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Table of Contents

Articles

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THE JOURNEY OF JOURNAL ON MATHEMATICS EDUCATION: FROM LOCAL TO GLOBAL

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Abstract

One of indicators to determine the quality of a journal can be observed from how many indexing institutions recognized it nationally and globally, such as Sinta, Scopus, and ScimagoJR. Furthermore, the rank of a journal in the indexing institution will add more value to the reputation of a journal. However, few journal editors are willing to share their experiences in managing a journal, from the beginning to getting recognition at the international level. Thus, this article describes the journey of the first journal in the mathematics education field from Indonesia called the Journal on Mathematics Education (JME), which is globally recognized and indexed on Scopus. JME's journey to gain global recognition is narrated in a structured way, starting from the history of journal formation, recognition at the local level, strategies to get authors from various countries, promotional activities to get credit, until finally getting a global position. In addition, this article also describes many contributions from world-class Mathematics Education researchers who have published their research results in JME. Finally, this article also describes the position of JME at national and international levels based on the data of several indexing institutes and JME's future targets.

Keywords: Journal on Mathematics Education, International Reputable, Indexing, CiteScore, Sinta, Scopus, Scimagojr

Abstrak

Salah satu penentu kualitas suatu jurnal dapat dilihat dari seberapa banyak Lembaga pengindeks yang diakui secara nasional maupun global, seperti Sinta, Scopus, dan ScimagoJR. Selanjutnya, peringkat suatu jurnal dalam Lembaga pengindeks tersebut akan menambah nilai lebih terhadap reputasi suatu jurnal. Namun, masih sedikit editor jurnal yang bersedia menceritakan pengalaman mereka dalam mengelola suatu jurnal, mulai dari awal hingga mendapatkan pengakuan di level internasional. Oleh karena itu, artikel ini menceritakan perjalanan Jurnal pertama dan satu-satunya bidang Pendidikan matematika dari Indonesia bernama Journal on Mathematics Education (JME) yang diakui secara global dengan terindeks di Scopus. Perjalanan JME untuk mendapatkan pengakuan secara global diceritakan dengan terstruktur mulai dari sejarah terbentuknya jurnal, pengakuan di level local, sejumlah strategi mendapatkan penulis dari berbagai negara, sejumlah aktivitas promosi untuk mendapatkan rekognisi, sampai akhirnya mendapatkan posisi di level global. Selain itu, artikel ini juga mendeskripsikan sejumlah kontribusi dari para peneliti Pendidikan matematika level dunia. Mereka telah mempublikasikan hasil risetnya di JME. Terakhir, artikel ini juga menjelaskan tentang posisi JME di level nasional dan internasional berdasarkan data dari sejumlah Lembaga pengindeks, dan target JME di masa yang akan datang.

Kata kunci: Journal on Mathematics Education, Reputasi Internasional, Indeksasi, CiteScore, Sinta, Scopus, Scimagojr

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Journal on Mathematics Education (JME) or known as the Indonesian Mathematical Society Journal on Mathematics Education (IndoMS-JME), is the first international journal in the field of mathematics education in Indonesia that indexed in Scopus (Zulkardi et al., 2019). This journal provides a venue for the publication of original research articles, review articles by invited experts, and novel technology news in the mathematics education field. This journal is intended for and dedicated not only to members of the Indonesian Mathematics Society (IndoMS) but also to lecturers, researchers, mathematics school teachers, teacher educators, and university students (Master and Doctoral) who wish to publish their research reports or literature review articles (exclusively for invited contributors), as well as brief communications about mathematics education and its implementation. Apart from regular contributors, each volume's contents will be contributed by invited experts in mathematics education from Indonesia and abroad.

The focus and scope of this journal include Realistic Mathematics Education (RME), Design/Development Research in Mathematics Education, PISA Task, Mathematics Ability, ICT in Mathematics Education, and Ethnomathematics (Zulkardi, 2019). Currently, JME is one of the top journals in Indonesia that Sinta or Science and Technology Index created and developed by The Ministry of Research and Technology /National Agency for Research and Innovation of Indonesia. JME has been indexed in the S1 category, Scopus in the Q1 category, and Scimagojr in the Q2 category. However, this has, of course, gone through a journey which is not short and easy.

The History of Journal on Mathematics Education

JME was initiated by the Vice President of IndoMS at that time, who worried that there was no Journal of Mathematics Education field managed under the auspices of IndoMS. At that time, IndoMS only had one journal called the Majalah Ilmiah Himpunan Matematika (MIHMI), which has now changed its name to the Journal of the Indonesian Mathematical Society (JIMS). This journal focuses on pure and applied mathematics, even though most of the members of IndoMS come from the Lembaga Pendidikan Tenaga Kependidikan (LPTK), an Educational Institute for Educators which focuses on the field of Mathematics Education. Therefore, it was necessary to make a journal in the area of Mathematics Education published by IndoMS.

JME's journey began in 2010. At that time, the President and Vice President of IndoMS for Mathematics Education, Professor Widodo and Professor Zulkardi succeeded in launching JME at the Konferensi Nasional Matematika (KNM) opening at Universitas Negeri Manado (UNM) (Zulkardi, 2019). Universitas Sriwijaya collaborates with IndoMS in managing JME, where the core team, secretariat, and journal management processes are carried out at Universitas Sriwijaya.

Not long after its launching, JME published a volume consisting of five articles. One article was authored by Professor Lee Peng Yee, expert mathematicians from the National Institute of Education (NIE), Singapore. He wrote about designing a mathematics curriculum in Singapore (Yee, 2010). The other articles were authored by Professor Robert K Sembiring, a mathematician of Bandung Institute of Technology (ITB). He wrote about the developments and challenges of implementing the Indonesian Realistic Mathematics Education (PMRI) approach in Indonesia (Sembiring, 2010). At that time, JME was still in the form of a local journal that had not been indexed by any indexing agency and was still using WordPress for its website system, which could still be accessed https://jims-b.org/, as shown in Figure 1.

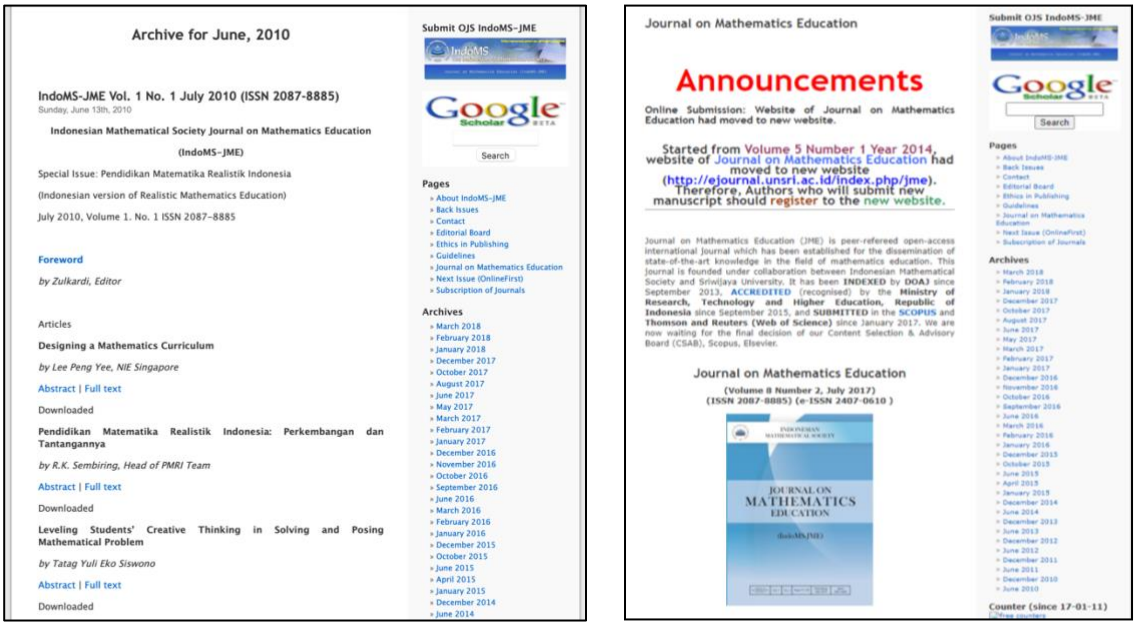

Figure 1. The first issue of JME and the first JME's website using WordPress

In 2011, JME succeeded in becoming a national journal by the efforts and hard work of the team in building a better JME, expanding networking and trying to get a journal governance grant from the Ministry of Research and Higher Education (Kemenristekdikti). JME managed to publish two issues with a total of 15 articles, of which world-class researchers wrote three papers. An expert on Realistic Mathematics Education (RME), Professor Koeno Gravemeijer from the Eindhoven University of Technology, Netherland, published his thoughts with the title How Concrete is Concrete? (Gravemeijer, 2011). Furthermore, Professor Kaye Stacey, the chairman of the Mathematics Expert Group for the OECD survey PISA from Melbourne University, Australia, wrote an article entitled The PISA View of Mathematical Literacy in Indonesia, which currently has the highest citations on Google Scholar as well as at Scopus (Stacey, 2011). Lastly, Professor Christa Kaune, a metacognitive expert from the Institut für Cognitive Mathematics, Universität Osnabrück, Germany, shared her idea by writing an article entitled Development of Metacognitive and Discursive Activities in Indonesian Maths Teaching (Kaune, Cohors-Fresenborg, & Nowinska, 2011). In addition, in this second volume, JME was fully supported by research outputs from the International Master Program on Mathematics Education (IMPoME), a collaboration between Sriwijaya University, Surabaya State University, and Utrecht University. Eleven of the fifteen articles published in the second year of JME publication came from the results of thesis research by IMPoME students from various regions in Indonesia related to design research methodology using the PMRI approach.

Over the next 3 years, the main contributors to JME would have been from IMPoME alumni with their results of their primary thesis research as many as 22 articles, students of the mathematics education doctoral program from the Indonesian Education University with their results of dissertation research as many as 5 articles, and invited authors who are Mathematics Education experts from around the world, including Berinderjeet Kaur from the National Institute of Education, Nanyang Technological University, Singapore, with her article entitled Mathematics Education in Singapore - An Insider's Perspective (Kaur, 2014), Edyta Nowinska from the Institute for Didactics of Mathematics, A. Mickiewicz University, Poland, with the article title A Cognitive Theory Driven New Orientation of Indonesian Lessons (Nowinska, 2014), Frans Van Gallen and Dolly van Eerde from Utrecht University, Netherlands, with the article Solving Problems with the Percentage Bar (van Galen & van Eerde, 2013), Wanty Widjaja from Deakin University, Australia, with the article title the Use of Contextual Problems to Support Mathematical Learning (Widjaja, 2013), Esther Yook-Kin Loong from Deakin University, Australia, regarding the use of the internet in learning mathematics for high school students (Loong, 2014), Caroline Bardini, Robyn Pierce, Jill Vincent, & Deborah King from Melbourne University, Australia, regarding understanding students majoring in mathematics on the concept of function (Bardini et al., 2014), and Fou-Lai Lin who was President of the International Group for the Psychology of Mathematics Education (PME) in 2007-2010, with his article entitled Designing Teacher Professional Development for Mathematics Teaching with Variation Theory, which was written with her doctoral students (Ekawati & Lin, 2014).

The JME team has been working hard to continuously improve and maintain the quality of its publications, one of which is 2014; JME successfully migrated by changing its website system from WordPress to the Open Journal System (OJS), seen in Figure 2.

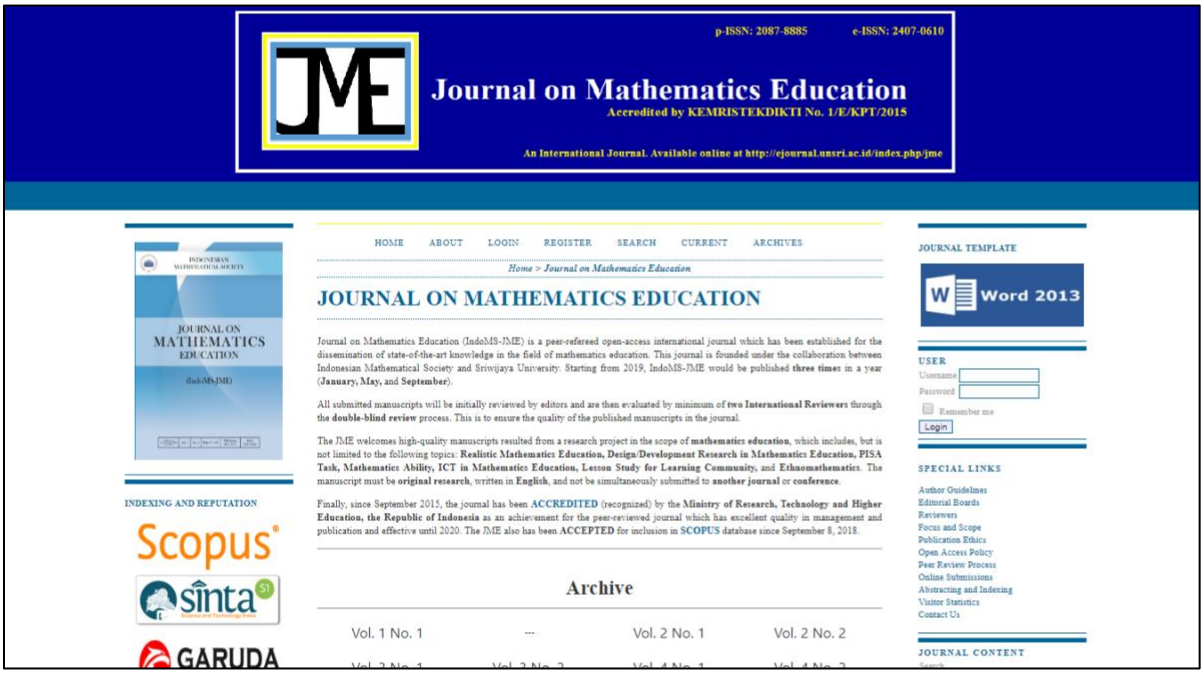

Figure 2. Display of JME in the first form with OJS

Furthermore, the JME OJS display has undergone a few changes to the current display, as shown in Figure 3. Every change that occurs on the website makes it easier for authors to complete the submission process and for readers to access every published article. The public can access it at https://ejournal.unsri.ac.id/index.php /jme/.

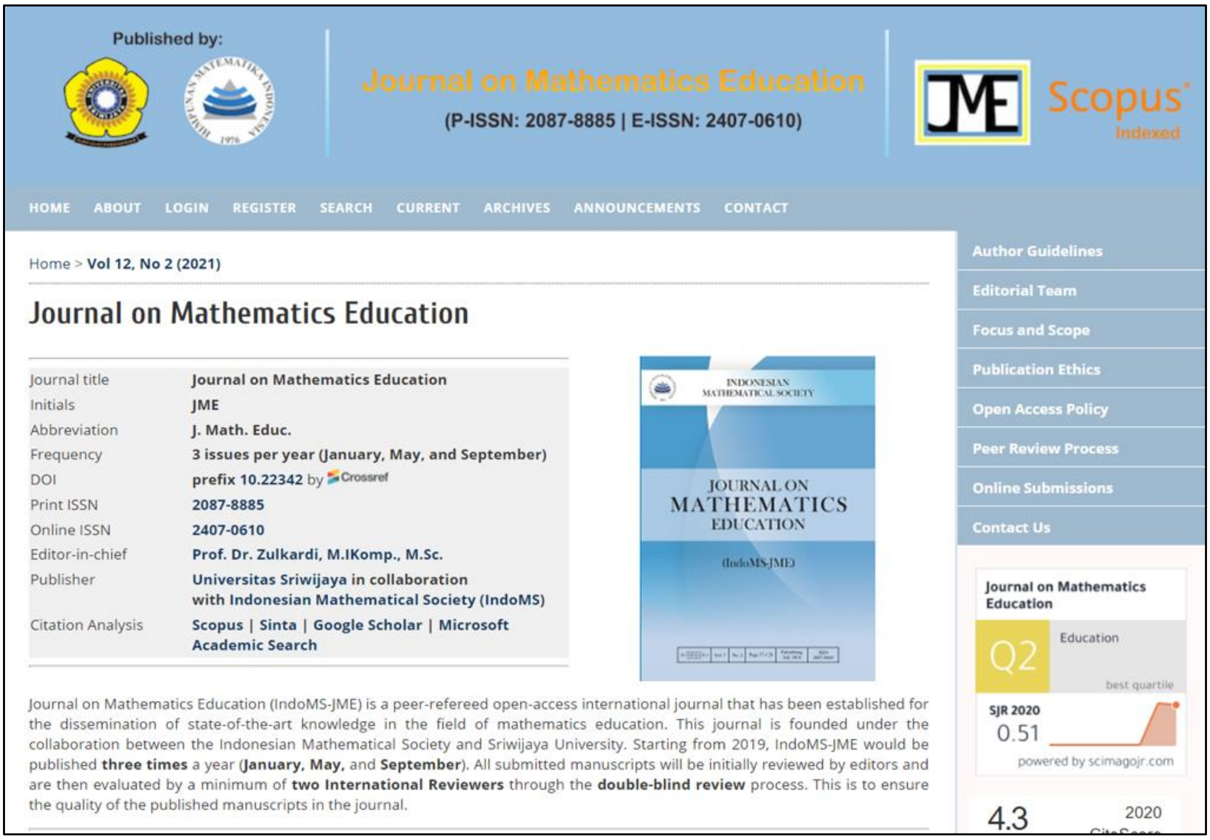

Figure 3. Display of JME in the current website with OJS

The First Recognition of Journal on Mathematics Education

In 2015, for the first time, JME was recognized at the national level. At that time, JME became the first mathematics education journal accredited by the Ministry of Research, Technology and Higher Education (Figure 4).


KEMENTERIAN RISET, TEKNOLOGI, DAN PENDIDIKAN TINGGI
DIREKTORAT JENDERAL PENGUATAN RISET DAN PENGEMBANGAN
DIREKTORAT PENGELOLAAN KEKAYAAN INTELEKTUAL

Sertifikat

Kutipan dari Keputusan Direktur Jenderal Penguatan Riset dan Pengembangan Kementerian Riset, Teknologi, dan Pendidikan Tinggi Republik Indonesia
Nomor: 1/E/KPT/2015, Tanggal 21 September 2015
Tentang Hasil Akreditasi Terbitan Berkala Ilmiah
Periode I Tahun 2015

Nama Terbitan Berkala Ilmiah
Indonesian Mathematical Society Journal on Mathematics Education (IndoMS-JME)
ISSN: 2087-8885
Penerbit: Indonesian Mathematical Society (IndoMS)

Ditetapkan sebagai Terbitan Berkala Ilmiah

TERAKREDITASI

Akreditasi sebagaimana tersebut di atas berlaku selama 5 (lima) tahun sejak ditetapkan.

Jakarta, 1 Oktober 2015
Direktur Pengelolaan Kekayaan Intelektual,
Direktorat Jenderal Penguatan Riset dan Pengembangan

Dr. Sadjuga, M.Sc
NIP. 195901171986111001

Figure 4. JME's first National Accreditation Certificate by Kemenristekdikti

This achievement was remarkable because, it was challenging to get recognition from the Ministry of Research, Technology and Higher Education in the form of accreditation in a journal. There were only two accreditation ratings for scientific journals in Indonesia, namely accreditation A and B, with a very rigid number of assessment indicators. Furthermore, in the same year, JME was also indexed by the Directory of Open Access Journals (DOAJ) and by the Education Resources Information Center (ERIC) from the United States. Not only satisfied with that, the team then focused on getting JME to be recognized globally by being indexed by the Scopus, by continuing to make guerrillas improve quality and expand JME promotions and networks both at home and abroad.

The Activities of JME's Team for Getting International Recognition

The JME team succeeded in promoting JME at significant events in the field of mathematics education, including at the 13th International Congress on Mathematics Education (ICME) in Hamburg, Germany, the 17th Konferensi Nasional Matematika (KNM) in Pekanbaru, and the 4th South East Asian-Design Research (SEA-DR) at Universitas Negeri Padang, as shown in Figure 5.

Figure 5. Teams promoting JME at significant events both at home and abroad

In addition, the team also succeeded in promoting JME to researchers from Utrecht University, the Netherlands, who were visiting Indonesia. As a result, great researchers from the university published their articles at JME. There are several researchers from Utrecht University who have published their research paper in JME, including Prof. Mieke Abels, Prof. Maarten Dolk, Prof. Frans van Galen, Prof. Dolly van Eerde, Prof. Michiel Doorman, and Prof. Jan de Lange (Murdiyani et al., 2013; van Galen & van Eerde, 2013; Putri, Dolk, & Zulkardi, 2015; Tanudjaya & Doorman, 2020; Apsari et al., 2020;

Wijaya, Elmaini, & Doorman, 2021; de Lange, 2021). Finally, when the Chief Editor of JME visited that university, unexpectedly, they recognized JME by placing the journal in their campus library by aligning the journal side-by-side with other Top Journals in the field of Mathematics Education, such as ZDM-Journal on Mathematics Education, Educational Studies in Mathematics, Journal of Mathematics Teacher Education, Journal for Research in Mathematics Education, and Mathematics Education Research Journal, as shown in Figure 6.

Figure 6. JME recognition in the Utrecht University Library

The Recognition as A Reputable International Journal

Finally, in 2018, JME was successfully accepted for indexation by Scopus and succeeded in asking Scopus to inclusion all JME articles starting from when they were first published or beginning in 2010. Its achievements have come from the hard work and prayers of the entire JME's team, including contributions from the JME Editorial Board members, reviewers, authors, the leader of IndoMS, and Universitas Sriwijaya. In that year, the Ministry of Research, Technology and Higher Education also awarded JME first rank accreditation (Sinta 1) to become one of the journals indexed by Scopus, as shown in Figure 7.

Figure 7. Submission of JME accreditation certificate rank 1

Since its inception until 2018, JME has published two issues every year. However, since 2019, the JME's Team has made a policy to increase the number of publications to 3 issues per year with a total number of publications of about ten articles per issue to maintain the articles' quality. This policy was a recommendation of the Ministry of Research, Technology and Higher Education to consider several issues related to the addition of good and quality articles to be displayed in the international arena as part of the nation's competitiveness. Currently, JME is in Q1 based on CiteScore 2020 data on Scopus and Q2 based on SJR 2020 data on Scimago Journal and Country Rank, as shown in Figure 8.

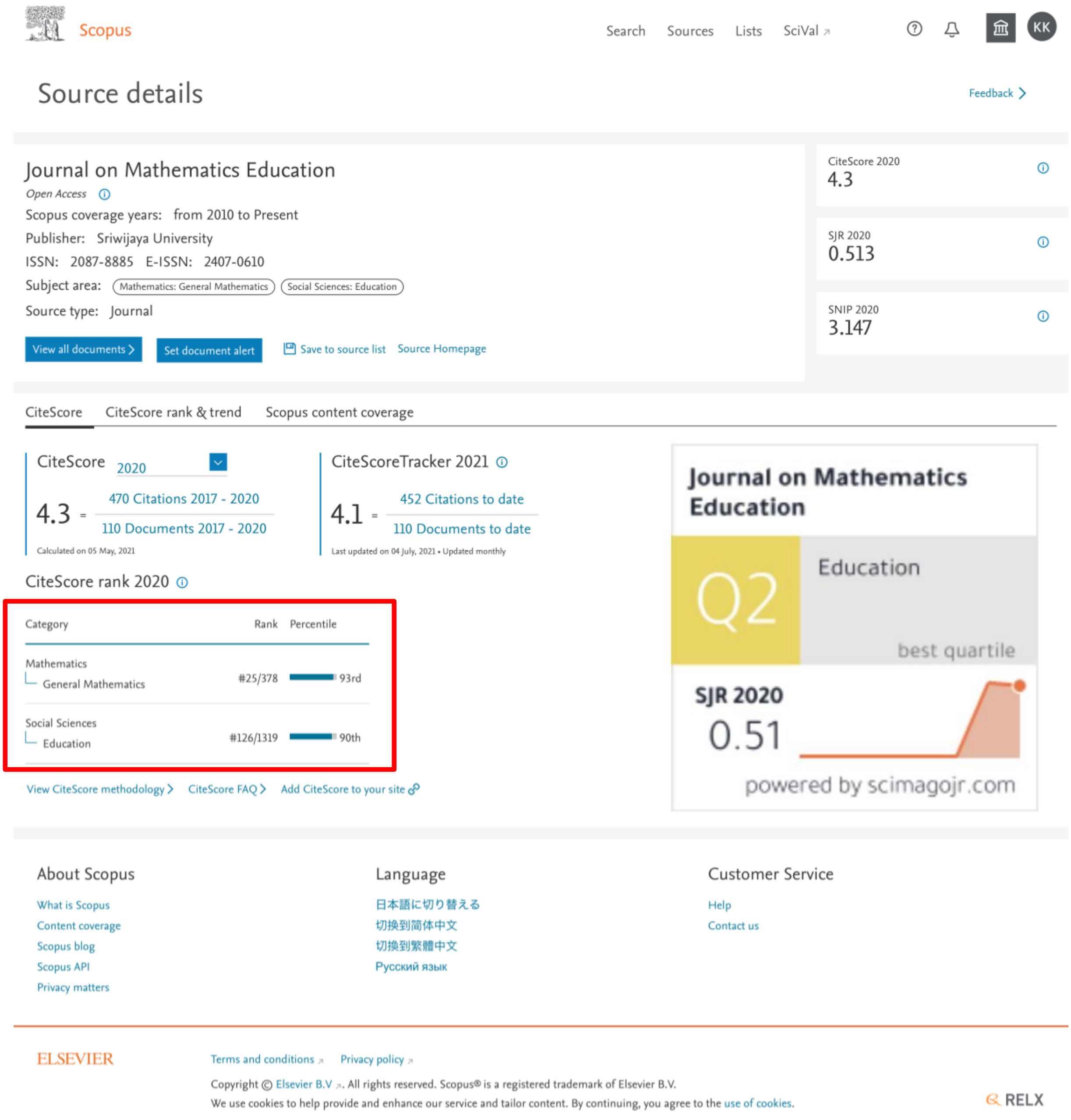

Figure 8. JME Profile on Scopus

Furthermore, in 2019, the JME team succeeded in holding the anniversary of the decade of JME's journey at the Institut Keguruan dan Ilmu Pendidikan (IKIP) Siliwangi, Cimahi, Bandung, which is the home of the Indonesia Mathematics Educators Society (I-MES), as shown in Figure 9. At the meeting, acting as the keynote speaker, there are Prof. Heris Hendriana (Chancellor of IKIP Siliwangi and Chair of I-MES), Prof. Zulkardi (Chief Editor of JME), Prof. Ratu Ilma Indra Putri (Editor of JME), Dr Wahyu Hidayat (Chief Editor of Infinity Journal and Secretary-General of I-MES), and Dr. Rully Charitas Indra Prahmana (Reviewer of JME). In addition, this activity was also attended by managing editors of mathematics education journals from various regions in Indonesia. Furthermore, to commemorate the 10th anniversary of JME, the chief and managing editors of the Mathematics Education journal collaborated to write down their experiences in managing the Mathematics Education journal and documented them in the form of a book entitled Kiat Mengelola Jurnal Pendidikan Matematika or How to Manage Mathematics Education Journals (Curahan Hati Para Editor/ Editors' Thought Sharing) (Zulkardi et al., 2019), shown in Figure 9.

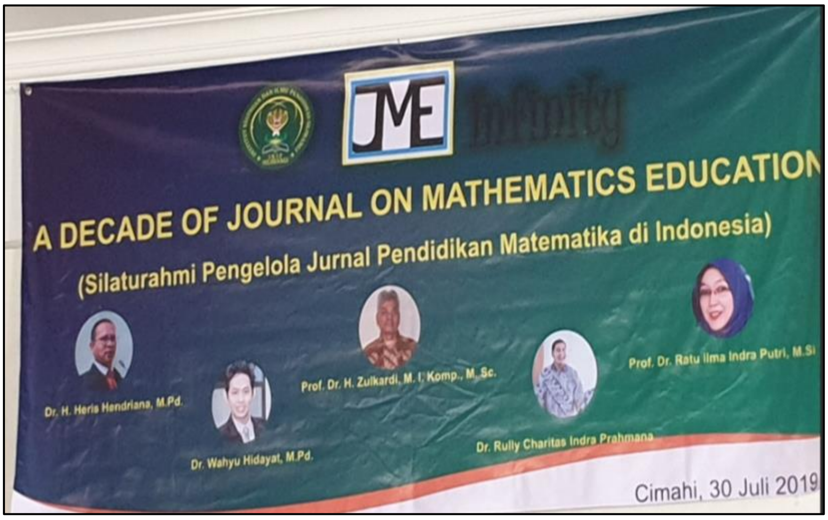

Figure 9. JME One Decade Commemoration (left) and the book cover of journal managers (right)

Since JME was accepted for indexation by Scopus, the JME team tried to give a certain portion by publishing articles with the characteristics of international collaboration while maintaining the quality of the research produced in their articles. This is important to maintain the reputation and readability of articles published by JME at the global level. A number of articles that have the characteristics of international collaboration include an article entitled the Development of A Student Survey on Attitudes Towards Mathematics Teaching-Learning Processes, which is a collaboration between Indonesian and Australian researchers, one of the authors is Tom Lowrie, who is the Program Director for the Early Learning STEM Australia (ELSA) project (Mutohir, Lowrie, & Patahuddin, 2018); an article entitled the Integration of A Problem-Solving Framework for Brunei High School Mathematics Curriculum in Increasing Student's Affective Competency, which is a collaboration between researchers at the Universiti Brunei Darussalam, Brunei Darussalam and the University of Edinburgh, United Kingdom (Chong, Shahrill, & Li, 2019); an article entitled A Comparative Study of Quadrilaterals Topic Content in Mathematics Textbooks Between Malaysia and South Korea, which is a collaboration between researchers at Universiti Teknologi Malaysia (UTM), Malaysia, and Chonnam National University, South Korea (Abdullah & Shin, 2019); an article entitled Elementary Preservice Teachers' Knowledge, Perceptions and Attitudes Towards Fractions: A Mixed-Analysis, which is a collaboration between researchers at Universiti Kebangsaan Malaysia, Malaysia, Texas A&M University, Sam Houston State University, and the University of Houston, United States (Rosli et al., 2020); an article entitled Student Engagement and Math Teachers Support, which is a collaboration of researchers at Imam Abdulrahman Bin Faisal University, Saudi Arabia, and Saint Louis University, United States (Alrajeh & Shindel, 2020).

Furthermore, an article entitled Learning Mathematical Modeling with Augmented Reality Mobile Math Trails Program: How Can It Work?, which is a collaboration of researchers at the State University of Semarang, Indonesia, and Goethe-Universität Frankfurt, Frankfurt, Germany (Cahyono et al., 2020); an article entitled Partitive Fraction Division: Revealing and Promoting Primary Students' Understanding, which is a collaboration of researchers at Mataram State Islamic University, Mataram University, Indonesia, and Technical University Dortmund, Germany (Wahyu et al., 2020); an article entitled Using Robotics And Engineering Design Inquiries to Optimize Mathematics Learning for Middle Level Teachers: A Case Study, which is a collaboration of researchers at West University, South Africa, University of Massachusetts Lowell, Illinois Mathematics and Science Academy, and Georgia State University, USA (Chahine, Robinson, & Mansion, 2020); an article entitled A Comparison of Mathematical Tasks Types used in Indonesian and Australian Textbooks based on Geometry Contents, which is a collaboration of researchers at SEAMEO QITEP in Mathematics, Indonesia, and Monash University, Australia (Hidayah & Forgasz, 2020); and an article entitled An Analysis of Learners' Solution Strategies in the Context of Modeling Tasks, which is a collaboration of researchers at the University of Duisburg, Germany, and Rhodes University, South Africa (Reit & Schäfer, 2020).

Finally, in the last edition of 2020, JME succeeded in publishing a scientific article entitled Learning Geometry and Values from Patterns: Ethnomathematics on The Batik Pattern of Yogyakarta, Indonesia (Prahmana & D'Ambrosio, 2020), a collaboration of article writing between researchers at Universitas Ahmad Dahlan, Indonesia, and a mathematician, the initiator of Ethnomathematics, and the founder of the Brazilian Society for Mathematics and History of the International Group of Ethnomathematicians from Brazil, Prof. Ubiratan D' Ambrosio. He died on May 12, 2021, 8 months after his collaborative article was published at JME and this is his last article published in a Reputable International Journal.

Journal on Mathematics Education in National and International Level

JME's ranking among journals in Indonesia can be seen from the ranking order of journals on the SINTA Kemenristekdikti website (https://sinta.ristekbrin.go.id/journals), which is based on the Impact value in each journal. Figure 10 shows that JME ranks first out of a total of 5990 journals in the Sinta database in Indonesia with the highest impact value; the difference in impact values is quite significant compared to the journals listed after.

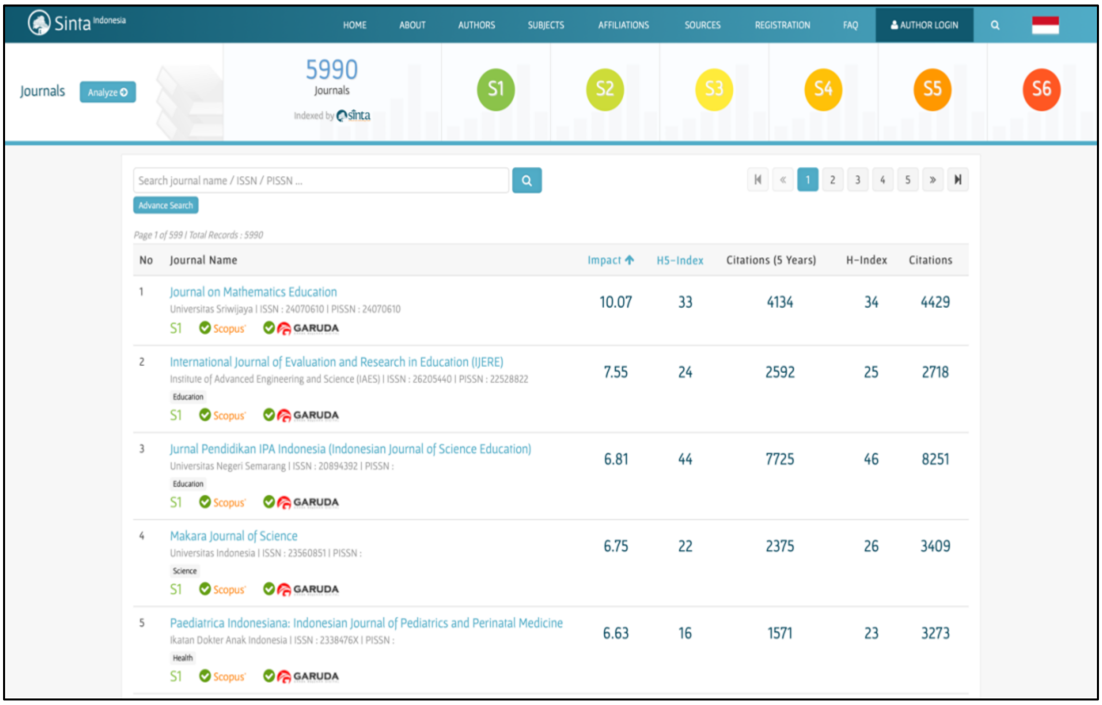

Figure 10. JME's position on the Sinta Kemenristekdikti website on Friday, 23 July 2021

The reputation and ranking of a journal at the Scopus indexing agency are determined by CiteScore data or the calculation of the 3-year average number of citations for the latest published articles divided by the number of documents in those three years in a journal. Based on these data, in 2021, JME is in the first quartile or Q1 with a CiteScore value of 4.3. This result puts JME in first place out of a total of 95 Indonesian journals indexed in the Scopus database, as shown in Figure 11.

