per bulan dan harga air per meter kubik!



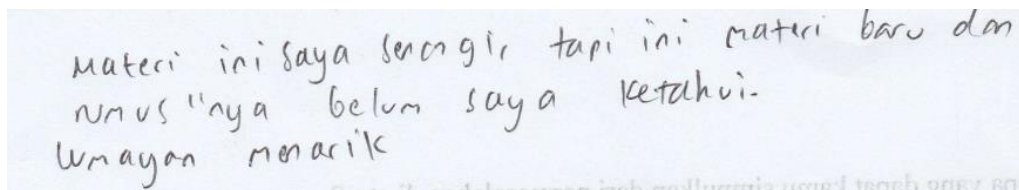
Model Matematika
 Bulan Oktober $\rightarrow 13a + b = 37.300$
 Bulan September $\rightarrow 12a + b = 35.200$

Kita dapatkan nilai a dan b sebagai berikut:
 $a = \text{Rp. } 2.600 \rightarrow \text{Biaya air per meter kubik}$
 $b = \text{Rp. } 4000 \rightarrow \text{Biaya tetap setiap bulan}$

Make a mathematical model to calculate the cost and cost of clean water per cubic meter using the variable you created. Determine the monthly cost and water price per cubic meter!
 Mathematical model
 October: $13a + b = 37.300$
 September: $12a + b = 35.200$
 We get the values of a and b as follows
 $a = \text{Rp. } 2.600 \rightarrow \text{cost of water per cubic meter}$
 $b = \text{Rp } 4000 \rightarrow \text{fixed costs per month}$

Figure 13. Context Mathematical Model PDAM

The comments of students after small group were that they were happy and the material was new and interesting. Student comments are shown in Figure 14.



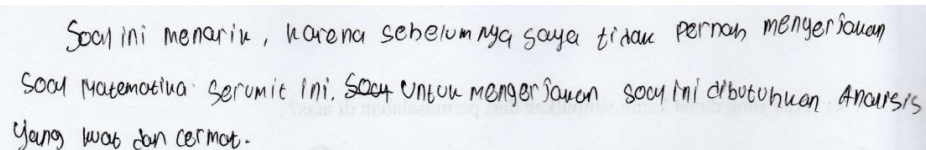
materi ini saya senang, tapi ini materi baru dan rumus-rumusnya belum saya ketahui. Umaman menarik

I enjoy this material and this material is new, and the formulas are unknown to me. Pretty interesting

Figure 14. Student's comment on small group

After that, the study continued to conducting a field test to Grade XI IPA 3 using modeling learning with the context of the PDAM. Then, field test was conducted to Grade XI IPA 3 and IPA 1 with modeling learning of the context of the musu 2 bridge and the toll roads fee. Then, it was conducted to Grade XI IPA 3 using the context of PDAM Fee. Then, it was conducted to Grade XI IPA 5 using the modeling learning the context of Musu 2 Bridge. Then, on Tuesday, April 9, 2019, it was carried out to Grade XI IPA 3 with the modeling learning of the context of the Toll Road Fee.

For the field test, students commented that this modeling task was interesting and they never studied modeling tasks like this before and this task required strong and careful analysis. Figure 15 is the student's comment after the learning process, i.e.

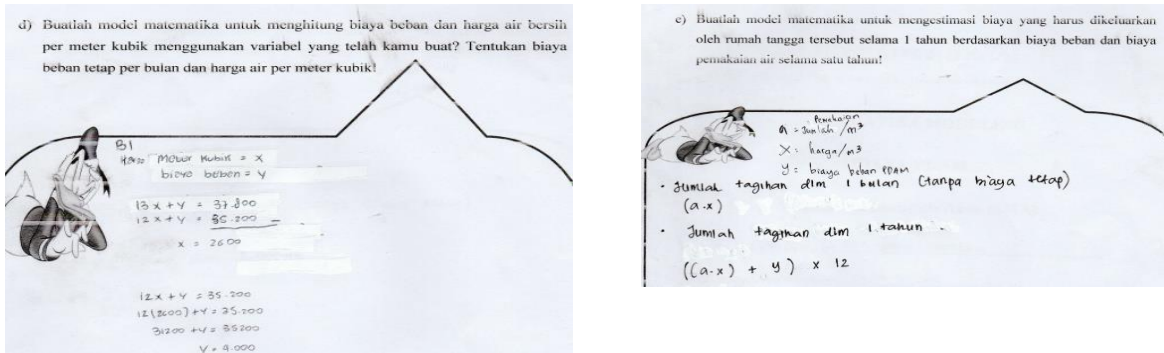


Soal ini menarik, karena sebelumnya saya tidak pernah mengerjakan soal matematika seperti ini. Soal untuk mengerjakan soal ini dibutuhkan analisis yang kuat dan cermat.

This problem is interesting, because I have never done math problems this complicated before. To do this problem requires a strong and careful analysis.

Figure 15. Students comment's on one-to-one and small group

The modeling learning activities indicate that students are very enthusiastic about learning mathematics which can be seen in the videos and observations. Student solutions in the modeling process also show that students could to create mathematical models for the five contexts of modeling learning that are done even though students have not been able to evaluate the mathematical models they have found. Figure 16 shows the solutions of students for modeling tasks with the context of the PDAM.



Make a mathematical model to calculate the cost and cost of clean water per cubic meter using the variable you created. Determine the monthly cost and water price per cubic meter!

Cost per cubic meter = x , Fixed costs per month = y .

$$13x + y = 37.800$$

$$12x + y = 35.200$$

$$12x + y = 35.200$$

$$12(2.600) + y = 35.200$$

$$x = 2.600$$

$$31.200 + y = 35.200$$

$$y = 4000$$

Make a mathematical model to estimate the cost that must be incurred by the household in one year based on the cost of water costs and costs for one year!

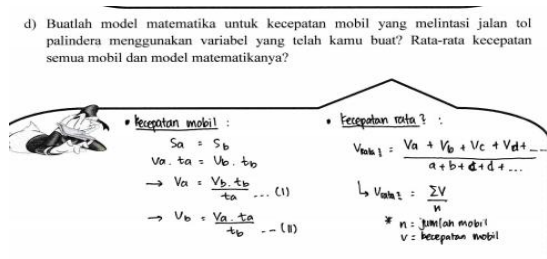
Usage: a = amount per cubic meter, x = price per meter cubic, y = expense per month

bill amount in one month (no expense) = (ax)

bill amount in one year (no expense) = $(ax + y)12$

Figure 16. Mathematical model of fee PDAM in one year

The same results also show the context of the Car Speed on the toll road. The students could to create mathematical models of problems given. Figure 17 is a student solution.



Make a mathematical model for the speed of a car that crosses the motorway using the variable you have created! The average speed of all cars and their mathematical models.

Car speed:
 $S_a = S_b \rightarrow v_a \cdot t_a = v_b \cdot t_b$
 $\rightarrow v_a = \frac{v_b \cdot t_b}{t_a} \dots (1)$ $\rightarrow v_b = \frac{v_a \cdot t_a}{t_b} \dots (2)$

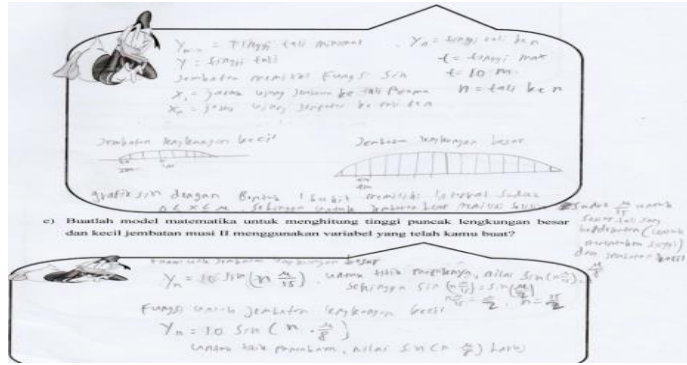
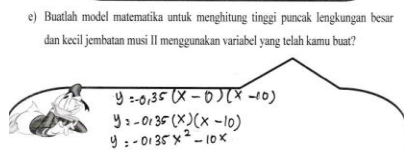
Average speed:

$$v_{rata} = \frac{v_a + v_b + v_c + \dots}{a + b + c + \dots} \rightarrow v_{rata} = \frac{\sum v}{n}$$

 n = many cars, and v = car speed

Figure 17. Mathematical model of the “car speed on toll road” context

For the context of Musi 2 Bridge, the students were able to produce two models, namely assuming the arch of the Musi’s bridge 2 is a quadratic function and a trigonometric function. Figure 18 is a solution to the mathematical model they found.



Create a mathematical model to calculate the height of the big and small arches of the musi bridge 2 using the variables you have created!
 $y = -0,35(x - 0)(x - 10)$
 $y = -0,35 x(x - 10)$
 $y = -0,35x^2 - 10x$

$y_n = nth$ string height, $y =$ tinggi maksimum
 $x =$ distance end to end horizontally
 The bridge arch is a function of sin
 Sin graph with a hill shape has angle intervals $0 \leq x \leq \pi$. So, for this bridge

has a difference in the angles $\frac{\pi}{15}$

for each adjacent rope (to determine height) and a small arc.
 Create a mathematical model to calculate the height of large and small arches of the musi bridge 2 using the variables you have created!

Large arch height: $y = 10 \cdot \sin\left(n \cdot \frac{\pi}{15}\right)$ for its breaking point

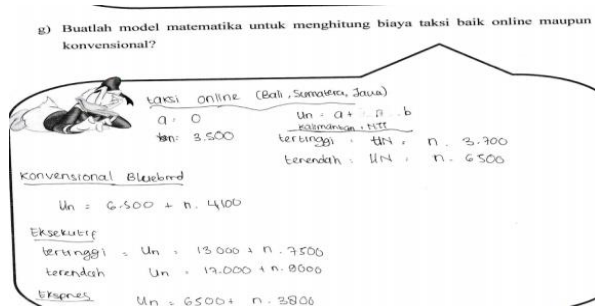
The value of $\sin\left(n \cdot \frac{\pi}{15}\right) = 1$, so, $\sin\left(n \cdot \frac{\pi}{15}\right) = \sin\left(\frac{\pi}{2}\right)$

So, $n = 15/2$. Function for small arches, i.e.

$$y_n = 10 \cdot \sin\left(n \cdot \frac{\pi}{8}\right)$$

Figure 18. Mathematical model for “Musi 2 Bridge” context

The context of online and conventional taxi show that students could produce mathematical model derived from the given problem. Figure 19 is the mathematical model found by the student.



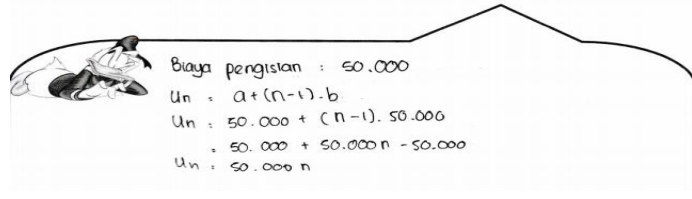
Make a mathematical model to calculate the cost of online and conventional taxis!

Online taxis (Jawa, Sumatera, Bali):
 $a = 0, b = 3.500, U_n + a + nb$
 Kalimantan, NTT: The highest: $U_n = n \cdot 3.700$
 The lowest: $U_n = n \cdot 6.500$
 Bluebird conventional: $U_n = 6500 + n \cdot 4100$
 Executive: The highest: $U_n = 13.000 + n \cdot 7.500$
 The lowest: $U_n = 17.000 + n \cdot 9.000$
 Express: $U_n = 6500 + n \cdot 3.800$

Figure 19. Mathematical model of online and conventional taxi

For the context of the online and conventional taxi fee also shows that students could make mathematical models of the problems given. Figure 20 is a mathematical model found by students.

e) Buatlah model matematika untuk biaya yang harus dikeluarkan oleh pengguna jalan tol selama satu tahun dan saldo pada kartu tol pada penggunaan terakhir jalan tol di akhir tahun menggunakan variabel yang telah kamu buat?



Biaya pengisian : 50.000
 $U_n = a + (n-1) \cdot b$
 $U_n = 50.000 + (n-1) \cdot 50.000$
 $= 50.000 + 50.000n - 50.000$
 $U_n = 50.000n$

Make a mathematical model for the costs that have to be incurred by toll road users for one year and sal do on the toll card at the last use of the toll road at the end of the year using the variable you have made!
 Charging Fee = 50.000 rupiah
 $U_n = a + (n - 1)b$
 $U_n = 50.000 + (n - 1) 50.000$
 $U_n = 50.000n$

Figure 20. Mathematical model for “Toll Road Fee” context

After doing field test potentially effect was looked. This can be seen from the comments of the model teacher, the observer teacher and the students themselves as research subjects stating that modeling learning is good to apply in the future because it is interesting, challenging, and meaningful for students and makes students motivated to learn. The learning process of mathematical modeling tasks can produce good mathematical models as expected and there is a validation process as in the modeling cycle. The mathematical model that has not been generated is in the context of the Musi 2 Bridge. The quadratic function model that still has parameters appears. Whereas for trigonometric functions mathematical models have emerged but there are no arguments or validations from the students. Whereas for the contexts of conventional online and taxi taxis, the toll road fee, PDAM and car speed on toll road produced the correct model. Therefore, results of the study shows the learning process on field test is good in accordance with the expectations in accordance with the characteristics of the task and the modeling learning process.