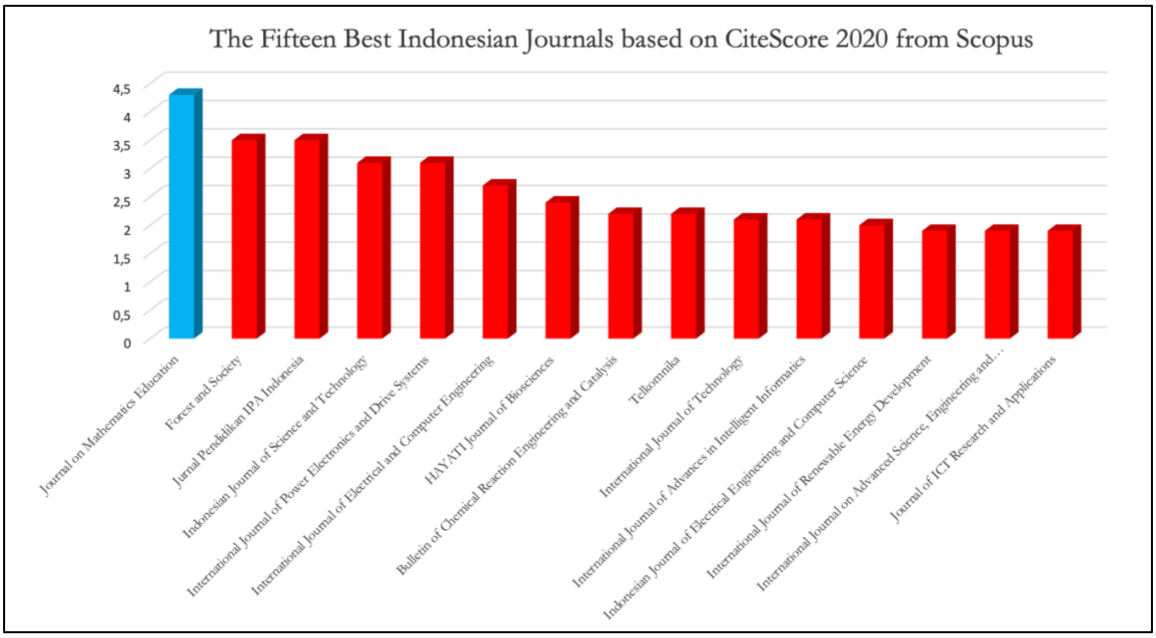

Figure 11. JME's position among the 15 best Indonesian Journals indexed by Scopus (Appendix 1)

In addition, JME's CiteScore 2020 score put JME in second place of 35 Scopus indexed journals in Mathematics (General Mathematics or Mathematics (miscellaneous)) and Education. It indicated that JME is a journal in the field of Mathematics Education whom a Scopus indexes globally. The best 15 positions for Mathematics Education journals in the Scopus database based on CiteScore 2020 scores is provided in Figure 12.

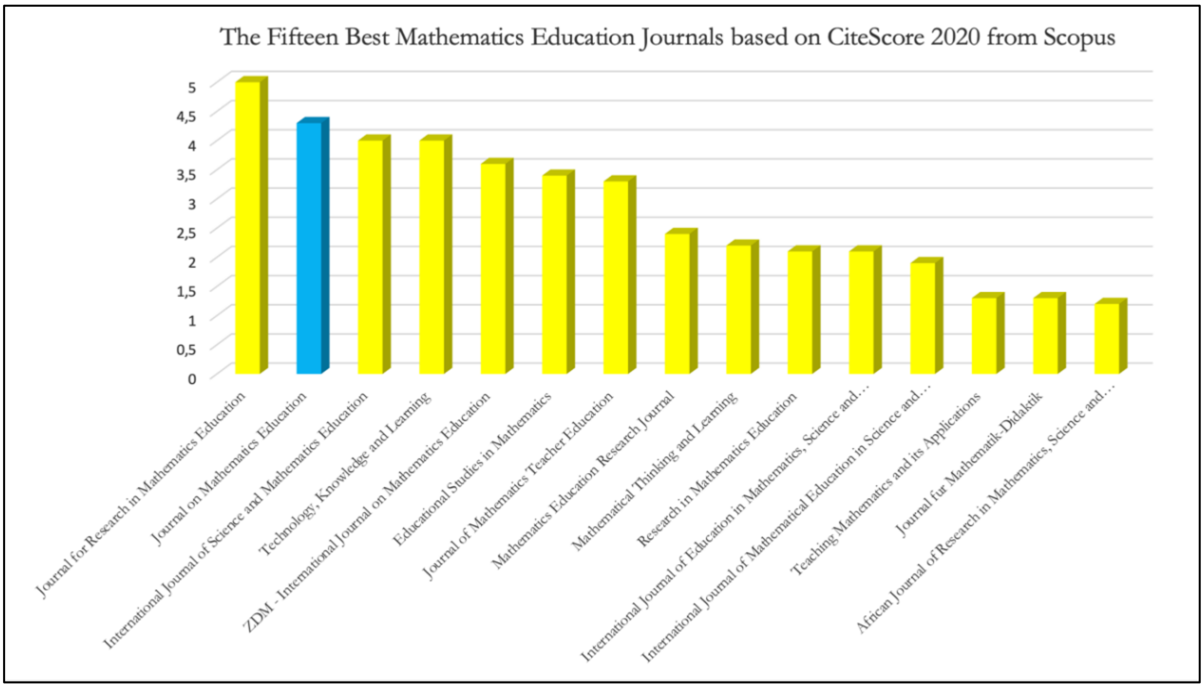

Figure 12. JME's position among the 15 best Mathematics Education Journals indexed by Scopus (Appendix 2)

Furthermore, if we look at the journal data on the Scimago Journal and Country Rank website, JME is in the Q2 quartile, which is determined from the Scimago Journal Rank (SJR) score. Based on these data, JME ranked 3rd out of a total of 95 Indonesian journals indexed at ScimagoJR, as shown in Figure 13.

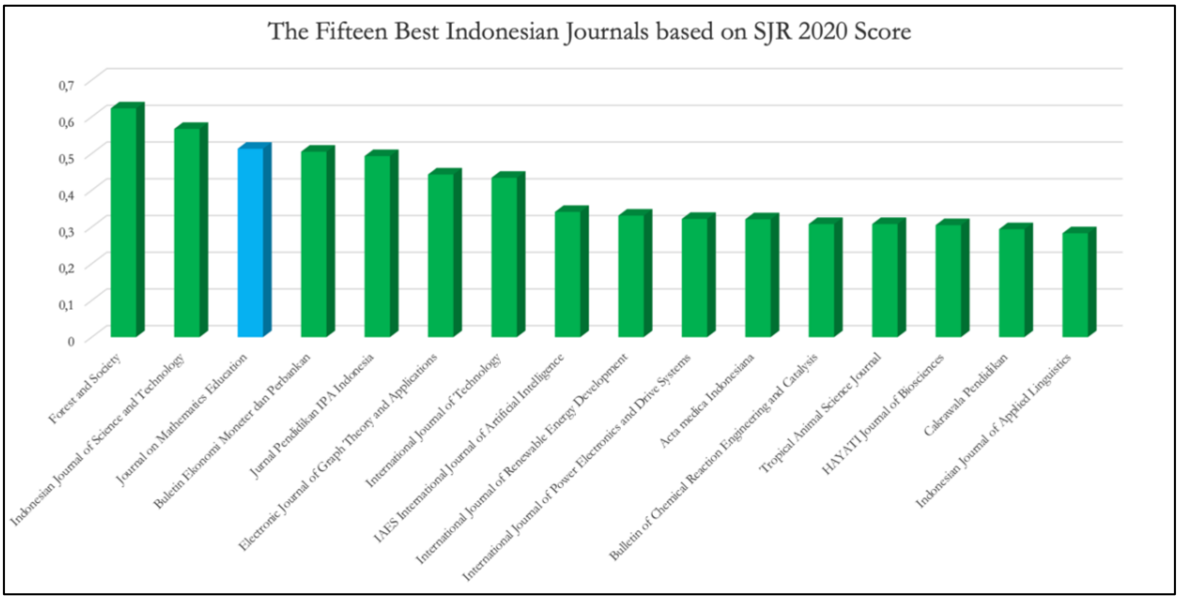

Figure 13. JME's position among the 15 best Indonesian journals indexed by ScimagoJR (Appendix 3)

In addition, JME's SJR 2020 puts the in rank 11th out of 35 journals indexed by ScimagoJR within the area of Mathematics (General Mathematics or Mathematics (miscellaneous)) and Education. It indicated that JME is a journal in the field of Mathematics Education whom a Scimago Journal Rank and Country. The complete data for the 15 best journals in the field of Mathematics Education according to the ScimagoJR version can be seen in Figure 14.

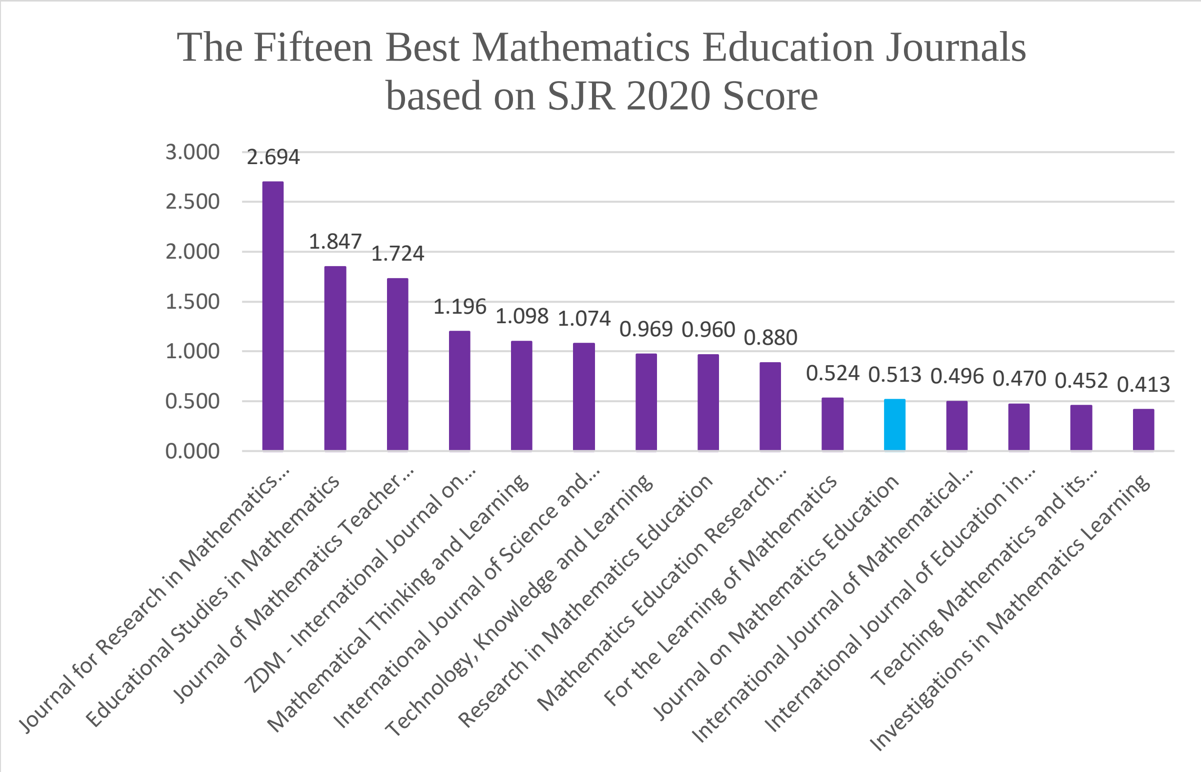

Figure 14. JME's position among the 15 best journals in the field of Mathematics Education, according to ScimagoJR (Appendix 4)

Journal on Mathematics Education in the Future

Currently, there are hundreds of researchers from dozens of countries who publish their scientific works at JME. They come from Australia, Brazil, Brunei Darusallam, Germany, Ghana, Indonesia, India, Israel, Japan, Malaysia, Mexico, Nepal, Netherlands, Palestine, Philippines, Poland, Portugal, Russian Federation, Rwanda, Saudi Arabia, Singapore, South Africa, South Korea, Spain, Taiwan, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Vietnam, Zambia and Zimbabwe, as shown in Figure 15.

By 2021, the Journal on Mathematics Education is increasingly improving itself to publish quality articles written by world-class researchers by the focus and scope of JME and following trends in that year, including collaborative writings between researchers from Universitas Ahmad Dahlan, SEAMEO QITEP in Mathematics, Indonesia, and two world-class Ethnomathematics experts, Prof. Daniel Clark Orey and Prof. Milton Rosa from Universidade Federal de Ouro Preto, Brazil, entitled Ethnomathematics: Pranatamangsa System and The Birth-Death Ceremonial in Yogyakarta (Prahmana et al., 2021).

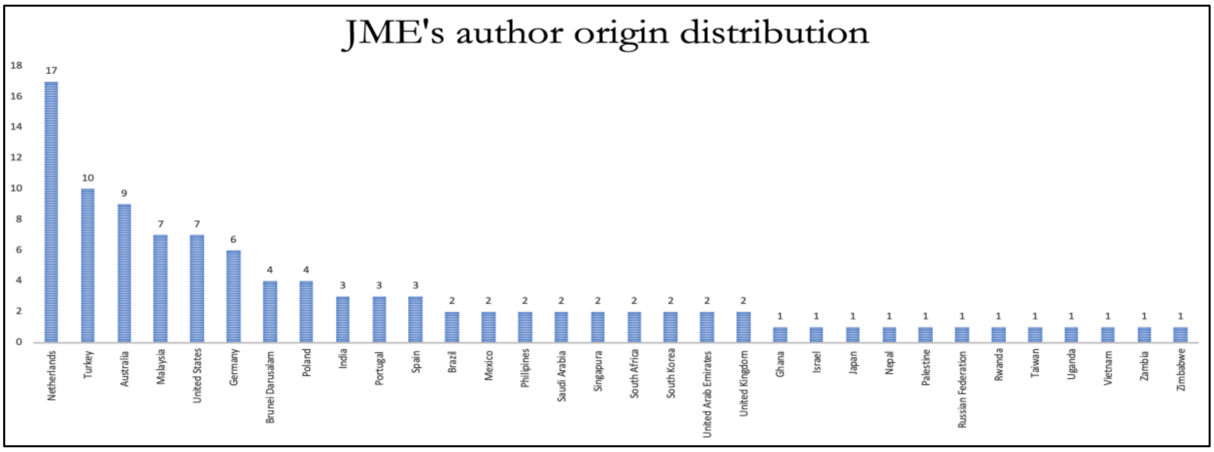

Figure 15. Distribution of origin of JME authors from various countries outside Indonesia (Appendix 5)

Furthermore, the article with the title Designing PISA-Like Mathematics Task using A COVID-19 Context (PISACOMAT) written by researchers from Sriwijaya University, Indonesia, is an article on the development of PISA type questions using the context of the COVID-19 pandemic that is spreading in the world, including in Indonesia (Nusantara, Zulkardi, & Putri, 2021). Lastly, JME also has published articles written by Prof. Jan de Lange entitled There Is, Probably, No Need for A Design Framework, as an invited author in this issue. He was a director of the Freudhental Institute and chairman of The Expert Group for Mathematics of OECD's new Program for International Student Assessment (PISA). In this article, he presented his personal perspective on designing a mathematics lesson with several examples of his work at the Freudenthal Institute, which focused on student learning and slow design derived from knowledge of mathematics and its support system in the learning process (de Lange, 2021). In addition, the distribution of authors since the beginning of 2021 has significantly varied with more than 15 countries (excluding Indonesia), as shown in Figure 16.

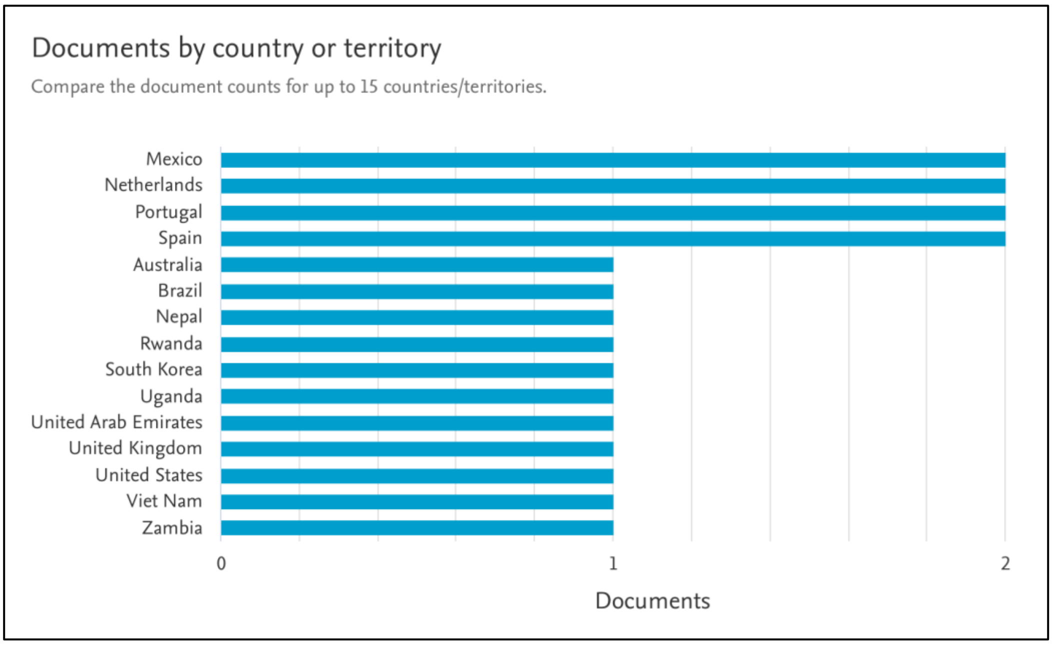

Figure 16. Distribution of the author's country of origin outside Indonesia in Vol. 12, No. 1 and 2, 2021

Final Remarks

The JME's team continues to strive for JME at the international level. They continue to improve the quality of articles published on JME. Furthermore, they maintain the integrity of conflicts of interest with the authors. They also continue to enhance the editing and display quality of articles and the JME website, making them more interesting to read. Lastly, they facilitate authors from Indonesian researchers to exist by maintaining the research quality and authoring excellent article at the international level.

In addition, JME's team are committed to maintaining the quality of all indicators provided by internationally reputable indexing institutions. JME needs to continue to the highest level in the reputable indexing institutions in future. To do so, it starts with increasing its number of citations each year, number of international collaboration and number of articles which potentially give big impact to scientific development of mathematics education in the world. Therefore, in the future, the target of being indexed by the Web of Science (WoS) Core Collection, Social Science Citation Index (SSCI), which is one of the best indexing institutions globally, by 2025 can be achieved.

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Appendix 1. The Fifteen Best Indonesian Journals based on CiteScore 2020 from Scopus (https://www.scopus.com)

Rank	Title	CiteScore 2020	Scopus Best Quartile
1	Journal on Mathematics Education	4,3	Q1
2	Forest and Society	3,5	Q1
3	Jurnal Pendidikan IPA Indonesia	3,5	Q1
4	Indonesian Journal of Science and Technology	3,1	Q2
5	International Journal of Power Electronics and Drive Systems	3,1	Q2
6	International Journal of Electrical and Computer Engineering	2,7	Q2
7	HAYATI Journal of Biosciences	2,4	Q2
8	Bulletin of Chemical Reaction Engineering and Catalysis	2,2	Q3
9	Telkomnika	2,2	Q3
10	International Journal of Technology	2,1	Q2
11	International Journal of Advances in Intelligent Informatics	2,1	Q3
12	Indonesian Journal of Electrical Engineering and Computer Science	2,0	Q3
13	International Journal of Renewable Energy Development	1,9	Q3
14	International Journal on Advanced Science, Engineering and Information Technology	1,9	Q2
15	Journal of ICT Research and Applications	1,9	Q2

Appendix 2. The Fifteen Best Mathematics Education Journals based on CiteScore 2020 from Scopus (https://www.scopus.com)

Rank	Title	CiteScore 2020	Scopus Best Quartile
1	Journal for Research in Mathematics Education	5,0	Q1
2	Journal on Mathematics Education	4,3	Q1
3	International Journal of Science and Mathematics Education	4,0	Q1
4	Technology, Knowledge and Learning	4,0	Q1
5	ZDM - International Journal on Mathematics Education	3,6	Q1
6	Educational Studies in Mathematics	3,4	Q1
7	Journal of Mathematics Teacher Education	3,3	Q1
8	Mathematics Education Research Journal	2,4	Q1
9	Mathematical Thinking and Learning	2,2	Q1
10	Research in Mathematics Education	2,1	Q1
11	International Journal of Education in Mathematics, Science and Technology	2,1	Q1
12	International Journal of Mathematical Education in Science and Technology	1,9	Q2
13	Teaching Mathematics and its Applications	1,3	Q2
14	Journal fur Mathematik-Didaktik	1,3	Q2
15	African Journal of Research in Mathematics, Science and Technology Education	1,2	Q2

Appendix 3. The Fifteen Best Indonesian Journals based on SJR 2020 Score from ScimagoJR (https://www.scimagojr.com)

Rank	**Title**	**SJR 2020 Score**	**SJR Best Quartile**
1	Forest and Society	0,623	Q1
2	Indonesian Journal of Science and Technology	0,567	Q1
3	Journal on Mathematics Education	0,513	Q2
4	Buletin Ekonomi Moneter dan Perbankan	0,505	Q2
5	Jurnal Pendidikan IPA Indonesia	0,493	Q2
6	Electronic Journal of Graph Theory and Applications	0,443	Q3
7	International Journal of Technology	0,434	Q2
8	IAES International Journal of Artificial Intelligence	0,341	Q2
9	International Journal of Renewable Energy Development	0,331	Q3
10	International Journal of Power Electronics and Drive Systems	0,322	Q2
11	Acta medica Indonesiana	0,321	Q3
12	Bulletin of Chemical Reaction Engineering and Catalysis	0,308	Q3
13	Tropical Animal Science Journal	0,308	Q2
14	HAYATI Journal of Biosciences	0,305	Q2
15	Cakrawala Pendidikan	0,294	Q3

Appendix 4. The Fifteen Best Mathematics Education Journals based on SJR 2020 Score from ScimagoJR (https://www.scimagojr.com)

Rank	Title	SJR 2020 Score	SJR Best Quartile
1	Journal for Research in Mathematics Education	2,694	Q1
2	Educational Studies in Mathematics	1,847	Q1
3	Journal of Mathematics Teacher Education	1,724	Q1
4	ZDM - International Journal on Mathematics Education	1,196	Q1
5	Mathematical Thinking and Learning	1,098	Q1
6	International Journal of Science and Mathematics Education	1,074	Q1
7	Technology, Knowledge and Learning	0,969	Q1
8	Research in Mathematics Education	0,960	Q1
9	Mathematics Education Research Journal	0,880	Q1
10	For the Learning of Mathematics	0,524	Q2
11	Journal on Mathematics Education	0,513	Q2
12	International Journal of Mathematical Education in Science and Technology	0,496	Q2
13	International Journal of Education in Mathematics, Science and Technology	0,470	Q2
14	Teaching Mathematics and its Applications	0,452	Q2
15	Investigations in Mathematics Learning	0,413	Q2

Appendix 5. Distribution of origin of JME authors from various countries outside Indonesia

Rank	JME's Author Origin	Quantity
1	Netherlands	17
2	Turkey	10
3	Australia	9
4	Malaysia	7
5	United States	7
6	Germany	6
7	Brunei Darussalam	4
8	Poland	4
9	India	3
10	Portugal	3
11	Spain	3
12	Brazil	2
13	Mexico	2
14	Philippines	2
15	Saudi Arabia	2
16	Singapura	2
17	South Africa	2
18	South Korea	2
19	United Arab Emirates	2
20	United Kingdom	2
21	Ghana	1
22	Israel	1
23	Japan	1
24	Nepal	1
25	Palestine	1
26	Russian Federation	1
27	Rwanda	1
28	Taiwan	1
29	Uganda	1
30	Vietnam	1
31	Zambia	1
32	Zimbabwe	1

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CULTURAL HISTORICAL ANALYSIS OF IRANIAN SCHOOL MATHEMATICS CURRICULUM: THE ROLE OF COMPUTATIONAL THINKING

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Abstract

In this paper, six mathematics curriculum changes in Iran will be reviewed, spanning from 1900 until the present time. At first, change forces, barriers, and the main features of each curriculum reform will be represented. The first five curriculum changes are described briefly and the sixth and most recent curriculum reform will be elaborated. In this paper, we call the last reform as contemporary school mathematics curriculum change. This recent (contemporary) curriculum reform will be explained in more detail, followed by a discussion of the effect of globalization and research finding in the field of mathematics and mathematics education (in the Iranian mathematics curriculum). In total, three key ideas are distinguished as an effect of globalization which is "New Math", "International Comparative Studies", and "Computational Thinking". Finally, the paper comments on the necessity of paying more attention to information and communication technology as part of globalization; in particular, recall policy-makers to consider "Computational Thinking" as an important component of future curriculum design.

Keywords: Computational Thinking, School Mathematics Curriculum, Globalization, Curriculum Change

Abstrak

Pada artikel ini, enam perubahan kurikulum matematika di Iran akan dibahas, mulai dari tahun 1900 hingga saat ini. Pada awalnya, kekuatan perubahan, hambatan, dan fitur utama dari setiap reformasi kurikulum akan terwakili. Pada lima perubahan kurikulum pertama dideskripsikan secara singkat dan reformasi kurikulum keenam dan yang terbaru akan diuraikan. Pada tulisan ini, kami menyebut reformasi kurikulum yang terakhir sebagai perubahan kurikulum matematika sekolah kontemporer. Reformasi kurikulum (kontemporer) yang terbaru ini akan dijelaskan secara lebih rinci, diikuti dengan diskusi tentang pengaruh globalisasi dan temuan penelitian di bidang matematika dan pendidikan matematika (dalam kurikulum matematika di Iran). Secara keseluruhan, tiga gagasan utama yang dibedakan sebagai efek globalisasi, diantaranya "Matematika Baru", "Studi Perbandingan Internasional", dan "Berfikir Komputasi". Terakhir, makalah ini mengomentari pentingnya memberikan perhatian lebih pada teknologi informasi dan komunikasi sebagai bagian dari globalisasi; khususnya, mengingatkan para pembuat kebijakan untuk mempertimbangkan "Berfikir Komputasi" sebagai komponen penting dalam desain kurikulum di masa depan.

Kata kunci: Berpikir Komputasi, Kurikulum Matematika Sekolah, Globalisasi, Perubahan Kurikulum

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Iran has a centralized educational system, so the mathematics curriculum and textbooks are designed at the national level and distributed around the country by the Ministry of Education and all schools, teachers, and students have to use the same mathematics textbook for teaching and learning of mathematics (Kiamanesh, 2005). Also, the main resource in all sorts of exams (like classroom assessment and national external exams) is the same math textbook which all students have access to. According to TIMSS math teachers' questionnaire, most Iranian math teachers in grades 4 and 8 reveal that they use mathematics textbooks as the main source for their teaching in classrooms (Mullis et al., 2008; 2012). So, in a centralized educational system, textbooks have an important role. The first Iranian National Curriculum was prepared and published by 2010 (Gooya, 2010). Therefore, in this paper through a *cultural historical review*, we will illustrate the Iranian mathematics curriculum reforms traced through Iranian mathematics textbook changes. Indeed, mathematics textbook changes show the reflection of new aims, scopes, and direction of the mathematics curriculum.

Iranian mathematics curriculum has experienced six reforms from 1900 until now which five of them occurred before launching the National Curriculum in Iran and one of them occurred subsequently and upon the direction of the National Curriculum (Rafiepour, 2018). Indeed, the Iranian national curriculum was prepared and announced by the Ministry of Education in 2010 (Gooya, 2010). After launching the Iranian national curriculum, recent reforms in all mathematics textbooks began, and gradually all of the math textbooks at primary and secondary levels changed upon new direction. In this paper, the recent reform (post-2009) will be known as contemporary mathematics curriculum reform.

Despite the main focus of this paper being an analysis of the contemporary reform in the Iranian mathematics curriculum, using an application and modeling point of view, past reforms dating from 1900 to 2009 shall also be reviewed. This will allow a better understanding and a more comprehensive picture to be gleaned of historic Iranian mathematics curriculum reform. Therefore, in this study, we will explore the following research questions through cultural historical perspective, as follow:

1. What were the key ideas which guided the changes in school mathematics over time?
2. What are the main ideas for future change?

In this paper, Computational Thinking will be introduced as an effect of globalization which will be considered for future educational changes. We have to distinguish between internationalization as cultural imperialism and globalization as global collaboration for sustainable development. This paper discusses curriculum changes in the Iranian context, but there are at least two international contributions which would inform scholars from different contexts and courtiers around the world. Firstly, the Iranian context as a developing country was analyzed so the ideas which are discussed can help other scholars from developing countries learn how they can integrate them globalization effect into their official curriculum. Secondly, discussion about how the Iranian school mathematics curriculum provides an in-depth insight into mathematical content and process in which Iranian migrant students learned during their study in Iran (Farsani, 2015; 2016). At this time there are 7 million Iranian migrants which live abroad where the majority (around 1.4 million) live in the USA. The findings of this retrospective cultural historical review can therefore help mathematics teachers and curriculum developers in developing countries for devising better programs for migrant students.

METHOD

In this paper, we use a cultural historical approach for analysing mathematics curriculum change in Iran from 1850 until now (170 years). For this purpose, we collected all kinds of materials (e.g.

academic articles, books, and reports) which refer to Iranian mathematics curriculum changes, and reforms during this period of time, both English and Persian. In order to address our first research question, we reviewed and thematically analyzed all materials (papers, books, reports, and mathematics textbooks) to determine the main features of each mathematics curriculum reform in Iran. Subsequently, then we extracted the key ideas which guided the changes in school mathematics over time. For pursuing our second research question, we reviewed official documents of developing countries (e.g., USA, UK, Canada, Japan, etc.) for presenting main ideas in future mathematics curriculum reforms.

RESULTS AND DISCUSSION

In this section all, educational changes and mathematics curriculum reforms from 1900-2009 will be briefly reviewed and change forces and barriers of past reforms will be discussed. Some of these reforms were previously distinguished and explained in national journals and magazines by other Iranian scholars in Persian (Farsi) language which is the official language in Iran (e.g. Jalili, 2016; Rezaie, 2016).

The first mathematics curriculum reform started after establishing a new type of school in Iran to look like a European school style. In 1851, the first Iranian school (namely 'Dar Ul-Funun') was established in Tehran (Capital city of Iran from 1788). The key ideas of the mathematics curriculum in the first curriculum reform were preparing students for solving real world problems. So, mathematical concepts taught through practical application and joined with other discipline such as geography. In first Iranian school, foreign teachers were employed to teach modern knowledge to Iranian students. Gradually, this school published some textbooks on a different subject. These textbooks continued to be used until 1938 where the Ministry of Education tried to unify them (include mathematics textbooks) by 1938. So, the second mathematics curriculum reform starts from 1938 and continues until 1962, during which time all mathematics textbooks orchestrated solidarity all over the country. The key framework of mathematics curriculum in the second curriculum reform was designing core mathematics ideas for all students in the country. In this reform content of mathematics contain geometry, algebra, and arithmetic (Rezaie, 2016).

In 1962, the third reform in the mathematics curriculum started after the White Revolution (Revolution of Shah and Peoples). This curriculum changes as third reform stimulated with educational system changes by 1967 and the educational system was divided into three sections: Primary school (5 years), Intermediate or guidance school (3 years), and Secondary school (4 years). All textbooks include mathematics textbooks changed during this reform (Rostami, 1978; Orton, 1981). According to Orton (1981) the key ideas of the third curriculum reform were related to educational system changes and mathematics textbooks designed and published by an organization which works under ministry of education.

The fourth reform starts in 1975 and continued until 1992. In this time "New Math" was introduced to the Iranian school mathematics curriculum. Traces of "New Math" could be seen all over the mathematics textbooks during this period from primary to secondary level. At that time, students were divided into two types of school (theoretical and vocation) after grade 8. In theoretical school, students have divided again into three different groups: Mathematics and Physics, Experimental Sciences, and Human Sciences. The key ideas of mathematics curriculum in the fourth curriculum reform were adding some component of “New Math” into the math textbooks specially for students studying Natural Sciences, such as Mathematics, Physics, and the Experimental Science branches. As a result of integrating “New Math” into the math textbooks, mathematics became meaningless for students. So, only small numbers of students (about nine percent) chose math and physics for their future studies (Rejali & Parvaneh, 2019). Therefore, there would be a future shortage of candidates in math and science-related roles. As a reaction to this social phenomenon, the High Commission of Fundamental Changes in Educational System decided to launch new reform.

The fifth reform started from 1992 until 2009, as a reaction to new math and this curriculum reform influenced just secondary level (grade 9-12) and this reform coincided with educational system changes (Gooya, 2007). The educational system is divided into four sections: Primary (5 years), Intermediate (3 years), Secondary (3 years), and Pre-University (1 year). Another change in this reform; the school year starts from September to June, which is divided into two parts (as a semester) instead of three parts (as before). There is special attention to vocational education at the secondary level during this reform. In this reform, all mathematics textbooks in grades 9-12 were changed. During this reform research finding in the mathematics education domain was used widely and in some of the mathematics textbooks at the time, there are some features of constructivism point of view and problem-solving activities which would draw upon new findings in the field of mathematics education (Gooya, 2007). Mathematics changes also considered, titled “discrete mathematics”, was added to grade 12 as a separate mathematics textbook for students in the mathematics and physics branch.

Contemporary Mathematics Curriculum Changes

The Iran National Curriculum project started in 2006 in the High Commission of Fundamental Changes in Educational System (under High Commission of Cultural Revolution) and the first edition of this document was published in 2009. After approval of this national curriculum in 2010, the sixth and most recent mathematics curriculum reform started and continues until now. In this Office for Developing National Curriculum (2010) there are eleven learning domains which Iranian students have to study during their formal education. Mathematics is one of these learning domains which is defined as a science of pattern, asymmetry, art, and finally precise language. In the Iranian national curriculum document, several roles for the necessity and function of mathematics are considered, as follow:

1. Understanding the laws of nature (anticipating and controlling different natural situation)
2. Solving real-world problems
3. Developing a method of thinking in other natural and human science enhances rational reasoning.

There are four content topics (Number and Operation, Algebra and Symbolic Representation, Geometry and Measurement, Data and Statistics and Probability), and seven process topics (Problem Solving, Modelling Real Data, Reasoning, Visual Thinking, Creative Thinking, Connection, Communication) in the Iranian national curriculum document.

In this document, there is an emphasis to express the role of Iranian mathematicians in developing mathematics in the Golden Islamic Age (Europe's Dark Ages). In some of the mathematics textbooks which were published after the approval of the Iran national curriculum, there are substantial historical references to the works of Iranian mathematician and scientists in the Golden Islamic Age which have a major impact on Muslim culture and civilization. In Iran's national curriculum, there is also an emphasis on the use of technology (such as calculators and computers) in mathematics. Not, however, seen in new mathematics textbooks!

After the enactment of the Iranian National Curriculum, contemporary mathematics curriculum reform started in Iran and two mathematics textbooks (one from the primary level and another from secondary level) were changed in each school year. Now, almost all school textbooks were changed or modified upon the national curriculum.

The sixth curriculum reform is also stimulated with educational system changes. The educational system is divided into four sections: Junior Primary (3 years), Senior Primary (3 years), Junior Secondary (3 years), and Senior Secondary (3 years).

More Details of Contemporary Mathematics Curriculum Changes

It seems that new contemporary mathematics curriculum changes have small influences on the process of teaching and learning mathematics in Iran. Several studies show that there is still plenty of work to do. As an example, TIMSS 2015 reveals that Iranian students' performance in mathematics in both grades 4 and 8 increased, but this result shows that Iranian students' performance is still below the international average and still not good (Mullis et al, 2016). There is a public opinion among Iranian people about Iranian students' performance in mathematics. Most Iranians think Iranian students had or must have good performance in mathematics because of their good record in the mathematics Olympiad contest where they have finished in the top 10. However, when the first results of TIMSS were published around 2000, Iranians were shocked and after that, the poor performance of Iranian students became one of the researchers' and policy-makers' main concerns.

After analysis of some new mathematics textbooks published during recent, contemporary curriculum reforms, Gholam-Azad (2015) mentioned some of the challenges of new textbooks. It could be observed upon that the importance of these challenges is an instrumental understanding of recent research findings, instead of a rational and deep understanding of them. Gholam-Azad (2015) said instrumental (superficial) understanding of recent research findings cause to reverse outcome. To clarify superficial understanding of research findings, we will focus on application and modeling (which is one of the mathematics processes in the Iranian National Curriculum). This will allow for a discussion through the systematic literature review of research studies related to mathematics textbooks.

The modeling approach means a process that starts with a problem situated in the real world. The modeling process continues with formulating real-world problems in mathematical terms. When this process is complete, the mathematical problem can be solved by the application of mathematical concepts and solution processes. Finally, the mathematical solution must be interpreted to provide an answer to the real-world problem and checked for its adequacy in answering the original question. A new cycle of the formulation may then begin for improving the model. In Figure 1, a simple diagram of the modeling cycle is presented.

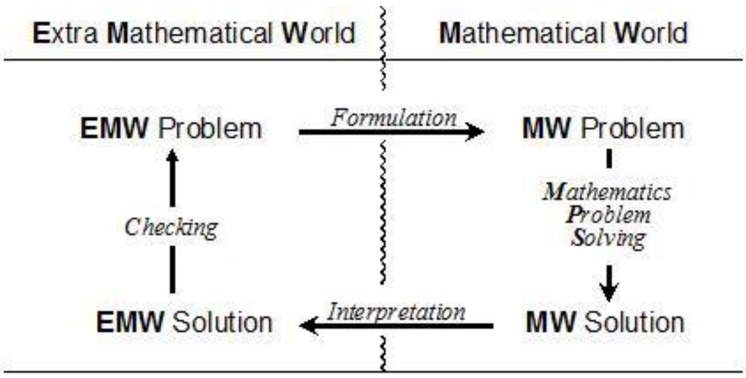

Figure 1. A model of the modeling cycle (Verschaffel, 2002)

In all of the new mathematics textbooks which were written after a re-enactment of the national curriculum, authors of textbooks mentioned their loyalty to the national curriculum in the preface of each textbook. So, there are some parts in these mathematics textbooks related to the application of mathematics in the real world, but almost all of them are of superficial usage of mathematics in the real world, and not real modeling. An example of modelling problem from literature review mentioned in Figure 2.

How can different rates of various mobile phone contracts be compared depending on customers' habits?

Figure 2. An example for modelling problem (Kaiser & MaaB, 2007)

A study of Rafiepour, Stacey & Gooya (2012) shows that modeling activities are absent in the new mathematics textbook in grade 9, while there are some standard application tasks included. Several research studies show that in new Iranian mathematics textbooks series, at the primary and secondary level, there isn't actual modeling activity (e.g. Rafiepour, 2012; 2014; Khani & Rafiepour, 2015).

The contemporary mathematics curriculum integrates different modalities for meaning-making such as the use of technology, diagrammatic representations, and symbolic notations. These modalities are perceived to be rich resources in mathematical contexts particularly when the medium of instruction may not be the learners' native tongue (Farsani et al., 2020; Rosa et al., 2020). By integrating these visual modalities in the curriculum would empower students who are not communicative competent in the language to be more engaged with the mathematical content (Krause & Farsani, In Press). Therefore, the phase transitions of this contemporary curriculum can enable mathematics teachers and curriculum developers in developing countries for devising better programs for migrant students.

Impact of Globalization in School Mathematics Curriculum

Several curriculum changes can be distinguished as an effect of globalization in Iranian mathematics curriculum history. In this section, these curriculum changes will be reviewed through historical order. The Office for Developing National Curriculum (2010), explicitly does not mention globalization and its position in the national curriculum, with just a definition of globalization expressed in the appendix of the national curriculum document. However, investigation of Iranian mathematics textbooks shows that recent research findings in the fields of mathematics and mathematics education were to be used for shaping new changes in math textbooks during that time. For example, during 'Dar Ul-Funun' time (1851-1938), European teachers jointly with Iranian partners, try to develop new textbooks for updating Iranian students with new knowledge. As another example, during the fourth reform (1975-1992), "New Math" was introduced to the Iranian school mathematics curriculum. In more recent reforms in the Iranian mathematics school curriculum, writers of textbooks try to use the recent findings of mathematics education. For example, in the fifth Iranian mathematics textbooks reform (1992-2009), there is an emphasis on constructivism and problem-solving in some of the mathematics textbooks. In contemporary and sixth Iranian mathematics textbooks reforms, writers continue to emphasize problem-solving skills and cover some application of math through the math textbooks in all grades.

Another context for the effect of international experiences on changing the Iranian mathematics curriculum related to Trend International Mathematics and Science Study (TIMSS) data will follow. Iranian students participated in the TIMSS study from 1995 until the present day, in different grades. Iranian students' performance in mathematics was not good and in all TIMSS studies (1995, 1999, 2003, 2007, 2011, and 2015) was below the international average (TIMSS, 2016). Although in grade 4, Iranian students' performance increased from 387 in TIMSS 1995 to 431 in TIMSS 2015, and in grade 8, Iranian students' performance increased from 418 in TIMSS 1995 to 436 in TIMSS 2015; but this situation is still not perceived to be desirable. Educational policy-makers frequently ask researchers and curriculum developers to reform the Iranian school mathematics curriculum toward enhancing Iranian students' performance in the TIMSS study. In contemporary school mathematics textbooks reform, writers try to direct change in such a way to focus on problem-solving in all mathematics textbooks and to respond to educational policy-makers' concerns concerning Iranian students' performance in TIMSS.

Results of another large-scale international assessment, namely the Programme for International Students Assessment (PISA) which is organized by OECD, influence the mathematics curriculum in many countries (de Lange, 2021). The Iranian mathematics curriculum was not an exception. Although Iran didn't participate in PISA until now, PISA has had an implicit effect on Iranian school mathematics (Stacey et al., 2015). PISA was introduced to the Iranian community through mathematics educators' research, at first. Following this, several teachers of mathematics who had started their master's degree researched mathematical modeling and applications, which is one of the focal points of PISA. The results of this research were published in national and international conferences and journals. Through this sharing, other mathematics teachers familiar with PISA can benefit and improve their practice of teaching and learning mathematics in Iran. Aforementioned, there is some sort of application of mathematics in all-new versions of mathematics textbooks to reflect the passion of the community.

Finally, the last and recent globalization effect in many curricula changes around the world related to Computational Thinking (CT). Indeed, human beings in society have become more and more technology-based in the 21st century and must have appropriated knowledge to perform their work in a more efficient manner, in which such a society is heavily integrated with technology (Bocconi et al., 2016). One of the important aspects that everyone has to know is CT as Wing (2006) suggested, "to reading, writing, and arithmetic, we should add CT to every child's analytical ability" (p. 33). Shodiev (2014) defines CT as a way of thinking algorithmically using design trees from computer science as a guiding structural, and sometimes metaphorical, framework. Wing (2014) also defines CT as "the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer-human or machine—can effectively carry out" (para. 5). Hoyles and Noss (2015) consider CT as abstraction, algorithmic thinking, decomposition, and pattern recognition. CT involves concepts (e.g. loops, conditions) and practices (e.g., abstraction and debugging) (Lye & Koh, 2014; Kafai & Burke, 2013). However, CT is not simply programming and rote skill, but it is a conceptualizing and fundamental skill; a way that humans think (Wing, 2006).

CT changes the nature of some contemporary researches in the mathematics domain. For example, we can see the computer-based proof in mathematics (e.g. four-color theorems). For another example, a new domain of research related to mathematics and computation such as Bioinformatics emerged recently. In this regard, the European Mathematical Society (2011) recognized an emerging way of engaging in mathematical research: "Together with theory and experimentation, the third pillar of scientific inquiry of complex systems has emerged in the form of a combination of modeling, simulation, optimization, and visualization" (p. 2). Weintrop et al. (2016) try to address CT as a more sophisticated concept upon literature review and interview experts who use CT in their carrier. They develop a taxonomy of CT skills which has close relation with the third pillar of scientific inquiry. This taxonomy contains four main categories: data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices. In each category, Weintrop et al. (2016), explain what sort of activities are used by mathematicians and scientists related to CT, and based on this they “provide a roadmap for what CT instruction should include in the classroom” (p. 128).

This effort for combining CT in mathematics education is not new and investigation on the historical origin of CT in mathematics education shows a legacy of over 53 years in which begins by designing LOGO programming language in the theory of constructionism (Papert & Harel, 1991). Studies of constructionism at higher-level mathematics education show how programming supports students’ understanding of mathematical concepts (e.g., Leron & Dubinsky, 1995; Wilensky, 1995) and how it contributes to the development of critical thinking skills (e.g., Abrahamson et al., 2006; Marshall, 2012). Kaufmann and Stenseth (2020) demonstrate in a case study, lower secondary school-level pupils use programming to solve a mathematical problem.

CT activities can enhance the learning of difficult mathematical concepts through low floor high ceiling (Gadanidis et al., 2017). Weintrop et al. (2016) explain the mutual relationship between CT and mathematics education in which CT activities can deepen the learning of math and vice versa, with mathematics providing a meaningful context for CT. In Buteau et al. (2020) paper, the concept of “legitimate peripheral participation” was used, which is defined by Lave and Wenger (1991) for describing how learners enter into a community and gradually take up its practices. In this paper, legitimate peripheral participation was used to understand how undergraduate students learn mathematics through CT activities. Through these activities, students encounter mathematical ideas and problems; they then try to use CT as a means for constructing an “object to think with”. This is in line with Papert (1980), who believes the computer provides the learner a means for constructing “objects to think with” (p.204). Indeed, this view on learning concords with the constructionism paradigm.

At this time, there is nothing about computational thinking (CT) in Iran's National Curriculum and school mathematics textbooks. However, there is a chapter (10 pages long) about building a computer game using Scratch software in the grade nine “work and technology” textbook (Esmaili et al., 2020). This chapter starts with a discussion about the advantages and limitations of computer games and then expresses the disadvantages of using a computer game. The chapter continues with introducing Scratch software components, how to change the language of the program, and so on. Then devises a simple project for drawing a rectangle through the representation of the Scratch software cat. It finally asks students to complete a real project related to design the “balls and rocket” game (see Figure 3).

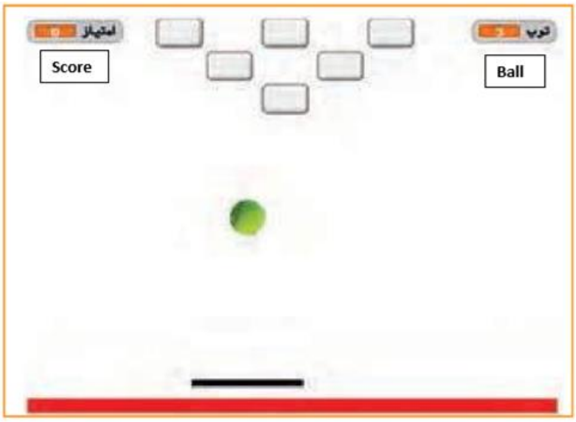

Figure 3. “balls and rocket” game page 48 of grade nine “work and technology” textbook

In this final project, students will become familiar with the concept of variables (in terms of computer science which is a place for saving dynamic information) and roles of that in programming and they start to have experience with writing scenarios for designing a game. In the 10-page unit in the grade 9 “work and technology” textbook, there is a short comment about debugging a computer program. Besides, there is an after-school program in which students in some provinces participate in and learn computational thinking skills.

CONCLUSION

A review of mathematics curriculum reforms in Iran shows that every 15-20 years, Iranian mathematics textbooks were changed. In all of these reforms, some barriers hinder progression. It seems one of the important barriers is related to the teacher education program. For example, Gooya's (2007) research shows that traditional mathematics teachers didn’t believe in the constructivist point of view and they are opposed to geometry curriculum change in direction to constructivism. Several researches show that teachers of mathematics haven’t adequate knowledge for implementing contemporary curriculum change which related to modelling and application (Rafiepour, 2014). Indeed, teachers of mathematics hadn’t access to good resources to improve their knowledge and skills in line with the direction of new educational reforms regard to modelling and application (Rafiepour & Molaie, 2020). They didn’t receive suitable content and pedagogical knowledge during their pre-service and in-service program concerning modelling and application (Rafiepour, 2016). In such a situation, teachers stand alone with their problems and they didn’t receive suitable and adequate support. It is more and more important to support math teachers for future mathematics curriculum reform, especially in the 21st century where changes occur quickly, and the school mathematics curriculum must reflect these new changes in new reforms. Teachers are the most important and smallest loop in the curriculum chain. If teachers are properly supported through pre-service and in-service programs, then we can expect improved results after any educational reform.

A review of the history of education reforms in Iran reveals that there were several different reasons for different educational changes such as:

1. Varying goals, perspectives, and educational expectations upon social changes
2. Assessment of implemented curriculum
3. New research finding in the field of mathematics and mathematics education
4. Widespread usage and pervasiveness of technology such as a computer, Internet, smart boards, calculators, and so on.

This latter reason is the contemporary concern of almost all educational systems around the world and one that has been neglected. It should be given more attention in the Iranian educational system. Society has become more and more technology-based in the 21st century and must have the appropriate knowledge to perform their work in a more efficient manner, in which such a society is heavily integrated with technology. Gardner (2006) mentioned the future world with a search engine, robots, and other computer-based instruments will demand capacities that until now have been mere options. To meet the new world demand capacity, one of the necessary skills that every student should learn is CT, which Wing (2006) suggested "to reading, writing, and arithmetic, we should add CT to every child's analytical ability" (p. 33). CT will change the paradigm of many businesses in the future and change the job environment.

Today, many countries around the world address computational thinking (CT) in their school curriculum as part of the mathematics curriculum or separately (Bocconi et al., 2016). In the context of Iran, except the "work and technology" textbook in grade 9 which has one module (10 pages), there is nothing about computational thinking (CT) in k-12 textbooks in Iran. Although, there is an after-school program in which some students can participate to learn computational thinking skills. So, it can be said computational thinking (CT) is neglected in the school curriculum in Iran and can be considered as a "null curriculum" in terms of Eisner (2004). Therefore, it seems necessary to consider computational thinking (CT) as a component across curriculum and textbooks in k-12. Although different countries have different issues and problems to each other and curriculum developers must keep in mind local issues and local considerations (such as cultural components, environmental challenges, natural disasters, and so on), but they also have to prepare their citizens for 21st-century requirements. In this regard, it seems CT is a necessary skill for Iranian students in the 21st-century.

Finally, the findings of this retrospective cultural historical review and its practical implications can inform international scholars in two ways. Firstly, this paper informs the bigger mathematics education community with the Iranian community as developing countries to integrate CT into the official curriculum. This can encourage other developing countries to start to teach CT for their future generation. Secondly, the finding of this study can help scholars in developed countries for designing a better program for migrant students who spent some part of their studies in developing countries.

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MOTIVATION AND MATHEMATICS ACHIEVEMENT: A VIETNAMESE CASE STUDY

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Abstract

Motivation is key to engaging students in studying mathematics and in improving their mathematics achievement. Although the related literature has explored the correlation between motivation and mathematics achievement, a research gap remains in terms of the empirical testing of these variables in the context of mathematics education in Vietnam. Thus, the current study aims to fill this gap by empirically testing the correlation between mathematics motivation and mathematics achievement among high school graduate students in Vietnam, using a quantitative approach to test hypotheses. The study adopted the Academic Motivation Toward Mathematics Scale for collecting data from students and received 680 responses. The main study findings are that amotivation negatively correlates with mathematics achievement, whereas introjected regulation, identified regulation and intrinsic motivation positively correlate with mathematics achievement. These findings provide a strong theoretical foundation for improving mathematics achievement by encouraging teachers to improve motivational conditions in mathematics classes in Vietnam.

Keywords: Mathematics Motivation, Mathematics Achievement, Introjected Regulation, Identified Regulation, Intrinsic Motivation

Abstrak

Motivasi memiliki peran penting dalam melibatkan siswa untuk belajar matematika dan meningkatkan prestasi matematika mereka. Hubungan korelasi antara motivasi dan prestasi matematika telah dieksplorasi dalam penelitian sebelumnya; namun, terdapat kesenjangan penelitian pada hal uji empiris untuk elemen utama dalam konteks pendidikan matematika di Vietnam. Oleh karena itu, penelitian ini bertujuan untuk memecahkan masalah penelitian dengan menguji secara empiris korelasi antara motivasi matematika dan prestasi matematika di kalangan siswa lulusan sekolah menengah di Vietnam. Penelitian ini menggunakan pendekatan penelitian kuantitatif untuk menguji suatu teori. Penelitian ini mengadopsi tes *Academic Motivation Toward Mathematics Scale* (AMTMS) untuk mengumpulkan data. Enam ratus delapan puluh tanggapan diterima siswa sekolah menengah. Terdapat beberapa poin utama yang dihasilkan dari penelitian ini. Motivasi telah terbukti berkorelasi negatif dengan prestasi matematika. Namun, regulasi yang diintrojeksi, regulasi yang teridentifikasi dan motivasi intrinsik berkorelasi positif dengan prestasi matematika. Temuan yang dihasilkan pada penelitian ini memberikan landasan teoritis yang kuat untuk meningkatkan prestasi matematika dengan mendorong guru untuk meningkatkan kondisi motivasi di kelas matematika di Vietnam.

Kata kunci: Motivasi Matematika, Prestasi Matematika, Regulasi Introjeksi, Regulasi Teridentifikasi, Motivasi Intrinsik

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Mathematics education has attracted considerable attention from educators and scholars (Attard et al., 2016). The literature has shown that individuals' attitudes, beliefs and emotions play a vital role in their responses to, and interest in, mathematics, as well in their application of mathematics in real-word situations (OECD, 2018). In particular, enhancing the motivational conditions in their classroom for students is crucial for improving mathematics teaching and learning owing to its relationship with their behaviour and achievement (Pantziara & Philippou, 2015). Students who feel more confident about their mathematical skills are more likely than their less-confident counterparts to apply mathematics in different situations (OECD, 2018). Consequently, in mathematics education, developing students' attitudes, beliefs and emotions is a main objective. Aeschlimann et al. (2016) stated that enhancing motivational conditions in mathematics and science classes is a promising intervention strategy to ensure that more individuals participate in science, technology, engineering and mathematics occupations.

In this regard, motivation and engagement seem to be attracting increasing research attention in the mathematics education field (Attard et al., 2016). Motivation is among the most affective domains identified to strongly relate to mathematics achievement (Lim & Chapman, 2015b). The role of motivation in teaching and learning mathematics has been widely reported. Hannula (2006) revealed that motivation can be used for directing behaviour to control emotion and that individuals' motivation is revealed through three aspects: their cognition, behaviour and emotion. Pantziara and Philippou (2015) provided insight into motivation through these three aspects: Cognition occurs when people believe in the essence of a task. Behaviour occurs when they have a consistent strategy for solving problems. Emotion occurs when they feel disappointed on failing to address problems.

Moreover, students' perceptions about their classroom and school environment affect their motivation to study mathematics (Polychroni et al., 2012). The achievement goal theory has suggested that a likely characteristic that motivates students to learn mathematics is the classroom goal structure (Maehr & Zusho, 2009). A goal structure is defined as the way that achievement goals are designed through education policies and practices (Wolters, 2004; Skaalvik et al., 2017). It has two general forms: the mastery goal structure and the performance goal structure (Patrick et al., 2011; Skaalvik et al., 2017). The mastery goal structure focuses on developing competence, whereas the performance goal structure focuses on demonstrating competence (Patrick et al., 2011; Skaalvik et al., 2017). Skaalvik et al. (2017) emphasised that it is vital to identify the types of elements that affect students' motivation.

In Vietnam, mathematics is one of the most important subjects at school. It is one of three compulsory subjects in the National High School Graduation Examination. Parents usually encourage their children to study mathematics as they believe that it will assist in further studies. Further, people who are good at mathematics are often viewed as smart. Therefore, there are many interventional activities undertaken by both educators and parents that seek to improve mathematics education in Vietnam. Schools organise a formal class outside of the official school timetable to teach students. Parents often seek a private tutor for their children to foster mathematical achievement. These activities are criticised by society because they may place even more pressure on students. However, as suggested by Aeschlimann et al. (2016), one emerging intervention strategy to improve students' mathematics achievement is to improve the motivational conditions in the mathematics classroom. In addition, Lim and Chapman (2015b) discovered that motivation is one of the most affective domains that has strong correlation with mathematics achievement. Further, Aeschlimann et al. (2016) emphasised that the enhancement of motivational conditions in mathematics classes has emerged as an encouraging intervention strategy to improve mathematics achievement. Therefore, mathematics motivation can be a potential intervention strategy for improving mathematics achievement in Vietnam. Consequently, understanding the correlation between mathematics motivation and achievement in the Vietnamese context is vital for improving mathematics achievement. However, until date, no empirical study has focused on this issue. Therefore, this study aims to address this gap and to identify the related affective factors by answering the following research question: What is the correlation between mathematics motivation and mathematics achievement in Vietnam?

Mathematics Motivation

According to the self-determination theory, the three types of academic motivation are amotivation, extrinsic motivation and intrinsic motivation (Deci & Ryan, 1985). Figure 1 illustrates the self-determination continuum. Amotivation can be defined as people's lack of interest in performing a task because they do not perceive its value or feel unable or incompetent to achieve the expected results (Deci & Ryan, 2000). Intrinsic motivation can be defined as the internal demand to accomplish a task owing to the satisfaction it offers (Deci & Ryan, 2000). In contrast, extrinsic motivation—which is situated between the other two types on the continuum—includes external, introjected, identified and integrated regulation. As regards intrinsic motivation, it has been found to positively correlate with achievement (Ahmed et al., 2010; Woolley et al., 2010). Further, intrinsic motivation is associated with autonomous forms of extrinsic motivation and positive academic achievement (Deci et al., 1991; Grolnick et al., 1991).

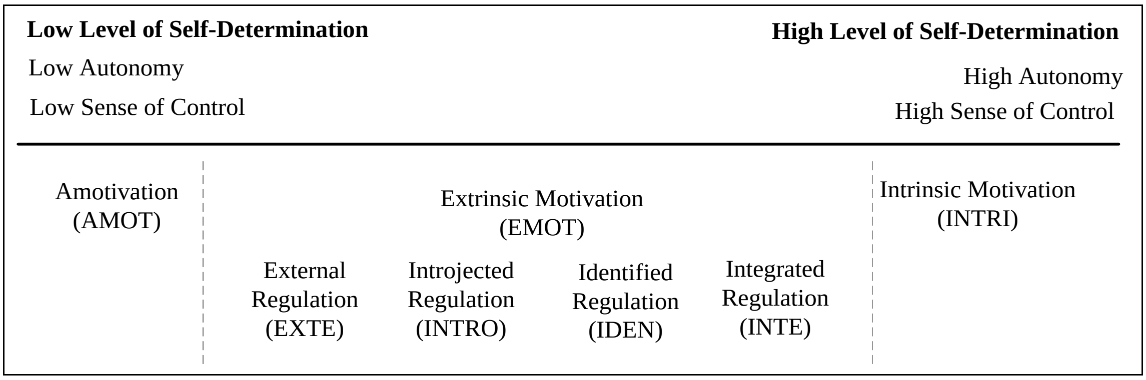

Figure 1. Self-determination continuum (adapted from Deci and Ryan (2000))

As regards the types of extrinsic motivation, the first, external regulation causes people to act to obtain a desirable outcome (Deci & Ryan, 2000). The second, introjection, can occur when individuals feels that they 'ought to' participate in an activity, and it is a more internal type of extrinsic motivation than external regulation (Wang et al., 2009; Lim & Chapman, 2015b). The third, identified regulation, is the most internal type of extrinsic motivation and occurs when people identify with the justifications for their actions (Lim & Chapman, 2015b).

Several studies have discovered the correlations between mathematics motivation and achievement. Mathematics motivation can be measured through intrinsic motivation, identification, introjection, external regulation and amotivation (Lim & Chapman, 2015a). Research has indicated that motivation positively correlates with the desired outcome, such as good academic performance (Gottfried et al., 2007). However, the lack of academic motivation negatively correlates with educational results (Barkoukis et al., 2008). Several highly negative outcomes are associated with amotivation (Deci & Ryan, 2000). Moreover, intrinsic motivation and amotivation were statistically associated with mathematics achievement (Lim & Chapman, 2015a). Amotivation was found to be negatively correlated with mathematics achievement among Years 11 and 12 mathematics students in Singapore (Lim & Chapman, 2015a). Herges et al. (2017) found that intrinsic motivation positively associates with mathematics achievement for middle school students in Years 6, 7 and 8. Further, mathematics achievement has a positive correlation with identification (Lim & Chapman, 2015a). However, Lim and Chapman (2015b) noted that future studies on students from different cultures and countries were warranted because their findings were based on a sample of Chinese students in Singapore.

In addition, studies have shown that students' motivation and performance are influenced by their learning environments, such as through the classroom goal structure, namely, the performance goal and mastery goal structures (Maehr & Zusho, 2009). A teacher who applies a mastery goal structure can emphasise students' effort, understanding and improvement and view mistakes as part of the learning environment (Patrick et al., 2011). The success of students is assessed based on their improvements and their ability to judge. In contrast, when using a performance goal structure, a teacher may promote comparison or competition among students and make public the achievement of all students, rather than emphasising effort and improvement (Patrick et al., 2011). Students' success in the performance goal structure is defined by surpassing targets or outperforming others (Patrick et al., 2011).

Thus, the discussion thus far reveals that through identifying effective elements in mathematics education, mathematics teachers may apply valuable intervention strategies for assisting students to improve their self-confidence in mathematics. This approach may result in improved mathematics performance. However, the correlation between mathematics motivation and achievement in Vietnam is yet to be empirically tested. Therefore, an empirical study on this correlation is necessary.

METHOD

Conceptual Model and Hypothesis Development

Figure 2 presents the current research model. The direct correlations of (1) amotivation, external regulation, introjected regulation, identified regulation and intrinsic motivation with (2) mathematics achievement are empirically tested. Obviously, one indicator of the self-determination continuum was not included in the indicators of motivation in the present study (i.e., integrated regulation). The main reason was that the current study adopted the Academic Motivation Toward Mathematics Scale (AMTMS) which was developed by Lim and Chapman (2015a) based on the Academic Motivation Scale (AMS) (Vallerand & Blssonnette, 1992). Please note that the AMS was developed based on the self-determination theory (Deci & Ryan, 1985) and it measures academic motivation in general, while the AMTMS measures academic motivation in mathematics. The AMTMS includes amotivation, external regulation, introjection, identification and intrinsic motivation to accomplish.

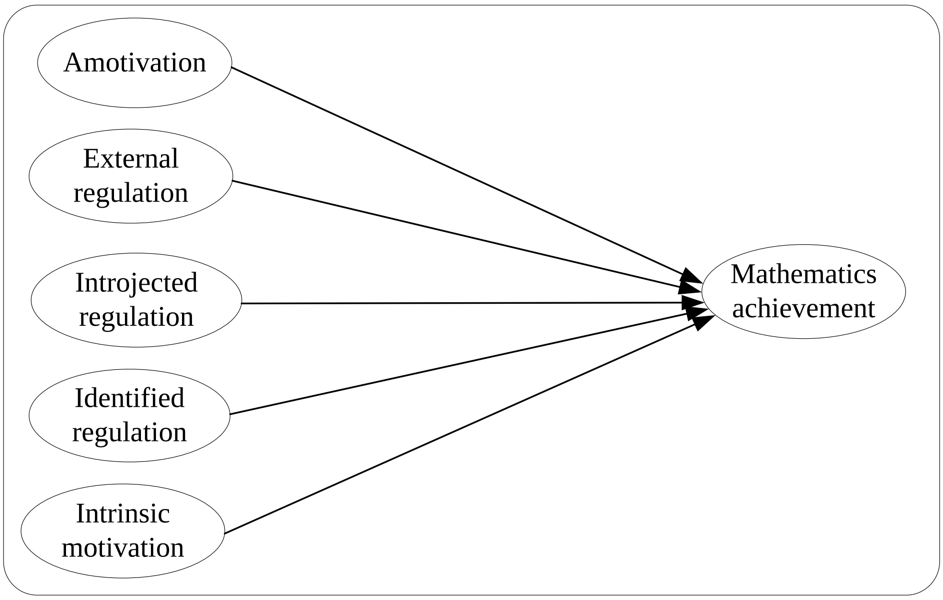

Figure 2. Research model

The literature has indicated that mathematics motivation correlates with academic achievement. Amotivation is correlated with negative outcomes (Deci & Ryan, 2000). Further, Flammer and Schmid (2003) found that extrinsic motivation positively influences students' academic achievement. Identified regulation, introjected regulation and external regulation are different forms of extrinsic motivation. Moreover, many studies have discovered the correlation between intrinsic motivation and positive academic performance (Deci et al., 1991; Grolnick et al., 1991). Therefore, the following hypotheses were developed:

Hypothesis 1 (H1): Amotivation negatively correlates with mathematics achievement.
Hypothesis 2 (H2): External regulation positively correlates with mathematics achievement.
Hypothesis 3 (H3): Introjected regulation positively correlates with mathematics achievement.
Hypothesis 4 (H4): Identified regulation positively correlates with mathematics achievement.
Hypothesis 5 (H5): Intrinsic motivation positively correlates with mathematics achievement.

Data Collection Instrument

Mathematics Motivation

This study adopted the AMTMS (Lim & Chapman, 2015a). The AMTMS has 21 measurement items, including four for each latent variable—amotivation, external regulation, introjection and identification—and five for intrinsic motivation (Table 1). The main question of the AMTMS is ‘Why do you spend time studying mathematics?’. The participants were asked to rank the AMTMS items using a 5-point Likert scale (1 = ‘strongly disagree’ to 5 = ‘strongly agree’).

Mathematics Achievement

The study used the mathematics results of the National High School Graduation Examination, 2019. The results from 2019 were used because, at the time of the current survey, they were the most recent. The pen-and-paper mathematics examination ran for a period of three hours and students used the results to apply for entry into college or university. To obtain its data, the study utilised the self-reporting of grades by participants. A ten-point grading scale is used in the Vietnamese education system.

Participants

The target participants were students who undertook the National High School Graduation Examination, 2019. The final sample ($N = 680$) comprised 367 females, 305 males and eight others. The self-reported grades (mathematics achievement) ranged from 0 to 10, in which the mean values were 7.28 (out of 10) with a standard error of 0.051 and standard deviation of 1.343. In all, 93.8% of the participants were enrolled in a higher education course, among whom 92.2% were enrolled at a university for a bachelor’s degree, and the remaining had opted for vocational training or a diploma course. Surprisingly, 85.9% of participants informed that their mathematics scores had been used in their combination of subject scores to apply for entry to university.

Data Analysis Method

Descriptive data analysis was undertaken to assess the fundamental characteristics of the data. A visual inspection and statistical assessment confirmed that the data were normally distributed. Thereafter, structural equation modelling was used for data analysis, in which the maximum likelihood estimation method was used. Confirmatory factor analysis (CFA) was applied to the measurement model for confirming the reliability and fitness of the factor structures of the latent variables. Structural equation modelling was used to test the hypotheses on the correlations between mathematics motivation and achievement. The following indicators were adopted to evaluate the model fit: the chi-squared test (χ^2), the comparative fit index (CFI), the Tucker–Lewis index (TLI), the standardised root mean square residual and the root mean square error of approximation (RMSEA) (Kline, 2015). The model is considered to have an acceptable fit when these indices satisfy the following conditions: CFI and TLI > 0.90 (Hoyle & Panter, 1995) and RMSEA < 0.08 (Hair et al., 2014).

RESULTS AND DISCUSSION

Validity and Reliability Analysis

In this study, IBM SPSS AMOS Graphics Version 26 was used for CFA, for all measurement items. Convergent validity was assessed using the factor loadings, *t*-values and *p*-values of each indicator. As presented in Table 1, the factor loadings for all the measurement variables were statistically significant at 0.001 (the unstandardised estimates are reported). The correlation values, average variance extracted (AVE) scores and the AVE square root scores were used for assessing discriminant validity.

Table 1. Measurement variables and their factor loadings

Label	Variable	β	$t - value$
Question: Why do you spending time studying mathematics?			
Amotivation (AMOT)			
AMOT1	Honestly, I don't know; I feel that it is a waste of time studying mathematics.	0.948***	22.098
AMOT2	I can't see why I study mathematics and frankly, I couldn't care less.	1.000	
AMOT3	I don't know; I can't understand what I am doing in mathematics.	0.930***	21.132
AMOT4	I am not sure; I don't see how mathematics is of value to me.	0.882***	19.875
External regulation (EXTE)			
EMER1	Because without a good grade in mathematics, I will not be able to find a high-paying job later on.	0.800***	18.180
EMER2	In order to obtain a more prestigious job later on.	0.931***	27.565
EMER3	Because I want to have "the good life" later on.	0.941***	29.192
EMER4	In order to have a better salary later on.	1.000	
Introjected regulation (INTRO)			
EMIN1	Because of the fact that when I do well in mathematics, I feel important.	0.515***	12.987
EMIN2	Because I want to show to others (e.g., teachers, family, friends) that I can do mathematics.	0.821***	19.498
EMIN3	To show myself that I am an intelligent person.	1.000	
EMIN4	Because I want to show myself that I can do well in mathematics.	0.985***	23.356
Identified regulation (IDEN)			
EMID1	Because I think that mathematics will help me better prepare for my future career.	1.000	
EMID2	Because studying mathematics will be useful for me in the future.	0.966***	26.546
EMID3	Because I believe that mathematics will improve my work competence.	0.907***	24.144
EMID4	Because what I learn in mathematics now will be useful for the course of my choice in university.	0.966***	23.501
Intrinsic motivation (INTRI)			
IMTA4	Because I want to feel the personal satisfaction of understanding mathematics.	0.943***	28.935
IMTK2	For the pleasure I experience when I discover new things in mathematics that I have never learnt before.	0.976***	31.862

Label	Variable	β	$t-value$
IMTK3	For the pleasure that I experience in broadening my knowledge about mathematics.	1.000	
IMTS2	For the pleasure that I experience when I learn how things in life work, because of mathematics.	0.929***	25.235
IMTS3	For the pleasure that I experience when I feel completely absorbed by what mathematicians have come up with.	0.945***	23.087

Figure 3 illustrates the CFA model and Table 2 shows the test results. As shown, the composite reliability (CR) values for all latent constructs exceeded 0.80 (0.823–0.913), which proves strong reliability. Convergent validity was confirmed as the AVE values were above 0.50 (0.547–0.677). The square roots of the AVE were higher than its correlation; thus, discriminant validity was confirmed. The TLI (0.090) and CFI (0.922) were higher than 0.90 and RMSEA (0.078) was smaller than 0.08; thus, the CFA model had an acceptable fit.

Table 2. Validity and reliability analysis results

Model Validity Measures	CR	AVE	AMOT	EXTE	INTRO	IDEN	INTRI
AMOT	0.860	0.606	0.779				
EXTE	0.879	0.649	-0.344***	0.806			
INTRO	0.832	0.547	-0.355***	0.475***	0.740		
IDEN	0.891	0.671	-0.587***	0.718***	0.523***	0.819	
INTRI	0.913	0.677	-0.581***	0.416***	0.606***	0.673***	0.823

Note: $*p < 0.10$, $**p < 0.05$, $***p < 0.001$. Model fit indices: $\chi^2 = 914.868, df = 179, p = 0.000, \chi^2/df = 5.111, TLI = 0.909, CFI = 0.922$, RMSEA = 0.078.

Figure 3 presents the CFA of a five-factor model comprising AMOT, EXTE, INTRO, IDEN and INTRI. In addition, the figure shows the correlation results between latent variables. AMOT has a significant and negative correlation with EXTE, INTRO, IDEN and INTRI, whereas EXTE, INTRO, IDEN and INTRI have a significant and positive correlation with each other.