This study produced mathematical tasks, lesson plan, and student worksheets for senior high school mathematics modeling. As a results, the valid, practical and potentially effective senior high school mathematics task of mathematics modeling, lesson plan, and student worksheet were designed valid, practical, and potentially effective to be implemented for more interesting and meaningful mathematics learning to better the quality of learning and mathematical achievement.

The first research problem, how senior high school mathematics modeling tasks, lesson plan, and student worksheet were designed valid?. This research conducted experts review from mathematicians Prof. Hendra Gunawan, experts of mathematical modeling Prof. Edi Cahyono, expert of realistic mathematics education Al Jupri, S.Pd., M.Sc., Ph.D and expert of mathematics education Dr. Rusdy A. Siroj. They state that mathematical modeling tasks in this reseach is good and need revision. After experts review, one-to-one was conducted. Students say that this tasks was interesting and make student to think. Student could make mathematical model in one-to-one. This show that this tasks was valid.

In this research of designing modeling tasks based on the characteristics of the modeling task, the problem comes from the real world. Sullivan et al. (2015) state that designing tasks should consider three aspects of pedagogy, namely, student motivation, task recognition to students, which relates to the teacher wanting learners to have the ability to interpret the demands of the task, and

accessing assignments by all students, on which this issue causes the teacher to think of student motivation, the level of initial knowledge to push with assignments, the current mathematics culture class, expanding tasks that could distinguished to allow all learners to push appropriately. To implement this designed task the teacher can make modeling tasks as an effective way of learning. This modeling tasks can motivate students; with mathematical modeling tasks the students will carry out the process of interpreting assignments and modeling tasks which are valid for all students who are weak, moderate and high in their mathematical abilities.

In desiging tasks of mathematical modeling learning, teachers must make use of everyday contexts. The function of a context is very essential in mathematical modeling because the modeling needs a context for problem frame, and mathematical applications (Lingefjard, 2006; Mousoulides, 2007). A focus characteristic of traditional mathematics learning in many countries is the activity of 'solving story problems' (Schoenfeld, 1992; Mousoulides, 2007).

Saxena et al. (2016) state that sometimes they find it difficult to solve problems and learn formulas and prove theorems. Although a lot of effort and research were carried out in this field but there was a lot of work done at the basic level. However, the level of difficulty gets high at the middle and high levels because we have a lot of content in grades 11 and 12 as well as many new things for students and many formulas, theorem relationships and a few parts of the application. In research it shows that it is very essential to understand the contexta in modeling.

The second research problem is how senior high school mathematics modeling tasks, lesson plan, and student worksheet were designed practical?. This research was conducted small group. In this small group students very active in learning. Students could learn mathematical modeling to get mathematical model. Students also state that this learning is very interesting. This show that mathematical modeling tasks and student worksheet is practical.

It is not easy to implement mathematical modeling lessons in schools due to time constraints (Hino, 2007). Research in mathematics education emphasizes the essential of teacher knowledge about learner thinking (Ball & Cohen, 1999; Ball et al., 2008; Even & Tirosh, 1995; Stillman et al., 2010). But teacher knowledge about students' thinking is not easy matter because it needs them to see, hear, understand, and interpret learners' thinking when driven by mathematics tasks (Ball, 1997). Teachers must be able to estimate difficulties, lack of understand, and predict whether students are appealing in choosing examples (Ball et al., 2008; Hill et al., 2008). It shows the importance of the task of modeling school mathematics. Modeling learning place new challenges on the teacher (Doerr, 2006).

The study conducted by Riyanto et al. (2017) indicates that junior high school learners are very extracted in learning mathematical modeling. Likewise, in the study conducted by Riyanto et al. (2018), students are also interested in learning mathematical modeling, and the research in elementary schools shows the same thing that elementary school pupils can produce good mathematical models and are interested in learning mathematical modeling (Riyanto et al., 2019). Based on the Eric's

(2010) research results, he recommend that future studies be required in the area of mathematical modeling to link theory and practice, to fit between contemporary and approaches of traditional, and role of the teacher as a facilitator. The study conducted by Anthony and Hunter (2010) show that mathematical modeling and application tasks that are discussed in group settings to give a solid basis to learner development of argumentation because they are inherently social experiences with support of effective teacher and communication, collaboration, teamwork, and reflection effectively.

The results of the study conducted by Biccard (2010) show that modeling competencies can not develop by implementing traditional approach and essential teaching to develop mathematics as their own competence. Development of modeling competencies requires support by wise task selection, heterogeneous group formation, and effective teacher knowledge about modeling. The study findings of Wong (2008) can have implications to training of teacher on modeling tasks to improve quality of mathematical achievement. The suppression on mathematical modeling in school level is to focus on the process rather than on the product (Balakrishnan et al., 2010).

The third research problem is how senior high school mathematics modeling tasks, lesson plan, and student worksheet were designed potentially effective?. This study conducted field tests for PDAM, Fee Tol, Musi 2 Bridge, Online and Conventional taxi, and Car speed on Tol road. From field tests, students was very enthusiastic and enjoyed learning mathematics with mathematical modeling and students could make mathematical models using their own methods. From 36 students only one student didn't like this tasks. On field tests also students could make mathematical model but students couldn't validate the mathematical model, so that couldn't do iteration in modeling process. It is caused that learners are rarely to learn mathematical modeling.

In this study, students studied mathematical modeling to solve real-world problem with empty topic in mathematics. In learning mathematical capital, students must make assumptions, make selections, make variables, mathematization, working mathematically and validating the obtained results both mathematically and in the real world. Similar, the study results of Stillman and Brown (2014) show that the inability to mathematize in the classroom is connected to the inability to apply relevant mathematical skill in the context of modeling than lack of relevant mathematical skill per person, or an orientation view of mathematical applications or persistence on tasks.

Creativity is needed to face an unknown future and the education should complement learners for this purpose (Wessels, 2014). Several skills, such as creativity, spatial, and problem solving, are generally associated with art, but it is also a basic part of mathematics, technology, economics and politics - forming an integral part of a person's daily life (Robinson, 1999; Hendroanto et al., 2018; Ahamad et al., 2018). In mathematical modeling, it is very important for creativity, fluency, flexibility, and novelty. It is very important in facing an increasingly complex life. Furthermore, Saxena et al. (2016) state that the students' performance has not been at the desired level, the students did not solve mathematical problems interestingly. The importance of mathematical modeling in schools is at all levels (Mousoulides, 2007). In this study, the creativity of students in learning

mathematical modeling rises. In addition, metacognitive development and critical thinking skills are seen as importance and must help learners reflect on their answers to modeling tasks (Garofalo & Lester, 1985; Schoenfeld, 1992; Mousoulides, 2007).

In the study conducted by Eric (2013) it concludes that encouraging students in mathematical modeling opens up a strategy to teachers to redesign strategies in which learners be able solve problems in real-world problems. Modeling tasks of "travel plan" context take a dimension for students to look at agreed phenomena for mathematical thinking, asking them to explain the purpose of modeling situations, evaluating these amounts and variables leads relevance to work with manipulating, developing, and interpreting the model in many strategies to get solutions (Eric, 2013).

In mathematical modeling approach, students should be able to use the many stages in modeling cycle to many open-ended tasks (Haines & Crouch, 2010; Knott, 2014). This approach can be applied to solve other problems successfully and to develop a critical, reflective individual (Knott, 2014). The teachers in his study comment that it is essential for learners to learn mathematics is helpful in extra-mathematical situations (real-world) and one approach to make students learn mathematics with this purpose in thinking must be integrate more modeling activities in mathematics learning (Frejd, 2012; Muhtadi et al., 2017; Mumu et al., 2018).

Good modeling tasks invite learners to encounter the whole modeling process (Balakrishnan et al., 2010). In this study, student was given the entire modeling process. In this study, it can be seen from experts review, student's opinion and student's solution of mathematical modeling tasks. The same results occur in the study using the contexts of Musi 2 Bridge, Toll Road Fee, Car Speed on Toll Roads, online and conventional taxis, and the context of the PDAM whose contexts originating from the real world situation in which the learning uses authentic, opened-ended problems, students assuming, doing election. In this learning, the students produced good mathematical models, active, effective learning processes and they were interested in learning mathematics. The results supported to several research that shown students are interested in realistic material use real-time daily applications for them (Zulkardi, 2002; Saleh et al., 2018; Nuari et al., 2019).

Zulkardi and Putri (2019) said that some new components of the 2013 curriculum have the same characteristics as PMRI. *PMRI* and Learning mathematical modeling are not much different (Zulkardi, 2017). It indicates the importance of modeling learning in the 2013 curriculum. Consequently, teachers should design learning tasks based on the characteristics of mathematical modeling tasks. This statement also correspondends to Blum et al. (2019) stating that the design of activities in mathematical didactics can be in the form of design of tasks, lessons (lesson plan), sequences of teaching, textbooks for mathematical leaning, curriculum, assessments, and ICT-based teaching material or programs for teacher education and must be done by teachers, educators, book writers, curriculum and assessment developers, designer of ICT, and researchers. This statement is in accordance with this study, namely designing modeling tasks, student worksheet, and lesson plan.

CONCLUSION

This study produces the valid and practical high school mathematics modeling tasks, lesson plan, and student worksheet. The products also have potentially effective. The findings in this study were that students were very enthusiastic and enjoyed learning mathematics with mathematical modeling and students could make mathematical models using their own methods. Student can apply modeling process, so this promoting student's modeling literacy. Mathematics learning should implement mathematics modeling learning starting from elementary school to college level. Then students individually and in groups solve this mathematical modeling problem by using suitable mathematical topics. There needs to be a change in curriculum that uses real-world problems in mathematics learning. There needs to be a training for teachers in designing modeling tasks, implementing mathematics learning, and evaluating mathematical modeling learning in schools and colleges.

REFERENCES

- Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, 943(1), 012008. <https://doi.org/10.1088/1742-6596/943/1/012008>.
- Akker, J., Bannan, B., Kelly, A.E., Nieveen, N., & Plomp, T. (2006). *Educational Design Research*. Enschede: SLO
- Anderson, J. (2010). Collaborative problem solving as modelling in the primary years of schooling. In: Kaur, B., & Dindyal, J., (Eds.). *Mathematical Applications and Modelling*. Singapore: World Scientific Publishing Co. Pte. Ltd.
- Anthony, G., & Hunter, R. (2010). Communities of mathematical inquiry to support engagement in rich tasks. In: Kaur, B., & Dindyal, J., (Eds.). *Mathematical Applications and Modelling*. Singapore: World Scientific Publishing Co. Pte. Ltd.
- Arseven, A. (2015). Mathematical modelling approach in mathematics education. *Universal Journal of Educational Research*, 3(12), 973-980.
- Balakrishnan, G., Peng, Y.Y., & Eng, E.G.L. (2010). Mathematical modelling in the Singapore secondary school mathematics curriculum. In: Kaur, B., & Dindyal, J., (Eds.). *Mathematical Applications and Modelling*. Singapore: World Scientific Publishing Co. Pte. Ltd.
- Ball, D.L. (1997). What do students know? Facing challenges of distance, context, and desire in trying to hear children. In: Biddle, B., Good, T., & Goodson, I., (Eds.). *International Handbook on Teachers and Teaching 2*. Dordrecht: Kluwer Press.
- Ball, D.L., & Cohen, D.K. (1999). Developing practice, developing practioners: Toward a practice-based theory of professional education. In: Sykes, G., & Darling-Hammond, L., (Eds.). *Teaching as Learning Profession: Handbook of Policy and Practice*. San Fransisco: Jossey Bass.
- Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What make it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>.

- Biccard, P. (2010). An investigation into the development of mathematical modelling competencies of grade 7 learners. *Unpublish Doctoral Dissertation*. Stellenbosch: University of Stellenbosch.
- Bliss, K., Libertini, J., Levy, R., Zbiek, R.M., Galluzzo, B., Long, M., Teague, D., Godbold, L., Malkevitch, J., Kooij, H.V.D., Giardano, F., Fowler, K., Pollak, H., Gould, H., Montgomery, M., & Garfunkel, S. (2016). *GAIMME: Guidelines for Assessment & Instruction in Mathematical Modelling Education*. Philadelphia: COMAP & SIAM.
- Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education discussion document. *Educational Studies in Mathematics*, 51(2), 149-171. <https://doi.org/10.1023/A:1022435827400>.
- Blum, W., Artigue, M., Mariotti, M.A., Sträßer, R., & Heuvel-Panhuizen, M.V.D. (2019). European didactic traditions in mathematics: Introduction and overview. In: Blum, W., Artigue, M., Mariotti, M. A., Sträßer, R., & Heuvel-Panhuizen, M.V.D. *European Traditions in Didactics of Mathematics*. Switzerland: Springer.
- Bonotto, C. (2007). How to replace word problems with activities of realistic mathematical modelling. In: Blum, W., Galbraith, P., Henn, H.W., & Niss, M. (Eds.). *Modelling and Applications in Mathematics Education: The 14th Icmi Study*. New York: Springer.
- Cobb, P., Wood, E., Yackel, E., & Mcneal, B. (1992). Characteristics of classroom mathematics traditions: an interactional analysis. *American Educational Research Journal*, 29(3), 573-604. <https://doi.org/10.3102/00028312029003573>.
- Doerr, M.H. (2006). Examining the tasks of teaching when using student's mathematical thinking. *Educational Studies in Mathematics*, 62(1), 3-24. <https://doi.org/10.1007/s10649-006-4437-9>.
- Eric, C.C.M. (2010). Tracing primary 6 students' model development within the mathematical modelling process. *Journal of Mathematical Modelling and Application*, 1(3), 40-47.
- Eric, C.C.M. (2013). Students' designing an ideal tourism route as mathematical modelling. In: Stillman, G. A., Kaiser, G., Blum, W., & Brown, J.P. *Teaching Mathematical Modelling: Connecting to Research and Practice*. Brazil: Springer.
- Even, R., & Tirosh, D. (1995). Subject-matter knowledge about students as sources of teacher presentations of the subject-matter. *Educational Studies in Mathematics*, 29(1), 1-20. <https://doi.org/10.1007/BF01273897>.
- Ferri, R.B. (2018). *Learning How to Teach Mathematical Modeling in School and Teacher Education*. Kassel: Springer International Publishing.
- Frejd, P. (2012). Teachers' conceptions of mathematical modelling at Swedish upper secondary school. *Journal of Mathematical Modelling and Application*, 1(5), 17-40.
- Garofalo, J., & Lester, F. (1985). Metakognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16(3), 163-176. <http://dx.doi.org/10.2307/748391>.
- Haines, C., & Crouch, R. (2010). Remarks on a modelling cycle and interpretation of behaviours. In: Lesh, R., Galbraith, P.L., Haines, C.R., & Hurford, A. (Eds.). *Modeling Students' Mathematical Modeling Competencies*. New York: Springer. <http://doi.org/10.1007/978-1-4419-0561-1>
- Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial

- visualization. *Journal of Physics: Conference Series*, 943(1), 012029. <https://doi.org/10.1088/1742-6596/943/1/012029>.
- Hill, H.C., Ball, D.L., & Schilling, S.G. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.
- Hino, K. (2007). Toward the problem-center classroom: Trends in mathematical problem solving in Japan. *ZDM Mathematics Education*, 39(5-6), 503-514. <http://doi.org/10.1007/S11858-007-0052-1>.
- Kantowski, M.G. (1981). Problem solving. In: Fenemina, E. (Ed.) *Mathematics Education Research: Implication for the 80's*. Reston, VA: National Council for the Teaching of Mathematics (NCTM).
- Kawasaki, T. (2012). The problems of mathematical modelling introduction on mathematics education in Japanese school. *Journal of Mathematical Modelling and Application*, 1(5), 50-58.
- Kilbane, C.R., & Milman, N.B. (2014). *Teaching Models Designing Instruction For 21st Century Learners*. New York: Pearson.
- Knott, A. (2014). The Process of Mathematisation in Mathematical Modelling of Number Patterns in Secondary School Mathematics. *Unpublish Dissertation*. Stellenbosch: Stellenbosch University.
- Lingefjard, T. (2006). Faces of mathematical modeling. *Zentralblatt Fur Didaktik Der Mathematik*, 38(1), 96-112. <https://doi.org/10.1007/BF02655884>.
- Maaß, J., O'meara, J., & Patrick, O. (2018). *Mathematical Modelling for Teacher: A Practical Guided to Applicable Mathematics Education*. Switzerland: Springer.
- Martinez-Luaces, V.E. (2005). Engaging secondary school and university teachers in modelling: Some experiences in South American countries. *International Journal of Mathematical Education in Science and Technology*, 36(2), 193-205. <https://doi.org/10.1080/0020739051233134003>.
- Mousoulides, N. (2007). The Modeling Perspective in the Teaching and Learning of Mathematical Problem Solving. *Unpublish Dissertation*. Cyprus: University of Cyprus.
- Muhtadi, D., Sukirwan, Warsito, & Prahmana, R.C.I. (2017). Sundanese ethnomathematics: mathematical activities in estimating, measuring, and making patterns. *Journal on Mathematics Education*, 8(2), 185-198. <https://doi.org/10.22342/jme.8.2.4055.185-198>.
- Mumu, J., Prahmana, R.C.I., & Tanujaya, B. (2018). Construction and reconstruction concept in mathematics instruction. *Journal of Physics: Conference Series*, 943(1), 012011. <https://doi.org/10.1088/1742-6596/943/1/012011>.
- Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. *Journal on Mathematics Education*, 10(1), 127-142. <https://doi.org/10.22342/jme.10.1.6913.127-142>.
- Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2017). Mathematical modeling in realistic mathematics education. *Journal of Physics: Conference Series*, 943(1), 012049. <https://doi.org/10.1088/1742-6596/943/1/012049>.

- Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2018). Mathematical learning through modeling task in senior high school: Using nutrition context. *Journal Physics: Conference Series*, 1097(1), 012102. <https://doi.org/10.1088/1742-6596/1097/1/012102>.
- Riyanto, B., Zulkardi, Putri, R.I.I., & Darmawijoyo. (2019). Learning mathematics through modeling tasks in elementary school: Using growth of population context. *Journal of Physics: Conference Series*, 1166(1), 012033. <https://doi.org/10.1088/1742-6596/1166/1/012033>.
- Robinson, K. (1999). *Creativity, Culture and Education. Report*. London: National Advisory Committee on Creative and Cultural Education.
- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education*, 9(1), 41-54. <https://doi.org/10.22342/jme.9.1.5049.41-54>.
- Saxena, R., Shrivastava, K., & Bhardwaj, R. (2016). Teaching Mathematical Modeling in Mathematics Education. *Journal of Education and Practice*, 7(11), 34-44.
- Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In: Grouws, D. (Ed.) *Handbook of Research on Mathematics Teaching and Learning*. New York: Mcmillan.
- Stillman, G., & Brown, J.P. (2014). Evidence of implemented anticipation in mathematising by beginning modellers. *Mathematics Education Research Journal*, 26(4), 763-789. <http://doi.org/10.1007/S1339-014-0119-6>.
- Stillman, G., Brown, J., & Galbraith, P. (2010). Researching application and mathematical modelling in mathematics learning and teaching. *Mathematics Education Research Journal*, 22(2), 1-6. <https://doi.org/10.1007/BF03217561>.
- Sullivan, P., Knott, L., & Yang, Y. (2015). The relationships between tasks design, anticipated pedagogies, and student learning. In: Watson, A., Ohtani, M. *Task Design in Mathematics Education: an ICMI Study 22*. New York: Springer.
- Tessmer, M. (1993). *Planning and Conducting Formative Evaluation*. Philadelphia: Kogan Page.
- Wessels, H. (2014). Levels of mathematical creativity in model-eliciting activity. *Journal of Mathematical Modelling and Application*, 1(9), 22-40.
- Wong, K.Y. (2008). An extended Singapore mathematics curriculum framework. *Maths Buzz*, 9(1), 2-3.
- Yanagimoto, T. (2005). Teaching modelling as an alternative approach to school mathematics. *Teaching Mathematics and its Applications*, 24(1), 1-13. <http://doi.org/10.1093/teamat/hrh011>.
- Zulkardi & Putri, R.I.I. (2019). New school mathematics curricula, PISA, and PMRI in Indonesia. In: Vistro-Yu, C.P., & Toh, T.L. (eds.). *School Mathematics Curricula, Mathematics Education – An Asian Perspective*. Singapore: Springer.
- Zulkardi. (2002). Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers. *Unpublished Dissertation*. Enschede: University of Twente.
- Zulkardi. (2006). *Formative Evaluation: What, Why, When, and How*. Retrieved Juni 2019. from <http://www.reocities.com/zulkardi/books.html>.
- Zulkardi. (2017). Pembelajaran pemodelan matematika. *Seminar Nasional Matematika dan Pendidikan Matematika (SNMPM)*. Palembang: Universitas Sriwijaya.



REFLECTIVE THINKING SKILLS OF ENGINEERING STUDENTS IN LEARNING STATISTICS

Rindu Alriavindra Funny¹, Muhammad Abdul Ghofur², Wuli Oktiningrum³, Ni Luh Sakinah Nuraini⁴

¹Sekolah Tinggi Teknologi Adisutjipto, Jl. Janti Blok R, Lanud Adisutjipto, Yogyakarta 55198, Indonesia

²Akademi Angkatan Udara, Jl. Laksda Adisucipto, Berbah, Sleman, DIY 55281, Indonesia

³Universitas Islam Raden Rahmat, Jl. Raya Mojosari No 2 Kepanjen, Malang, Jawa Timur, Indonesia

⁴Universitas Negeri Malang, Jl. Semarang No.5, Kota Malang, Jawa Timur 65145 Indonesia
Email: rindualri@stta.ac.id

Abstract

Reflective thinking is known as assessing what they know, what they need to know, and how they bridge that gap during learning situations. Relating what students knew with what they learn today is not easy, sometimes they forget or just cannot make a connection between it. This study will analyze how the students actively participating in reflective thinking by creating project-based learning on statistics lesson based on design research. Thirty-nine engineering students, in a group of four, were asked to make their own data, analyze and transfer it into a poster to inform the readers about the use of statistics in daily life. This study used the measurement of reflective thinking scales by Kember to analyze students answer through the worksheet and instructional activity. The instructional activity succeeds to reveals students thinking of all four skills; habitual action, understanding, reflection, and critical reflection. Furthermore, it also discovered the misuse of some terms when used in daily life.

Keywords: reflective thinking, engineering students, statistics, design research

Abstrak

Berpikir reflektif dapat diartikan sebagai mengevaluasi apa yang sudah mahasiswa ketahui, apa yang harus mahasiswa ketahui dan bagaimana mahasiswa menjembatani perbedaan informasi tersebut selama proses pembelajaran. Proses penghubungan informasi lama dan informasi baru tidaklah mudah, terkadang mahasiswa lupa atau tidak dapat menghubungkannya. Penelitian ini akan menganalisa bagaimana mahasiswa secara aktif berpartisipasi dalam berpikir reflektif dengan mendesain suatu pembelajaran berbasis proyek pada mata kuliah Statistika yang didasari oleh desain riset. Tiga puluh Sembilan mahasiswa teknik, yang dibagi dalam kelompok beranggotakan empat mahasiswa, diminta untuk membuat atau mencari data mereka sendiri, kemudian menganalisis dan merepresentasikannya ke dalam poster. Poster tersebut ditujukan untuk memberikan informasi kepada pembaca tentang penggunaan statistika di kehidupan sehari – hari. Skala berpikir reflektif yang dilakukan oleh Kember digunakan untuk menganalisis jawaban mahasiswa melalui lembar kerja mahasiswa dan kegiatan pembelajaran. Kegiatan pembelajaran yang dilakukan telah berhasil untuk memunculkan 4 kemampuan berpikir reflektif mahasiswa yaitu, *habitual action*, *understanding*, *reflection* dan *critical reflection*. Lebih lanjut, penelitian ini juga menemukan kesalahan penggunaan istilah statistika ketika digunakan dalam kehidupan sehari-hari.

Kata kunci: berpikir reflektif, mahasiswa teknik, statistika, desain riset

How to Cite: Funny, R.A., Ghofur, M.A., Oktiningrum, W., & Nuraini, N.L.S. (2019). Reflective Thinking Skills of Engineering Students in Learning Statistics. *Journal on Mathematics Education*, 10(3), 445-458. <https://doi.org/10.22342/jme.10.3.9446.445-458>.

Learning nowadays is becoming more complex, information is becoming available and changing more fast that promote learners to constantly rethink, switch directions, and change problem-solving strategies (Porntaweekul, Raksasataya, & Nethanomsak, 2015; Shahrill et al., 2018). These support the suggestions to need learning which implemented to train the students think (Palinussa, 2013). It indicates that during the process of thinking students should reflected by empower and allow the skills which he has, so that he could understood and mastered what he does. Therefore, it is important to reveal reflective

thinking during learning to help learners develop strategies to apply new knowledge to the complex situations in their day-to-day activities.