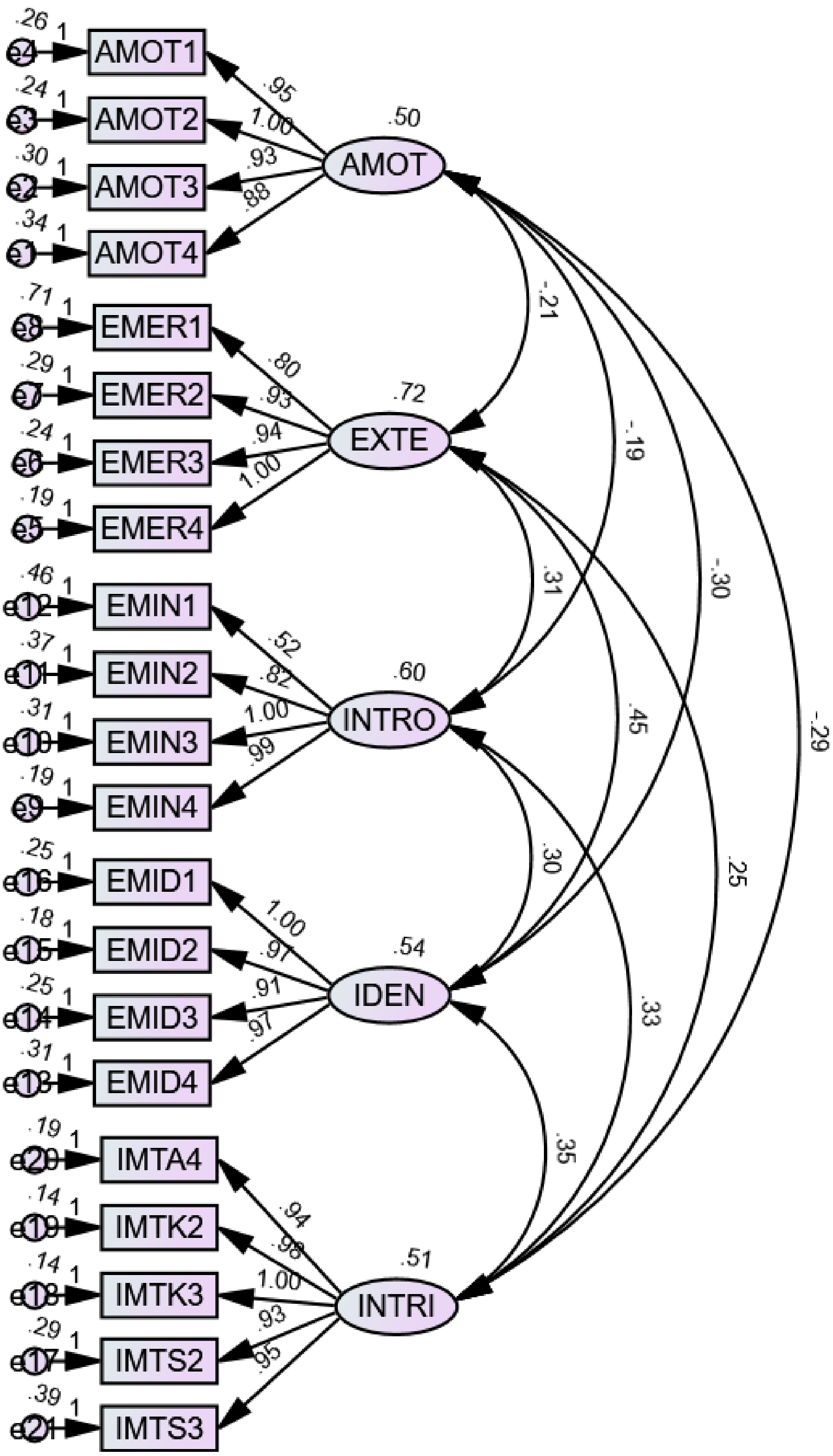

Figure 3. CFA model

Relationship between Mathematics Motivation and Mathematics Achievement

Figures 4–8 present the relationships between (1) AMOT, EXTE, INTRO, IDEN and INTRI and

(2) MA, respectively. The test results are presented in Table 3. Table 3 provides information on the dependant variable, independent variable, model fit indices and path indices for each testing model. The results are unstandardised estimations. The path models representing these relationships were run separately to test the hypotheses.

Amotivation

Figure 4 presents a model for testing the direct relationship between AMOT and MA, in which AMOT negatively correlates with MA.

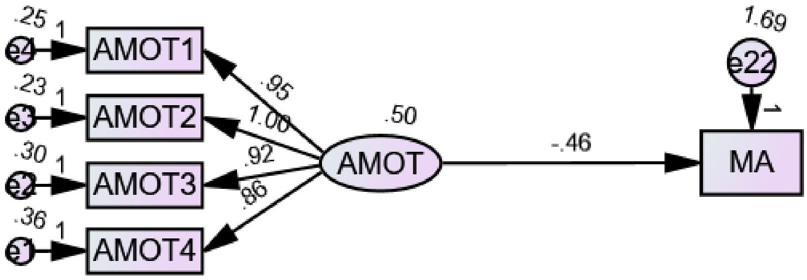

Figure 4. Relationship between amotivation and mathematics achievement

The results in Table 3 suggest a significant and negative correlation between AMOT and MA ($\boldsymbol{\beta} = -\mathbf{0.462}, \boldsymbol{t} - \boldsymbol{value} = -\mathbf{5.999}, \boldsymbol{\rho} < \mathbf{0.001}$), thus supporting H1—'Amotivation negatively correlates with mathematics achievement'.

External Regulation

Figure 5 shows the model used to test for a direct relationship between EXTE and MA.

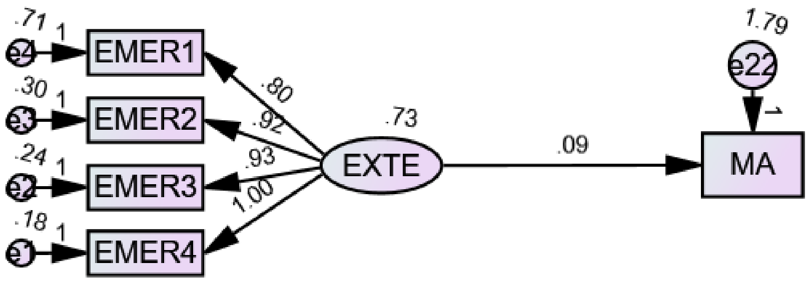

Figure 5. Relationship between external regulation and mathematics achievement

The results in Table 3 show there is no significant correlation between EXTE and MA ($\boldsymbol{\beta} = \mathbf{0.088}, \boldsymbol{\rho} = \mathbf{0.164}$). Therefore, H2 was rejected.

Introjected Regulation

Figure 6 presents a model for testing the direct relationship between INTRO and MA, in which INTRO positively correlates with MA.

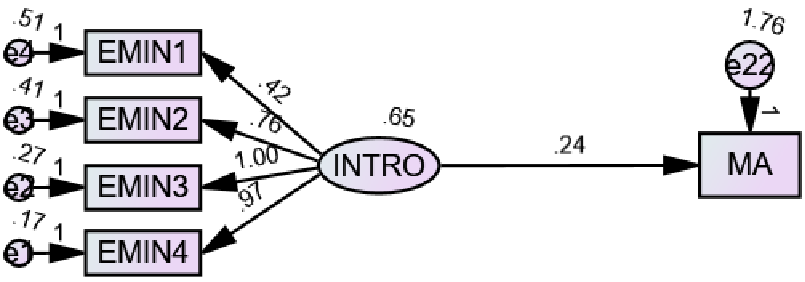

Figure 6. Relationship between introjected regulation and mathematics achievement

The results in Table 3 illustrate a significant and positive correlation between INTRO and MA ($\boldsymbol{\beta = 0.236, t - value = 3.418, \rho < 0.001}$), thus supporting H3—'Introjected regulation positively correlates with mathematics achievement'.

Identified Regulation

Figure 7 shows a model for testing the direct relationship between IDEN and MA, in which IDEN positively correlates with MA.

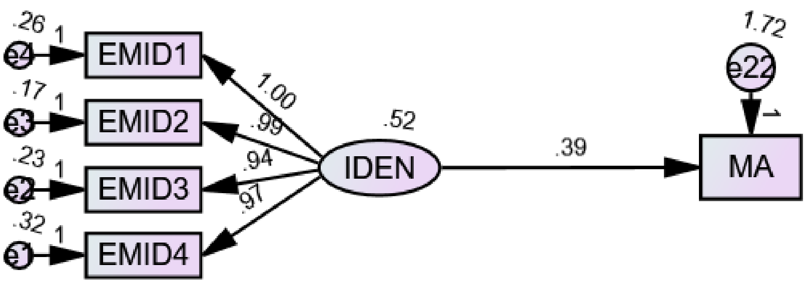

Figure 7. Relationship between identification and mathematics achievement

The Table 3 results show a significant and positive correlation between IDEN and MA ($\boldsymbol{\beta = 0.389, t - value = 5.217, \rho < 0.001}$), thus supporting H4—'Identified regulation positively correlates with mathematics achievement'.

Intrinsic Motivation

Figure 8 presents a model for testing the direct relationship between INTRI and MA, in which INTRI positively correlates with MA.

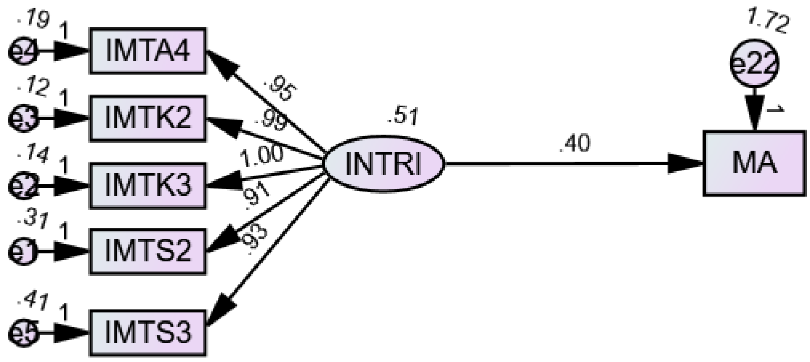

Figure 8. Relationship between intrinsic motivation and mathematics achievement

As the results in Table 3 show, there was a significant and positive correlation between INTRI and MA ($\boldsymbol{\beta = 0.402, t - value = 5.433, \rho < 0.001}$). Therefore, H4—'Intrinsic motivation positively correlates with mathematics achievement'—was supported.

Table 3. Relationship between mathematics motivation and mathematics achievement

Model	**Variables**		**Model Fitting Indexes**				**Path Indexes**		
	DV	IV	$\boldsymbol{\rho}$	$\boldsymbol{\chi^2/df}$	CFI	RMSEA	$\boldsymbol{\beta}$	t-value	P
1	AMOT	MA	0.000	9.757	0.966	0.114	-0.462^{***}	–5.999	0.000
2	EXTE	MA	0.000	15.893	0.952	0.148	0.088^{n}	1.392	0.164
3	INTRO	MA	0.000	13.937	0.941	0.138	0.236^{***}	3.418	0.000
4	IDEN	MA	0.044	2.279	0.996	0.043	0.389^{***}	5.217	0.000
5	INTRI	MA	0.000	7.099	0.976	0.095	0.402^{***}	5.433	0.000

Note: $^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.001$; n: non-significant; DV: dependant variable; IV: independent variable.

Table 3 presents a summary of the research outcomes. As seen in H1, the study revealed that amotivation negatively correlates with mathematics achievement. This result suggests that higher levels of amotivation may lead to decreased mathematics performance in an individual. This finding is in line with Lim and Chapman (2015a) research, whereby amotivation is negatively associated with mathematics achievement among Year 11 and 12 mathematics students in Singapore. Amotivation was characterised by four measurement indicators: 'It is a waste of time studying mathematics' (AMOT1), 'I cannot see why I study mathematics' (AMOT2), 'I cannot understand what I am doing in mathematics' (AMOT3) and 'I do not see how mathematics is of value' (AMOT4). Deci and Ryan (2000) stated that amotivation is correlated with many highly negative outcomes. Further, according to self-determination theory, amotivation results in negative consequences (Tiwari et al., 2014). Thus, the present study's finding is in line with that of prior research, which refer to amotivation as a state in which people lack the aim to perform a particular action and either act without intent or do not act at all (Ryan & Deci, 2000b).

As anticipated in H3, the findings of the present study found that introjected regulation positively correlates with mathematics achievement. Although Lim and Chapman (2015a) did not find any statistically significant correlation between introjection and mathematics achievement, the current study revealed a positive correlation between introjected regulation and mathematics achievement. Introjected regulation was characterised by four measurement items: 'When I do well in mathematics, I feel important' (EMIN1), 'I want to show others that I can do mathematics' (EMIN2), 'I want to show myself that I am intelligent person' (EMIN3) and 'I want to show myself that I can do well in mathematics' (EMIN4). Introjected regulation is one kind of extrinsic motivation (Deci & Ryan, 2000). Students with an extrinsic orientation see tasks as a means to an end and complete tasks for a reason, for instance, for grades, performance or rewards that are evaluated by others (Pintrich, 1991; Herges et al., 2017). Studies prove that extrinsic motivation affects adolescent academic performance (Herges et al., 2017). Flammer and Schmid (2003) discovered that extrinsic motivation has a positive influence on students' academic achievement. It is critical to note that different age groups of adolescent students are motivated differently (Herges et al., 2017). Further, Woolley et al. (2010) assumed that early adolescents may identify extrinsic motivation as essential as students try to please their parents and teachers.

As anticipated in H4, identified regulation positively affects mathematics achievement. This substantiates previous findings in the literature, whereby identification positively correlates with mathematics achievement. Identified regulation was characterised by four items: 'I think that mathematics will help me better prepare for my future career' (EMID1), 'Studying mathematics will be useful for me in the future' (EMID2), 'I believe that mathematics will improve my work competence' (EMID2) and 'What I learn in mathematics now will be useful for the course of my choice in university' (EMID4). This concurs with Lim and Chapman (2015a) research, wherein identification significantly and positively affects mathematics achievement.

As seen in H5, intrinsic motivation positively affects mathematics achievement. The result suggests that the greater the inner desire to study the subject, the better the mathematics performance. Our results share a number of similarities with those of Lim and Chapman (2015a), for whom intrinsic motivation has a positive correlation with mathematics achievement in Grades 11 and 12 in Singapore. In this present study, intrinsic motivation was measured by five items (Table 1): 'I want to feel the personal satisfaction of understanding mathematics' (IMTA4), 'For the pleasure I experience when I discover new things in mathematics that I have never learnt before' (IMTK2), 'For the pleasure that I experience in broadening my knowledge about mathematics (IMTK3), 'For the pleasure that I experience when I learn how things in life work, because of mathematics' (IMTS2) and 'For the pleasure that I experience when I feel completely absorbed by what mathematicians have come up with' (IMTS3). Ryan and Deci (2000a) stated that intrinsic motivation may create high-quality learning and creativity; thus, supporting school environments in which intrinsic motivation is promoted is vital for ensuring this outcome. Ryan and Deci (2000a) cautioned that people may overestimate the degree of extrinsic motivation and undermine intrinsic motivation, such as through external regulation that results in a low level of autonomy. However, other extrinsic motivations, such as integrated regulation, introjected regulation and identification, are considered to have the same positive effects as intrinsic motivation (Lim & Chapman, 2015a). The current study results reinforce this argument, in which introjected regulation and identification positively affect mathematics achievement.

Interestingly, as seen in Table 3, the regression coefficients between i) amotivation (AMOT), external regulation (EXTE), introjection (INTRO), identification (IDEN) and intrinsic motivation (INTRI) and ii) mathematics achievement increase by –0.462***, 0.088n, 0.236***, 0.389*** and 0.402***, respectively. These results can be explained by self-determination theory (Ryan & Deci, 2000b), whereby people with higher levels of self-determination tend to perform better. Individuals are more motivated when they feel autonomous (Svinicki, 2010). However, the positive relationship between introjected regulation and mathematics achievement differs from that found in previous studies, in which no significant relationship was observed between introjected regulation and mathematics achievement (Lim & Chapman, 2015a). The difference can be explained by the role of mathematics in the overall curriculum in Vietnam. Mathematics is one of three compulsory subjects in the National High School Graduation Examination. Further, in Vietnam, people who are good at mathematics are often viewed as smart. Therefore, students are encouraged to prove ability in studying mathematics. There is some internal driver; however, 'introjected behaviours still have an external perceived locus of causality and are not really experienced as part of the self' (Ryan & Deci, 2000b, p. 72).

External regulation, introjected regulation, identified regulation and integrated regulation are types of extrinsic motivation. Not all extrinsic motivations positively affect mathematics achievement and the regression coefficients between the latent variables and mathematics achievement were smaller than those between intrinsic motivation and mathematics achievement. Therefore, the role of mathematics teachers is critical and teachers' instructional practices play an important role in encouraging students in learning mathematics. Moreover, Skaalvik et al. (2017) discovered that intrinsic values are positively correlated with the mastery goal structure. They noted that a combination of mastery goal and performance goal structures is often applied in teachers' instructional practices. A mastery goal structure aims at emphasising students' effort, understanding and improvement and considers mistakes to be part of the development process. Thus, students' improvement is used to define their success (Sproule et al., 2007; Skaalvik et al., 2017).

There are several methods that can be applied to foster mastery orientation. These include (i) giving students choices; (ii) modelling a mastery approach; (iii) emphasising learning from mistakes; (iv) providing positive, diagnostic feedback that focuses on personal development; (v) minimising comparisons with other students and emphasise comparisons with previous performance; and (vi) fostering a community within the classroom (Svinicki, 2010). Giving students a choice allows them some control over their own fate (Deci & Ryan, 1985). Regarding the modelling of a mastery approach, Svinicki (2010) suggested that a teacher needs to inform students that if they make a mistake, they can view it as a learning experience, rather than try to avoid or hide it. Teachers should show their students strategies that involve successfully coping with failure so that they can learn ways to handle their own failures (Bandura, 1986). It is also noted that when people know how to cope with failure successfully, they are more confident and less likely to fear or avoid it. Regarding emphasising learning from mistakes, Svinicki (2010) suggested that teachers should allow students to have a chance to correct and learn from their mistakes. For example, if students lose marks in an exam, they should have a chance to redo it; however, they should also explain why they were wrong, and why their current answers are right. If they are able to do this, they should be awarded half a mark for each of the marks that they lost. Regarding giving positive, diagnostic feedback that focuses on personal development, Svinicki (2010) recommended that teachers should not only point out that something is wrong, but also reveal how to correct it. When providing positive feedback, teachers should compare students' current levels to their previous performances and emphasise areas that show improvement. Regarding minimising comparisons with other students and emphasising comparisons with previous performance, one strategy is to make individual performance outcomes more private (Elliot & Murayama, 2008). Regarding fostering community within the classroom, organising a classroom a safe place is very important because a good learning environment encourages students to perceive others as resources and supports rather than as competitors (Svinicki, 2010). For instance, teachers may encourage students to work in groups and then to consult with other groups while working through problems. Importantly, teachers should encourage all students in the class to treat individuals with respect (Svinicki, 2010).

Further, Tran and Nguyen (2020) stated that using technology that is effective in improving mathematics engagement plays a critical role in teaching and learning mathematics. They argued that such application of technology may encourage more collaboration between educators and learners. Thus, the engagement facilitates problem-solving and flexible thinking. They suggested that applications that allow interactive options, such as number lines, number frames and geoboards, should be developed based on advanced technologies and the Internet of Things. Moreover, the use of virtual whiteboards and websites may enable students to share and link ideas as well as visualise concepts. Virtual reality should be applied in teaching and learning mathematics to enable students to observe three-dimensional geometry, which would thus assist and encourage them to learn mathematics.

CONCLUSION

Several key findings were revealed by the current study. Amotivation was proven to negatively correlate with mathematics achievement. However, introjected regulation, identified regulation and intrinsic motivation were shown to positively correlate with mathematics achievement. To the best of the authors' knowledge, this is the first study to have examined the correlation between mathematics motivation and mathematics achievement among graduate high school students in Vietnam. The analysis results proved that both internal and extrinsic motivation (introjected regulation and identified regulation) positively affect mathematics achievement. The findings provide a strong theoretical foundation to improve mathematics achievement by encouraging teachers to develop motivational conditions in mathematics classrooms in Vietnam. The study discussed certain strategies for improving mathematics achievement, such as ways to foster mastery orientation and apply effective technology.

Nevertheless, the study has some limitations. However, it could be a starting point for investigating mathematics affective domains in developing countries and, in particular, in Vietnam. The key limitation results from the fact that most of the participants had studied at a university, even though the study sought to target a wider range of respondents by considering those who had undertaken the National High School Graduation Examination in 2019. Thus, the results may be valid only for this particular group—those who have studied at a university—and generalisation to a wider group should be undertaken with care. In addition, the current study was not specifically designed to collect data on students' learning environments. These limitations may leave room for further studies.

In particular, studies need to be conducted to investigate the relationship between other affective domains (e.g. engagement, anxiety, identity, attitudes and beliefs) and mathematics achievement in Vietnam. With the advancement of technology, future studies should also explore the efficiency of incorporating technology to motivate and engage students in mathematics education. Further, the moderating effect of students' learning environment on the correlations between affective domains and mathematics achievement should be explored.

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EXPLORING FIRST YEAR UNIVERSITY STUDENTS' STATISTICAL LITERACY: A CASE ON DESCRIBING AND VISUALIZING DATA

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Abstract

Statistical literacy, which is the ability to use statistics in daily life, is an essential skill for facing society 5.0. This study aims to explore first-year university students' ability to properly use simple descriptive statistics and data visualization. Qualitative data were collected using a set of questions from 39 undergraduate students. Many students were able to calculate various descriptive statistics, but some of them were still unable to determine suitable statistics to describe the data clearly. Related to data visualization, many students failed to provide a meaningful chart that effectively shows the difference between two groups of data. Students with higher statistical literacy tend to use comparison or variability reasoning to determine the usage of descriptive statistics, and use data-based reason in visualizing the data. Improvement in statistical teaching – both in the university and the secondary school – is needed so that the students can use descriptive statistics and data visualization correctly.

Keywords: Statistics Education, Data Visualization, Descriptive Statistics, Qualitative

Abstrak

Literasi Statistika, kemampuan untuk menggunakan statistika dalam kehidupan sehari-hari, merupakan salah satu kemampuan penting untuk menghadapi masyarakat 5.0. Penelitian ini bertujuan mengeksplorasi kemampuan mahasiswa tahun pertama dalam mempergunakan statistik deskriptif sederhana dan visualisasi data yang telah mereka pelajari pada jenjang sekolah menengah. Data kualitatif dikumpulkan dari 39 mahasiswa menggunakan beberapa soal, dan diperoleh hasil sebagai berikut. Mahasiswa mampu menghitung bermacam statistik deskriptif, namun sebagian diantaranya tidak mampu menentukan statistik yang tepat untuk menggambarkan data secara jelas. Berkaitan dengan visualisasi data, sejumlah mahasiswa tidak mampu menggambar diagram yang bermakna untuk menunjukkan perbedaan antara dua kelompok data. Siswa dengan literasi statistika relatif tinggi cenderung menggunakan alasan terkait perbandingan dan keragaman untuk menentukan penggunaan statistik deskriptif, serta alasan berdasar-data untuk dalam menyajikan visualisasi data. Diperlukan peningkatan kualitas pengajaran statistika, baik pada jenjang pendidikan tinggi maupun sekolah menengah, agar mahasiswa dapat mempergunakan statistik deskriptif maupun visualisasi data dengan tepat.

Kata kunci: Pendidikan Statistika, Visualisasi Data, Statistik Deskriptif, Kualitatif

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The 4.0 industrial revolution has occurred around the world, as indicated by the rapid growth of big data, the Internet of Things (IoT), and artificial intelligence. Gandomi and Haider (2015) stated that big data is characterized by several properties namely the data volume, velocity, variety, veracity, and value. These types of data can be obtained from various sources such as the usage of applications, social media, and the internet of things (IoT) (Yaqoob et al., 2019). As the volume of data grows at a rate of 50% per annum (Waal-Montgomery, 2016), understanding data is an essential skill for any data-driven society. Analysis of data using statistics is needed to filter massively available information and separate them from opinions (Widjajanti et al., 2017). These situations increase the need for statistical literacy competence (Frost, 2013).

What is statistical literacy? The definition of statistical literacy varies across literatures (Sharma, 2017). In the beginning, statistical literacy is defined as the ability to "... comprehend text and the meaning and implications of the statistical information in it, in the context of the topic to which (it) pertains" (Rumsey, 2002). It is also defined as "trans-numerative thinking" where people will be capable of making sense of and use a different representation of data to make sense of the situation among them (Chick et al., 2005). In a broader view, statistical literacy consists of statistical understanding required in modern democracies as well as peoples' dual role of statistical producer and consumer (Gould, 2017). Statistical literacy is also related to statistical reasoning, which can be defined as the way people reason with statistical ideas and make sense of statistical information (Ben-Zvi & Garfield, 2004).

To describe someone's statistical literacy, some frameworks have been proposed, for example by Wild and Pfannkuch (1999), Gal (2004), Watson and Callingham (2003), and Sharma (2017). These frameworks can be used to classify students' statistical literacy into several level or categories. Based on these frameworks, a high level of statistical literacy can be represented by the ability to critically question engagement with context, use proportional reasoning, appreciate the need for uncertainty, as well as understand the purpose of data, data analysis, and data representation.

In general, testing students' statistical literacy can be done at any level. However, measuring their statistical literacy on their first-year university study has several benefits. First, statistical literacy should become the goal for the Introductory Statistics course (Rumsey, 2002). Therefore, students' statistical literacy can be used as a diagnostic tool before learning the introductory statistics course at the university. Proper ability to use statistics, as well as the result of the university statistical course (if any) should enable students to work with data and do meaningful research for their thesis. Secondly, statistics is widely known as an awkward course in the university and sometimes causes statistical anxiety (Hedges, 2017). One of the factors affecting statistical anxiety is the students' previous knowledge of statistics (Sutarso, 1992). Understanding students' previous knowledge might help the instructor to adjust the statistics course and prevent statistical anxiety. Third, this result may represent how students have learned statistics in secondary school. It is widely known that statistical literacy competence is a part of the school curriculum in many countries (Watson, 2003; Garfield & DelMas, 2010), such as the USA (Weiland, 2017), Brazil (Campos et al., 2011), and Indonesia (Setiawan, 2019). Therefore, any information about first-year university students' statistical literacy might be insightful for the improvement of statistical teaching at the secondary school level. Lastly, the result can help the statistics educator to ensure that students' statistical literacy competence is sufficient for their life in their workplace and in their society.

As listed in Ziegler and Garfield (2017), various instruments had been proposed to evaluate students' statistical literacy by measuring many aspects on statistical literacy. Most of them was an objective test with focus on a broad competence of statistical literacy rather than a specific one. Using some of these instruments, many studies have been done to evaluate the statistical literacy of university students. Yotongyos et al. (2015) presented that students from a university in Thailand have a moderate level of overall statistical literacy. Kim et al. (2019) did a test and observed the lesson plan to measure pre-service mathematics teachers' statistical literacy in Korea. In Pakistan, a survey revealed that BS students have a low level of statistical literacy (Hassan et al., 2020). However, there are several limitations on such studies that should be anticipated. For example, studies by Khaerunnisa and Pamungkas (2017), Takaria and Talakua (2018), as well as Jatisunda et al. (2020) give emphasis on students' ability to calculate statistics or follow a prescribed statistical procedure. Although proposing to describe statistical literacy, these studies did not measure students' critical reasoning which is the indicator of statistical literacy. These studies also measure wide aspects of statistical procedures, from data tabulation up to the statistical hypothesis testing, and analyze them as a whole as the statistical literacy competence. Hence, it is difficult to identify the ability that is not yet mastered by the students.

Following Gould (2017), knowing how to analyze data and create a basic representation of data is a part of the minimum level of statistical literacy. These abilities are essential to understand what the data have to say. Therefore, students must be able to do these procedures correctly before learning more advanced topics such as hypothesis testing, regression analysis, etc. For educational purposes, higher-order thinking skills (HOTS) should be encouraged so that the student can use the proper statistics and data visualization. Moreover, they must be able to give critical justifications on the usage of statistics and data visualization.

This study aims to present undergraduate students' statistical literacy in terms of their ability to use descriptive statistics and visualize data appropriately. We identify whether they can select and calculate correct descriptive statistics which can represent the data. Similarly, we examine how the students visualize the data using their way, without any guidance on which types of diagrams should be used. Simply speaking, this study focuses on how students can produce meaningful statistics and data visualization based on the data.

METHOD

Approach and Subject

To extensively describe students' abilities in descriptive statistics and data visualization, we used a qualitative approach. Following Creswell (2014), this type of study examines a natural situation and is suitable to describe the actual result from the subjects. We provided no treatment or manipulation to the respondents before and during the study.

The subjects of this study were 39 students in the first semester of the undergraduate program in statistics at a public university in Indonesia. Therefore, we expected that they would have some interest in statistics and want to learn more about it.

The profile of the respondents is as follows. Most of them were 17-18 years old, and 9 (23.1 %) were male. One student had graduated from the pharmacy stream of a vocational school (SMK), whereas the rest had graduated from the general or Islamic high school (SMA/MA) in Indonesia.

Data Collection

The data were collected using a test for students. In preparing the test, we assumed that students had already known several descriptive statistics and data visualization since these concepts were studied in primary and secondary school. Although some students had learned statistical inference in the twelfth grade of high school (Setiawan, 2020), we did not explore this topic further since it was taught only in the mathematics and natural science stream.

Formulation of the questions was inspired by a list of questions for assessing statistical education presented by Garfield and Ben-Zvi (2007) as well as Sharma (2017), with a focus on using descriptive statistics and visualizing data. The questions given to the subjects are presented in Figure 1.

You are familiar with descriptive statistics (eg. mean, mode, standard deviation, etc.). Suppose that we have the data of age (in years) of people with disease X from two different villages.

Village A: 10, 12, 14, 35, 56, 58, 60

Village B: 32, 34, 35, 36, 38

a) A researcher reported that the mean and median age of peoples with disease X in village A are the same as people in village B. Do you agree with that statement? Explain.

b) Suppose you are asked to give other descriptive statistics to explain the report given by the researcher mentioned in previous question, what statistics will you give? Calculate it and give your reason.

c) Someone asks you to present the above data in form of a chart/diagram. What type of chart you will use? Draw the chart and give your reason.

Figure 1. Translation of questions used in this study

As shown in Figure 1, these questions have a context, namely the ages of the patients, which means that the data must be positive. The dataset consists of one variable with two categories or groups. Students with a higher level of statistical literacy would be aware of the presence of two groups and be able to show the correct comparison between them.

The first question ensures that the respondents can calculate the mean and median from raw-ungrouped data. Since this is a closed question, students' answers can be classified only into two groups, namely the correct and incorrect answers. The correct answer for the median and mean of these two groups is 35, which implies that they are equal.

The second question moves to the proper usage of descriptive statistics, while the last question checks the ability to visualize the data in a suitable form. In answering these questions, students were allowed to use a calculator but not open any textbooks or references. Different from the test arranged in Garfield and Ben-Zvi (2007) or Jatisunda et al. (2020), we did not give any specific descriptive statistics nor chart type. Therefore, these questions can identify how the respondents understand the proper usage of descriptive statistics and chart types. Validity of the above questions was examined by consulting an expert in mathematics education.

Data Analysis

Following Mayring (2000), this study used two approaches to analyze the data. The deductive and inductive approaches were used for classifying students' answers and their reasons, respectively.

Regarding the level of students' statistical literacy, we found that the framework by Jones et al. (2000) or Watson and Callingham (2003) was somewhat abstract and difficult to follow. On the other hand, Sharma (2017) introduced four stages of students' statistical literacy, with informal/idiosyncratic as the lowest level, followed by consistent non-critical, early critical, and advanced critical. A description for each level was available and applicable to our problems. However, since it was rather difficult to separate the answers into four categories, we proposed a similar framework with three different levels of statistical literacy, as shown in Table 1.

Table 1. Framework for classifying students answer in question (b) and (c)

Level	Answer of question (b)	Answer of question (c)
Low	Student provide non-sense statistics or repeat calculation of statistics that already used in (a).	Student provide incorrect chart or use wrong data to create the chart.
Middle	Student calculate descriptive statistics other than mean/median correctly, but failed to show the difference between the two groups *or*	Student create a chart but unable to show the difference between the two groups *or*
	Any descriptive statistics other than mean/ median were calculated incorrectly.	Student failed to use a proper scale on the chart.
High	Student calculate descriptive statistics other than mean/median correctly and correctly presents the difference between the two groups.	Student create a chart using proper scale and clearly show the difference between the two groups.

In reference to Sharma (2017), the low level on Table 1 corresponds to the informal level, in which students provide random or inappropriate explanations. The middle level on Table 1 represents the consistent-non-critical level since students in this level are able to use simple statistics and graphs. Since the high-level indicates that students are able to present the difference between the two groups, it can be classified into the early or advanced critical on Sharma (2017) model. The samples of students' answers from each level are presented in the next section.

The application of the framework on Table 1 is as follows. First, we checked whether each student's calculation of descriptive statistics in questions (a) and (b) was correct. These statistics used in (b) and charts used in (c) were then classified using the above framework. The classifying and coding process were done by two researchers independently, with 94.9% inter-coder agreement.

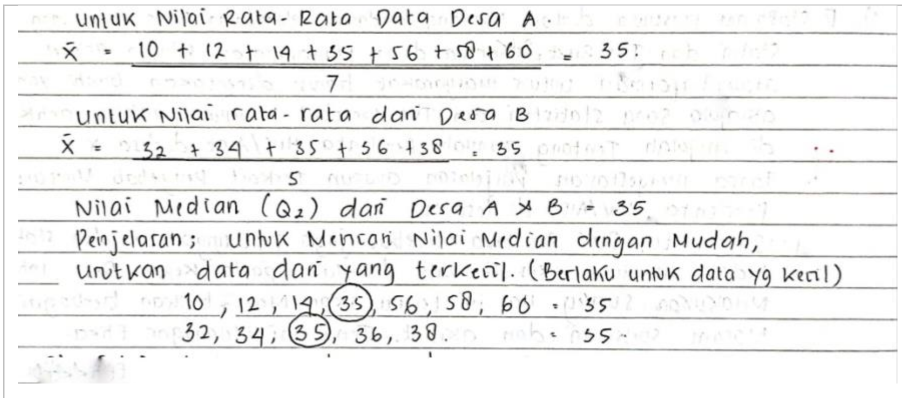
For the classification of students' reasoning, this study wants to present the original reason given by the students. Therefore, we did the data analysis procedure using inductive category development as follows. First, we list the reasons given by the students and group them to produce the initial coding. Later, a revision was done on the initial coding to produce the final coding criteria. Obtained result from the final coding then presented as the result of this study.

RESULTS AND DISCUSSION

Students' Ability on Using Descriptive Statistics

In general, any descriptive statistics can be calculated based on quantitative or numerical data manually or using an electronic calculator. Following the Indonesian mathematics curriculum, students learn several descriptive statistics from primary school up to high school. In primary school, they study how to calculate the mean, mode, and median, whereas, in secondary school, they explore the quartile(s) and range. In high school, students learned the absolute deviation, variance, and standard deviation. They also calculate each descriptive statistic studied before for the grouped data. Consequently, when starting their study at the undergraduate level, students are familiar and should be able to calculate and use various descriptive statistics.

From the answers to the first question, we find that almost all respondents can calculate the mean and the median. They can show that both the mean and median of the patients' age from village A are equal to the mean and median of patients' age from village B. As seen in Figure 2, the calculation of these statistics is quite simple.



For the average of the village A data ....

For the average of the village B data ...

The median from village A and B ...
Explanations: to find the median, sort the data from the smallest

Figure 2. Sample correct answer for the first question

If the students recognize the difference between the data from these two villages, they should note that the second question asks them to give statistics that represent these differences. Based on this idea, any statistics that the calculation based on the data from each village yields a different value is a correct answer.

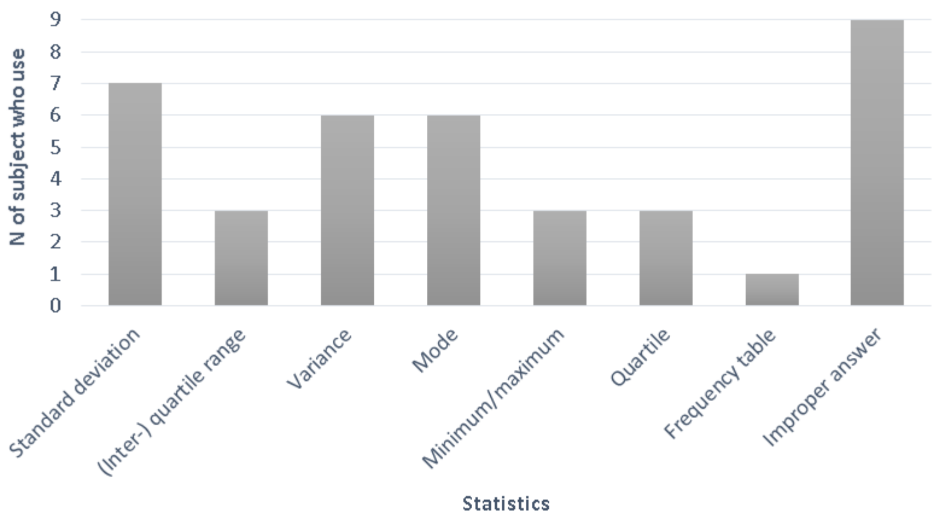


Figure 3. Descriptive statistics given by the respondent to answer the second question

Figure 3 shows that there are seven descriptive statistics given by the respondents to answer the second question, which was dominated by dispersion measures (i.e. standard deviation and interquartile range). Since all the data are different, the value of any dispersion measure between these two villages is different. Similarly, the first quartile, third quartile, minimum, and maximum of data from these two villages are not equal. Based on Table 1, more than half of the students belong to the high literacy group since they show the difference between these two groups of data using correct statistics.

> *"Berdasarkan perhitungan dapat kita ketahui simpangan baku usia penderita penyakit X di desa A dan B tidak sama.* (From our calculation, we know that the standard deviation of age of patients with disease X in the village A and B are not equal)" (Student #9)

> "*Karena menunjukkan tingkatan keberagaman dari data tersebut.* (Because it presents the degree of variability of the given data)" (Student #30)

> "*Dengan adanya Q1, Q3 dan IQR maka akan dapat mendukung informasi dalam sajian data.* (By the presence of Q1, Q3, and IQR will support information in the data presentation)" (Student #25)

> "*Berdasarkan data yang diberikan, kita dapat menghitung simpangan baku dari kedua desa tersebut.* (Based on the given data, we can calculate the standard devation from these two villages)" (Student #34)

It can be seen that the reason given by students from the high group were related to various aspects, namely the difference between the two groups (e.g. Student #9), the variability of the data without mentioning the difference (e.g. Student #30), the goal of giving information based on the data (e.g. Student #25), and the possibility of calculation (e.g. Student #34).

Seven students in the medium level of statistical literacy can use other descriptive statistics but fail to show the difference between these groups. This group is dominated by students that (incorrectly) calculate the mode of the data, while the data for each village have no mode. The samples of reasons given by students in this group were as follows.

> "*Rata-rata, median, dan modusnya sama.* (The mean, median, and mode are equal)" (Student #18)

> "*Karena nilai yang sering muncul hanyalah 35. 35 muncul paling banyak 2 kali, yang lain hanyalah sekali.* (Because the most frequent data is 35. 35 was appeared two times, while the others are only once)" (Student #21)

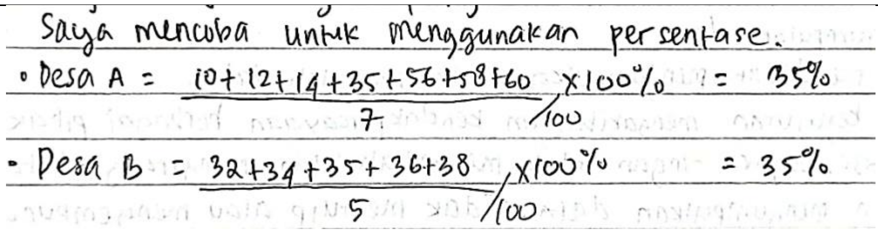
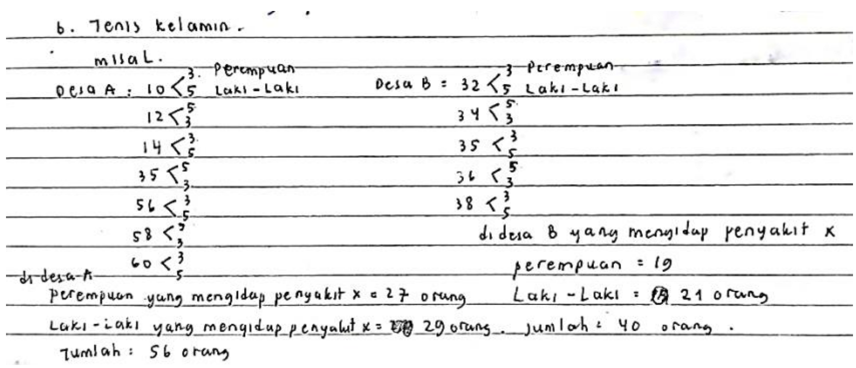
> "*Karena dengan banyaknya modus dapat mengetahui nilai yang sering keluar dan bisa memperjelas informasi.* (Because by using mode we are able to know the most occurred value and can get clearer information)" (Student #3).

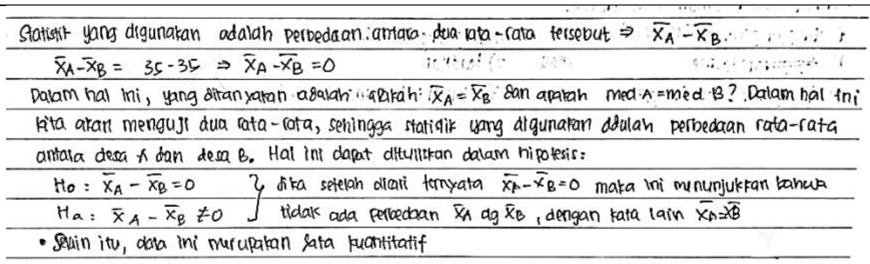
Both Student #18 and Student #21 incorrectly calculated the mode, which caused them stated that the mean, median, and mode were the same. On the other hand, Student #3 did not calculate the mode so that he/she might not realize that there was no mode in the data. Another type of student in this group provided a wrong calculation of variance (i.e. did not take the square of difference) resulting in zero variance and did not provide other statistics. As a consequence, this student claimed that the variances between these two villages were the same.

In the third group, namely the lowest statistical literacy, we found several students that repeated the calculation of mean or median or transforming the data into a table. These students might not be aware of several descriptive statistics mentioned in the questions, as presented by Student #28. Further classification of the improper answer is presented in Table 2.

> *"Memberikan rata-rata merupakan informasi yang tepat dan pasti karena sudah diketahui berapa rata-ratanya.* (Present the mean is a correct and certain information because its value has been known)" (Student #28, present the mean which already calculated).

Table 2. Incorrect answer of question (b) represent low statistical literacy

Types of improper answer	Sample answer
Non-sense statistics: calculating the mean and present them as a percentage.	 Saya mencoba untuk menggunakan persentase. • Desa A = (10+12+14+35+56+58+60)/7 x100% / 100 = 35% • Desa B = (32+34+35+36+38)/5 x100% / 100 = 35%
Assuming presence of another information (*jenis kelamin* = sex) that did not given nor asked in the question.	 b. Jenis kelamin. misal. Desa A : 10 < 3 Perempuan, 5 Laki-Laki; 12 < 5, 3; 14 < 3, 5; 35 < 5, 3; 56 < 3, 5; 58 < 3, 5; 60 < 3, 5 Desa B = 32 < 3 Perempuan, 5 Laki-Laki; 34 < 5, 3; 35 < 3, 5; 36 < 5, 3; 38 < 3, 5 di desa B yang mengidap penyakit x: perempuan = 19, Laki-Laki = 21 orang, jumlah = 40 orang. Perempuan yang mengidap penyakit x = 27 orang Laki-laki yang mengidap penyakit x = 29 orang Jumlah : 56 orang

Doing hypothesis testing.	 Statistik yang digunakan adalah perbedaan antara dua rata-rata tersebut ⇒ $\bar{x}_A - \bar{x}_B$ $\bar{x}_A - \bar{x}_B = 35 - 35 \Rightarrow \bar{x}_A - \bar{x}_B = 0$ Dalam hal ini, yang ditanyakan adalah apakah $\bar{x}_A = \bar{x}_B$ dan apakah med A = med B? Dalam hal ini kita akan menguji dua rata-rata, sehingga statistik yang digunakan adalah perbedaan rata-rata antara desa A dan desa B. Hal ini dapat dituliskan dalam hipotesis: Ho : $\bar{x}_A - \bar{x}_B = 0$ } Jika setelah dicari ternyata $\bar{x}_A - \bar{x}_B = 0$ maka ini menunjukkan bahwa Ha : $\bar{x}_A - \bar{x}_B \neq 0$ } tidak ada perbedaan $\bar{x}_A$ dg $\bar{x}_B$, dengan kata lain $\bar{x}_A = \bar{x}_B$ • Selain itu, data ini merupakan data kuantitatif

From Table 2, a notable result is that several students were unable to get the information from the dataset. Surprisingly, they asked for more information or add some information that was not presented nor asked by the question, with some reason as follows.

> "*Statistik yang perlu ditambahkan seperti: jumlah penduduk desa A dan desa B; jumlah penduduk desa A dan desa B yang sehat* (Added statistics should be: the number of population in village A and village B; number of healthy people in village A and village B)" (Student #30)

> "*Dengan adanya jumlah penderita di tiap umur, maka kita dapat mencari nilai dan membandingkannya* (If the number of patients in each level of age was available, we can find the value and compare them)" (Student #33)

The reason given by Student #33 shows that she did not understand that the presented data was the overall raw data instead of arranged data in the frequency distribution. Student #10, who wrote a hypothesis testing procedure to answer this question, might think that hypothesis testing was part of descriptive statistics instead of inferential statistics.

Based on these results, we can infer that most students know and are able to calculate various descriptive statistics. Meanwhile, some of them may have low competence in determining suitable statistics to describe and compare the raw data, or may be unaware that not all descriptive statistics (i.e. mode) can be used in any dataset.

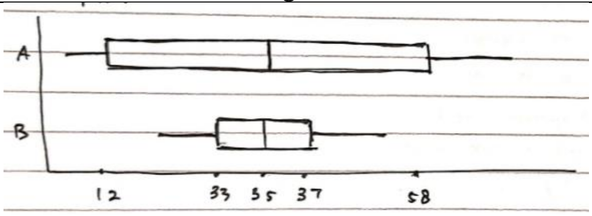
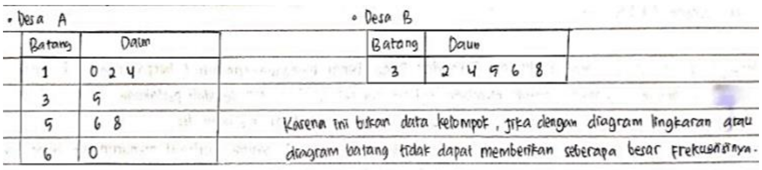
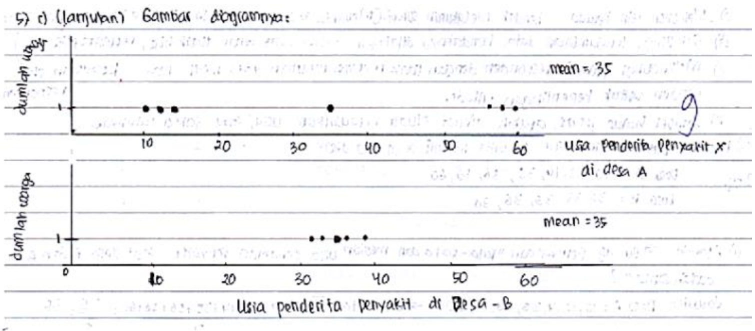
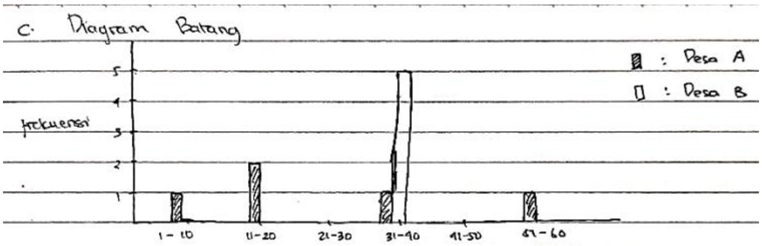
Students Ability on Using Data Visualization

As presented in Figure 1, the data in the question is about the ages of people with a specific disease, which is on an interval or ratio scale. Theoretically, the suitable data visualization might be bar charts, dot plot, histograms, box (-and whisker-) plot, or stem-and-leaf plot.

Of 39 participants of this study, only 37 participants answered this question. Most of the students (40%) use a bar chart to display the data, which seems reasonable since this type of data visualization is taught from the primary school level up to the higher secondary school level. The box plot and the stem-and-leaf plot are used by one student each, whereas the dot plot is created by four students. Most of the students create two separate charts, or one chart for each village, instead of combining this information into one chart. As a consequence, the same chart type (e.g. bar chart) may look very different: one presents the difference between the two villages clearly, whereas the other contains various mistakes and difficult to understand.

Following the classification on Table 1, students' answers that exhibit a high level of statistical literacy are displayed in Table 3. These charts clearly show the difference between the patients' ages from these two villages and use a correct scale in the axes. Some of the students in this group present all data in one chart, whereas the others use two separate charts. When the latter is used, statistical-literate students must create the same scale for each chart so that the reader can compare the data between these two groups easily.

Table 3. Sample proper chart types with clear/correct graphing

Figure	Explanations
	A boxplot clearly show the same median and the different variability of the data between these two groups.
	A stem-and-leaf plot was good for presenting the difference of variability between the two villages.
	A dot plot comparing the patients age, clearly show the variability of age between these two villages. The central tendency can be simply guessed from the display.
	A bar chart made by grouping the patients' age into five classes. The difference of patients' age among the two groups can be clearly seen.

In the group of students with a medium level of statistical literacy, we find two kinds of answers as follows. Despite presenting the data in a suitable chart, several students did not give much attention to the scale on the axis. As presented in Figure 4, they only put the value of the data below the axis without seeing the difference between them. We can read this chart, but the comparison between those two groups would be difficult to observe. Compared to the charts in Table 2, these charts did not represent the difference of variability between these two groups.

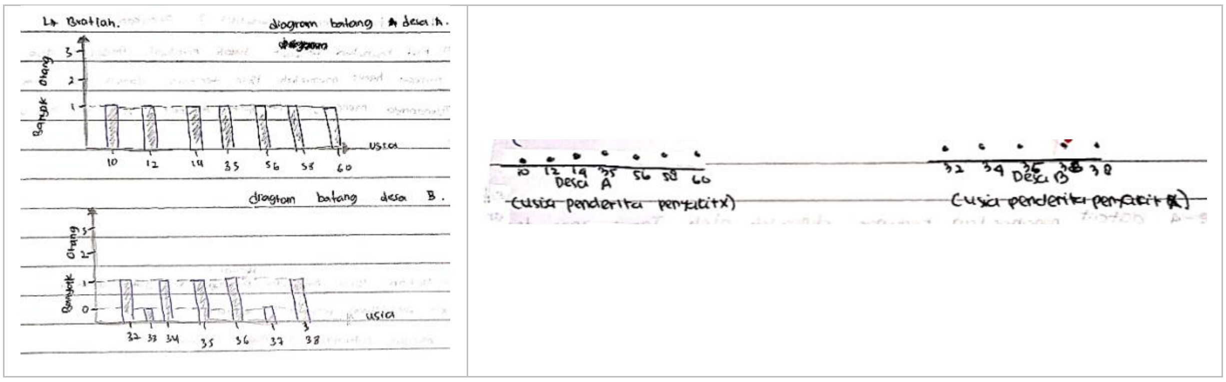


Figure 4. Correct data visualization in wrong scale. Note the equal distance between each bar or each dot, despite the difference of the level.

Secondly, we found that several students use improper chart types to display the data, as presented in Table 4. Similar to Figure 4, these charts were unable to show the different variability of the data between these two villages. The correct parts of charts in Table 4 were only the variable and groupings of data. Instead of making the reader directly understand the data, these chart types might cause the reader to feel confused. Difficulties in creating and reading would arise when the number of the subject becomes larger.

Table 4. Sample improper chart representing middle level of statistical literacy

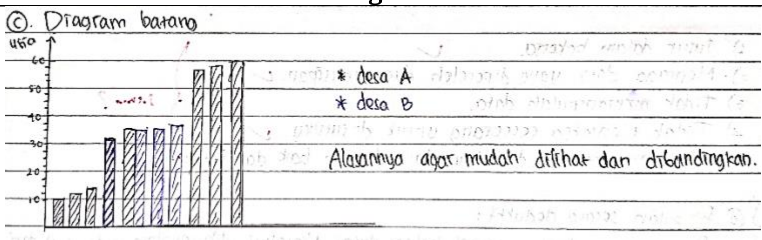
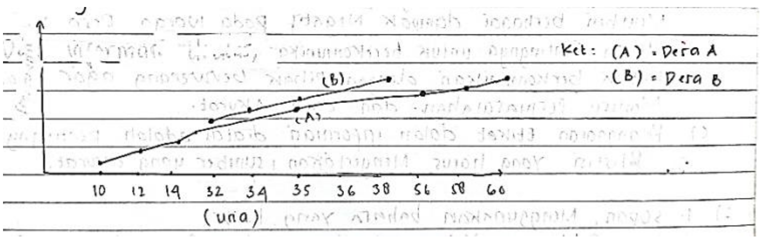
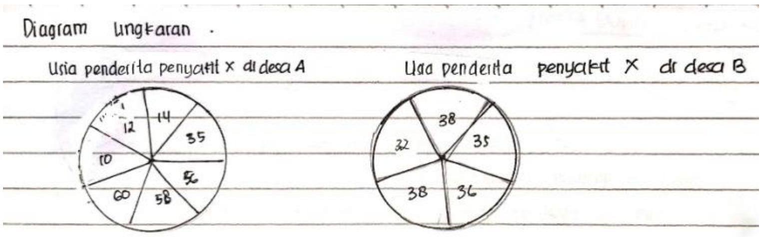
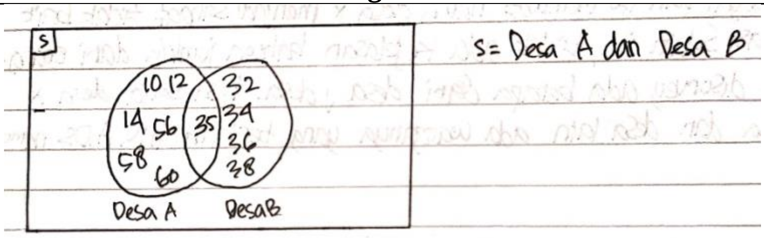
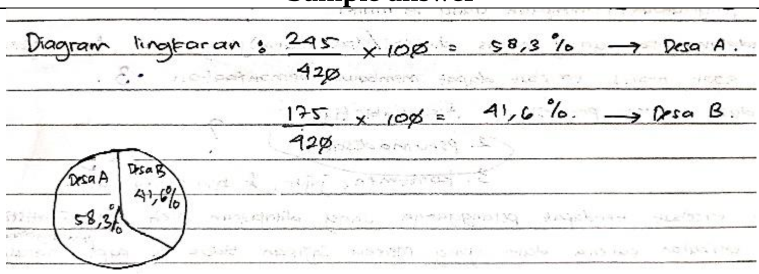
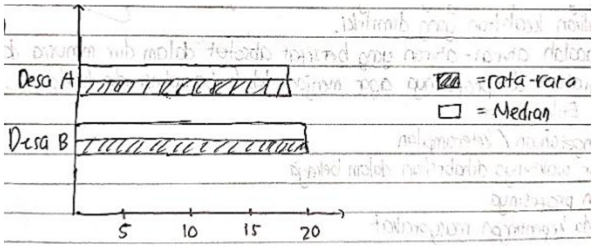
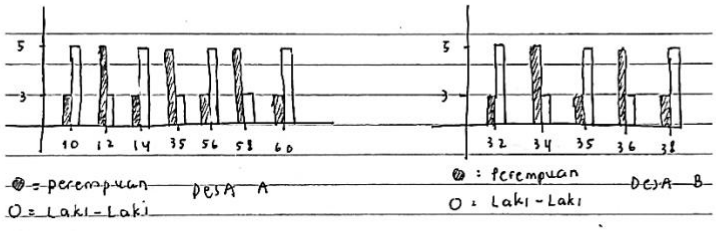
Figure	Explanations
	Only convert the raw data into bars; no explanations on the horizontal axis. Although the difference between the two villages is represented by the colour, this chart is unacceptable.
	Since the data consists of only one variable, there was no reason to made two dimensional plot.
	A pie chart might represent the grouped data, but it is very difficult to compare the patients' age between these two groups.

Figure	Explanations
	Although the content is correct, Venn diagram is used in set theory and not proposed to display numerical data. The variable also not mentioned in the diagram.

Students with a low level of statistical literacy created several charts that did not represent the original data. These charts might use irrelevant data or unimportant variable. Such types of chart, which is displayed in Table 5, represent the lowest competence in data visualization among all students who participated in this study.

Table 5. Sample improper chart representing low statistical literacy on data visualization

Types of mistakes	Sample answer
Failed to identify the proper variable	
Wrongly present the descriptive statistics (which were same) instead of the original data.	
Add more information, namely the patients' sex that were not available in the original data.	

Why do students choose a chart type to represent the dataset? Our study finds that seven respondents give no reason for their data display types. However, various reasons given by the students can be classified into four categories, as shown in Table 6.

Among these types, more than half of the participants wrote people-based reason, while the rarest was the purpose-based reason. How are students' reasons related to data visualization? Almost all students that draw a correct diagram (performing a high level of statistical literacy) wrote data-based reason, which were sometimes combined with other types. In contrast, most students with incorrect diagrams were only able to give a people-based reason or no reason at all.

Table 6. Classification of student reason when visualizing the data

Types of reason	Explanation	Sample answer
Data-based reason	Mention data properties, eg. number of variables, groups, types, etc.	"*Karena data yang diberikan merupakan data tunggal dan terdiri dari dua kelompok A dan B* (Because the data are single-valued and consist of two groups A and B)" (Student #16)
Chart-based reason*	Related to the chart types or chart usage.	"*Informasi dari data kuantitatif dapat terangkum dalam diagram ini.* (Information from quantitative data can be presented in this diagram)" (Student #10)
Purpose-based reason	Mention the purpose, ie. what are need to show or present from the data.	"*Dapat menampilkan hubungan jumlah penderita dengan umurnya.* (Can represent the relations between the number of patients and their age) (Student #31).
People-based reason	Related to the people who make and/or will read the diagram.	"*Agar mudah terbaca sehingga dapat dengan mudah menentukan simpulan mana yang mudah diambil.* (Will be easier to read, so that the conclusion can be taken easier)" (Student #13)

*In this study, all chart-based reason were combined to the data-based reason.

Descriptive statistics is the procedure used to organize and describe the characteristics or factors of a given sample to understand it (Fisher & Marshal, 2009). Data visualization can help the human eye see things that are difficult to understand in large datasets (Rodríguez et al., 2015). The learning of these topics frequently precedes the discussion on inferential statistics procedures such as hypothesis testing, estimation, and data analysis. Despite its simplicity, improper use of descriptive statistics or data visualization may lead to serious problems in understanding the data. A small book entitled 'How to Lie with Statistics' (Huff, 1954) and a textbook entitled 'Statistics, Concept and Controversies' (Moore & Notz, 2009) present various problems related to the wrong use of descriptive statistics and data visualization. For example, using an improper scale on a bar chart may cause wrong interpretation of the data. Usage of various descriptive statistics such as mean and median may yield very different results especially when the data contains one or more outliers.

In this study, we present the statistical literacy of undergraduate students in terms of their ability to use descriptive statistics and visualize data. The use of ill-structured essay questions yields some benefit over the multiple-choice question test like the ARTIST test (Garfield et al., 2002). First, various types of students' answers can be found and classified to measure their ability to use descriptive statistics and visualize data. Compared to Sharma (2017), this study not only measures students' ability to read and interpret the statistics or chart but also measures their ability to use proper statistics as well as meaningful charts. We can say that students need to apply higher-order thinking skills (HOTS) to create an understandable representation of the data. Secondly, these questions can identify their' ability to work with more than one group of data that represent their level of statistical literacy. Lastly, the reasons given by the students might be used to identify their statistical reasoning ability, which needs to be confirmed using other tests (Sabbag et al., 2018).

Respondents of this study were first-year students from an undergraduate program in statistics, whom we expected to have more awareness (and maybe more interest) on statistics. As expected, in the test, almost all students could easily show that the mean and median between the two villages were equal. We also find that most of them have a medium to a high level of statistical literacy in using proper descriptive statistics, as represented by their ability to provide other statistics that represent the difference between two groups of data. Similarly, there is only a small number of subjects that exhibit a high level of statistical literacy on visualizing data. By examining various types of charts, we find that students are unfamiliar with histograms, boxplots, stem-and-leaf plots, and dot plots, which are more suitable for continuous data such as age. Many subjects failed to identify that a pie chart, line chart, and even a Venn diagram, are not suitable to display this type of data.

The low ability on statistical descriptive and/or data visualization indicate a problem on statistical education. As mentioned by Ismail and Chan (2015), many studies show that there is misconception about the usage of descriptive statistics on students from various level. In addition, our study show that these problems also presented on data visualization. Since the problems did not suggest any type of data visualization, students must use their knowledge to decide which type that can be used. These approaches were not used by previous statistical literacy studies on Indonesian undergraduate student (Khaerunnisa & Pamungkas, 2017; Jatisunda et al., 2020; Tiro et al., 2020). As a consequence, none of these studies' present students' ability on visualizing data properly.

This study also explores various students' reasoning on using descriptive statistics and data visualization. Various reasons can be found on the high-level of statistical literacy regarding the use of descriptive statistics, namely (1) comparison between the groups, (2) variability measurement, (3) additional information, and (4) possibility of calculation. Among the middle-level and low-level groups of statistical literacy, most of the reasons were based on calculation, and none of them were related to the comparison of variability. Based on these results, we can say that students with a higher level of statistical literacy would be more likely to obtain the idea of comparing the groups of data and measuring the variability. This result confirms the frameworks from Jones et al. (2000) that place the ability to make a comparison on the high level of statistical reasoning. Similarly, understanding the variability or variation is somewhat complex and difficult, so not all students and even teachers can understand this concept well (Sánchez et al., 2011). Regarding the data visualization, we find that students' reasons for creating them can be divided into four groups, as presented in Table 6. Data-based reason becomes the most frequent reason used by students with a higher level of statistical literacy, i.e. those who can create correct and meaningful visualization.

This study provides more insight on understanding how students develop their reasoning when summarizing data (using descriptive statistics) and visualizing data with minimal guidance. It can be used to identify how students will act when facing a real situation related to data, namely when there is nobody that asks them to make a specific chart or calculate specific statistics. Therefore, it can be seen as an alternative to the framework on statistical reasoning by Chan and Ismail (2013) as well as Chan et al. (2016) which seems to be more rigid.

Even though this study only accounts for several undergraduate students in statistics, the result might be applied for any undergraduate students especially those that come from the mathematics and natural science stream in secondary school. Further studies with a qualitative approach should be done to profile another concept related to statistical literacy and its reasoning. Suitable concepts for the study may include the idea of sampling, design of experiments, statistical inference, and many more. To ensure generalizability, more respondents from various undergraduate programs can be chosen to participate in a similar study.

Implications on Teaching Statistics

This study raises a question: How can we increase the statistical literacy of undergraduate students, especially in the term of using descriptive statistics and data visualization? In Indonesia, it has been known that some of these concepts were introduced in primary and secondary schools (Setiawan, 2019; Funny et al., 2019). Therefore, an improvement on these two topics should be carried out on the university level as well as on the secondary school level.

At first, statistical learning in schools, which is dominated by computational aspects of statistics instead of conceptual understanding (Tiro, 2018), should be synchronized (Ridgway et al., 2011). Statistical literacy should become an important part of the multi-literacy model used in developing primary and secondary school curricula (Abidin, 2017; Nurgiyantoro et al., 2020). Data – and statistics – should become used in various subjects outside mathematics. As an example, in Geography students can learn how to represent spatial datasets, while in Economy student may visualize and identify the pattern on time series data. This approach may help students to obtain the conceptual understanding of the data beside the mathematical procedures of calculation.

We should realize that sufficient ability in statistical literacy could not be developed by statistical teaching that focused on gathering statistical knowledge, learning facts and formulas, and obeying standard procedures (Schield, 2004). As a consequence, more innovation on the strategies used for teaching statistics is needed. One of them, for example, is the guided discovery learning (Hariyanti & Wutsqa, 2020). The use of various modern technology (Suhermi & Widjajanti, 2020) should help the students increase their statistical literacy and statistical reasoning.

Usage of real datasets in the statistics classroom also encouraged in which open data can be used (Ridgway, 2016; Rivera et al., 2019). This approach will help students to face the emergence of big data, data science, and data analytics.

Moving into the data visualization, it is noted that teaching strategy using various modern tools has been developed, for example, by Nolan and Perret (2016) or Gelman and Nolan (2017). Following

Wolfe (2015), several textbooks on communication courses can help us to find completed guidelines for determining the visualization types. Numerous literatures on student difficulties related to data visualization (Boels et al., 2019; Dewi et al., 2020), misconceptions (Zaidan et al., 2012; Chan & Ismail, 2013; Ismail & Chan, 2015; Yusuf et al., 2017), as well as learning obstacles (Sotos et al., 2007) can be used as references on improving the course of data visualization.

Drawing data visualization by hand might be irrelevant for undergraduate students. When a software is used, the focus on teaching data visualization needs to be put on the usage of proper diagram instead of the steps to produce it. Recent types of data visualization such as *heatmap* and *violin plot* can be introduced. On the other hand, since drawing charts still become a competence for students in primary and/or secondary school (Setiawan, 2019; 2021), this suggestion is relevant for (mathematics) teacher in these levels. Similar study can be carried out to identify whether mathematics teacher and/or pre-service teacher are able to teach the correct usage of data visualization.

CONCLUSION

This study analyzed the statistical literacy of first-year students from the undergraduate program on statistics, with a focus on their ability to use descriptive statistics and to visualize the data. Half of the students participated in this study exhibit high level of literacy on using descriptive statistics, but with middle level of literacy on visualizing the data.

Related to the usage of descriptive statistics, almost all students are able to calculate the mean and the mode, which are central measure. We find that students with a higher level of statistical literacy become able to use comparison and variability reasons when choosing descriptive statistics that lead to the usage of a dispersion measure. On the contrary, students with lower level of statistical literacy are unable to realize the different variation between the groups of data.

Students with higher level of statistical literacy can visualize the data in the way so that the difference between these two groups are clear. They also more likely to give data-based reason, which mentions data properties such as the number of groups, types, etc. Sometimes these reasons are combined with chart-based reason or purpose-based reason. In contrast, students with low level of statistical literacy only able to give people-based reason such as ‘easier to read’ or ‘easier to understand’. Further study with a similar approach should be done to profile the other components of statistical literacy and its reasoning.

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TURKISH PRE-SERVICE MATHEMATICS TEACHERS' BELIEFS IN MULTIPLICATION

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Abstract

Mathematics teachers' beliefs play an important role in the mathematics teaching practices. However, the instruments used to measure the mathematics on certain contents are still limited. Thus, this study was conducted to develop a Multiplication Beliefs Questionnaire (MBQ) to identify and examine the profile of Turkish pre-service mathematics teachers' beliefs. The samples of this study consisted of 414 four-year pre-service primary mathematics teachers from 18 different universities in Turkey collected using a convenience sampling technique. The validity of the questionnaire was analyzed using an exploratory factor analysis (EFA). We obtained four components of beliefs in multiplication covering, remote belief in multiplication (C1), multiplication operation belief in mathematics textbooks (C2), dynamic belief in multiplication learning (C3), and self-efficacy belief in multiplication problems (C4). The results showed that the pre-service mathematics teachers' beliefs in components C1, C3, and C4 were positive, while component C2 was neutral. This study had an essential contribution to the mathematics literature since developing a questionnaire on multiplication distributed to the pre-service teachers. The previous studies showed that belief was subjective yet objectively influenced knowledge. Thus, identifying the pre-service teachers' beliefs in teacher education may provide various benefits in reforming mathematics teaching.

Keywords: Beliefs, Beliefs in Multiplication, Pre-Service Mathematics Teachers, Questionnaire Development

Abstrak

Keyakinan guru matematika berperan penting dalam praktik mengajar matematika. Namun instrumen untuk mengukur keyakinan pada konten tertentu masih terbatas. Sehingga penelitian ini dilakukan untuk mengembangkan kuesioner keyakinan perkalian (KKP) untuk mengidentifikasi dan memeriksa profil keyakinan calon guru matematika di Turki tentang topik ini. Penelitian ini menggunakan *convenience sampling* yang terdiri dari 414 calon guru matematika pada tahun keempat dari 18 universitas berbeda di Turki. Validitas kuesioner dianalisis dengan menggunakan *exploratory factor analysis* (EFA). Kami memperoleh empat komponen keyakinan tentang perkalian: keyakinan terasing tentang perkalian (C1), keyakinan tentang operasi perkalian dalam buku teks matematika (C2), keyakinan dinamis tentang belajar perkalian (C3), dan keyakinan *self-efficacy* tentang masalah perkalian (C4). Hasil penelitian menunjukkan bahwa keyakinan calon guru matematika positif untuk komponen C1, C3 dan C4 dan netral untuk komponen C2. Studi ini memiliki kontribusi penting untuk literatur karena mengembangkan kuesioner tentang perkalian yang diberikan kepada calon guru matematika. Berdasarkan penelitian sebelumnya, keyakinan berpengaruh pada pengetahuan, meskipun keyakinan bersifat subjektif dan pengetahuan bersifat objektif. Sehingga mengidentifikasi keyakinan calon guru dalam pendidikan guru dapat bermanfaat dalam reformasi pengajaran matematika.

Kata kunci: Keyakinan, Keyakinan Tentang Perkalian, Calon Guru Matematika, Pengembangan Kuesioner

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Belief has become an interesting topic to study shown by many disciplines which have already concerned on belief, such as social psychology which studies the belief structure and content influencing a person's thinking (Bar-Tal, 1990, Wilson & Cooney, 2002). Education is also considered interesting in which belief highly influences the learning practices (Cormas, 2020; Liljedahl, Rösken, & Rolka, 2021; Sevgi et al., 2021). In addition to the teachers' knowledge, belief can also influence the students' mathematical success (Ball, 1991; Hamukwaya & Haser, 2021). Muhtarom, Juniati, and Siswono (2019) found that pre-service teachers' beliefs on the nature of mathematics supported the beliefs in the teaching-learning process in the mathematics classrooms.

Changing the teachers' beliefs could be started from the education programs. Developing beliefs in mathematics and its teaching is one objective in the teacher preparation programs (Cross 2009; Zuljan, Valenčič, & Pejić, 2021). Thus, belief reformation in teacher education programs is greatly required (Szydlik, Szydlik, & Benson, 2003; Geisler & Rolka, 2021). The pre-service teachers participating in the programs have different initial beliefs in teaching and learning mathematics influenced by their previous experiences when studying at schools (Richardson, 1996). The study conducted by Liljedahl et al. (2021) revealed that the pre-service teachers' beliefs might change the course methods in teacher education. However, reforming the pre-service teachers' beliefs is not an easy nor simple process (Grootenboer, 2008).

Dealing with education in Turkey, Dede (2012) revealed that mathematics teachers at both primary and secondary schools have implemented a constructivism approach since 2004. Meanwhile, its implementation in higher education, especially in the faculty of education, has been started in 1997. In the constructivism approach, students independently constructed and developed their knowledge based on their levels of competency. Constructivism theories claimed that humans have their own understanding and knowledge related to the world through their first experiences and reflections (Reigeluth, 1999). The study conducted by Zuljan et al. (2021) showed that constructivism as a scientific way of teaching can improve the teachers' competencies and professional developments.

Constructivism approach applied in education is expected forming the desired mathematical beliefs. Many previous studies on beliefs have been conducted, including those on beliefs in mathematics nature and structure (Szydlik et al., 2003), beliefs in mathematics nature and teaching (Dede & Uysal, 2012), relationship between teachers' beliefs and teaching practices (Polly et al., 2013; Ren & Smith, 2018; Yang et al., 2020). Most conducted studies have discussed beliefs in mathematics nature, teaching, and learning, yet rarely referred to certain mathematical domains, processes, or topics (Zhang & Morselli, 2016). Therefore, this research then developed instruments meeting the requirements to measure the pre-service teachers' beliefs by focusing on certain mathematical contents, such as multiplication.

This study specialized in a multiplication content, that is, belief in multiplication as each mathematics term and object or procedure may become a belief object (Törner, 2002). The belief object on multiplication is greatly interesting to review since no research on how pre-service teachers view this content has been previously conducted. This content was chosen to review the pre-service teachers' beliefs as multiplication as one of difficult concepts at schools (Kennedy & Tipps, 1997) and there are various concepts and strategies possibly applied in multiplication (Simon, Kara, Norton, & Placa, 2018).

Belief plays an important role to the pre-service teachers. Consequently, various studies in this field have discussed various topics related to the influence of beliefs on knowledge and factors influencing the development of beliefs. The topics on beliefs are recently popular to discuss including beliefs in mathematics, learning mathematics, and teaching mathematics depending on the pre-service teachers' belief types. Studies on pre-service teachers' mathematical beliefs generally identify the belief types. Although many studies have been conducted in the mathematical beliefs, only few have developed instruments to assess beliefs in certain countries. This study aimed to identify the measured belief variables on multiplication using EFA equipped with SPSS software to construct each component in developing the questionnaire and examining the profile of Turkish pre-service mathematics teachers' beliefs in multiplication.

METHOD

Samples

Since 2000s, mathematics curriculum in Turkish elementary, junior high, and senior high schools (including the university's primary and secondary mathematics education programs) have been partly improved based on the constructivism philosophy. Thus, the research participants were also educated using this philosophy. In Turkey, the elementary and Junior high Mathematics Teaching Programs were conducted in 4 years. After the completion, the students then took some national examinations (for example, Professional Teaching Knowledge Test in Turkey: ÖABT). Those passing this exam then became the mathematic teachers for the junior high schools (for 5-8 grade levels) and senior high schools (for 9-12 grade levels). The research samples consisted of 414 fourth-year pre-service primary mathematics teachers (117 males and 297 females) from 18 different universities from several provinces in Turkey collected using a convenience sampling technique. The data showed that the female pre-service teachers participated more in this study, due to the increasing interest of Turkish female students in mathematics teaching programs when compared to the male students. Convenience sampling technique was employed in this research since the subjects were selected based on the researchers' interest in the related participants (Gravetter & Forzano, 2009). The research samples were conducted on the four-year pre-service primary mathematics teachers generally taking the mathematics courses on mathematical education and educational sciences. Some courses included Introduction to Algebra, Real Analysis, Geometry, Measurement and Evaluation, Mathematical Teaching Methods, Instructional Technology and Material Design, Introduction to Educational Sciences and Psychology Development.