Reflective thinking is a skill which will help learners to express tacit learning habits (Kizilkaya & Askar, 2009) which is an inability to actually explain what they know because they just know but not understand (Kuswandono, 2017). These skills also have significant role in the development of critical thinking (Kizilkaya & Askar, 2009) and critical thinking is one of the criteria that support high-order thinking (Ahmad et al., 2018). Then reflective thinking will help learners to develop higher-order thinking skill by encourage students to connect new knowledge to prior understanding, to think in both abstract and conceptual terms, to demonstrate specific strategies in novel tasks, and to understand their own thinking and learning strategies

Reflective thinking could support students to experience meaningful learning process (Van Es, 2006). Since students usually seeing a way to take on the future challenges by reflecting on their accomplishments and the opportunities for refinement. Ambrose (2004) and Gelter (2003) stated that reflective thinking gives opportunities for students to improve their weakness such as justifying misconceptions by helping students to think about what they did and why they did it. It indicates that the reflective thinking can occurs after mathematical problem solving conducted with the aim checking the error of the concepts used and try to justify these misconceptions, so that it can give meaningful learning process on mathematical concepts (Betne, 2009). In addition, Agustan (2017) stated that reflective thinking can be investigated after someone did a mathematical problem solving. Furthermore, reflective thinking can rise the accuracy and concentration when the students solve a mathematical problem.

Practicing reflective thinking in the learning process can be seen by where one question is answered by another question since it is a deliberate process of undertaking or cycles of inquiry (Ramsey, 2006). It will easily be followed under discussion process. Reflective practice has also become part of the movement for 'active learning' (McCoy, 2013). Project based learning, or PBL, is one of the best way to ginger students to have active learning. It is because the instructional approach in PBL built upon learning activities and real tasks that have brought challenges for students to solve. Moreover, these activities generally reflect the types of learning and work people do in the everyday world outside the classroom. Problem-based learning engages students with complex, real-world projects that teach significant subject content and require the use of a variety of 21st century skills (Grant, 2002; Ahamad et al., 2018). PBL also allows students to reflect upon their own ideas and opinions, and make decisions that affect project outcomes and the learning process in general. Thus, the model of project based learning accordance to the concept of reflective thinking.

In the college, statistics course is the development of statistical material that students got in the Senior High School. This development will be used for research analyzing to do thesis. Therefore, if students could not get the essential knowledge of the concept so they will tend to just follow the formula without a proper understanding of the meaning (Palinussa, 2013; Tanujaya et al., 2018). It is still could

be categorized as accomplishment of meaningful understanding in students (Dündar & Gündüz, 2017; Kusumaningsih et al., 2018). Usually, in this case students do not know what they are doing, why they are doing this and what is they doing for. Like when students who can cite the definition of a parallelogram but they could mention that rectangle is not a parallelogram is still categorized as meaningful understanding (Muhtadi et al., 2017; Saleh et al., 2018). One solution to help students have better understanding on statistics is to integrate active-learning strategies that grants students to apprehend what they have heard and read about statistics (Smith, 1998; Tanujaya et al., 2018).

Finally, reflective thinking answers the need of incorporate of what students' have heard, read and learn with what they will learn today. Therefore, the purpose of this study was to develop an instructional activity which can support students' reflective thinking skills and to measure their reflective thinking by using the instructional activity. Investigate subject' thinking can give description and insight how reflective thinking can be applied in learning mathematical concept. Since when students doing the project, the teacher can observe students' processes of learning and student's mathematical reflective thinking (Anwar et al., 2012). As a result, we can develop the best way to teach the students in providing a chance to reflect their prior knowledge and connect it with the future.

METHOD

This study designed instructional activity based on design research, an approach that envisions a tighter, more rigorous connection between learning principles and feature of the educational innovation (Walker, 2006), to reveal students' reflective thinking skills. The aims of this study in line with design research is to discover ways to develop a design, such as learning activity, based on theory and to determine the effectiveness of this design in practice (Funny, 2019). Hence, the design of instructional activity of this study will help to improve the mathematics education toward the reflective thinking skills. Combining with project based learning, the innovative design will give an intervention for the students to learn statistics while reveal their reflective thinking skills.

The participants of this study consisted of 39 of second year students in Adisutjipto College of Technology majoring on Aerospace Engineering who take statistics and probability course in the academic year of 2018/2019. Participants were divided into groups of four students to do the instructional activity based on project based learning.

Instruments to measure the reflective thinking skills when the students learning statistics was adopted from Kember et al. (2000). The worksheet as this instrument was the opposite of Kember Reflective Thinking Scale' Questionnaire. If the questionnaire provides a statements that should be answered in five degree of Likert Scale, our worksheet will ask questions with various answers. The main idea of the worksheet both for poster maker and visitors is writing questions to reveal the reflective thinking of learning process of statistics. The students' answers which will be examined using reflective

thinking scales by Kember that consists of four constructs or skills (Habitual Action (HA), Understanding (UN), Reflection (RE) and Critical Reflection (CR)).

The experiment was administered to the participants in their regular classrooms by the researcher. The project was delivered to the students begin on the second meeting of the course then it will continue until the Exhibition Day on the seventh meeting. This project consumed 6 weeks of the course. On the fourth until sixth meeting participant was having group discussion to analyze the data. This group discussion was paper-based recorded in the worksheet and by the researcher. In the exhibition day, the participants were divided into two roles, poster maker and visitor. The worksheet has to be completed while they were playing their roles.

The participants' responses were recorded by the researcher both in paper-based and digital-based like taking a photo and recording a video. These data were analyzed using qualitative approach in order to describe the implementation of learning activities and processes to reveal the reflective thinking skills clearly.

RESULT AND DISCUSSION

The Instructional Activity in Learning Statistics which Promote Students' Reflective Thinking

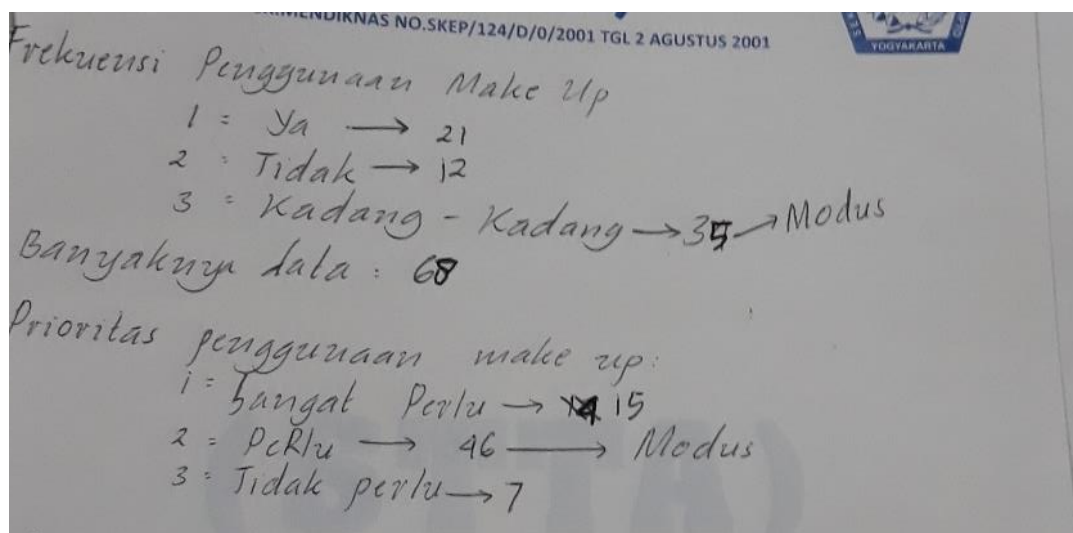
Table 1 shows the instructional activity which is designed in this study. It consists of three phase which are planning, analysing and displaying. This activity referred to project-based learning method as a small project in a short period. The focus on this project is to reveal students' reflective thinking skills. Therefore, statistics is selected as the teaching material since students have already learn the basic skills of statistics since Elementary Schools. So, it is expected to ease students recalling their prior knowledge and connecting with their problem now. This project call "Say it with Data". Furthermore, the instructional activity which was designed by this study is briefly summarised in Table 1.

Table 1. Main Instructional activity in the project 'Say it with Data'

Phase	Duration	Teaching Activities
Planning	Week 1 – Week 2	<ol style="list-style-type: none"> a. Group Formation b. Observe environment c. Identify issues or problem in their daily school activity d. Determine the title of the data e. Pose questioner to collect a data (Data Collection) f. Taking a photo while collecting data as a proof that the data is collected by their own
Analysing Data	Week 3 – Week 5	<ol style="list-style-type: none"> a. Writing all info that you want to know from the data b. Discussing how to answer questions (a) c. Find the answer of questions (a) d. Summarize it all in a poster
Displaying the Data	Week 6	<ol style="list-style-type: none"> a. Display all the posters in Exhibition Day of "Say it with Data". b. One students each group have to be a poster maker and have to answer all visitor questions. c. Others are being the visitor to examine every poster d. Each student, while doing his role, will complete the worksheet. e. Submit the worksheet.

Table 1 shows the results of the instructional activity which could reveal students' reflective thinking skills. Although the students' response shows misconception of their prior knowledge but it can prove that students reflect the problems in the project with their existing knowledge. Like the concept of Mode. It is known in school that mode means the value that appears most often.

For example, the mode of a set [2, 4, 6, 6, 6, 6, 9, 9, 11, 11, 15] is 6. This concept has already learnt by students in Junior High School. They also learnt that mode could not just one number, can be two numbers or three numbers that appears with same often. Such as, given a set of data [3, 3, 5, 7, 7] the mode is not unique – the data set may be said to be bimodal, while a set with more than two modes may be described as multimodal. This will not be a problem until one group in the project come up with the data 'The importance of using Makeup' (see Figure 1). The data shows that 15 students said 'Very Important', 46 argued 'Just Important' while just 7 students considered 'It does not necessary'. Looking the set of number [15, 46, 7], students easily conclude that the 46 is the mode of this data. Some students argued that 46 is the Maximum Value of the data, not the Mode. Others also have different opinion that said this data does not have Mode since all the number appears once. It leads students to have class discussion. They recalled all their prior knowledge about the Mode and connected into their current problems (Figure 1).



Frequency using make-up:

1. Yes = 21
2. No = 12
3. Sometimes = 35 (means "modus")

The number of all data = 68

Make-up utilization priority:

1. Very important = 15
2. Important = 46 (means "modus")
3. Not Important = 7

Figure 1. Data of 'The importance of using Makeup'

After several debates, the discussion drawn a conclusion that Mode in discrete data is the value that appears most often. Meanwhile, Mode in the continuous data is a locally maximum value, so any peak is a mode. In order to estimate the mode of the underlying distribution, the usual practice is to discretize the data by assigning frequency values to intervals of equal distance, as for making a histogram, effectively replacing the values by the midpoints of the intervals they are assigned to. The mode is then the value where the histogram reaches its peak (Rumsey, 2006)

This discussion shows that the activity in the project could reveal reflective thinking skills of the students. It has been successfully indicated students of four skills of Reflective Thinking by Kember et al. (2000), namely Habitual Action, Understanding, Reflection and Critical Reflection (see Table 2). The Habitual Action (HA) addressing the students who disagree that 46 is the Mode as their dependency on what their School teachers says as the only source of learning. While Understanding (UN) is addressed by students' opinion that 46 is Maximum Value or the data does not have Mode as their continuous thinking of the concept and content of the problem. Furthermore, the Reflection (RE) can be showed in the class discussion which students concern on questioning the way other argued, thinking of alternative ways of argument, reflecting and re-appraising experience within others. Last, the Critical Reflection (CR) pointed out by the conclusion that the "Importance of using Makeup" cannot be seen as a usual discrete data. The CR that shown in this case deal with changing students' outlook and challenging their firmly held ideas. Actually, there were several cases in applying this instructional activity which can reveal students' reflective thinking skills. Parsimony, we leave the other cases in this paper.

Table 2. Construction of Reflective Thinking Process and Questionnaire (Kember et al, 2000)

Dewey (1998)	Lee (2000)	Agustan et.al (2017)	Kember et.al (2000)
<ul style="list-style-type: none"> • An experience • Spontaneous interpretation of the experience • Naming the problem • Generating possible explanations for the problem • Ramifying the explanations into full-blown Hypotheses • Experimenting or testing the selected hypotheses 	<ul style="list-style-type: none"> • Problem context • Problem definition/reframing • Seeking possible solution • Experimentation • Evaluation • Acceptance / rejection 	<ul style="list-style-type: none"> • Formulation and synthesis of experience • Orderliness of experience • Evaluating the experience • Testing selected solution based on the experience 	<ul style="list-style-type: none"> • Habitual Action (HA): behaviour knowing-in action • Understanding (UN): makes use of existing knowledge • Reflection (RE): engage to explore their experiences in order to lead to new understanding • Critical Reflection (CR): becoming aware of why we perceive, think, feel or act as we do

Hence, the result of the worksheet that given in the Exhibition Day shows the effect of instructional activity into reflective thinking skills of the students. Moreover, students were very

enthusiastic in playing their roles. The visitors truly bombarded the poster maker with many questions in the middle of hot class air as seen in the Figure 2. The poster maker should be able to explain all about the poster of their group made, the content, the process, the analysis or the conclusion.