Research Design

The design of this research consisted of two phases: 1) developing and validating the questionnaire items; and 2) implementing EFA on the questionnaire items. Due to the long procedures, we first developed a framework to compile the teachers' beliefs on multiplication. We compiled the questionnaire items referring to Ernest's (1989), McLeod's (1992), and Beswick's (2012) theories on beliefs summarized in Table 1.

Table 1. Questionnaire development framework on beliefs in multiplication

	Instrumentalist	**Platonist**	**Problem-solving**
Beliefs on multiplication	Believing that multiplication is related to rules and procedures; the implementation of multiplication can solve the problems found in real-life.	Believing that multiplication is a rule and strategy existing for a long time; representing the existing multiplication number.	Believing that multiplication ideas can be developed following individual knowledge; representation of the created multiplication number
Self–efficacy beliefs	Posing or solving multiplication problems using strategies remembered	Posing or solving multiplication problems using the existing strategies	Posing or solving the multiplication problems using various strategies
Beliefs about teaching multiplication	Mastering multiplication by memorizing	Mastering multiplication by understanding	Mastering multiplication by developing ideas on multiplication
Beliefs about social context	Following and remembering multiplication strategies in textbooks	Understanding multiplication strategies in textbooks and problems of multiplication numbers which has a solution in the textbooks	Multiplication strategies are not only in textbooks but can also be created independently

After developing the framework above, we then arranged the questionnaire items through the following phases:

Phase 1: Developing and Validating the Questionnaire Items

This study aimed to develop a questionnaire on multiplication beliefs, including its construction and validation for Turkish pre-service mathematics teachers on multiplication. The construction of questionnaire was measured by selecting and developing the appropriate items reflecting the respondents' literature, context, and language. Several items were constructed and translated. The validity of items was assessed by experts and revised based on their inputs. A trial version was then tested and resulted to construct the experimental version of Multiplication Beliefs Questionnaire (MBQ) for this study.

54 Likert-scale items were constructed based on the available sources/theories to assess the belief statements, ranging from "strongly disagree" to "strongly agree". The instruments consisted of 30 positive statement items and 24 negative statement items implying the characteristics of 3 belief types consisting of Instrumentalist; Platonist; and Problem Solving. The belief items in multiplication were structured based on the beliefs in multiplication; self-beliefs in multiplication; multiplication teaching beliefs; and multiplication beliefs in social context. Each MBQ item gave five points, indicating the agreement levels on the related statements, in which positive statements were scored ranging from 1 to 5, while negative statements were ranging from 5 to 1, and high score indicated that the respondent had

a very strong positive belief. The following multiplication beliefs (MB) items were presented as examples:

MB1 : Multiplication operation is only dealing with calculation (Instrumentalist)

MB10 : Strategies in multiplication operation is fixed (Platonist)

MB13 : Every individual may have different multiplication definitions (Problem solving).

The instruments' psychometrics and language verifications were involved in investigating the items' definition and understanding. The questionnaire items were developed from English and then translated into Turkish to ensure that the questionnaire appropriately measured the pre-service teachers' multiplication beliefs. The questionnaire draft content validity was evaluated by three university experts experienced in Mathematics Education and Educational Evaluation to assess the conformity of items to the indicators formulated in accordance with the theory. In terms of language validity, this research applied the translation guideline stages developed by Beaton et al. (2000) to the questionnaire. The first stage was Adaptation. The questionnaire was given to two Turkish experts with different educational backgrounds (Mathematics education specialist and education specialist) to independently translate from English to Turkish. The second stage was synthesizing and then retranslating the questionnaire into English by those mastering English to examine the validity and consistency. The English translation results were then reviewed by the authors/writers as the questionnaire developers. In retranslation or third stage, synonymous terms were found and the authors mastering Turkish then chose the appropriate terms. Finally, in the fourth stage, some items written in Turkish got a minor revision. Based on the experts' opinion, no item was removed, but item number 22 was divided into 2 items (MBN) consisting of:

MB22 : Using a standard multiplication is better than using a risky uncommon strategy,

This item was divided into:

MBN22 : The best way in multiplication process is using standard formula.

MBN23 : Alternative strategy is best in multiplication process.

This process resulted in 55 MBQ items consisting of 31 positive statement items and 24 negative statement items. In the last stage, the MBQ was then given for a piloting study. The piloting study was conducted after some corrections based on the experts' suggestions. The Turkish version of MBQ was given to 25 pre-service teachers to answer and show that there was no non-understandable statement. The piloting study found some difficult-to-understand statements, such as:

M15 : The definition of multiplication concept may differ.

Phase 2: Exploratory Factor Analysis (EFA)

A factor analysis was conducted to explore a strong correlation between variables inside a group, yet variables outside a group poorly correlated. Meanwhile, EFA was then employed by using the

Principal Axis Factoring (PAF) method. According to Howard (2016), PAF method can present more accurate factor analysis results based on EFA. This study used 414 data of respondents considered adequate (Thompson, 2004) with a rule of thumb with a loading factor of at least 0.32 for a sample size of 414 (Tabachnick & Fidell, 2013). The first response to those 55 items were analyzed using the Kaiser-Mayer-Olkin Approach (KMO) and Barlett's Sphericity Test (BTS), resulting in KMO (0.842) and BTS (6687.397; 1485, $p<0.001$), which were considered sufficient to justify the obtained four components. According to Kaiser (1974), the minimum acceptance of KMO was 0.5. Meanwhile, Cronbach's Alpha was calculated to determine the questionnaire's internal consistency.

RESULTS AND DISCUSSION

Exploratory Factor Analysis

Based on the method used in factor analysis, 55 items with the loading factors of less than 0.32 were removed. From 414 respondents of pre-service teachers, 14 MBQ items with Cronbach's alpha (α) of 0.771 were obtained. Referring to Hair, Anderson, Tatham, and Black (1998), scores between 0.60 and 0.70 were considered as the minimum acceptance limit for the internal consistent reliability coefficient. Those 14 items consisted of 6 negative statement items and 8 positive statement items. The maximum standard deviation score for MBQ items was approximately 1, indicating that the data distribution tended closing to the average value.

To obtain the framework of MBQ components, the responses given to the MBQ items were analyzed using both PAF and Varimax rotation method. The Varimax method developed by Kaiser (1958) was recognized as a good and widely used method. The communality items showed the variation ranging from 0.402 to 0.710 was considered having a high communality (Tabachnick & Fidell, 2013). The procedures to identify factors in MBQ items used the eigen value of greater than 1. The number of the obtained factors was four as MBQ components seen in Table 2. The KMO value calculated in the final MBQ was 0.782, while the BTS was 1487.397; 91; $p < 0.001$. With the KMO value of > 0.50, the BTS factor analysis could be then proceeded (Kaiser, 1974).

All MBQ components were calculated at 51.293% of the variance and Merenda (1997) stated that ''for the number of 'real' factors and components, the proportion [of variance accounted for] should be at least 0.50" (p. 158). The first, second, third, fourth component was respectively calculated at 17.747%, 11.708%, 11.453%, and 10.385%.

As presented in Table 2, 14 MBQ items were distributed into four components consisting of MBQ item 19, 20, 27, 28, 43 and then loaded with the highest in component 1 (C1), interpreted as remote beliefs in multiplication (RBM). C1 might be classified as a negative belief statement as mathematics was separated from the real life. Only mathematicians know that multiplication representation and multiplication problems can only be settled through the already-known strategies. The questions in C1 were different from those in the instrumentalist belief viewing that mathematics is useful in real life (Ernest, 1989), Grootenboer and Marshman (2016) called it as a utilitarian belief. It is previously explained that negative statements were scored from 5 to 1. Figure 1 showed that the mean score of C1 was close to 4, Turkish pre-service teachers commonly did not agree with this remote belief view.

Table 2. Rotated structure matrix of PAF method

MBQ Items		Component				α
		1	2	3	4	
M20	Decimal numbers of multiplication operation cannot be implemented in daily life.	0.748				0.818
M28	Only mathematicians can find multiplication operation representation.	0.689				
M27	The fact that fraction multiplication has different representation is not found in some sources and shows that there is no different representation of multiplication.	0.666				
M19	Integer multiplication operation cannot be implemented in daily life.	0.655				
M43	Only known strategies can be used to solve the multiplication problems.	0.495				
M48	Solution for all multiplication problems can be found in the mathematics textbooks.		0.701			0.712
M47	Multiplication problems forms can be easily found in mathematics textbooks.		0.700			
M29	All strategies of multiplication operation can be found in school mathematics textbooks.		0.578			
M31	Some strategies in multiplication operation can be explored.			0.812		0.715
M12	Multiplication operation procedure can be re-found by the students.			0.550		
M35	New strategies can be generated from previous multiplication strategies in mathematical sources.			0.549		
M55	I can pose multiplication problems in many ways.				0.735	0.697
M54	I know how to pose difficult multiplication problems.				0.676	
M53	I can solve difficult school multiplication problems.				0.520	

*Note: Only the loading scores greater than 0.32 was presented

The second component (C2) was interpreted as the pre-service teachers' view related to the beliefs in multiplication operation on mathematics textbooks (BOMT). With a mean closing to 3, which was considered neutral, the pre-service teachers commonly had a neutral view on all problems related to the multiplication operations found in the mathematics textbooks. Ernest (1994) supported that mathematics materials should be enriched by adding the mathematical problems and activities into textbooks.

In addition to the previous two components, three items (M12, M31, and M35) were loaded into component 3 (C3) with a general characteristic of dynamic beliefs in multiplication learning (DBLM). C3 was classified as a component showing that in mathematics learning, pre-service teachers had a dynamic belief. Ernest (1989) viewed that learning dynamics was the problem-solving type of belief learning, in which the students might explore the strategies to independently find a solution to the multiplication problems. Figure 1 showed that the mean score was approaching to 4, explaining that the pre-service teachers commonly agreed with the multiplication learning through exploration, or commonly called as a dynamic belief.

The fourth component (C4) included three items consisting of 53, 54, and 55. The three components tended to be self-beliefs or by Bandura (1986) called self-efficacy. McLeod (1992) designed the belief categories, one of which, self-belief, for example, a belief that ‘I can solve problems”. Op 'T Eynde, De Corte, & Verschaffelg (2002) exemplified self-efficacy belief as “I am confident, I can understand the most difficult material presented in the readings of this mathematical course”. In this C4, the pre-service teachers believed in their ability to settle the mathematical problems. C4 could be then called as self-efficacy beliefs in multiplication problems (SEMP). C4 showed the mean score was approaching to 4, considering that the pre-service teachers agreed with their self-efficacy belief in multiplication problems. Finally, those 14 MBQ items containing 4 components in multiplication obtained in this study were used to explore the Turkish pre-service teachers' multiplication beliefs.

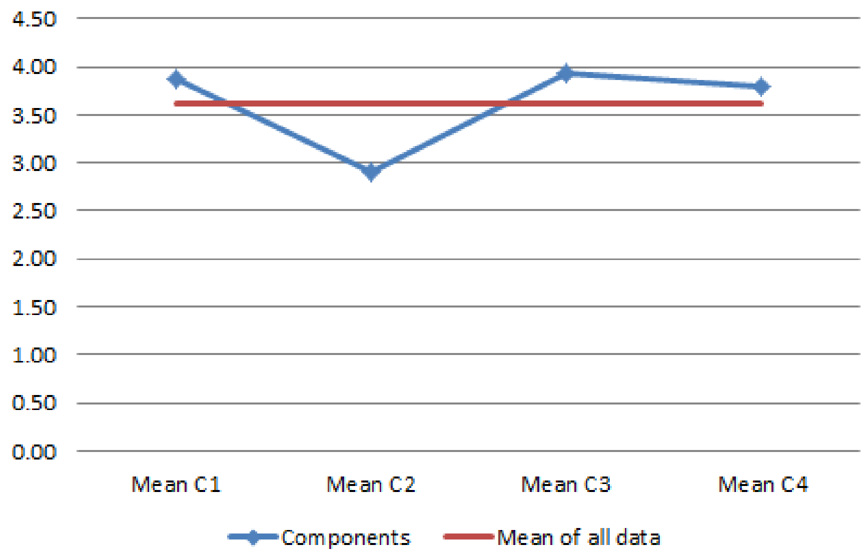

Figure 1. Mean score of each component

Summary of Questionnaire Responses

The summary of four component pre-service teachers' responses is shown in the following tables.

Those tables show the percentage of responses which were based on the categories of strongly agree, agree, neutral, disagree, and strongly disagree.

Table 3. The percentages of Turkish pre-service teachers' responses on RDM items

RDM Items	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Decimal numbers multiplication operation cannot be implemented in daily life.	2.7%	6.5%	14.7%	17.6%	58.3%
Only mathematicians can find multiplication operation representation.	3.9%	9.4%	14.5%	36.1%	35.9%
The fact that fraction multiplication has different representation is not found in some sources and shows that there is no different representation of multiplication.	5.8%	11.1%	15.9%	26.1%	41.1%
Integer of multiplication operation cannot be implemented in daily life.	3.4%	5.8%	11.6%	15.9%	63.3%
Only known strategies can be used to solve the multiplication problems.	5.3%	18.1%	22.7%	33.8%	20.0%

Table 3 shows that most pre-service teachers did not agree with the given negative statements. Pre-service strongly disagreed with the statement that integer, fractions, and decimal multiplications were separated from the real life. In addition, they also strongly disagreed with the statement that only mathematicians could find multiplication operation representation. Meanwhile, they disagreed with the statement in which only the known strategies could be used to solve the multiplication problems. Therefore, it revealed that most Turkish pre-service teachers' responses strongly disagreed with the items related to the remote beliefs in multiplication.

Table 4. The percentages of Turkish pre-service teachers' responses on BOMT items

BOMT Items	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
Solution for all multiplication problems can be found in mathematics textbooks.	10.4%	25.4%	38.9%	21.7%	3.6%
Multiplication problems form can be easily found in mathematics textbooks.	10.6%	29%	39.1%	16.7%	4.6%
All strategies of multiplication	8.9%	19.6%	32.9%	26.8%	11.8%

operation can be found in the school mathematics textbooks.

The highest percentage of pre-service teachers' responses was on the 'Neutral' BOMT positive statement (Table 4). Most pre-service teachers gave neutral responses on the use of textbooks in the multiplication operation learning. The neutral view on solution for all multiplication problems were found in the mathematics textbooks.

Table 5. The percentages of Turkish pre-service teachers' responses on DBLM items

DBLM Items	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
Some strategies in multiplication operation can be explored.	1.7%	2.7%	16.7%	49.8%	29.2%
Multiplication operation procedure can be re-found by the students.	1.4%	1.7%	13.8%	46.1%	37.0%
New strategies can be generated from previous multiplication strategies in mathematical sources.	1.7%	5.1%	24.6%	46.1%	22.5%

Table 5 shows that most pre-service teachers agreed with the positive statement of DBLM items. They agreed that the multiplication operation could be explored and re-founded by the students. In addition, they also agreed that the new strategy could be generated from the previous multiplication strategy. Therefore, most Turkish pre-service teachers supported the dynamic multiplication operations.

Table 6. The percentages of Turkish pre-service teachers' responses on SEMP items

SEMP Items	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
I can pose multiplication problems in many ways.	7%	4.3%	27.1%	45.9%	22.0%
I know how to pose difficult multiplication problems.	2.2%	8.7%	35.3%	37.7%	16.2%
I can solve difficult school multiplication problems.	1.0%	4.6%	19.8%	38.2%	36.5%

Table 6 shows that most pre-service teachers' responses agreed with the positive SEMP statement items showing positive self-efficacy belief to pose with the mathematical problems in many ways and knew how to pose with the difficult mathematical problems. They also believed that they could solve the difficult school-grade multiplication problems. Thus, most Turkish pre-service teachers in this study had positive self-efficacy beliefs in the multiplication problems.

This study used EFA to analyze the developed questionnaire. Of 55 items, the analysis resulted in 14 items. We developed the questionnaire based on the belief theory in multiplication. Furthermore, the reason analysis using strict criteria excluded some items not meeting the requirements. In this EFA process, the researchers decided to issue several MBQ items with low-loading, cross-loading, or free-standing. However, in eliminating the MBQ items, the researchers also used the conceptual significance of the items. According to Beavers et al. (2013), before removing an item with a loading factor of less than 0.32, the researchers should analyze whether the items had too much conceptual vitality in the result part. It was eventually excluded that 41 items were in low-loading, cross-loading, or free-standing. In addition, the researchers assumed that those 14 items represented all developed items with a minimum of 3 items for 4 MBQ components. According to Costello and Osborne (2005), a component having a minimum of 3 to 5 items with a loading factor meeting the requirements could be considered qualified as a stable and solid component.

One of the examples is statement M10: The strategy in the multiplication operation is in the finished product. statement M10 is in contradiction with statement M31. Statement M31: Strategies in multiplication operations can be explored. Statement M31 expresses a problem-solving belief type which has a dynamic nature (Ernest, 1989). Meanwhile, M10 represents the instrumentalist belief type viewing that mathematics is considered collecting rules and procedures. According to Callejo and Vila (2009), problem-solving beliefs viewed closer to a positive view. The instrumentalist viewed beliefs as the opposite of problem-solving beliefs. So, the value of M10 is in contradiction with that of M31. If M31 answers "strongly agree" then DBLM tends to be positive and "strongly disagree" and has a negative DBLM tendency. The researchers also did the same thing, by removing several other items with low-loading, cross-loading, or free-standing. The selected items consisting of only 14 items had already represented all items.

The first component was related to the pre-service teachers' view that multiplication did not have its implementation in real life; the mastery of multiplication was limited, as a value, only to mathematicians; and that only familiar strategies could be used in multiplication. These three arising problems were identified as remote belief in multiplication. In fact, this remote belief saw mathematics in a small and rigid scope. Different from the opinion delivered by Ernest (1989), mathematics was considered unable to be implemented in real life as instrumentalist type. This research found that most Turkish pre-service teachers did not have remote belief in multiplication.

As mentioned above, Turkish mathematics curriculum had been updated since 2004 based on the constructivism philosophy. The participants in this study were in facts educated based on this philosophy and possibly resulted in the present outcomes. This was supported by the research conducted by Grootenboer and Marshman (2016), showing high results of survey on secondary school students' beliefs and stating that mathematics was useful, important, and could be implemented in daily life. These results were different from the findings of research conducted by Gómezescobar and Fernández (2018), mentioning that pre-service teachers viewed that mathematics was useless since they had low confidence on their abilities or low self-efficacy beliefs. In this study, the Turkish pre-service teachers had a positive view on their self-efficacy beliefs in multiplication. It meant that they had a negative view on RDM.

Furthermore, the Turkish pre-service teachers in this research tended to have neutral view, believing that strategies and solutions of mathematics problems were available in mathematics textbooks. In line with the theory of McLeod (1992), social context affected the students' beliefs, the constructivism learning was possibly implemented in the Turkish curriculum and led the pre-service teachers have neutral view on the mathematics textbooks that multiplication learning sources could be obtained from everywhere. Referring to Cobb (1986), the social context, in this case the learning interaction, might be in the form of cognitive activity with meaningful activities constructing new knowledge or only passively received from the teachers and other learning sources, such as textbooks (Ernest, 1994). Dede (2006) found that Turkish mathematics textbooks in the elementary schools were isolated from the real world and written in abstract style. These results were in line with the findings of research conducted by Nicol and Crespo (2006) showing that the pre-service teachers viewed textbooks as useful for guidance at the beginning of teaching. In addition, textbooks were also flexible to adapt and change to meet the students' diverse needs in the classroom. However, the findings of research conducted Kılıç (2011) were different from those resulted in this study. The research showed that the Turkish pre-service teachers had a view that using textbooks encouraged the students to meet the expected learning objectives. There were different views related to the findings on the use of textbooks according to the pre-service teachers. The development of technology has shifted due to the use of printed textbooks. The shift in the use of textbooks had caused the pre-service teachers taking a neutral view. According to Robb (2019), students abandoned the use of printed textbooks due to the easy digital access.

Another finding showed that environment highly affected the development of pre-service teachers' beliefs. The second component respondents believed that rules, procedures, and strategies varied and could be obtained through invention and exploration. This result was supported by the research conducted by Liljedahl, Rolka, and Rösken (2007) stating that the pre-service mathematics teachers' beliefs might change when involved in the constructivism environments and in mathematics discovery. Grootenboer and Marshman (2016) also confirmed that the students could enjoy mathematics when their learning environment gave them opportunities to cooperate in solving the mathematical problems and engaged them in investigations. The research conducted by Şahin (2009) stated that in Turkey, curriculum with constructivism approach affected the teachers in teaching not by using the transmission model, but they emphasized more on implementing the student-centered model supported by Uysal and Dede (2016) who identified that the Turkish pre-service teachers had child-centeredness and problem solving-beliefs. The effect was that the teachers with constructivism beliefs could help the students improve their performance for the advanced mathematics assignments (Staub & Stern, 2002). Based on the research conducted by Yang (2020), it found that the dynamic belief had a positive effect on the pre-service teachers' mastery on mathematics.

This constructivism-based teaching experience urged the Turkish pre-service teachers to get used to finding something new in their teaching activities. The result obtained from such process was something new. In this research, the belief was identified as a dynamic belief in multiplication learning. The term dynamic referred to Ernest (1989), stating that mathematics was a dynamic perspective on the problem solving-belief, in which the conducted constructivism learning was considered to establish the connection of procedure/rule/strategy. The research conducted by Geisler and Rolka (2021), showed that dynamic beliefs could change from school to university. In the first year at university, the dynamic belief of students tended to decrease. However, if the university uses the constructivism-based learning, it is possible for the students to have positive dynamic beliefs. The curriculum in Turkey was based on constructivism and supported the development of positive DBM as found in this study.

This research identified self-efficacy belief in multiplication as the fourth component. The pre-service teachers in this research had positive self-efficacy belief in multiplication that they could design and solve the difficult multiplication problems in various ways. According to Hailikari, Nevgi, and Komulainen (2008), self-belief influenced the students' ways in accessing and using their prior understanding on the new learning situations and their learning outcomes. This finding supported that reported by Sevgi et al. (2021), mentioning that Turkish mathematics teachers had high self-efficacy on strategies to support the students' learning activities. Thus, both in-service teachers and pre-service teachers may have their beliefs on new ideas and methods in mathematics. This was also supported by the findings of research conducted by Yılmaz and Turan (2020), mentioning that the Turkish pre-service teachers had high self-efficacy in teaching mathematics and the results of study conducted by Aydın, Sevimli, and Abed (2019) also showing that self-efficacy level indicated the pre-service teachers' knowledge level. The Turkish pre-service teachers' self-efficacy beliefs in this research showed that they had positive beliefs to use the constructivism-based learning strategies in multiplication. They greatly believed in using the problem-solving learning in multiplication.

CONCLUSION

This study aimed to formulate the MBQ instruments and then describe the profile of Turkish pre-service teachers based on the resulted instruments. The process started from the EFA iteration, items with low-loading, cross-loading, or free-standing, and conceptual similarity with higher loading factor to the analysis excluding the items. At the end of EFA, 14 MBQ items were obtained and represented four components of Turkish pre-service mathematics teachers' beliefs. This study found that the Turkish pre-service teachers had a positive DBLM in learning multiplication which required exploration activities to find the procedures or strategies independently performed by the students. This belief had a negative impact on RBM. It meant that the pre-service teachers did not believe that multiplication was useless in everyday life. The positive belief was also obtained in SEMP, which then explained that Turkish pre-service teachers had a belief to be able to solve the multiplication problems. Last but not least, the Turkish pre-service teachers had a neutral view on BOMT, which mentioned that textbooks could improve multiplication activities.

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COUNTEREXAMPLES: CHALLENGES FACED BY ELEMENTARY STUDENTS WHEN TESTING A CONJECTURE ABOUT THE RELATIONSHIP BETWEEN PERIMETER AND AREA

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Abstract

One pedagogical approach to challenge a persistent misconception is to get students to test a conjecture whereby they are confronted with the misconception. A common misconception about a 'direct linear relationship' between area and perimeter is well-documented. In this study, Year 4-6 students were presented with a conjecture that a rectangle with a larger perimeter will always have a larger area. Eighty-two (82) students' written responses from three elementary schools in Victoria, Australia were analyzed. The findings revealed that Year 4-6 students could find multiple examples to support the conjecture but they struggled to find counterexamples to refute the conjecture. The findings underscored the importance of developing elementary school students' capacity to construct counterexamples and recognize that it is sufficient to offer one counterexample in refuting a conjecture about all cases. Implications for teaching practice to support investigating and testing a conjecture are discussed.

Keywords: Counterexamples, Conjectures, Perimeter, Area, Elementary Students, Justifying

Abstrak

Salah satu pendekatan pedagogis untuk menantang miskonsepsi yang terus-menerus adalah membuat siswa menguji dugaan yang mana mereka dihadapkan pada suatu miskonsepsi. Kesalahpahaman umum tentang 'hubungan linier secara langsung' antara luas dan keliling didokumentasikan dengan baik. Dalam penelitian ini, siswa Kelas 4-6 disajikan dengan dugaan bahwa persegi panjang dengan keliling yang lebih besar akan selalu memiliki luas yang lebih besar. Delapan puluh dua (82) tanggapan tertulis siswa dari tiga sekolah dasar di Victoria, Australia dianalisis. Temuan mengungkapkan bahwa siswa Kelas 4-6 dapat menemukan banyak contoh untuk mendukung dugaan tersebut, namun mereka berjuang untuk menemukan contoh tandingan untuk membantah dugaan tersebut. Mengembangkan kapasitas siswa sekolah dasar untuk membangun contoh tandingan dan menyadari bahwa cukup menawarkan satu contoh tandingan untuk menolak dugaan tentang semua kasus menjadi perhatian utama dalam penelitian ini. Penelitian ini juga membahas terkait implikasinya pada praktik pengajaran untuk mendukung penyelidikan dan pengujian dugaan.

Kata kunci: Contoh Pembanding, Dugaan, Keliling, Luas, Siswa Sekolah Dasar, Pembenaran

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Generating counterexamples is challenging for students (Zaslavsky & Ron, 1998; Zazkis & Chernoff, 2008) and its role to refute a conjecture might not be recognized. The majority of studies investigating the use of counterexamples and examples in refuting a conjecture involved secondary or university students (Yopp, 2013; Zazkis & Chernoff, 2008). A small number of studies have focused on elementary students' capacity to work with examples and counterexamples (Knuth, Zaslavsky, & Ellis, 2019; Komatsu, 2010; Markovits, Brisson, de Chantal & St-Onge, 2016). Mathematical reasoning is one of the proficiencies in the *Australian Curriculum Mathematics* (ACARA, nd) and in addition to analysing and generalising, students are expected to learn to justify that is, "to prove that something is true or false."

Watson and Mason (2005) assert that having students search for and construct counterexamples deliberately to explore the limitations of a relationship might lead to a better understanding and a deeper appreciation of conjectures and properties. Creating a cognitive conflict by presenting a situation where students are confronted with a common misconception is recognized as a pedagogical strategy to help learners recognize and rectify their misconception (Limon, 2001; Tirosh & Graeber, 1990; Watson, 2007).

A common misconception about a 'direct proportional relationship' between area and perimeter was reported among students of different ages (Cavanagh, 2007; De Bock, Verschaffel, & Janssens, 1998; Tan Sisman, & Aksu, 2016; Tirosh & Stavy, 1999). Tirosh and Stavy (1999) reported that a high proportion of students assumed that a linear relationship exists between area and perimeter and envisaged when the area of a figure decreases or increases, the perimeter will also decrease or increase. They linked this phenomenon to student use of intuitive rule 'more A – more B'. Similarly, De Bock, Verschaffel, and Janssens (1998) observed this phenomenon among lower grades of secondary students and referred to this as 'the illusion of linearity'. Fernández, De Bock, Verschaffel, and Van Dooren (2014) extended earlier studies by De Bock and colleagues (e.g., De Bock, Verschaffel, & Janssens, 1998; Van Dooren, De Bock, Janssens, & Verschafell, 2008) by making a distinction between dimensionality and "directionality".

This study aims to examine upper elementary school students' capacity to generate examples and counterexamples to test the conjecture of a linear relationship between perimeter and area of a rectangle. The following research questions were addressed:

a. What understanding do elementary school students have of the roles of examples and counterexamples in the process of testing a conjecture?
b. How do elementary school students use examples and counterexamples to test a conjecture?
c. What levels of justifying are evident when testing a conjecture?
d. How does the use of a task to test a conjecture reveal elementary students' understanding of the relationship between perimeter and area?

Use of Counterexamples and Examples in Refuting A Conjecture

Earlier studies (Goldenberg & Mason, 2008; Pedemonte & Buchbinder, 2011; Watson & Mason, 2005) ascertain different roles and uses of counterexamples and examples in mathematics learning. Effective construction and use of counterexamples and examples require strategic thinking beyond algorithmic or procedural thinking. It is vital for students to learn about the limitation of the scope of examples in proving. That is, examples could not be counted as proof because it violates the intellectual-honesty principle of proof (Buchbinder & Zaslavsky, 2019; Stylianides, 2007). Pedemonte and Buchbinder (2011) recognized different levels of efficacy in example use. They argue it is necessary to have a cognitive unity and structural unity between the argumentation leading to a conjecture and its subsequent proof in order for examples to be productive in proving.

There is prevalent use of examples as a form of justification in elementary school (e.g., Carpenter et al., 2003; Goldenberg & Mason, 2008; Martin & Harel, 1989; Mason, 2019). An explicit attention to build students' capacity in using and choosing examples and counterexamples is key in order to lay a solid foundation for a more formal mathematics, particularly in relation to proof (Martin & Harel, 1989, Mason, 2019; Stylianides, 2007). Martin and Harel (1989) stated "If [elementary] teachers lead their students to believe that a few well-chosen examples constitute a proof, it is natural to expect that the idea of proof in high school geometry and other courses will be difficult for the students" (pp. 41-42). Mason (1982) and Ellis et al. (2019) discuss ways that teachers might engage students in exploring examples and counterexamples.

Mason (1982) argued that conjecturing involves a cyclical process that requires verifying a conjecture, checking if the conjecture encompasses all identified cases and examples, and testing the conjecture by trying to refute it using a counterexample. Ellis et al. (2019) examined the use of examples in exploring conjectures and developing appropriate justifications and distinguished two different ways to view examples connected to different mathematical reasoning processes. In exploring conjectures, students might use examples to explore and make sense of the conjecture or use counterexamples to refute a conjecture. Secondly, examples might be used to form a new conjecture. In the justifying process, examples might be used to "convey the claim of the conjecture is true (or false), or to convey a general argument" (p. 269).

A counterexample is a mathematical concept that is used to test the limitation of a relationship between mathematical concepts or to contest a conjecture (Komatsu, 2016; Watson & Mason, 2005; Yopp, 2013; Zazkis & Chernoff, 2008). Counterexamples play a critical role to "delineate the example space… and to understand and appreciate conjectures more deeply" (Watson & Mason, 2005, p. 60). However, the efficacy of counterexamples relies upon a learner having a personal history of constructing counterexamples (Watson & Mason, 2005; Zazkis & Chernoff, 2008). Zazkis and Chernoff (2008) stated "Different counterexamples, while serving the same mathematical purpose of rejecting a conjecture, may not be equally effective in serving a pedagogical purpose of helping a learner recognize the faulty conjecture." (p. 206).

Research on secondary student difficulties with counterexamples revealed that students had trouble in accepting the logic that a counterexample refutes a rule (Stylianides & Al-Murani, 2010; Peled & Zaslavsky, 1997; Zaslavsky & Ron, 1998). Widjaja et al. (2021) previously reported Year 3 and 4 Australian and Canadian students' capacity to search for examples and counterexamples when testing a conjecture that was true for a task called "Magic V" (NRICH, 2018). They found that some students argued that because they could not find counterexamples then the conjecture that a Magic V using the numbers 1 to 5 could not have an even number in the vertex was true. These students used the absence of counterexamples, rather than a logical argument, to accept the conjecture. They did however believe that if they could find a counterexample they would be able to refute the conjecture. Zazkis and Chernoff (2008) attributed the challenges faced by pre-service elementary teachers in realizing the significance of a counterexample in refuting a conjecture to an assumption that students would follow a proof scheme similar to the expert's proof scheme. Furthermore, they argued that some students might not grasp the significance of counterexamples and dismiss them as an exception. Similarly, Stylianides and Al-Murani (2010) reported that some secondary students maintained that a true mathematical statement and a counterexample could co-exist together.

Factors Contributing to Misconceptions about Perimeter and Area

Several researchers (Grant & Kline, 2003; Kamii & Clark, 1997; Moyer, 2001) argued that the pedagogical approach in teaching measurement place too much prominence on the measurement procedures of 'how to measure' and not enough emphasis on the key attributes and ideas of measurement in order for students to attach meaning to the concept of area and perimeter. Lack of understanding of length and area and a hasty introduction of the formulas were attributed as possible reasons for students to overgeneralize the relationship between perimeter and area. This common misconception was also noted in The National Council of Teachers of Mathematics documents (NCTM, 1989), "Most students in grades 5–8 incorrectly believe that if the sides of a figure are doubled to produce a similar figure, the area and volume also will be doubled" (NCTM, 1989, pp. 114–115). Other researchers (Livy, Muir, & Maher, 2012; Yeo, 2008) observed a similar misconception among pre-service teachers and reported a strong reliance on procedural knowledge. This suggests that the confusion about the relationship between perimeter and area is persistent.

The *Australian Curriculum Mathematics* (ACARA, nd) for teaching measurement suggests a sequence that in Foundation year and Year 2 students estimate which one is bigger i.e., form a conjecture, and then use direct or indirect comparison to verify or refute the conjecture about which is bigger or to order of the size of objects. In Year 4 students are expected to explore the areas of different rectangles using concrete representations of metric units. They are expected to develop understanding of relationship between area and length and width of rectangle; not recognizing that a square is a rectangle may hinder development of this relationship. In Year 5 the focus is on formalizing formulae for calculating area and perimeter and in Year 6 they are expected to "solve problems involving the comparison of lengths and areas using appropriate units" (ACARA, nd). Whilst the curriculum does expect that students will begin to explore the relationship between area and perimeter in Year 4, the focus on understanding this relationship is not explicit in the curriculum statements for Years 5 and 6.

Mathematical Reasoning

Jeannotte and Kieran (2017) proposed two conceptual frameworks for mathematical reasoning - a process framework for reasoning in addition to a structural framework for reasoning. They distinguished mathematical reasoning processes into two broad categories of searching for similarities and differences, and validating. In their view, conjecturing fits under a reasoning process related to searching for similarities and differences whilst justifying is considered as a mathematical reasoning process related to validating. Jeannotte and Kieran (2017) argued that the process of justifying is associated with two epistemic paths and elaborated the distinction between the two as follows:

> The first is related to the justification of a conjecture that arises from the process of conjecturing. This passage allows for changing the epistemic value from likely to more likely… The second type of epistemic passage is related to a validation that changes the epistemic value from likely to true or false, without being considered necessarily as constituting the process of proving. (p. 12)

In their framework, Jeannotte & Kieren framework (2017) emphasise the importance of focusing on the processes aspect of reasoning and interrogating the connections between different reasoning processes of searching for similarities and differences, conjecturing and validating. Vale et al. (2017) introduced a framework called 'Mathematical Reasoning Actions and Levels (MRAL)'. This framework drew on earlier work (e.g., Carpenter et al., 2003; Ellis, 2007; Lobato, Hohensee, & Rhodehamel, 2013). It elaborated and extended the three 'reasoning actions': comparing and contrasting, generalising, and justifying by theorizing 'levels of reasoning using a generalising task of "What else belongs?" (Small, 2011). The MRAL framework was then used to map Year 3-4 and Year 4-5 students' reasoning when testing a conjecture that arose when exploring examples for the "Magic V" task (NRICH, 2018). Analysis of students' arguments led to revision of the levels of justification in the MRAL framework (Widjaja et al., 2021).

In the larger study, in order to support teachers to teach and assess elementary students' mathematical reasoning, we developed a generic assessment rubric (see Table 1) to assist teachers in developing awareness of students' reasoning actions and to assess their levels of reasoning (Loong et al., 2018). The rubric built on previous studies and was developed through an iterative design-based research process of design, testing with elementary school students, and getting feedback from teachers to refine the rubric. In this version of the rubric 'comparing and contrasting' was relabeled as 'analysing,' 'forming conjectures' was added to 'generalising', and 'logical argument' was included in the heading for 'justifying'. These terms were included to support teachers as they aligned with terms used in the *Australian Curriculum Mathematics* (ACARA, nd) to describe mathematical reasoning.