Figure 2. The poster maker answers visitor questions

While students got around the exhibition, they have to write down all the questions and answers they have done (see Figure 3). Since this study will examine student’s reflective thinking skills based on their answer in the worksheet.



Figure 3. The participants record the answer and the question on the worksheet

The design of the poster also become one of attractive point in the exhibition. The visitors usually come into their favorite poster first then asking some questions. In this phase, visitors tried to pay full attention into poster makers’ explanation (see Figure 4).



Figure 4. The visitors observe the poster maker explanation seriously

The Students' Reflective Thinking Analysis Using the Instructional Activity

The result of the worksheet shows all of reflective thinking skills appears on students. Refer to Table 2 of Construction of Reflective Thinking Process and the Questionnaire of Kember et.al (2000), this study is naming the categories emerging from the analysis with the four skills as described below:

- a. Questions that belong to Habitual Action (HA) appears when the questions are such a habit of repetition and their dependency on what the teacher says as the only source of learning. In other words, it is such a common questions or using questions that have been given as an example before.
- b. Questions that belong to Understanding (UN) appears when the questions show their understanding of the concept and content continuous thinking of material taught. We could say it such a developed question that has deep comprehension.
- c. Questions that belong to Reflection (RE) appears when the students' questioning the way others do, thinking of alternative ways, reflection on actions and re-appraising. Let say 'out of the box' questions.
- d. Questions that belong to Critical Reflection (CR) appears when the questions changing students' outlook, challenging their firmly held ideas, changing their routines as well as finding faults with their belief. This types of challenge question or badly called as deadly questions.

From all the participants we found more than 173 questions, 122 from visitors and 51 from poster maker. Since a visitor can visit more than one poster, so he will collect several questions. Here the briefly number about four s of questions that we have found in the worksheet.

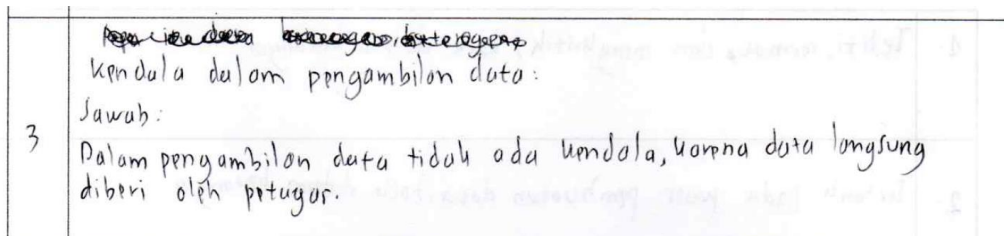
Table 3. The results of reflective thinking skills among students' questions

	Visitors (asking questions)		Poster maker (being questioned)	
HA	66	54%	29	57%
UN	29	24%	11	22%
RE	20	16%	7	14%
CR	7	6%	4	8%

Table 3 shows that more than half of students' questions still belongs to the first skill, HA. The questions that belonging to habitual action skills are listed below:

- 'How do you get the data?'*,
- 'What methods do you use to collect your data?'*,
- 'Is there any difficulties in collecting the data?'*,
- 'What is the aims of your data?' and etc.*

It shows that students just asking for an ordinary question, that they actually have already known the answer without asking to the poster maker. Since one students can ask several questions, it found that almost on every student question appeared this skills (see Figure 5), habitual action.



“Q: Did you find any problem in collecting the data? “

“A: No, I did not. Since it is given by the officer. “

Figure 5. Question belongs to Habitual Action

Furthermore, quarter of questions stand in UN skills by asking the material of the poster information related with statistics. It is questions about something deeper from the poster such as:

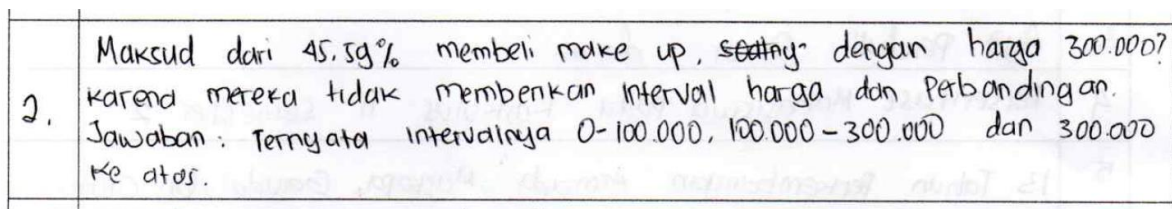
‘How long do you take the data?’

‘How many sample do you take to complete your data?’

‘Is the data homogenous?’

‘Did you taking a part in collecting the data?’ and etc.

These question not only asking about how do you get the data but also the story behind it. These students have applied their knowledge about collecting the data, types of data, and the roles of researcher and so on. Therefore, these kinds of questions (see Figure 6) are belongs to UN.



“Q: What is the meaning of 45.59% buying make up for Rp. 300.000? Since the poster did not give the price interval or comparison. “

“A: Actually, the interval used in this research are 0-100.000, 100,000- 300.00 and more than 300.000“

Figure 6. Questions belongs to Understanding

Meanwhile, the Reflection (RE) skills come from questions about relating students’ old knowledge to get new understanding like:

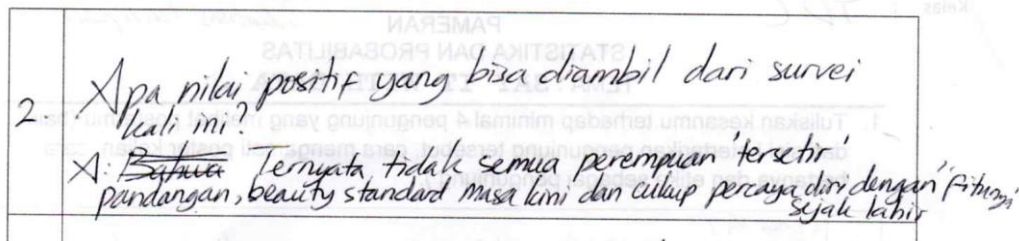
‘Why for boys, the class is distributed fairly?’

‘Why women using make up?’

‘Could blood being expired?’

‘Why the participant using social media?’ and etc.

The point is, the questions reflect information that not appear in the poster but they can consider it from the poster. They reflect the poster whether it could be improved or could be done in a better way with more information. Therefore, these types of questions are Reflection (see Figure 7).



“Q: What is the positive conclusion from your Survey? “

“A: Not all woman is driven by beauty stigma. Some of them are confident with their inner beauty. “

Figure 7. Questions belong to Reflection

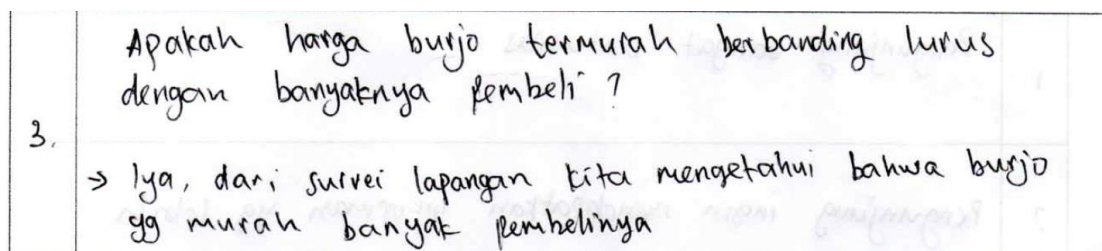
The last question skill that appears in the worksheet is CR skill. The questions that belongs this skill must go through various experience, reflection and thought also related with testing other knowledge, for example:

‘What is the meaning of deviation coefficient that you wrote on your poster?’

‘What is the meaning of heterogeneous and homogeneous from the library visitors?’

‘How does the way to improve passenger’ safety in the flight?’

This kind of questions often need additional explanation after the first answers. So, it could frighten the answerer or it could not be answered. Thus, sometimes it become a deadly question like the question on Figure 8.



“Q: Is the price of Burjo (Mung Bean Porridge) rise as long as the customer demand?”

“A: Yes, we had surveyed that the cheaper Burjo make more customer come to buy.”

Figure 8. Questions belong to Critical Reflection

In addition, the reflective thinking also found on the students’ opinion of the learning process like shows in Figure 9.

4. Tuliskan pendapatmu tentang pameran ini secara keseluruhan.

Menurut saya, masih ada data yang tidak pas dengan judul. Sebagai contoh, judul: Pengaruh tetapi tidak diperlihatkan pengaruhnya, hanya data dari survey. Ada juga yang sudah sangat bagus, dari segi design poster, segi isi poster. Tidak hanya data, ada fakta permasalahan atau adanya kalimat persuasif, yang mengajak untuk hal positif? Overall, pameran ini terkesan ajang pameran data saja. Sekian. Terimakasih.

"In my opinion, there is some data that insufficient with the title. For example, the title is "The impact of" but they did not put it out in the poster. The design of the poster was well -turned, while the content not only show data but also problem facts. Moreover, it has persuasive sentence which leads to positive things. Overall, this exhibition seemed a data showcase."

Figure 9. Students Opinion that shows Reflective Thinking

Figure 9 showed students notion about the exhibition they had. All of the students gave reflection on this part. Some of them just gave common reflection but many of them gave deeper reflection of what they like and dislike from this project, what they feel about the way of learning and also their recommendation to have better project next time. Critical reflection not only showed from the opinion about the project but also from the opinions of the poster maker about the visitor character. When they asked to give their impression from the visitor that came to their poster, they told us positive response. Most of them showed that the visitors were very interested to their poster by asking many questions that sometimes unpredictable for the poster maker (see Figure 10).

3.	Hizkia Timotius, sangat menarik pertanyaannya diluar dari pikiran saya, namun
----	---

"Hizkia Timotius have a catchy question which out of my mind."

Figure 10. Poster Maker Opinion That Belong to Critical Reflection.

Reflective thinking skill of this engineering students' mostly still in Habitual Action (HA) skill. Although, some of them success to reveal the other three skills, Understanding (UN), Reflection (RE) and Critical Reflection (CR). But it happened as the prompting of the instructional activity on the project designed. Reflective thinking usually is not spontaneous; it must be stimulated intentionally by the educational context (Khalid et al., 2015). Lecturers should be able to allow for approaches that put forward learners' mental activities like reflective and critical thinking now and later in their classroom learning process (Tuncer & Ozeren, 2012).

CONCLUSION

The Instructional Activities which made in this study were success to reveal students Reflective Thinking skills. But students still need encouragement to reflect spontaneously on their learning process. Therefore, lecturers and learning activities still play an important role in guiding students to develop better reflective thinking skill. Also, it found that the coding scheme of Kember et al (2000), with refinement, could be used to analyze and categorized types of questions in students' reflections thinking skills. Revealing student' reflective thinking skill is not an easy job. If it has appeared, try to maintain and improve it. Because we know that learning is a lifelong journey.

REFERENCES

- Agustan, S., Juniati, D., & Siswono, T.Y.E. (2017). Reflective thinking in solving an algebra problem: a case study of field independent-prospective teacher. *Journal of Physics: Conference Series*, 893(1), 012002. <https://doi.org/10.1088/1742-6596/893/1/012002>.
- Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, 943(1), 012008. <https://doi.org/10.1088/1742-6596/943/1/012008>.
- Ahmad, S., Prahmana, R.C.I., Kenedi, A.K., Helsa, Y., Arianil, Y., & Zainil, M. (2018). The instruments of higher order thinking skills. *Journal of Physics: Conference Series*, 943(1), 012053. <https://doi.org/10.1088/1742-6596/943/1/012053>.
- Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education*, 7(2), 91-119. <https://doi.org/10.1023/B:JMTE.0000021879.74957.63>.
- Anwar, L., Budayasa, I.K., Amin, S.M., & de Haan, D. (2012). Eliciting mathematical thinking of students through Realistic Mathematics Education. *Journal on Mathematics Education*, 3(1), 55-70. <https://doi.org/10.22342/jme.3.1.620.55-70>.
- Betne, P. (2009). Reflection as a learning tool in mathematics. *Transit: The LaGuardia Journal on Teaching and Learning*, 4, 93-101.
- Dündar, S., & Gündüz, N. (2017). Justification for the Subject of Congruence and Similarity in the Context of Daily Life and Conceptual Knowledge. *Journal on Mathematics Education*, 8(1), 35-54. <https://doi.org/10.22342/jme.8.1.3256.35-54>.
- Funny, R.A. (2019). Prompting the use of online application on smartphone (integral calculator) in learning integration techniques. *Journal of Physics: Conference Series*, 1200(1), 012017. <https://doi.org/10.1088/1742-6596/1200/1/012017>.
- Gelter, H. (2003). Why is reflective thinking uncommon. *Reflective Practice*, 4(3), 337-344. <https://doi.org/10.1080/1462394032000112237>.
- Grant, M.M. (2002). Getting a grip on project-based learning: Theory, cases and recommendations. *Meridian: A Middle School Computer Technologies Journal*, 5(1), 83-99.
- Kember, D., Leung, D.Y., Jones, A., Loke, A.Y., McKay, J., Sinclair, K., ... & Yeung, E. (2000). Development of a questionnaire to measure the level of reflective thinking. *Assessment & Evaluation in Higher Education*, 25(4), 381-395. <http://dx.doi.org/10.1080/713611442>.

- Khalid, F., Ahmad, M., Karim, A.A., Daud, M.Y., & Din, R. (2015). Reflective thinking: an analysis of students' reflections in their learning about computers in education. *Creative Education*, 6(20), 2160-2168. <http://dx.doi.org/10.4236/ce.2015.620220>.
- Kizilkaya, G. & Askar, P. (2009). The development of a reflective thinking skill scale towards problem solving. *Education and Science*, 34(154), 82-92.
- Kusumaningsih, W., Darhim, Herman, T., & Turmudi. (2018). Improvement Algebraic Thinking Ability Using Multiple Representation Strategy on Realistic Mathematics Education. *Journal on Mathematics Education*, 9(2), 281-290. <https://doi.org/10.22342/jme.9.2.5404.281-290>.
- Kuswandono, P. (2017). Reflective practices for teacher education. *LLT Journal: A Journal on Language and Language Teaching*, 15(1), 149-162. <https://doi.org/10.24071/llt.2012.150102>.
- McCoy, B. (2013). Active and reflective learning to engage all students. *Universal Journal of Educational Research*, 1(3), 146-153. <https://doi.org/10.13189/ujer.2013.010302>.
- Muhtadi, D., Sukirwan, Warsito, & Prahmana, R.C.I. (2017). Sundanese ethnomathematics: mathematical activities in estimating, measuring, and making patterns. *Journal on Mathematics Education*, 8(2), 185-198. <https://doi.org/10.22342/jme.8.2.4055.185-198>.
- Palinussa, A. L. (2013). Students' critical mathematical thinking skills and character: Experiments for junior high school students through realistic mathematics education culture-based. *Journal on Mathematics Education*, 4(1), 75-94. <http://dx.doi.org/10.22342/jme.4.1.566.75-94>.
- Porntaweekul, S., Raksasataya, S., & Nethanomsak, T. (2015). Development of the reflective thinking instructional model for student teacher. *International Forum of Teaching and Studies*, 11(1-2), 24-32.
- Ramsey, C. (2006). *Introducing Reflective Learning*. The Open University, UK: Thanet Press Limited.
- Rumsey, D. (2006). *Probability for Dummies*. Indiana Polis, Indiana: Wiley Publishing, Inc.
- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education*, 9(1), 41-54. <https://doi.org/10.22342/jme.9.1.5049.41-54>.
- Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings*, 1952(1), 020019. <https://doi.org/10.1063/1.5031981>.
- Smith, G. (1998). Learning statistics by doing statistics. *Journal of Statistics Education*, 6(3), 1-12. <https://doi.org/10.1080/10691898.1998.11910623>.
- Tanujaya, B., Prahmana, R.C.I., & Mumu, J. (2018). Designing learning activities on conditional probability. *Journal of Physics: Conference Series*, 1088(1), 012087. <https://doi.org/10.1088/1742-6596/1088/1/012087>.
- Tuncer, M., & Ozeren, E. (2012). Prospective teachers' evaluations in terms of using reflective thinking skills to solve problems. *Procedia-Social and Behavioral Sciences*, 5, 666-671. <https://doi.org/10.1016/j.sbspro.2012.08.221>.
- Van Es, J.M. (2006). Encouraging reflective thinking in the high school classroom: Effective use of questioning and wait time strategies. *Master Thesis*. Iowa: Dordt University.
- Walker, D. (2006). Toward productive design studies: Why now? In Akker, J. Van den et al. (Eds). *Educational Design Research*. London and New York: Routledge-Taylor & Francis Group.

AQUATIC IN ASIAN GAMES: CONTEXT OF PISA-LIKE MATHEMATICS PROBLEM

Levana Maharani, Ratu Ilma Indra Putri, Yusuf Hartono

Universitas Sriwijaya, Jalan Sriwijaya Negara, Bukit Besar, Palembang, Indonesia
Email: ratu.ilma@yahoo.com

Abstract

This study aimed to create mathematical problems of uncertainty and data contents in PISA using aquatic context that were valid, practical and had the potential effect. This study was design research of development study. The subjects in this study were 10th-grade students of Indo Global Mandiri senior high school in Palembang consisting of 20 students. Data were gathered through interviews, observation, and tests. The results of data analysis showed that there were eight valid and practical items of PISA type of uncertainty and data contents. The contents obtained using bicycles and aquatic contexts in Asian Games. The 11 out of 20 students showed reasoning skills and reasonable arguments, and 9 out of 20 students showed reasoning skills and discussions but incomplete. It was because they were not used to solving PISA type problems in learning.

Keywords: task design, PISA, aquatic context

Abstrak

Tujuan dari penelitian ini adalah menghasilkan soal matematika tipe PISA dengan konteks cabang olahraga akuatik yang valid, praktis, serta memiliki efek potensial. Penelitian ini merupakan penelitian pengembangan *design research* tipe *development study*. Subjek dalam penelitian ini adalah siswa kelas X IPA 1 SMA LTI-IGM Palembang yang berjumlah 20 siswa. Pengambilan data dilakukan dengan cara wawancara, observasi dan tes. Dari hasil analisis data penelitian ini menghasilkan 8 soal tipe PISA konten *uncertainty and data* menggunakan konteks sepeda dan akuatik yang valid dan praktis dan dapat disimpulkan bahwa 11 dari 20 siswa menunjukkan kemampuan penalaran dan argumen yang baik dan 9 dari 20 siswa menunjukkan kemampuan penalaran dan argumen tetapi kurang lengkap, hal ini dikarenakan tidak terbiasanya siswa dengan soal-soal tipe PISA dalam pembelajaran.

Kata kunci: *task design*, PISA, konteks cabang olahraga akuatik

How to Cite: Maharani, L., Putri, R.I.I., & Hartono, Y. (2019). Aquatic in Asian Games: Context of PISA-Like Mathematics Problem. *Journal on Mathematics Education*, 10(3), 459-470. <https://doi.org/10.22342/jme.10.3.5252.459-470>.

Statistics is part of the mathematical material consisting of ways of data collection, data processing, and drawing conclusions based on data collection and data analysis performed (Bakker, 2004; Chong, et al. 2017; Tanujaya, et al. 2018). Statistics is widely applied in various disciplines like in natural sciences, business, and industry, where almost every decision made in the field uses statistical reasons (Adams & Lawrence, 2018; Salkind & Frey, 2019). Therefore, statistics is one of the essential subject that must be to learn for students.

Many students face difficulties in solving statistical problems that result in low achievement. It happens because students have not been accustomed to solving problems with the characteristics of a real context, and only work on the issues that the teacher exemplifies without knowing the benefits in daily life (Muttuqin, et al. 2017; Saleh, et al. 2018; Cukurova, et al. 2018; Nasution, et al. 2018). It is in line with the results of PISA for uncertainty and data content where in 2003 PISA results, Indonesia was ranked 38 out of 40 countries with a score of 385 (OECD, 2004). While the results of the 2012 PISA, Indonesia was ranked 63 out of 65 countries because the score of Indonesian students, 384, was far below the average OECD score of

493 for uncertainty and data content (OECD, 2014). In addition, the Indonesian students were only able to resolve the matter of uncertainty and data up to level 5 of about 0.3% of students, far below the OECD average of 9.2% of students able to solve level 5 problems, even 3.2% students were able to complete up to level 6 (OECD, 2016). It showed that Indonesian students' mathematical literacy was still very low in resolving problems of PISA type.

The low ability of Indonesian students in PISA is because Indonesian students are not familiar with the contextual problems such as in the PISA problem, especially about the high level both in the process of learning and evaluation (Novita, et al. 2012; Ahyani, et al. 2014; Kamaliyah, et al. 2014; Stacey, et al. 2015; Oktiningrum, et al. 2016). On the other hands, there is the challenging to find contextual problems designed to hone student problem-solving skills and have PISA characteristics and frameworks on mathematics textbooks used by Indonesian students (Stacey, 2011; Wijaya, et al. 2014; Wijaya, et al. 2015). The problems should be the basis for the development of the 2013 curriculum, adjusting the learning in Indonesia with the questions tested on PISA so that the problems used must be adapted to the characteristics of PISA (Kemendikbud, 2014). Therefore, non-routine issues with PISA characteristics are required to familiarize students with the necessary procedures to solve the PISA problems.

In uncertainty and data content, researchers are more centered on its data content, where the content of this data is a statistical matter. Statistics learning in Indonesia is generally teacher-centered without the effort to develop students' math ideas through interaction or discussion (Widjaja, Julie, and Suryandari, 2010). Also, Groth (2006) also revealed that the learning was carried out by giving the formula directly without first learning about meaningful basic concepts and procedures for students. Shi, He, and Tao (2009) added one of the causes of students less interested in statistics is because statistics are taught theoretically and less connected to the real world. Thus, the students do not know the application on each of these materials. It has an impact on the decrease in students' motivation and achievement in learning statistics. Therefore, non-routine questions with PISA characteristics are required to make students accustomed to solving problems of PISA type.

Statistics is vital in the field of sports and have a close relationship (Escalante, et al. 2013; Bernards, et al. 2017). Several researcher already develop the PISA-task problem based on the sports context in the 2018 Asian Games (Gunawan, et al. 2018; Rawani, et al. 2019; Efriani, et al. 2019; Yansen, et al. 2019; Pratiwi, et al. 2019; Jannah, et al. 2019). Therefore, this research produce a problems of uncertainty and data contents in PISA using aquatic context that were valid, practical and had the potential effect of reasoning skills for students.

METHOD

This study was design research of development study (Akker, 1999; Bakker, 2004) aiming to produce valid and practical PISA mathematics problems for class X and to examine the potential effects of the issues developed on mathematical literacy skills of high school students. This study was conducted in two stages namely preliminary evaluation and formative evaluation (Tessmer, 1999; Zulkardi, 2006).

The preliminary evaluation stage consists of preparation where at this stage, it becomes the first step of the researchers in developing the problem of PISA type. At this stage, the researchers analyzed the

characteristics of the PISA problem and the essential competencies of the questions to be developed based on the PISA framework. Then, the designing was at this stage; researchers designed the problem device, including the question grids and the question card according to the characteristics of PISA problems.

The formative evaluation stage consists of self-evaluation where in this stage the assessment of the designing problem of PISA type was done by the researchers themselves then it was revised and prototype 1 problem obtained. Then proceed to the expert review stage where at this stage, prototype one made by researchers was consulted to experts to validate and evaluate based on the validation criteria of content, constructs, and language. Simultaneously the researcher performed one to one stage. At this stage, they asked three students who were not the subjects of study as testers and were asked to work on, observe, comment, and respond freely to the question of prototype 1. After the one to one stage, the researchers entered the small group stage where at this stage they tested prototype 2, the revision of prototype 1, to a group of students who were not the subject of study, so that prototype three was obtained.

Then the researcher enters the final stage of the field test where at this last stage the researcher tries the prototype three questions to the field where the situation is realistic where the test is carried out on the research subject. The subject in this study were 10th-grade students of Indo Global Mandiri senior high school Palembang consisting of 20 students aged 15 years old. The data were gathered through the walkthrough, observation, and interview and analyzed qualitatively.

RESULT AND DISCUSSION

Analysis Stage

One activity conducted at the stage of student analysis was to visit the place of research implementation at SMA LTI Indo Global Mandiri (IGM) Palembang. Furthermore, researchers discussed the subject of study with Siti Marfuah, S.Pd., as the classroom teacher. The purpose of this discussion was to explain the research procedure, to determine the subject of research on stage one to one, small group, and field test with 20 students of 10th-grade science 1 LTI-IGM senior high school Palembang.