Table 1. Levels of Mathematical Reasoning (Source: Loong et al., 2018)

	Analysing	Generalising	Justifying and Logical argument
Not evident	• Does not notice numerical or spatial structure of examples or cases. • Attends to non-mathematical aspects of the examples or cases.	• Does not communicate a common property or rule for pattern. • Non-systematic recording of cases or pattern. • Random facts about cases, relationships or patterns.	• Does not justify. • Appeals to teacher or others.

Beginning	• Notices similarities across examples • Recalls random known facts related to the examples. • Recalls and repeats patterns displayed visually or through use of materials. • Attempts to sort cases based on a common property.	• Uses body language, drawing, counting and oral language to draw attention to and communicate: ○ a single common property ○ repeated components in patterns. • Adds to patterns displayed verbally and/or visually using diagrams or through use of materials.	• Describes what they did and why it may or may not be correct. • Recognises what is correct or incorrect using materials, objects, or words. • Makes judgements based on simple criteria such as known facts. • The argument may not be coherent or include all steps in the reasoning process.
Developing	• Notices a common numerical or spatial property. • Recalls, repeats and extends patterns using numerical structure or spatial structure. • Sorts and classifies cases according to a common property. • Orders cases to show what is the same or stays the same and what is different or changes. • Describes the case or pattern by labelling the category or sequence.	• Communicates a rule about a: ○ *property* using words, diagrams or number sentences. ○ *pattern* using words, diagrams to show recursion or number sentences to communicate the pattern as repeated addition. • Explains the meaning of the rule using one example.	• Verifies truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods. • Refutes a claim by using a counter example. • Starting statements in a logical argument are correct and accepted by the classroom. • Detecting and correcting errors and inconsistencies using materials, diagrams and informal written methods.
Consolidating	• Notices more than one common property by systematically generating further cases and/or listing and considering a range of known facts or properties. • Repeats and extends patterns using both the numerical and spatial structure. • Makes a prediction about other cases: • with the same property • included in the pattern.	• Identifies the boundary or limits for the rule (generalisation) about a common property. • Explains the rule for finding one term in the pattern using a number sentence • Extends the number of cases or pattern using another example to explain how the rule works.	• Uses a correct logical argument that has a complete chain of reasoning to it and uses words such as 'because', 'if…then…', 'therefore', 'and so', 'that leads to' ... • Extends the generalisation using logical argument.
Extending	• Notices and explores relationships between: ○ common properties ○ numerical structures of patterns. • Generates examples: ○ using tools, technology and modelling ○ to form a conjecture.	• Communicates the rule for any case using words or symbols, including algebraic symbols. • Applies the rule to find further examples or cases. • Generalises properties by forming a statement about the relationship between common properties. • Compares different symbolic expressions used to define the same pattern.	• Uses a watertight logical argument that is mathematically sound and leaves nothing unexplained. • Verifies that the statement is true or the generalisation holds for *all* cases using logical argument.

METHOD

The study reported here is part of a larger research project to develop teaching resources to support elementary teachers' understanding, teaching, and assessing of mathematical reasoning. In the current study, we used a task for which a counterexample could be used to refute the conjecture, but given the documented misconception, discussed above, students are likely to initially believe the conjecture.

The task was presented to students as follows:

Nathan said: "When you increase the perimeter of a rectangle, the area always increases". Explain why or why not Nathan might be correct? Is this statement true for all cases?

The aim of the lesson was to engage students in testing a conjecture and to confront a potential misconception that a larger perimeter will always result in a larger area. In particular, students will learn that it is sufficient to offer one counterexample to refute a conjecture. In supporting and challenging students to test and justify a conjecture, students were expected to use their understanding of area and perimeter to select examples or counterexamples (analysing) and to use the results of their trials to refute the conjecture (justifying).

While all the schools and the teachers chose the same task for their students, they set out the task differently with their students. In School A, the teachers presented the conjecture and discussed/presented the rules for finding perimeter and area (see Figure 1a). The students worked in pairs to respond to the task using a blank sheet of paper. In school B, students recorded their exploration of examples and counterexamples in a blank sheet of paper and they mainly worked individually with some exception of a group of students worked in a small group of three. In School C, the teachers differed in their introduction of the task. One of the teachers introduced the task and then asked students to work in pairs to explore different rectangles using a geoboard on their iPad and then record their examples in a table (see Figure 1b). Another teacher from School C introduced the task and provided her students with concrete materials (tiles) to generate different rectangles and record their rectangles in a table.

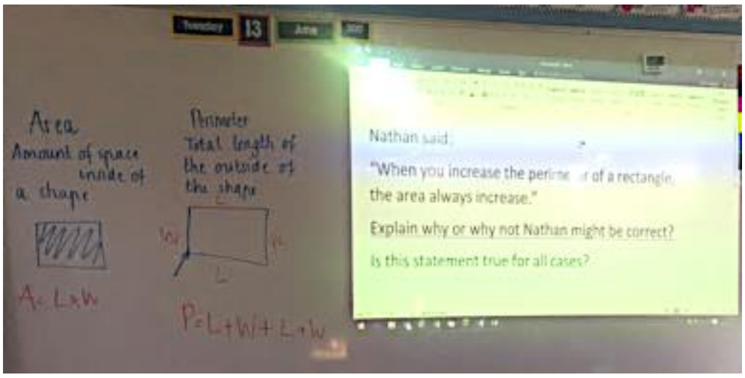

Figure 1a. A presentation of the task in School A

Figure 1b. A pair of students working on the task in School C

Participants

In total, 119 elementary school students from three elementary schools in Victoria participated in the project (see Table 2). Students typically worked in pairs on the task in Schools A and C and individually at School B.

Table 2. List of participating schools and year levels

School	Year level	Teachers	Students
A	4	2	28 (14 pairs)
B	5 & 6	2	51 (47 individuals, 2 pairs, 1 group of 3)
C	5	2	40 (2 individuals, 19 pairs)

Methods of Data Collection

Data were collected from three elementary schools in Victoria, Australia. The teachers participated in the professional development session delivered by the research team and were given resources to support the teaching of the task including suggested prompts to elicit, support and challenge reasoning, along with anticipated student solutions with examples of how to use the reasoning rubric to assess students' mathematical reasoning. Following the lesson, each pair of teachers participated in a discussion with the research team to examine samples of student written work, identified different levels of student reasoning actions based on their work samples and informed by classroom observations.

The task was taught by the classroom teacher in each school as a one-off mathematics lesson and not as part of a sequence of lessons on area and perimeter due to a logistical limitation. The lesson was observed by another teacher who taught the same year level from each participating school, and two members of the research team. In the lesson materials, there was a clear expectation for students to communicate their reasoning to one another and to the teacher. However, as the schools and the participating teachers were relatively new to mathematical reasoning, it was unclear if there was an established classroom culture that expected students to communicate their reasoning in their regular classroom practice. The teachers interacted with students during the lesson using prompts provided in the materials. The nature and content of these interactions was not a focus of this study. Rather, evidence of students' reasoning, both their analysing and justifying actions, was gathered from their written work as well as their verbal and non-verbal communication captured in the videos of paired and whole class discussions.

Data Analysis

Using the levels of reasoning framework (Loong et al., 2018), the levels of justifying of 82 work samples were classified (Table 2). Some work samples were from pairs of students and some were individual student's work. They were analysed for the reasoning processes of justifying in line with the focus of the task. Evidence of students' reasoning actions of analysing and justifying were gathered from their written work and were analysed using the rubric presented in Table 1. The first author used the levels of mathematical reasoning to classify the levels of justifying based on the written work collected which was then checked and verified by the second author using multiple rounds of coding and checking the coding (Corbin & Strauss, 2008). Categories of example usage (random, ordered, systematic) were generated in line with the levels of reasoning for analysing of levels of reasoning framework in Table 2. Selected written work from the three participating schools will be presented and discussed to elucidate elementary school students' use of examples and counterexamples to test Nathan's conjecture.

RESULTS AND DISCUSSION

Different levels of justifying were identified among students' written responses to the task and the analysis of these work samples also revealed the analysing processes students used to test the conjecture. We found that there were students at each school whose reasoning was classified at each of the justification levels, except School A, where no Year 4 students demonstrated reasoning at a level higher than developing. In examining the developmental aspect of learning, we analysed evidence of students' justifying levels based on their written work and cross-tabulated it with the year levels (See Table 3). Some examples of students' responses to a conjecture and the levels of justifying are included in Table 4. More than half of the work samples indicated that students could not justify or they did not provide a coherent argument in responding to Nathan's conjecture. There were variations in students' analysing the conjecture that is, the process of exploring examples and counterexamples. Some used random approaches in their search for examples and counterexamples. Other students were more systematic when generating examples, for example increasing perimeter by changing the length and/or width, or keeping the perimeter or area the same to explore the area and perimeter of other rectangles. Some students were prompted by the teacher to use a table for a more systematic recording of their examples and counterexamples.

Table 3. Frequency of justifying levels for work samples grouped based on schools and year levels (n=82)

Schools	**Year levels**	**Levels of justifying**				
		Not evident	Beginning	Developing	Consolidating	Extending
A	4	3	10	1	0	0
B	5	13	12	9	5	0
	6	3	3	1	0	1
C	5	5	4	10	1	1
Total		24	29	21	6	2

Table 4. Frequency and examples of justifying levels for work samples (n=82)

Levels	Frequency (%)	Example of responses to Nathan's conjecture
Not evident	24 (29%)	Nathan is incorrect because [blank]
		Nathan is not correct because Matt proved that he is wrong by showing us a rectangle 50 by 1 so not for all cases. (Year 4, School A)
		Nathan is correct. Ms Bree showed me the answer and I wouldn't know the right answer (Year 5, School B)
		Making a graph helped me to understand how Nathan was wrong. (Year 5, School C)
Beginning	29 (35%)	We believe Nathan is correct and incorrect because different examples are different (Year 4, School A)
		Yes, I think Nathan is correct if the rectangle increases, so will the area and perimeter because the rectangle is getting bigger so will the number. (Year 5, School B)
		Nathan is correct because it always increases. It is not always the case because sometimes the perimeter can be small. (Year 5, School C)
Developing	21 (26%)	We believe Nathan is incorrect because we found an example that the area stays the same but the perimeter got larger. (Year 4, School A).
		I think Nathan is correct because if you increase the outside of a rectangle, it will always increase the inside of a rectangle. The rectangle will always be bigger, so perimeter and area will both get bigger (if might be different for a square. I don't know). (Year 6, School B)
		I learnt what area and perimeter and that the area is not always bigger than the perimeter. I also learnt what Geoboard was. (Year 5, School C)
Consolidating	6 (7%)	Nathan is incorrect because the area can stay the same even if the perimeter increases. The statement is true in some cases but not all. (Year 5, School B)
		With 12 blocks, a block of 6 by 2, the perimeter is 16cm and the area is 12cm^2. With the same amount of blocks, a block of 3 by 4, the perimeter is 14 cm. (Year 5, School B)
		I learnt that if you get a rectangle and increase both sides the area will increase, but if you decrease one side and increase the other side, the area will not increase. (Year 5, School C)
Extending	2 (3%)	I believe this statement is false. However, this is not true in every case. In A (a 6 cm × 4cm rectangle), the perimeter is 20 cm and the area is 24 cm^2. In B (a 23cm × 1cm rectangle), I increased the perimeter to 48 cm but the area decreased (23 cm^2). (Year 6, School B)
		It [the conjecture] does not work unless you add more cubes to the rectangle. We have learnt how to disprove mathematical hypothesis by testing its area and perimeter. (Year 5, School C)

The work samples are classified as *Not evident* level of justifying when students did not offer justification or appealed to authority or others in their justification. As an example, one student provided a justification stating: "Nathan is incorrect because Mathew proved he was wrong" (Table 3, row 1). This response indicated that these students wrote their justification after the presentation by other students at the end of the lesson. The distinguishing features between *Beginning* and *Developing* levels of justifying are characterized by the fact that the work at the beginning level seemed to focus mainly on extending both sides of the rectangles as they explored the example space (see Table 4, row 2). This led them to arrive at the conclusion to support Nathan's conjecture. Work samples categorized as beginning failed to recognize other ways of increasing perimeter to generate cases where a counterexample could be found. The work samples that were classified as *Developing* showed evidence that they could generate examples to verify Nathan's conjecture and they could also find counterexamples to refute Nathan's conjecture (see Table 4, row 3 and Figure 2). However, they did not arrive at the logical conclusion that a counterexample refutes Nathan's conjecture.

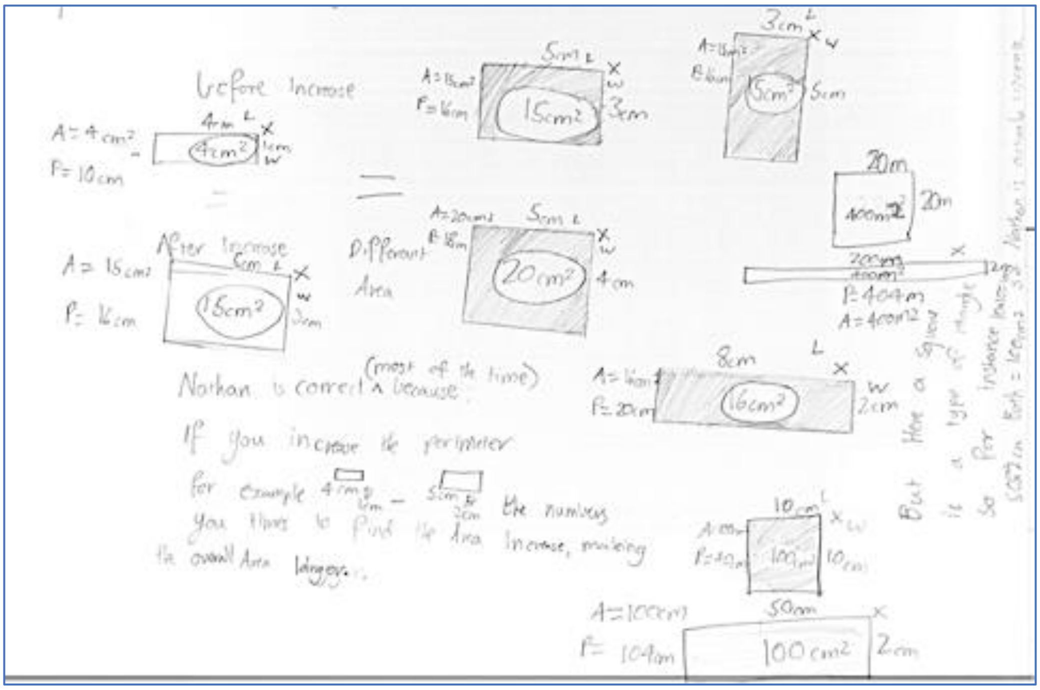

Figure 2. An illustrative work sample exploring area and perimeter of rectangles and squares (Yung Qi, Donald, &, Heather, Year 5, School B)

For instance, in Figure 2 the pair of students generated 10 rectangles. They initially compared 4 cm × 1 cm, 5 cm × 3 cm and 5 cm × 4 cm rectangles to show that Nathan was correct: "Nathan is correct because if you increase the perimeter for example [rectangle] 4 cm & 1 cm [and rectangle] 5 cm & 3 cm, the numbers you times to find the area increase, making the overall area bigger" (Yung Qi, Donald, &, Heather, Year 5, School B). They continued to search for examples to include an 8 cm × 2 cm rectangle with a perimeter of 20 cm and an area of 16 cm^2 but did not yet realize that this provided a counterexample when compared with the 5 cm × 4 cm rectangle. However, they continued their exploration and found a few counterexamples. They compared a 200 cm × 2 cm rectangle with a 20 cm × 20 cm square and a 10 cm × 10 cm square with a 50 cm × 4 cm rectangle. They changed their conclusion about the conjecture by including a qualification: "Nathan is correct (most of the time) because…" (Yung Qi, Donald, &, Heather, Year 5, School B). The fact that they found more than one counterexample might indicate that they did not realize that one counterexample would be sufficient to refute Nathan's conjecture.

As the previous work sample shows, while some students could find counterexamples, it was quite challenging for them upon finding counterexamples to refute a conjecture. This was evident in a work sample that was classified as *Developing* (Figure 3) where a systematic search for examples started by finding another rectangle with a longer length but a shorter width compared to the first rectangle. The third rectangle had the same area as the second rectangle with a larger perimeter. The students were able to identify this as a counterexample as evidenced by the asterisk. They went a bit further by finding another counterexample (marked by an asterisk), the fourth rectangle with a smaller area but a larger perimeter compared to the second rectangle. However, the reflection of what they have learnt did not show a coherent argument in response to Nathan's conjecture:

> That the area is not always bigger then [than] the perimeter… In some mathamaitals [mathematical] minet [minds] says it is bigger but in some ways it is not bigger…I in my way think that the area can be bigger but sometimes not. (Sarah & Lily, Year 5, School C)

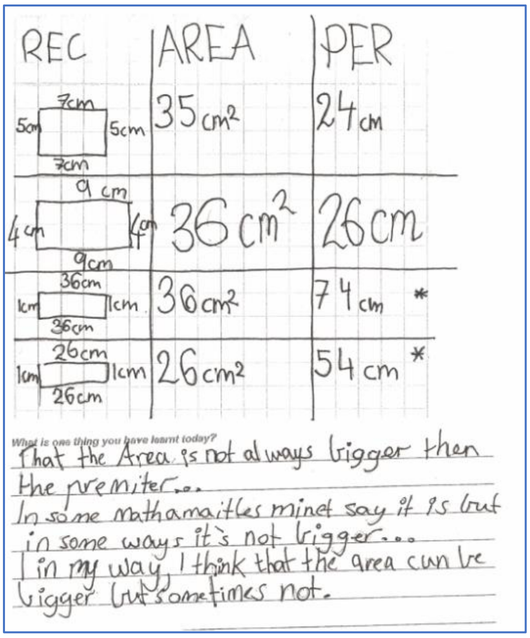

Figure 3. An illustrative work sample of developing level of justifying (Sarah & Lily, Year 5, School C)

The *Consolidating* level is characterized by evidence of a correct logical argument in refuting Nathan's conjecture. A work sample at the Consolidating level of justifying (Figure 4) showed evidence of a systematic search for examples and a clearer explanation about the process to reach a conclusion as recorded in their justification. "I learnt that if you get a rectangle and increase both sides the area will increase but if you decrease one side and increase the other side, the area will not increase". While this argument is logical and correct, it does not meet the requirement of a sound logical argument as decreases of either the width or length whilst increasing the other side do not always result in smaller area, as is claimed.

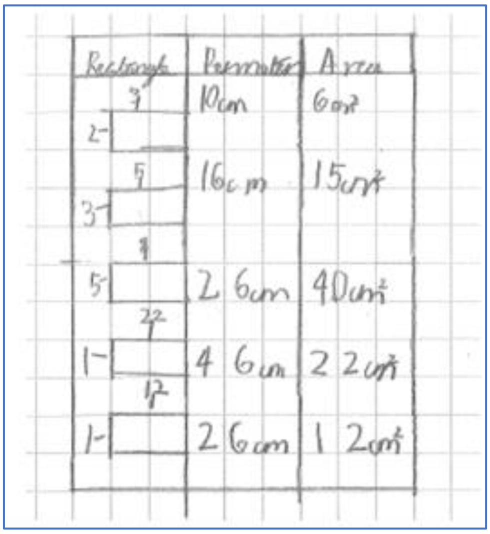

Figure 4. An illustrative work sample of consolidating level of justifying (Cody, Jay, & Neo, Year 5, School C)

Lastly, the *Extending* level is characterized by evidence of a sound logical argument in refuting Nathan's conjecture. A work sample of Extending level of justifying (Figure 5) showed evidence of a systematic search whereby students kept the area of the rectangles constant but altered the dimensions of the rectangles. As a result, they found different perimeters. Their justification "It does not work unless you add more cubes to the rectangles" (Archie & Scott, Year 5, School C) suggested that the students realized the power of counterexamples to refute a conjecture and a different outcome if they did not keep the area constant. They have identified that they have learnt "How to disprove mathematical hypothesis by testing its area and perimeter" (Archie & Scott, Year 5, School C). Hence, they have demonstrated evidence of an argument at the Extending level.

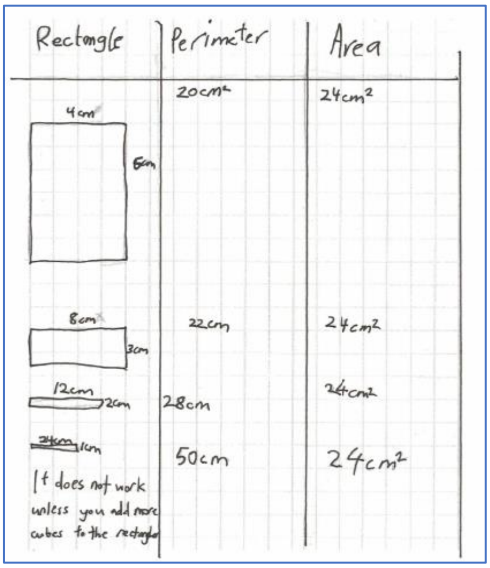

Figure 5. An illustrative work sample of Extending level of justifying (Archie & Scott, Year 5, School C)

The analysis of 82 work samples from Year 4-6 students revealed that refuting a conjecture using a counterexample was challenging for students at all year levels and that it requires a higher level of justifying compared to verifying truth of statements by using a common property, rule or known facts that confirms each case. In the reasoning framework (Loong et al., 2018), refuting a conjecture using a counterexample was classified as *Developing*. Findings from this study contend that it is necessary to revise the framework and include different levels for the way in which students use counterexamples as an argument to test and refute conjectures. The framework was subsequently changed so that "refutes a claim using a counter example" was included at the consolidating level in the Assessing Mathematical Reasoning Rubric (AAS, 2020) as shown in Table 5.

Our argument to reconsider the level of justifying in relation to refuting a conjecture using a counterexample is based on the perspective that the justifying process should be perceived not only as a *disciplinary practice* (Davis & Hersch, 1981; Lakatos, 1976) but it is important to also emphasize justifying as a *learning practice* (Cohen & Ball, 2001; Staples et al., 2012; Staples, 2014). Furthermore, Staples (2014) argued that "What "counts" as a justification is locally defined, and the nature of justification activity is locally constituted in the classroom through engagement of the members of the community. Hence we argue that 'refuting a conjecture using a counterexample' should be classified at the higher level of 'Consolidating' instead of 'Developing'. We posit the challenges evident in students' work samples relates to complexities related to aspects of proof, that needs to be a focus in the classroom when students communicate their argument (Stylianides & Ball, 2008).

Table 5. Assessing Mathematical Reasoning Rubric: Developing and Consolidating levels (Source: AAS, 2020; Loong et al., 2018)

	Analysing	**Generalising**	**Justifying**
Developing	• Notices a common property, or sorts and orders cases, or repeats and extends patterns • Describes the property or pattern.	• Generalises: communicates a rule (conjecture) using mathematical terms, and records other cases or examples.	• Attempts to verify by testing cases, and detects and corrects errors or inconsistencies. • Starting statements in a
Consolidating	• Systematically searches for examples, extends patterns, or analyses structures, to form a conjecture. • Makes predictions about	• Generalises: communicates a rule (conjecture) using mathematical symbols and explains what the rule means or explains how the rule	• Verifies truth of statements by confirming all cases or **refutes a claim by using a counter example.** • Uses a correct logical

Year 4-6 elementary students involved in this study were not accustomed to testing conjectures that were not true or realising that they only need to generate one counterexample to disprove a mathematical statement such as for Nathan's conjecture. Justification recorded in students' work samples at the 'Developing' level suggested that these students did not realize that a mathematics statement cannot be both true and false at the same time (Stylianides & Al-Murani, 2010; Zaslavsky & Ron, 1998; Zazkis & Chernoff, 2008). The analysis of the written work samples suggested that most students used a random strategy to generate examples that supported Nathan's conjecture. One approach to address the challenge of a widespread overreliance on examples as justification is "to help students to understand the limitations of examples (Sowder & Harel, 1998; Zaslavsky, Nickerson, Stylianides, Kidron, & Winicki, 2011).

One of the teachers in School C provided their students with tiles and asked them to work with the tiles to generate different rectangles after realizing that many of their students would benefit from exploring the conjecture using concrete manipulatives. This teaching action is consistent with findings from earlier studies (Chen & Herbst, 2013; Komatsu, 2010; Lin & Tsai, 2016; Schifter, 2009) about the importance of using appropriate modes of argumentation and choice of representations such as concrete manipulatives in elementary school particularly to reason and communicate justification effectively.

The findings from this study does show that directly challenging students' misconceptions of the relationship between area and perimeter provided students with the opportunity to develop some understanding of the relationship. Whilst only a few students rejected the conjecture based on counter examples, about a third of students in this study did find counterexamples and realized that the conjecture did not work for all cases. The different approaches used by teachers to introduce the problem did influence students' strategies for generating the examples and counter examples and comparing and contrasting. For example, the use of concrete materials such as tiles and digital technology such as *Geoboard* and *Show me* supported students to investigate and justify their reasoning. The lesson materials included examples of prompts for teachers to use in their interactions with students such as "Is it always true or just sometimes true?" and enabling prompts such as "Have you searched for examples that show Nathan is not correct" (AAS, 2020)? However, it is evident that teachers were not experienced in providing prompts to challenge the students' misconceptions about the relationship between area and perimeter that persisted in the approach that students took to generate and compare and contrast their examples.

In examining elementary school students' use of examples and counterexamples in testing a conjecture using only written work, we realized the limitation of only using written arguments to classify their justifying levels. Campbell, King, and Zelkowski (2020) compared written and oral arguments of 47 Year 8 students who worked in groups to solve proving tasks. They found that students' oral arguments often were at the higher level than their written arguments. Their finding concurred with Soto-Johnson and Fueller (2012) who recommended the potential benefit of student audio-recording their oral reasoning to improve the quality of their written reasoning. We acknowledge this as a limitation of this study as we did not capture recording of students' oral reasoning. As described when discussing the response from the student who said that they had been convinced by another student that the conjecture was not true, it is therefore possible that their justifying may have been more sophisticated as they orally argued about the significance of counterexamples.

CONCLUSION

Testing a conjecture about the relationship between the perimeter and area of a rectangle was challenging for most students in this study. The majority of students confirmed the misconception that a rectangle with a larger perimeter will also have a larger area using examples as evidence, even though many of these students did find counterexamples. About a third of the students did make a qualifying argument about this conjecture but only a few students were able to provide a logical argument using a counterexample to reject the conjecture. Rubrics for assessing reasoning need to include this trajectory of understanding the use of counterexamples when justifying. It needs to include identifying counterexamples, qualifying conjectures using counterexamples and refuting conjectures using counterexamples.

Prompts to support systematic exploration of examples and to challenge students to search for counterexamples were provided for teachers but it is not clear that all the teachers were prepared to use prompts to address students' misconceptions about the relationship between perimeter and area as their student generated examples. This means that teachers need to understand the relationship between area and perimeter and use of a counterexample to refute a conjecture, as particular to mathematical argumentation, if students' misconceptions are to challenged and argumentation developed. Many of the opportunities for developing reasoning proficiency in the elementary mathematics curriculum are focused on making arguments about common properties or relationships (generalizations). More opportunities are needed for students to identify what is different, and to appreciate that in mathematics using a counterexample is an acceptable means of disagreement. The teachers in the three schools in this study presented the task and supported their students differently. Further research of the role of teacher knowledge and teacher actions when introducing the task, supporting students to explore and to encourage argumentation during orchestrated whole class discussion is needed to develop coherent approaches for developing students' use of counterexamples.

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STUDENTS' GROWING UNDERSTANDING IN SOLVING MATHEMATICS PROBLEMS BASED ON GENDER: ELABORATING FOLDING BACK

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Abstract

Students' previous knowledge at a superficial level is reviewed when they solve mathematical problems. This action is imperative to strengthen their knowledge and provide the right information needed to solve the problems. Furthermore, Pirie and Kieren's theory stated that the act of returning to a previous level of understanding is called folding back. Therefore, this descriptive-explorative study examines high school students' levels of knowledge in solving mathematics problems using the folding back method. The sample consists of 33 students classified into male and female groups, each interviewed to determine the results of solving arithmetic problems based on gender. The results showed differences in the level of students' understanding in solving problems. Male students carried out the folding back process at the level of image having, formalizing, and structuring. Their female counterparts conducted it at image-making, property noticing, formalizing, and observing. Subsequently, both participants were able to carry out understanding activities, including explaining information from a mathematical problem, defining the concept, having an overview of a particular topic, identifying similarities and differences, abstracting mathematical concepts, and understanding its ideas in accordance with a given problem. This study suggested that Pirie and Kieren's theory can help teachers detect the features of students' understanding in solving mathematical problems.

Keywords: Characteristics, Folding Back, Gender, Mathematical Problems, Understanding

Abstrak

Pengetahuan siswa sebelumnya pada tingkat dangkal ditinjau ketika mereka memecahkan masalah matematika. Tindakan ini sangat penting untuk memperkuat pengetahuan mereka dan memberikan informasi yang tepat yang dibutuhkan untuk memecahkan masalah. Teori Pirie dan Kieren menyatakan bahwa tindakan kembali ke tingkat pemahaman sebelumnya disebut *folding back*. Oleh karena itu, penelitian deskriptif-eksploratif ini mengkaji tingkat pengetahuan siswa SMA dalam menyelesaikan masalah matematika dengan menggunakan metode folding back. Sampel terdiri dari 33 siswa yang dikelompokkan menjadi kelompok laki-laki dan perempuan, masing-masing diwawancarai untuk mengetahui hasil pemecahan masalah aritmatika berdasarkan jenis kelamin. Hasil penelitian menunjukkan adanya perbedaan tingkat pemahaman siswa dalam menyelesaikan masalah. Siswa laki-laki melakukan proses *folding back* pada level *image having*, *formalising*, dan *structuring*. Rekan-rekan perempuan mereka melakukannya pada level *image-making*, *property noticing*, *formalizing*, dan *observing*. Selanjutnya kedua peserta mampu melakukan kegiatan pemahaman, antara lain menjelaskan informasi dari suatu masalah matematika, mendefinisikan konsep, memiliki gambaran umum tentang topik tertentu, mengidentifikasi persamaan dan perbedaan, mengabstraksikan konsep matematika, dan memahami ide-idenya sesuai dengan yang diberikan. masalah. Penelitian ini menyarankan bahwa teori Pirie dan Kieren dapat membantu guru mendeteksi ciri-ciri pemahaman siswa dalam memecahkan masalah matematika

Kata kunci: Karakteristik, *Folding Back*, Jenis kelamin, Masalah Matematika, Pemahaman

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Problem-solving is an essential field in mathematics consisting of numerous requirements. In recent years, great attention has been paid to this field in education. The problem-solving process has always been the primary and fundamental area of study since the early 1980s (Schoenfeld, 2007; Bayat & Tarmizi, 2010). Its significance has been recognized at the international levels (NCTM, 2000). Problem-solving is the most significant cognitive activity in everyday life (Jonassen, 2000; Verschaffel et al., 2020). In addition, it is a cognitive process that requires a solution to a certain problem (Sweller, 1988; Holyoak, 1990; Jonassen, 2003; Düşek & Ayhan, 2014). Furthermore, this process is closely related to understanding students' concepts to solve the problems at hand and the basis of the associated mathematical concepts (Pape & Tchoshanov, 2001; Stylianides & Stylianides, 2007). Conversely, students' inability to understand mathematical concepts makes it difficult to solve problems (An, Kulm, & Wu, 2004). Therefore, they must have adequate understanding to solve problems, particularly regarding the resolution of those requiring 'understanding.'

Based on classroom observations, some students experience inconsistencies with problem-solving activities. They have difficulty restating and presenting concepts in various forms of mathematical representation. Students' inability to solve mathematical problems indicates they do not have an adequate understanding of the subject. Their inability to solve problems indicates that the implication of problem-solving in mathematical learning is not well educated.

One of the prominent factors that support problem-solving in practice is understanding and NCTM (2000) emphasized its importance as a fundamental aspect of learning mathematics. The process of studying to understand has become an overwhelming priority among educators and psychologists, as well as one of the most critical targets for students in all subjects because it is physically more rewarding and practical (Stylianides & Stylianides, 2007; Skott, 2019). Theoretically, the understanding is defined as a growth process that is complete, dynamic, layered, continuous, and not linear (Pirie & Kieren, 1994; Pirie & Martin, 2000). It is also a passionate and organized process needed to abstract mathematical concepts based on the properties that emerge and build new knowledge from previous experiences.

This study utilized the Pirie-Kieren theory and the associated model, which are well-established and recognized theoretical perspectives on the nature of mathematical understanding to understand growth (Pirie & Kieren, 1994; Martin & Towers, 2016). According to Martin (2008), this theory emphasizes the integration of mathematical understanding in more localized ways, such as intuitive ideas, concrete representations, specific aspects of action, as well as acts of generalization, formalization, and the repetition of less complex understandings. The Pirie-Kieren theory provides an insight into how knowledge is organized and reorganized, as well as the strategies used by learners to reflect upon and build on their understanding accordingly. The growth of understanding in this theory is a dynamic, active process that involves development and action. This involves a constant move among different levels of thought without the involvement of a straight-like system (Pirie & Kieren, 1994). The act of re-examining the existing understandings and ideas of a mathematical concept is called "folding back" in the Pirie-Kieren theory, which is the focus of this study.

In the problem-solving process, folding back the way in which learners work with and build on existing knowledge offers a potentially powerful tool to follow and characterize the process by which mathematical comprehension emerges and develops. However, there was a lack of substantial evidence showing how and why folding back occurred and its relationship with subsequent mathematical activities (Martin, 2008). Therefore, this study aims to closely explore the concept and nature of folding back, elaborate on the phenomenon, and understand more fully the part played by action in the development of mathematical understanding.

The Pirie-Kieren theory contains 8 potential levels of action to describe an individual's development of understanding and to describe a particular concept. Those levels are called Primitive Knowing, Image Making, Image Having, Property Noticing, Formalizing, Observing, Structuring, and Inventising (Pirie & Kieren, 1994; Pirie & Martin, 2000; Thom & Pirie, 2006; Martin, 2008; Martin & LaCroix, 2008). The eight levels provide a theoretical model or two-dimensional diagram, and each level includes all prior layers to emphasize the integrated nature of mathematical understanding. These levels are further elaborated as follows (1) primitive knowing, is the process of growing students' understanding of mathematical concept, (2) image-making, a level that enables students to have an understanding based on previous knowledge of mental and physical actions, (3) image having, a stage where students use mental images on a topic without taking specific actions that lead to the topic, (4) property noticing or the manipulation of a topic aspects to form related properties, (5) formalizing, enables abstract concept possession based on existing properties, (6) Observing, supports formal activities coordination to use them for the problem at hand, (7) structuring, a phase that facilitating students to relate the relationship between one theorem to another and prove it based on logical arguments, and (8) inventising, a period which signified by a structured and complete understanding, with the ability to create questions and grow into a whole new concept.

Folding back is the technique used by students to review their previous knowledge at a superficial level. This process helps them solve various mathematical problems (Gülkılık, Uğurlu, & Yürük, 2015; Yao & Manouchehri, 2020). Martin, Lacroix, and Fownes (2005) stated that folding back is an integral part of the learner's mathematical understanding, which helps students to develop the right knowledge that fits their task. It folds back the source, form, and outcome to expand students' mathematical understanding (Martin, 2008). According to Slaten (2010), students that fold back understand the development of mathematical concepts appropriately.

The description showed that folding back is essential in the growth of student understanding because it expands, sharpens, and strengthens their knowledge of the material while providing information that can be used to solve mathematical problems. Furthermore, this process allows students to renew their understanding and even replace their knowledge with new versions relevant to the math problem. Sagala (2017) stated that the structure of understanding the concept of derivative functions of student pre-service mathematics is based on gender and in accordance with Pirie & Kieren's (1994) theory. The result indicated that female and male subjects understood the basic knowledge layer in accordance with the folding back theory of Pirie-Kieren. Another essential aspect in the folding back theory is gender differences, which affect practical and theoretical issues in learning and solving math problems.

Over the last couple of decades, numerous studies have been conducted to solve mathematical problem resolution, considered an important factor in gender differences in education (Zhu, 2007). The meta-analyzes from 100 studies indicate that gender differences in mathematical performance of females in high school were minor (Royer et al., 1999; Gallagher et al., 2000). Multiple factors like cognitive ability, processing speed, styles of learning, and socialization contribute to gender differences in mathematical problem-solving. However, the contributions of some factors are still in doubt and are only applicable in some specific areas (Royer et al., 1999). Therefore, based on these findings, the authors can assume that female and male have various mathematical problem-solving patterns built on a multi-step approach. Furthermore, with standardized testing, students can come up with a correct solution by selecting and combining a set of appropriate strategies.