The subject matter was identified based on the curriculum used in the school where the study was conducted. The curriculum used in LTI-IGM senior high school Palembang is the 2013 curriculum. In this curriculum, the standard content of mathematics learning includes numbers, algebra, geometry, and data management. After analyzing the students and the curriculum, the researcher analyzed the PISA problem based on the PISA framework, then they also studied various things about the Asian Games that could be used as contexts in developing the math problem of PISA type, and three contexts were obtained namely bicycle, swimming, and synchronized swimming.

Design Stage

At this stage, researchers began designing and compiling the PISA math problems using the cycling road and aquatic context. These questions were designed for 10th-grade senior high school students. The result is obtained from developing in the form of instrument consisting of question grids of PISA type mathematics using bicycle and aquatic context, PISA type cards using bicycle and aquatic context for senior

high school level, rubric assessment of PISA mathematics problems using the bike and aquatic context for senior high school level, and lesson plan. The five items were generated in this designing stage, namely bicycle (2 items), swimming (1 item), and synchronized swimming (2 items).

Evaluation Stage

a. Self-Evaluation

At this stage, the PISA mathematical problems using a bicycle and aquatic context had been designed and then reexamined by the researchers. It aimed to find and correct errors or deficiencies in the design process. Supervisors also assisted in the examination of questions that researchers had designed during the consultation before conducting the research.

b. Expert Reviews

The validity of a PISA mathematical problem using a bicycle and aquatic context was carried out in terms of content, constructs, and language. Before giving to the expert, these questions were discussed previously with the supervisors. The experts who reviewed the problems were Hongki Julie, lecturer of mathematics education at Sanata Darma University, Yogyakarta, Zulkardi, professor of mathematics education at Sriwijaya University, Palembang, Somakim, lecturer of mathematics education at Sriwijaya University, Palembang, Rani, assistant professor of mathematics education at Sriwijaya University, Palembang, and Siti, high school math teacher in LTI-IGM Senior High School Palembang. The validation process by Hongki Julie was carried out via email. While the process of validation by Zulkardi, Somakim, and Ranni was executed through the item panel then validation with the model teacher was done face to face in the library LTI-IGM Senior High School Palembang. Table 1 explores a recapitulation of expert suggestions and comments.

Table 1. Recapitulation of Expert Suggestions and Comments

Unit Item	Suggestions and Comments
1	<p>Honki Item 1: Clarify again the words “grafik”. Item 2: Is the answer only on the track only?</p> <hr/> <p>Ranni Item 1: the picture needs to be clarified because the description in the image does not exist so it make the students difficult to draw the possibility of the graph. Item 2: The meaning of the question is still unclear. Can be slightly changed to “Based on the picture above, when will the players go at high speed? Give me your reasons”</p> <hr/> <p>Siti In general: Provide a description of the boundary on the picture, so that students can comprehend the meaning of the problem. Item 1: Typo</p>
2	<p>Ranni In general: if it can be made into two problems. Problem 3 completing the table and Problem 4 about the question.</p>

3	<p>Zulkardi In general: Do not camouflage if you can find the original data. Then avoid using the exclamation mark (!) in the problems. Item 4: a jury in a big game is never absent. Although unable to attend, certainly replace people. So there's no way there's no value at all</p>
4	<p>Ranni In general: Add captions when scoring is viewed from the highest point, and if there are similar points, then scoring is seen from goal difference. Then it's Asean Games data. If possible, because we use the Asian Games context so the data used is the Asian Games data</p>
5	<p>Zulkardi In general: The numbers and letters in the picture must be the same as the letters in the question, because they are illustrations, so it should be clear.</p> <hr/> <p>Somakim In general: Later there must be fooled students who think that the high bar diagram wins.</p> <hr/> <p>Ranni In general: The correct one is 4 x 100 m or 4 x 10 m, because in the title of relay swimming 4 x 100m but in explanation 4x10m relay swimming. Item 8: this can be added in question “Berdasarkan grafik di atas..... (Based on the graph above)” Item 9 questions can be changed to “berdasarkan grafik di atas, pemain manakah yang mempunyai pengaruh lebih besar agar negaranya menjadi emenang dalam perlombaan? (Based on the graph above, which player has greater influence to make his country win in the race?)”</p> <hr/> <p>Siti Item 8: Typo</p>

c. One-to-One

In this stage, the researcher used three levels of student ability, high-ability students (S.R), medium-skilled students (O.V.W) and low-ability students (J.C.T). The students were then asked to read the questions on prototype 1 with ten items and solve the problems. It aimed to observe students' responses and difficulties while working on each question, whether the student understood the intent of each item developed. Researchers here only act as facilitators who oversee and assist students if they have difficulty in answering questions.

d. Small Group

At this stage, researchers tested the revision of prototype 1 called prototype to 6 students of 10th one class of Year 10 students from one Senior High School at Palembang consisting of 2 high-ability students (IF and RH), 2 medium-skilled students (TW and DR), and 2 students low-performing (NF and APW). At this stage, researchers began learning by providing an apperception of the Asian

Games, sports at the Asian Games, statistical materials and links to sports at the Asian Games. Then the researchers distributed the activities of prototype 2 units 1 and 2 to students and students were required to understand in advance the activities that had been distributed. Then the students did the questions individually for 5 minutes, then discussed in groups where one group consisted of 3 high-ability students, moderate, and low. After the discussion, the researchers asked representatives of each group to present their work.

e. Field Test

The field test was conducted on November 20, 2018. At this stage, the prototype three questions were tested to the research subjects, i.e. students of class X IPA 1 SMA LTI-IGM, which in 1 class consisted of 20 students. In this field test, the data collection techniques are in the form of walkthrough, observation, interview, and analyzed qualitatively that can be seen in Figure 1.



Figure 1. Teacher guiding student discussion during field test

At the first meeting, Miss Siti began learning by giving apperception on the Asian Games, sports at the Asian Games, statistical materials and links to games at the Asian Games. Then the teacher distributed the activity in the form of prototype three units 1 and 2 to the students, and the students were asked to understand first the events that had been distributed. Then the teacher asked students to do the questions on the individual activities for 5 minutes, later discussed in groups where one group consists of 3-4 students. There were four groups in this class. After the discussion, the teacher asked representatives from each group to present their work. M.N. is one of the students who work on the activities of prototype three units 1 and 2 (Figure 2).

UNIT 1

Cycling Road

Cycling Road merupakan salah satu cabang olahraga yang ada di dalam *Asian Games* 2018. *Cycling road* merupakan balap sepeda dimana pembalap yang berkompetisi disini berangkat secara bersamaan dan pemengangnya adalah pembalap yang mencapai garis finish pertama kali.



<http://media3.architecturemedia.net>

Soal 1
 Dari gambar di atas, gambarkanlah diagram garis untuk peningkatan dan penurunan kecepatan pemain di lintasan tersebut! Berikan alasanmu.

Soal 2
 Berdasarkan gambar di atas, kapan pemain akan melaju dengan kecepatan tinggi? Berikan alasanmu.

UNIT 2

RENANG

Renang merupakan salah satu cabang olah raga di *Asian Games* 2018. *Gaya renang* yang diperlombakan adalah *gaya bebas, gaya kupu-kupu, gaya punggung, dan gaya dada*. Perenang yang memenangkan lomba renang adalah perenang yang menyelesaikan jarak lintasan *tercepat*. Pemenang babak penyisihan maju ke babak semifinal, dan pemenang semifinal maju ke babak final.



Tabel di bawah ini merupakan catatan waktu dan nilai gerakan dari perenang putra renang gaya bebas dengan jarak 50m.

Peserta Ke-	Nilai Gerakan	Waktu (detik)
1	97	22,37
2	94	22,93
3	95	22,91
4	100	21,95
5	96	22,60
6	98	22,16
7	99	22,11
8	93	23,11

(https://en.wikipedia.org/wiki/Swimming_at_the_2014_Asian_Games)

Soal 1
 Tentukanlah perenang di lintasan mana yang mendapatkan medali emas, perak, dan perunggu pada table di bawah ini.

Medali	Peserta ke-	Nilai gerak	Kecepatan
Emas			
Perak			
Perunggu			

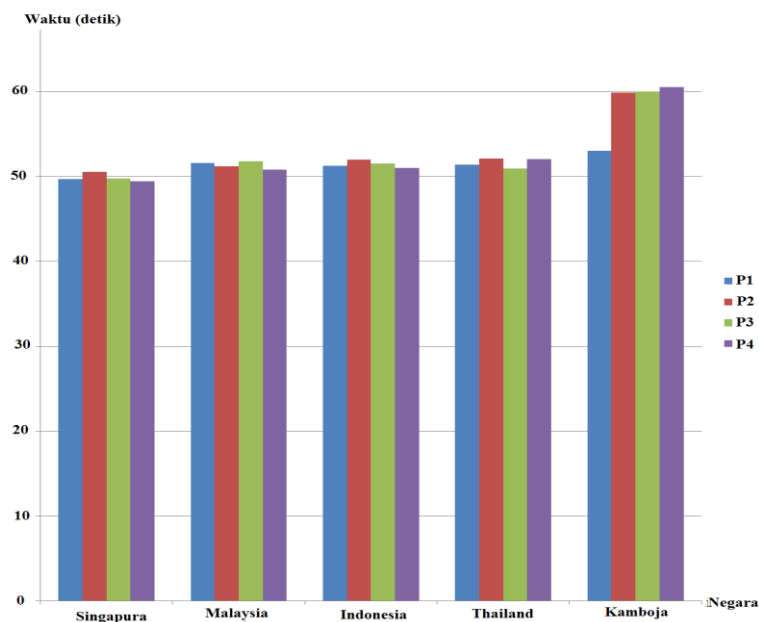
Soal 2
 Jika dilihat dari nilai gerakan, apakah yang meraih medali emas, perak dan, perunggu tersebut? Berikan alasanmu.

Figure 2. Prototype 3 Unit 1 (left) and Unit 2 (right)

At the second meeting on November 21, 2017, the teacher held a test, where the test questions are prototype three units 3 and 4. Here is a description of the students when conducting the test. The model teacher monitored the test for IPA 1 class X, so no students were allowed to discuss something. Students are given 90 minutes. Here is one of the student answers. Figure 3 is one of the problems that have been developed by researchers.

Renang Estafet 4 x 100 m

Diagram di bawah ini menunjukkan hasil perlombaan renang estafet 4 x 100 m untuk 5 besar Negara pemenang. Negara dengan pencetak waktu tercepat itulah yang mendapatkan medali emas.



Soal 7

Berdasarkan grafik di atas negara manakah yang menjadi pemenang dalam perlombaan? Berikan alasanmu.

Soal 8

Berdasarkan grafik di atas, pemain manakah yang mempunyai pengaruh lebih besar agar negaranya menjadi pemenang dalam perlombaan? Berikan alasanmu.

Figure 3. Prototype 3 Unit 4

Based on field test results, from 20 students, only one student answered wholly and correctly, 13 students responded incompletely, and six students reacted wrongly. Here are some students' answers on 1 unit about running relay.

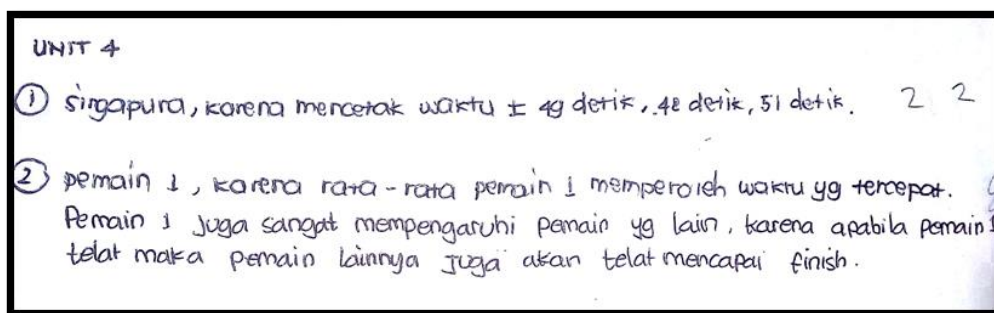


Figure 4. Student's Answer (H.A.)

In Figure 4, the student responded to player 1, a player who was very influential on the win of the relay swimming race. It was because player 1 was a player who started the start so players must start the race with high speed to win the swimming relay race.

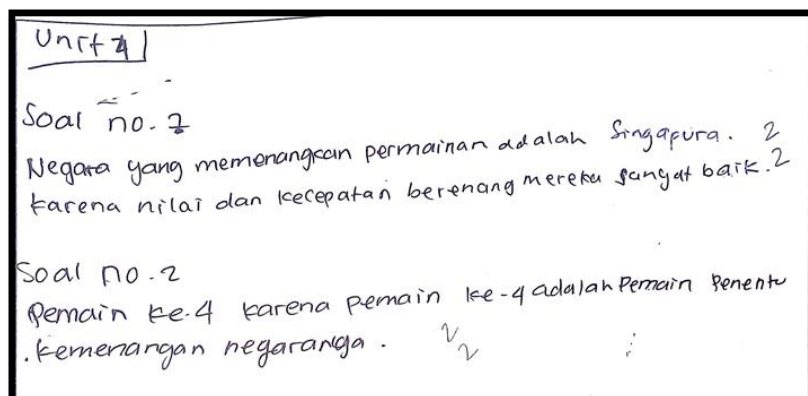


Figure 5. Student's Answer (S.A.D)

In Figure 5, S.A.D replied that the most decisive player of victory was player 4. It was because the 4th player was the last player to reach the finish. Then, the 4th player was the winner of every country.

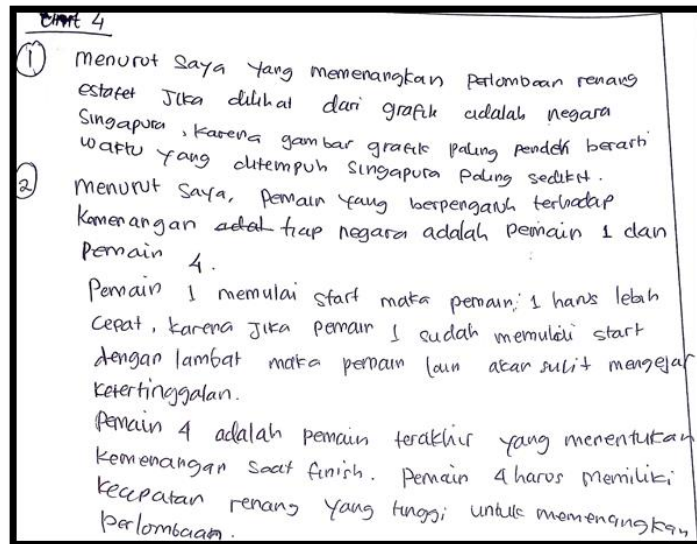


Figure 6. Student's Answer (B.P)

In Figure 6, B.P responded that the influential players were player one and player 4. It was because player 1 was a player who started the start while player 4 was a player who reached the finish line. Then the speed of player one should be superior so that other players, especially player four, do not have difficulty to achieve the finish. If player 1 starts to start slowly, then player four will be difficult to catch up. Besides, four players must also be faster; it is because four players are the winner of each country.

In unit 4, obtained mathematical literacy abilities that appear on students' answers, namely the strength of reasoning and argument. Based on the analysis of the solutions of students in this unit, it seems that the ability that is owned by students is the ability of reasoning and argument wherein the first problem students do not understand the available charts and have errors in answering which countries won the race. Furthermore, in the second question, there are still many students who answer the statement, but it is incomplete. The results supported from several research stated that some students also find it difficult to give statements that support or refute their arguments to answer given questions (Novita, et al. 2012; Kamaliyah, et al. 2014; Oktiningrum, et al. 2016; Wijaya, et al. 2014; Wijaya, et al. 2015; Mumu, et al. 2018; Gunawan, et al. 2018; Sukirwan, et al. 2018; Pratiwi, et al. 2019).

CONCLUSION

There are eight valid and practical items of PISA type of uncertainty and data contents obtained using bicycles and aquatic contexts in Asian Games. Characteristics constructed in the development of this problem were PISA characteristics and used the context within the scope of Asian Games, a personal context consisting of 1 issue of application ability and seven issues of interpretation. The validity of the questionnaire could be seen in terms of the content, whether the question was compatible with the dominant PISA literacy for context, content, and process capability. Construct, whether the problem was in line with the characteristics of PISA and the ability of the students of

class X, and language, was it a matter of using a style compatible with EYD and understood by students. It was done in expert reviews and one to one phase. The practicality criteria of the problem revealed from the results of the small groups where this type of PISA math problem used contexts known, understood, and applied in learning. The potential effects on PISA math problems can be obtained from the analysis of student responses in the field test phase in order to see the ability of mathematical literacy that appears in the student's answer. Judging from the issues discussed, it showed students' reasoning, and argumentation abilities, where 11 out of 20 students demonstrated reasoning skills and reasonable arguments and 9 out of 20 students showed reasoning skills and discussions but incomplete.

ACKNOWLEDGMENTS

The researchers would like to express their gratitude to those who have helped and given support in this study, such as Dr. Hongki Julie, M.Si., Prof. Dr. Zulkardi, M.I.Komp., M.Sc., Dr. Somakim, M.Pd., Elika Kurniadi, S.Pd., M.Sc., and Ranni Permatasari, S.Pd., as the validators in expert reviews. We also say thanks to Siti Marfuah, S.Pd., as the model teacher and the students of LTI-IGM Senior High School Palembang for participating in this study.

REFERENCES

- Adams, K.A., & Lawrence, E.K. (2018). *Research Methods, Statistics, and Applications*. California: Sage Publications.
- Ahyan, S., Zulkardi, & Darmawijoyo. (2014). Developing Mathematics Problems Based on Pisa Level of Change and Relationships Content. *Journal on Mathematics Education*, 5(1), 47-56. <https://doi.org/10.22342/jme.5.1.1448.47-56>.
- Akker, J.VD. (1999). Principles and methods of development research. In *Design approaches and tools in education and training* (pp. 1-14). Dordrecht: Springer.
- Bakker, A. (2004). *Design Research in Statistics Education on Symbolizing and Computer Tools*. Utrecht: Wilco, Amersfoorts/Freudenthal Institute.
- Bernards, J., Sato, K., Haff, G., & Bazyler, C. (2017). Current research and statistical practices in sport science and a need for change. *Sports*, 5(4), 87-96. <https://doi.org/10.3390/sports5040087>.
- Chong, J.S.Y., Chong, M.S.F., Shahrill, M., & Abdullah, N.A. (2017). Implementing inquiry-based learning and examining the effects in junior college probability lessons. *Journal on Mathematics Education*, 8(2), 157-164. <http://dx.doi.org/10.22342/jme.8.2.3964.157-164>.
- Cukurova, M., Luckin, R., & Baines, E. (2018). The significance of context for the emergence and implementation of research evidence: the case of collaborative problem-solving. *Oxford Review of Education*, 44(3), 322-337. <https://doi.org/10.1007/s11165-015-9465-y>.
- Efriani, A., Putri, R.I.I., & Hapizah. (2019). Sailing context in PISA-like mathematics problems. *Journal on Mathematics Education*, 10(2), 265-276. <https://doi.org/10.22342/jme.10.2.5245.265-276>.
- Escalante, Y., Saavedra, J.M., Tella, V., Mansilla, M., García-Hermoso, A., & Domínguez, A.M. (2013). Differences and discriminatory power of water polo game-related statistics in men in

- international championships and their relationship with the phase of the competition. *The Journal of Strength & Conditioning Research*, 27(4), 893-901. <https://doi.org/10.1519/JSC.0b013e318260ed85>.
- Groth. (2006). An exploration of students' statistics thinking. *Mathematics Education Research Journal*, 28(1), 17-21. <https://doi.org/10.1111/j.1467-9639.2006.00003.x>.
- Gunawan, M.S., Putri, R.I.I., & Zulkardi. (2018). Addition of fraction in swimming context. *Journal of Physics: Conference Series*, 943(1), 012035. <https://doi.org/10.1088/1742-6596/943/1/012035>.
- Jannah, R. D., Putri, R. I. I., & Zulkardi. (2019). Soft Tennis and Volleyball Context in Asian Games for PISA-like Mathematics Problems. *Journal on Mathematics Education*, 10(1), 157-170. <https://doi.org/10.22342/jme.10.1.5248.157-170>.
- Kamaliyah, Zulkardi, & Darmawijoyo. (2014). Developing the sixth level of PISA-like mathematics problems for secondary school students. *Journal on Mathematics Education*, 4(1), 9-28. <https://doi.org/10.22342/jme.4.1.559.9-28>.
- Kemendikbud. (2014). *Permendikbud No. 58 tahun 2014 tentang Kurikulum 2013 Sekolah Menengah Pertama/Madrasah Aliyah*. Jakarta: Kemendikbud.
- Mumu, J., Prahmana, R.C.I., & Tanujaya, B. (2018). Construction and reconstruction concept in mathematics instruction. *Journal of Physics: Conference Series*, 943(1), 012011. <https://doi.org/10.1088/1742-6596/943/1/012011>.
- Muttaqin, H., Putri, R.I.I., & Somakim. (2017). Design research on ratio and proportion learning by using ratio table and graph with Oku Timur context at the 7th grade. *Journal on Mathematics Education*, 8(2), 211-222. <https://doi.org/10.22342/jme.8.2.3969.211-222>.
- Nasution, M.F., Putri, R.I.I., & Zulkardi. (2018). Rowing sport in learning fractions of the fourth grade students. *Journal on Mathematics Education*, 9(1), 69-80. <http://dx.doi.org/10.22342/jme.9.1.4270.69-80>.
- Novita, R., Zulkardi, & Hartono, Y. (2012). Exploring primary student's problem solving ability by doing tasks like PISA's question. *Journal on Mathematics Education*, 3(2), 133-150. <https://doi.org/10.22342/jme.3.2.571.133-150>.
- OECD. (2004). *PISA Learning for Tomorrow's World: First Results from PISA 2003*. Paris: Simon and Schuster.
- OECD. (2014). *PISA 2012 Results in Focus: What 15-year-old know and what they can do with what they know*. Paris: OECD.
- OECD. (2016). *PISA 2015 Result: Excellence and Equity in Education*. Paris: OECD.
- Oktiningrum, W., Zulkardi, & Hartono, Y. (2016). Developing Pisa-like mathematics task with Indonesia natural and cultural heritage as context to assess students mathematical literacy. *Journal on Mathematics Education*, 7(1), 1-8. <https://doi.org/10.22342/jme.7.1.2812.1-8>
- Pratiwi, I., Putri, R.I.I., & Zulkardi. (2019). Long jump in Asian Games: Contexts of PISA-Like mathematic problems. *Journal on Mathematics Education*, 10(1), 81-92. <https://doi.org/10.22342/jme.10.1.5250.81-92>.

- Rawani, D., Putri, R.I.I., & Hapizah. (2019). PISA-like mathematics problems: Using Taekwondo context of Asian Games. *Journal on Mathematics Education*, 10(2), 277-288. <https://doi.org/10.22342/jme.10.2.5243.277-288>.
- Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education*, 9(1), 41-54. <https://doi.org/10.22342/jme.9.1.5049.41-54>.
- Salkind, N.J., & Frey, B.B. (2019). *Statistics for People Who (Think They) Hate Statistics*. California: Sage Publications, Incorporated.
- Shi, N.-Z., He, X., & Tao, J. (2009). Understanding statistics and statistics education: A chinese perspective. *Journal of Statistics Education*, 17(3), 1-8. <https://doi.org/10.1080/10691898.2009.11889538>.
- Stacey, K. (2011). The PISA view of mathematical literacy in Indonesia. *Journal on Mathematics Education*, 2(2), 95-126. <https://doi.org/10.22342/jme.2.2.746.95-126>.
- Stacey, K., Almuna, F., Caraballo, R. M., Chesné, J. F., Garfunkel, S., Gooya, Z., ... & Perl, H. (2015). PISA's influence on thought and action in mathematics education. In *Assessing Mathematical Literacy* (pp. 275-306). Springer, Cham.
- Sukirwan, Darhim, Herman, T., & Prahmana, R.C.I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series*, 943(1), 012026. <https://doi.org/10.1088/1742-6596/943/1/012026>
- Tanujaya, B., Prahmana, R.C.I., & Mumu, J. (2018). Designing learning activities on conditional probability. *Journal of Physics: Conference Series*, 1088(1), 012087. <https://doi.org/10.1088/1742-6596/1088/1/012087>.
- Tessmer, M. (1999). *Planning and Conducting Formative Evaluation: Improving the Quality of Education and Training*. London, Philadelphia: Kogan Page.
- Widjaja, W., Julie, H., & Suryandari, H.D. (2009). The nature of discourse in PMRI classroom: Exploring the notion of average. *Proceedings of IndoMS International Conference on Mathematics and Its Applications* (pp. 765-772). Yogyakarta: IndoMs.
- Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational studies in Mathematics*, 89(1), 41-65. <https://doi.org/10.1007/s10649-015-9595-1>.
- Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast*, 11(3), 555-584.
- Yansen, D., Putri, R. I. I., Zulkardi, & Fatimah, S. (2019). Developing PISA-like mathematics problems on uncertainty and data using Asian Games football context. *Journal on Mathematics Education*, 10(1), 37-46. <https://doi.org/10.22342/jme.10.1.5249.37-46>.
- Zulkardi. (2006). *Formative Evaluation: What, Why, When, and How*. Retrieved Juni 2019. from <http://www.reocities.com/zulkardi/books.html>.