In problem-solving, boys are seen to return to performing more image-making and are confronted with problems while working with sophisticated mathematics (Pirie & Kieren, 1994; Martin, 2008). Therefore, they fold back to a lower level of activity to extend their overall and formal understanding. However, the procedures not offered by the analysis and the framework developed in this study provide is a detailed examination of why and how females fold back and how their actions in the lower level could facilitate their continuous work, thereby increasing understanding. Therefore, the variety of results is an essential reason for conducting gender-related study.

Some of the literature described above indicates the diversity of examining gender-related concepts in solving mathematical problems. However, this study was designed to explore the characteristics of the level of understanding used by senior high school students to solve mathematical problems based on gender. In particular, it focuses on the "folding back theory” originally developed by Pirie & Kieren (1994).

METHOD

This is a descriptive-explorative study designed to explore the characteristics of high school students’ understanding of mathematical problems that focused on "folding back." The purposive sampling method was used to obtain data from 33 students at Public Senior High School, Bone, South Sulawesi, Indonesia. The students were grouped based on their gender and told to complete the Mathematical Ability Test question. Furthermore, to explore each group's characteristics, both participants were instructed to solve arithmetic sequences. Afterward, a task-based interview was conducted with a student from each group. They were both selected because (1) they both fulfill mathematical ability test results for the criteria based on the Minimum Exhaustiveness Criteria standard ≥ 75, (2) have good and qualified communication skills, and (3) are ready to participate in the study.

The questions used as a Mathematics Ability were adapted from the UN examination bank for the 2019/2020 high school year, which was modified into a description of 5 items by observing the process of students' understanding level (focus folding back) and recording interviews. This process also consists of 3 open-ended questions, which were used to explore students' understanding in solving mathematical problems (arithmetic). The instrument was also tested for validity and reliability to validate the questions, and 2 mathematicians and an educational expert carried out interview sheets. The instrument's validity criteria included the feasibility of the test questions, content, language, and appropriate instructions, which were used to reveal the understanding level process of student high school. Furthermore, these results were used to instruct participants on mathematical problems, such as arithmetic sequences. The pattern is based on the addition or subtraction of operations using fixed patterns. Therefore, it is very suitable to be used to explore the growth of students' understanding, as shown in Figure 1.

Given the quadratic equation $x^2 - 7x + (r + 2) = 0$, which has roots p and q, with $r \in R$. If p, q and r form an arithmetic sequence, then what is the arithmetic sequence!

Figure 1. Arithmetic Sequences Problems

To analyze data, each participant was thoroughly observed, based on their growing understanding of solving problems. Furthermore, the triangulation process was carried out to verify the data collected through interviews. This process was also used to confirm the findings of students' answers, which were coded as S (Students) and R (Researcher). Conclusively, the results of the folding back at each growth understanding of the 2 students' in solving mathematical problems were also summarized.

RESULTS AND DISCUSSION

Among the 33 students that carried out the Mathematics Ability Test, 7 comprising 2 male and 5 female had a score of ≥ 75. Among the 7 prospective potential participants that achieved these criteria, 1 male and female candidate with relatively similar ability were selected from the mathematical ability and gender. Furthermore, 26 students comprising 10 males and 16 females had a score of <75. The following are the interview results with 2 participants, namely Male Students (MS) and Female Students (FS), to obtain more information on the folding back at each level of student understanding growth.

Folding Back in Solving Mathematical Problems Process of MS

Primitive Knowing Level of MS

MS understood the given mathematical problem and provided detailed information, including known problems and commonly asked questions. Excerpts from interviews by MS on the level of primitive knowing are as follows.

R : What did you do after given the mathematical problems?
MS : I read the problem first to determine the information.
R : What is the information you got from the mathematical problem?
MS : Given that $x^2 - 7x + (r + 2) = 0$ is a quadratic equation with roots p and q,

where r∈R with p, q, and r are used to form an arithmetic sequence.
R : Why don't you write everything down on the answer sheet?
MS : I will do that right away, it actually skipped by memory

MS read the mathematical problem given in advance to determine the associated information without writing the answers to what is known and asked on the answer sheet. In addition, they understand the given mathematical problem due to its ability to explain the information. Therefore, by understanding, they can identify the information presented. However, without writing the information obtained during the problem-solving phase, the understanding activity carried out by male students is the ability to define concepts verbally based on the previous knowledge (Codes et al., 2013; Martin, 2008). At this level, there is no folding back activity because primitive knowing here does not imply low-level mathematics, rather it is the starting place for the growth of any particular mathematical understanding (Pirie & Kieren, 1994).

Image Making Level of MS

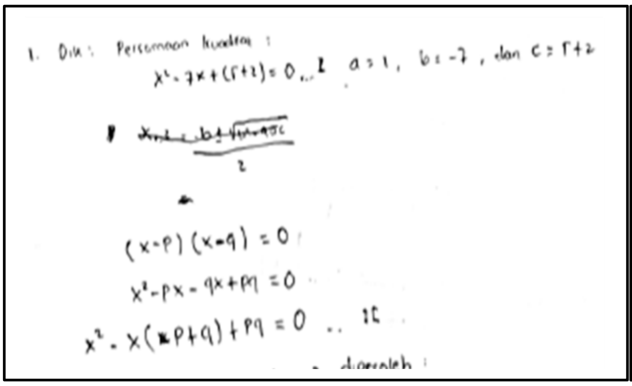
Quotations from the interview by MS regarding the level of image-making are as follows:

R : What is your opinion on quadratic equations?
MS : The general form of a quadratic equation is $ax^2 - bx + c = 0$ where $\alpha \neq 0$, where the highest power is 2, and the root is determined by factoring ABC.
R : What was it like?
MS : The form of factoring is $(x + x_1)(x + x_2)$ while the ABC formula is $x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.
R : What do you know about arithmetic sequences?
MS : The form of the arithmetic sequence is $U_1, U_2, U_3, \ldots, U_n$, each adjacent term has the same difference obtained using the following formula, $B = U_n - U_{n-1}$.

MS stated that the general form of quadratic equations and ways to determine the roots is by using the factoring method and the ABC formula. This is in addition to the general form of the arithmetic sequence and the conditions. They specified the arithmetic sequence difference formula and described a concept based on prior knowledge. Some of the procedures used in understanding activities by MS are developing specific ideas, making conceptual images, combining factoring methods, and using the ABC formula to solve arithmetic problems based on possessed knowledge. The understanding level shows that MS can make distinctions in the previous knowledge and use it in new ways. Its growth level in mathematical understanding strengthens Pirie & Kirien's theoretical model, especially at the image-making level (Gulkilik et al., 2020; Martin, 2008).

Image Having Level of MS

Figure 2 is a quotation of interview results carried out by MS on the level of an image having.



Translate Version

Asked: Quadratic equation?

$x^2 - 7x + (r + 2) = 0$, a = 1, b = -2, and c = r + 2.

$x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(x-p)(x-q) = 0

$x^2 - px - qx + pq = 0$

$x^2 - x(p + q) + pq = 0$ ……(II)

Figure 2. Problems Solving Activities Image Having Level by MS

R : Apart from the ABC formula, is there any other way to determine the roots of a quadratic equation?

MS : Yes, it can also be determined using the factoring method, $(x + x_1)(x + x_2) = 0$, although I utilized the ABC formula.

R : Why?

MS : At first, I calculated $b^2 - 4ac$ and got $(49 - 4r + 8)$, and due to my inability to obtain the root, I crossed the equation.

R : After crossing out the ABC formula, what did you do and think?

MS : I re-read the problem to determine what I know, and it turned out that the equation $x^2 - 7x + (r + 2) = 0$ already has roots p and q. I tried to think of another way to determine the roots and remembered solving a similar problem using quadratic equations.

MS explained the initial steps used to solve mathematical problems by determining the roots of the quadratic equation using the ABC formula. Furthermore, they utilized the folding back to the primitive knowing level to procedure and re-read mathematical problems and examine known procedures. The folding back result carried out by MS is to remember that the problem is like the given mathematical equation. Hence, they concluded that when a quadratic equation has roots such as p and q, the new formula can be searched using the sources, which shows that male students already had a mental picture of the topic. Therefore, by understanding, MS solves arithmetic sequence problems by folding back to primitive knowing based on Pirie & Kirien's theory (Martin, 2008). At the image having level, students use a mental image of a case without taking specific actions that lead to the topic. This means they have an idea of the concept through activities conducted at the previous level (Gokalp & Bulut, 2018; Gulkilik et al., 2020).

Property Noticing Level of MS

Quotations from the interview by MS regarding the level of property noticing are as follows.

R : You earlier stated that one way to determine the roots of the quadratic equation is by using the factoring method. Meanwhile, you used the ABC method. Is it the same?

MS : Yes, both methods are similar with different answers. This is because when the quadratic equation is determined using the factoring method, the roots will be obtained. However, supposing the roots are known, then the quadratic equation is obtained as shown in this study.

MS described the difference in using the factoring method based on what is known and stated that when the quadratic equation is determined using the factoring method, the roots are easily obtained. However, when the seeds are known, the quadratic equation is obtained as shown in the above excerpt. These results indicate that they achieved the property noticing level by checking for the similarity and difference of these descriptions and related to specific mathematical sentences. Students are able to recognize the properties of the different concepts that have been learned at the noticing level by having images. The activities carried out by MS are in accordance with the theory of mathematical understanding (Martin, 2008; Yao & Manouchehri, 2020). At this level, students can also notice the distinctions, combinations, or connections between multiple mental images. However, they do not conduct the folding back activities at this level. In mathematical understanding, there are 2 phases of folding back, the first is from image Having to Making and the second from property Noticing to Image Making (Pirie & Kieren, 1994; Thom & Pirie, 2006).

Formalising Level of MS

Following interviews with MS.

R : Educate me on the steps you used to determine the new quadratic equation!

MS : I used the factoring method, namely $(x - x_1)(x - x_2) = 0$, because the roots, which comprise p and q, are known, whereby $x_1 = p$ and $x_2 = q$. After that, I replaced x_1 and x_2 with p and q, to get $(x - p)(x - q) = 0$, then substituted the equation to become $x^2 - px - qx + pq = 0$, before simplifying it to obtain $x^2 - x(p + q) + pq = 0$.

R : I see that you did not immediately continue here (the data referred to is p, q, r forming an arithmetic sequence). Why?

MS : I had no idea on the procedure to utilize ma'am.

R : So, what did you do?

MS : I re-read the problem and realized that $x^2 - 7x + (r + 2) = 0$ has roots, namely p and q. Where p, q, and r form an arithmetic sequence.

MS used the factoring method to substitute the process's assumptions and the multiplication operations to obtain a new quadratic equation. This activity showed that they abstracted a mathematical concept based on related problems (Pirie & Kieren, 1994). However, at this level, male students had a hard time continuing their work, which led to folding back to rediscover the quadratic equation with p, q, and r as its roots in an arithmetic sequence. At this level, MS's folding back action contradicts Pirie & Kieren's theory of mathematical understanding. At the formalising level, the students are able to think consciously on the generalised properties and work with the concept as a formal object, without specific reference to a particular action or image (Pirie & Kieren, 1994; Pirie & Martin, 2000; Martin, 2008). When formalizing, students abstract the mathematical characteristics or properties of the image and create a concept written into a formal definition or algorithm. Therefore, at this level, students generalize statements on an idea and develop common concepts that are similar to the mathematical definition (Gulkilik et al., 2020).

Observing Level of MS

Following interviews with MS.

R : Please explain the factoring method you used!

MS : Here $q^2 - 4q - 5 = 0$, Therefore, I looked for numbers that tend to produce -5 when multiplied and -4 when added, namely $(q - 5)(q + 1) = 0$, q = 5, and q = -1.

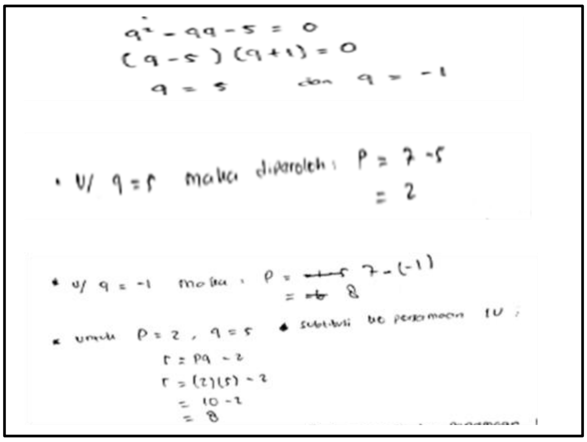
R : After getting a score of q, what did you do?

MS : I substituted the value of q into equation 3 $(p = 7 - q)$. For q = 5, it became $p = 7 - 5$, therefore, the p score equals 2. Whereas for q = -1 it became $p = 7 - (-1)$), hence the p score equals 8.

R : OK, how did you get the r score?

MS : I utilized the same procedure in determining p score, by substituting the p and q scores in equation 4, as follows $r = pq - 2$. For p = -2 and q = 5, $r = (2)(5) - 2$ equals r = 10-2, therefore r equals 8.

MS carried out the factoring process by thinking of numbers that correspond to the quadratic equation owned when multiplied and added together. The factoring method used shows that MS has a q score, therefore, it links the mathematical concepts with the problem at hand. MS substituted the q score with 3 to obtain a p score in the 4 equations to get an r score. Therefore, MS reached the level of observation where students are able to coordinate and use formal activities to solve problems (Figure 3).



Translate Version

$q^2 - 4q - 5 = 0$

$(q - 5)(q + 1) = 0$, q=5 and q=-1

For q=5, then obtained, p=7-5 = 2

For q=-1, then, p=7-(-1) = 8

For p=2, q=5, Substitution to equation IV

r = pq – 2

r = (2)(5) – 2 = 8

Figure 3. Activities Observing Level by MS

Furthermore, at the observation level, students are able to make formal statements on mathematical concepts and determine algorithm or theorem patterns (Gulkilik et al., 2020). Moreover, students at this level are able to observe, structure, and organize personal thought processes and recognize the ramifications of problem-solving. This activity is in accordance with the theory of mathematical understanding by Pirie & Kieren (Pirie & Martin, 2000).

Structuring Level of MS

The following interview was conducted with MS.

R : Okay, why did you cross this out? (($r = (-6)(-2) - 2 = 10$)

MS : At the beginning, I got the second p-score which was equal to -6, then I substituted it for the equation $r = pq - 2$, to obtain an r-score of 10.

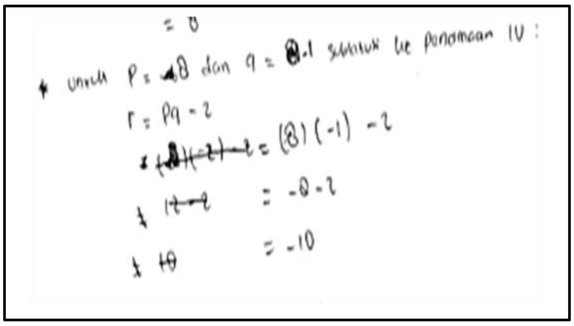
R : So, how did you find r?

MS : After, determining the various values of p, q, and r which equals 2, 5, and 8 in the first rows and -6, -1, and 10 in the second, I used the different arithmetic sequence formula to determine the difference in the first and second sequence. These are -1, -6, and -7, while the difference between the third and the second term is 10 - (- 1) = 11.

R : After you got the difference, what did you do?

MS : I used the factoring equation to determine the q and p scores. However, there was an error in the second p score, I wrote $p = -1 - 5$, which culminated in 6? Therefore, the difference between the second term and the first is not similar to the third and second.

MS made a mistake in creating a substitution; hence a write-off was found on the worksheet with folding back carried out on the observed level using the factoring method. This process was used to determine the error after checking the arithmetic sequence obtained using the difference formula. The activity further showed that male students make logical formal observations and verify previously developed ideas, presented in Figure 4.



Translate Version
For P = 8 and q = -1, substitution to equation IV
r = pq – 2
= (8) (-1) – 2 = -8 – 2
= - 10

Figure 4. Activities Structuring Level by MS

They are also able to link the relationship between one theorem and another at the structuring level and prove it based on rational arguments (Thom & Pirie, 2006; Martin & Towers, 2016). MS checked the factoring and p-score obtained previously and determined an error in substituting the second p. This error causes the difference between the 2 adjacent terms in the second arithmetic sequence, thereby indicating that MS Races its thought process into an axiomatic structure (Gülkılık, Uğurlu, & Yürük, 2015; Gulkilik et al., 2020).

Based on the interview results, an understanding map was created, as shown in Figure 5, which confirms that MS conducted the folding back process 3 times at the level of mathematical understanding growth.

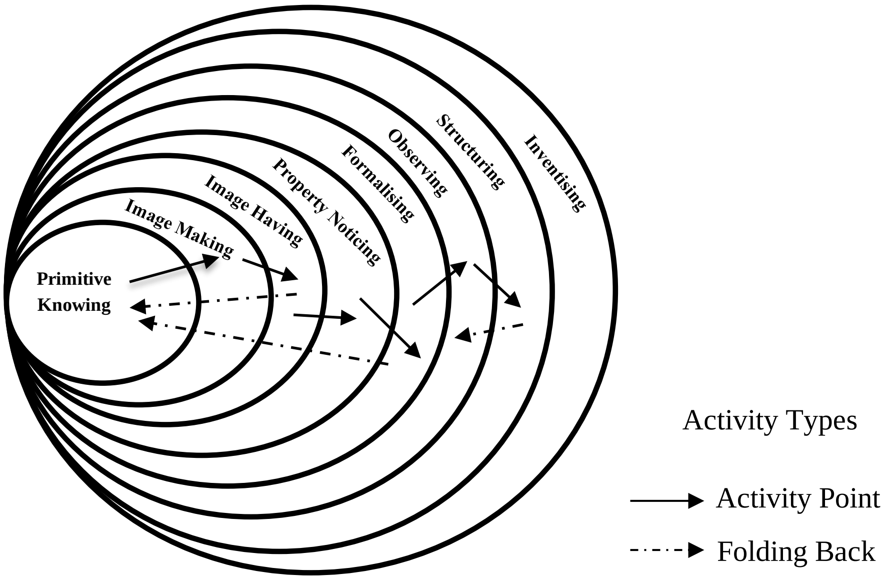


Figure 5. Folding Back in Solving Mathematical Problem of MS

Figure 5 showed the folding back activity of MS at the level of mathematical understanding. This activity suggests that students tend to return to a lower level of understanding when faced with problems that cannot be solved immediately. It also depicts 3 folding back phases, with the first from image Having to Primitive Knowing, followed by Formalising to Primitive Knowing, and the last is from Structuring to Observing. The outcome of folding back is that male students have the ability to expand their current inadequate and incomplete understanding by reflecting on and rearranging their former concepts. They can also achieve this by generating and creating new images, supposing the existing constructs are not sufficient to solve the problem.

Folding back in Solving Mathematical Problems Process of FS

Primitive Knowing Level of FS

The FS explained all the information on the mathematical problem, such as what is known and asked. The excerpt from interviews by the FS on the level of primitive knowing is as follows.

R : Did you ever get a question like this previously?
FS : No, I was never questioned on the model.
R : Now look at the answer sheet! Why didn't you write down the information obtained from this question?
FS : Sorry, ma'am. I was too excited.
R : Alright, so what information did you get from the questions given?
FS : Given the quadratic equation $x^2 - 7x + (r + 2) = 0$, which has the roots p and

q, where p, q, and r form an arithmetic sequence.

The FS failed to write down all the information obtained from the given mathematical problem despite having an adequate understanding of what was asked. This understanding activity shows that female students describe the initial thinking process and new concepts. At the primitive level, knowledge on concepts that students are assumed to have prior understanding was explored (Gülkılık, Uğurlu, & Yürük, 2015). At this level, students need to construct new ideas and information on the learning situation for further understanding (Yao, 2020a, 2020b; Yao & Manouchehri, 2020).

Image Making Level of FS

Quotations from the interview by FS regarding the image-making level are as follows.

R : What material is related to this math problem?
FS : Arithmetic sequences and quadratic equations.
R : What do you know about quadratic equations and arithmetic sequences?
FS : The general form of a quadratic equation is $ax^2 - bx + c = 0$ where $\alpha \neq 0$ and the highest power is 2. Meanwhile, the general form of an arithmetic sequence is $U_1, U_2, U_3, \ldots, U_n$ is significantly different from arithmetic sequence.
R : Ok, tell me what you did!
FS : Usually I am meant to factor $x^2 - 7x + (r + 2) = 0$, and determined the roots. However, I was not able to factor it due to the presence of (r + 2). Therefore, I re-read the problem again, and I got $x^2 - 7x + (r + 2) = 0$, it turns out that it already has roots, namely p and q, which motivated me to use quadratic equations.

The FS explained a concept based on prior knowledge by stating that arithmetic sequences and quadratic equations are closely related to the problem presented (Pirie & Kieren, 1994). FS also explained the quadratic equation problem in the form of $x^2 - 7x + (r + 2) = 0$ and found it difficult to determine its roots due to the constant (r + 2). Hence, the folding back to the primitive knowing level was used to re-read the problem by remembering the quadratic equations (Martin, 2008). Nonetheless, FS reached the image-making level where students try to picture the concept using prior knowledge with mental and physical actions (Gülkılık, Uğurlu, & Yürük, 2015).

Image Having Level of FS

Quotations of interview results by MS regarding the level image having are as follows.

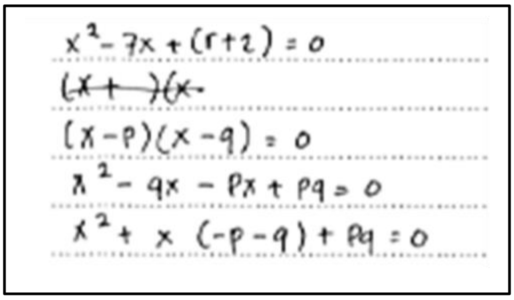
R : Apart from factoring, is there any other way to determine the new quadratic equation?
FS : That's all I remember Ma'am.
R : Okay, please explain what you did here?
FS : I determined the roots, namely p and q, using the factoring method by substituting $x_1 = p$ and $x_2 = q$, in$(x - x_1)(x - x_2) = 0$, and wrote it to be $(x - p)(x - q) = 0$. Furthermore, I multiplied $(x - p)$ by $(x - q)$, to obtain

$x^2 - qx - px + pq = 0$, and simplified it to $x^2 - x(p+q) + pq = 0$.

R : Why did you convert $x^2 - qx - px + pq = 0$ to $x^2 - x(p+q) + pq = 0$?

FS : I carried out the conversion process because I was trying to obtain a new quadratic equation, therefore, I changed the old form by collecting the variable x.

From the interview descriptions, FS provided an overview of a concept used to solve mathematical problems by explaining the factoring step procedure to determine quadratic equations. FS explained that the process design generates new quadratic equations, which means the FS understood the image-making process (Figure 6). This level indicates students' first abstraction to adjust and manipulate images without working on examples (Martin & Towers, 2014).



Translate Version

$x^2 - 7x + (r+2) = 0$

$(x-p)(x-q) = 0$

$x^2 - qx - px + pq = 0$

$x^2 - x(p+q) + pq = 0$

Figure 6. Activities Image Having Level by FS

Property Noticing Level of FS

Quotations of the interview results by FS regarding the property noticing level are presented as follows.

R : What is the difference between the factoring methods you want to apply here ($x^2 - 7x + (r+2) = 0$) and $(x-p)(x-q) = 0$?

FS : Both methods are similar, with differing results. For instance, in the question is $x^2 - 7x + (r+2) = 0$, I was able to obtain the roots using the factoring method. Meanwhile, in $(x-p)(x-q) = 0$), I used the factoring method to determine a new quadratic equation with known roots.

R : I notice you were silent for a few minutes before continuing. What was the problem?

FS : I obtained a new quadratic equation while solving the problem, which left me confused on how to operate the 2 equations.

R : So, what did you do?

FS : I related the 2 quadratic equations to the properties of the roots by substituting x_1 and $x_2$in $ax^2 - bx + c = 0$, hence the formula for the number of roots is $x_1 + x_2 = \frac{-b}{a}$, and the product is $x_1 x_2 = \frac{c}{a}$.

Based on the interview excerpt, FS explained the difference in the use of the factoring method based on $x^2 - 7x + (r+2) = 0$ and $(x-p)(x-q) = 0$. Conversely, FS has a slight difficulty with 2 equations of the previous operation, which led to the use of the folding back to connect the roots of the two new quadratic equations obtained (Gokalp & Bulut, 2018). The understanding activities carried out show that the FS understands the existence of a relationship between the description of a topic and suggests the right strategy for its verification (Martin & Towers, 2014; Yao & Manouchehri, 2020).

Formalising Level of FS

Quotations of interview results by the FS on the level of formalising are as follows.

R : Explain what you did with the 2 new equations created?

FS : In the quadratic equation $x^2 - 7x + (r + 2) = 0$, I substituted a, b, c, x_1 and x_2 with 1, -7, $r + 2$, p and q. Here I use the formula for the number of roots to determine the equation $p = 7 - q$.

R : Ok. How about $pq = r + 2$?

FS : For $pq = r + 2$, I used the formula for the product of roots and further substituted $x_1 = p$, $x_2 = q$, $c = r + 2$, and $a = 1$, therefore $r + 2 = 7q - q^2$.

R : Okay, why did you cross out $r + 2 = (7 - q)q$? I noticed that you stopped here for a while, why did you and what do you think?

FS : I thought of simplifying the equation, but it turned out to be back to the previous form, thereby leaving me confused on the process to utilize in continuing the operation. Therefore, I re-read the problem and thought of the relationship between the arithmetic sequence and p, q, and r.

The FS abstracts a mathematical concept based on a problem by describing the steps used to determine a new quadratic equation. The efforts used to solve the problem were described by making an example, substituting, and multiplying operations. After obtaining a new quadratic equation, they had difficulty continuing its work. Therefore, this leads to the folding back of the primitive knowing level by repeatedly re-reading the questions and thinking about the relationship between arithmetic sequences and quadratic equations whose roots are p, q, and r, presented in Figure 7.

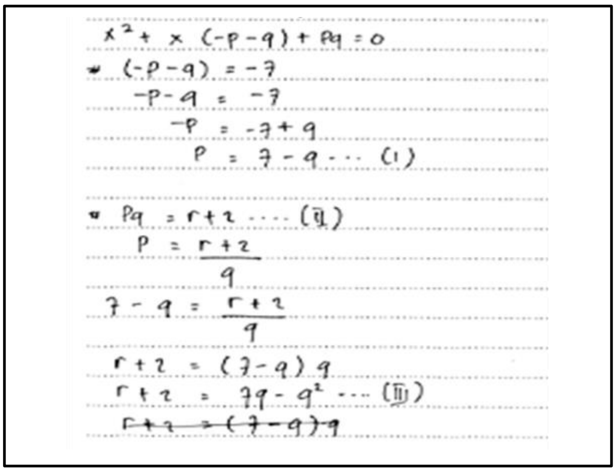


Figure 7. Problem Solving Activities Formalising Level by FS

The folding back activity by FS contrasts with Pirie & Kieren's theory of mathematical understanding (Pirie & Kieren, 1994; Martin, 2008). The activities carried out show that the FS abstracts a mathematical concept based on the properties that emerge with the ability to formalize prior understanding (Martin, Lacroix, & Fownes, 2005; Güner & Uygun, 2019).

Observing Level of FS

Following interviews with FS.

R : What do you know about arithmetic sequences?

FS : The general form of arithmetic sequences is $U_1, U_2, U_3, \ldots, U_n$ with a difference usually referred to as arithmetic sequence difference.

R : Okay, please explain the steps you used here!

FS : I determined the relationships p, q, and r that form an arithmetic sequence, by substituting $U_1 = p$, $U_2 = q$, and $U_3 = r$. Next, for B = $U_2 - U_1$, I substituted q to U_2 and p to U_1, to obtain B = q − p. Similarly, with B = $U_3 - U_2$, I substituted r to U_3 and q to U_2, therefore, it became B = r − q.

R : Why didn't you continue with the work in this section?

FS : I wondered about the right steps to use and thought of relating it to the arithmetic sequence difference formula. Furthermore, I remembered that the difference between the two adjacent terms was the same.

FS stated that the general form of an arithmetic sequence with an edge can be used to determine the roots of the quadratic equations p, q, and r. MS substituted the q score for the equality of 3 to get a p score, while FS substitutes p and q into the equation to obtain B. This activity shows that MS linked the mathematical concept understood with the problem using new knowledge structures (Gülkılık, Uğurlu, & Yürük, 2015). However, FS has difficulty continuing its work at this level of understanding; therefore, folding back to the image-making level was used to determine the previous knowledge. The folding back is that FS gets the difference between the 2 adjacent terms, which is similar to ($U_2 - U_1 = U_3 - U_2$). Based on this, it shows that the FS has reached the observing level.

Structuring Level of FS

The following interview was carried out with FS.

R : Is the factoring method the same as the ABC formula?

FS : The result is the same with varying steps and a limited factoring method.

R : Fine, please explain the method!

FS : From the quadratic equation $q^2 - 4q - 5 = 0$, I supposed that $a = 1, b = -4$, and $c = -5$, then I substituted the values for the formula ABC. I carried out further operation in order to obtain the value of q, namely $q_1 = 5$ and $q_2 = -1$. Next, I substituted the values for q_1 and q_2, into the first equation $p = 7 - q$, to obtain $p_1 = 7 - 5 = 2$. Similarly, with p_2, I wrote $p_2 = 7 - (-1) = 8$.

R : Ok. what about r?

FS : I utilized a similar method for an equation with variables r and p or q. I used Equation 5, which is $r = -7 + 3q$, and substituted the value for q_1, for $r_1 = -7 + 3(5) = 8$. Similarly, with r_2, I substituted the value for q_2, which led to $r_2 = -7 + 3(-1) = -10$. Therefore, this led to the formation of 2 arithmetic sequences, namely 2, 5, 8, and 8, -1, -10.

FS logically related one concept to another based on argument and stated that the results obtained by the factoring method with the ABC formula are the same. FS performed a substitution at each step of the ABC formula taken to get the roots of the quadratic equation in the form of arithmetic sequences with the same selection of 2, 5, 8, and 8, -1, -10 (Figure 8).

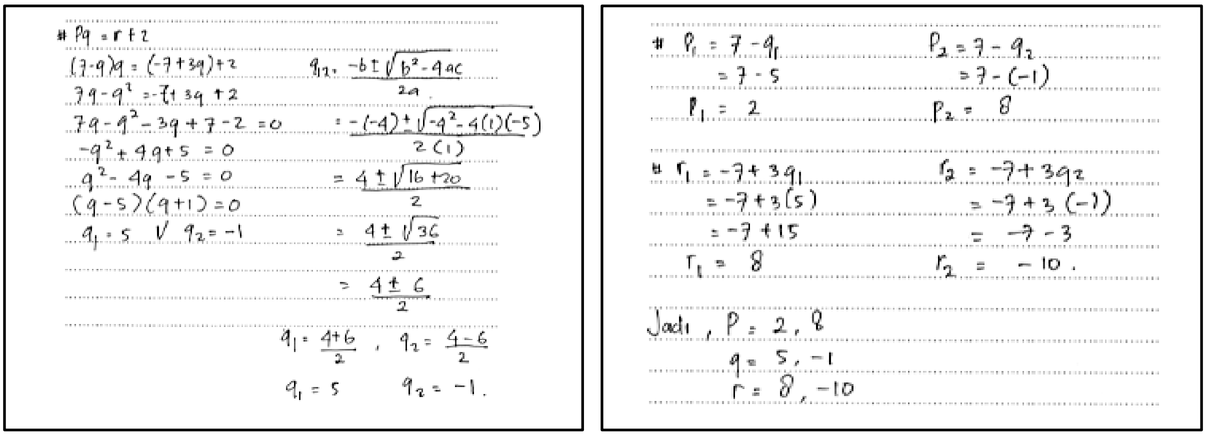


Figure 8. Activities Structuring Level by FS

The understanding activities carried out show that FS explained the formal observations logically and considered the observations as theories with relationships between theorems. At this level, students are aware of the interrelationship between a collection of theorems and demand that statements be justified or verified through logical or meta-mathematical argument (Pirie & Kieren, 1994; Codes et al., 2013; Yao, 2020b).

Based on the interview results, FS folding back is 4 times at the level of mathematical understanding growth, as shown in Figure 9.

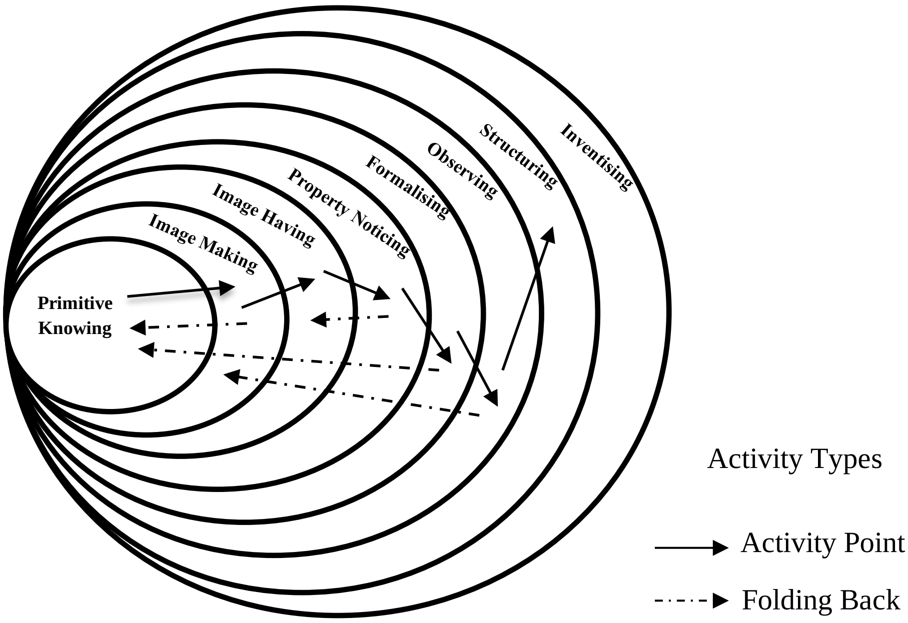


Figure 9. Folding Back in Solving Mathematical Problem of FS

Figure 9 demonstrated FS's folding back activity at the level of mathematical understanding, which suggests that students use lower-level understanding when confronted with any problem. This is shown in Figure 9, comprising of 4 folding back phases, namely Image-Making to Primitive Knowing, Property Noticing to Image Having, Formalising to Primitive Knowing, and Observing to Image Making. Folding back results fall into 3 categories, including returning to an outer level with/without an external prompt and effective folding back.

The first form is ensuring that female students are aware of the limitations of their existing understandings at the outer level and decide to shift to work at a lower level. The less sophisticated lower-level understanding activities are informed by what is already understood at an outer level. The second form is folding back to collect, which consists of female students' involvement in retrieving previous knowledge for a specific purpose and reviewing it considering the needs of current mathematical actions. Moving out of topic and working there is the third form of folding back, which enables them to develop the concept from a different mathematical area. The discussion has focused on the definitions of the levels and their embedded nature, which are necessary and structurally essential to the theory's mathematical understanding. However, a more vital issue is folding back, and according to Martin (2008), it is an important stage in the dynamical growth of mathematical understanding.

Data analysis in this study documents 7 levels of understanding growth based on mathematical problems by Pirie & Kieren theory, namely primitive knowing, image-making, image having, property noticing, formalizing, observing, and structuring, without describing the level of inventising understanding. An explanation of the characteristics of each type based on gender is shown in Table 1.

Table 1. Characteristics of Students (Male and Female) in Growth Understanding

Types of Growth Understanding based on Mathematics Problems	**Description of Characteristic based on gender**	
	Male Student (MS)	**Female Student (FS)**
Primitive Knowing	• Describing information obtained from mathematical problems. • Stating concepts related to mathematical problems.	• Describing information obtained from mathematical problems. • Stating concepts related to mathematical problems.
Image-Making	• Explaining the description of a concept based on previous knowledge develops a specific picture based on prior knowledge.	• Explaining the description of a concept based on previous knowledge develops a specific picture based on prior knowledge • Carrying out folding back to primitive knowing level

Types of Growth Understanding based on Mathematics Problems	**Description of Characteristic based on gender**	
	Male Student (MS)	**Female Student (FS)**
Image Having	• Having an overview of a concept used in solving mathematical problems. • Conducting folding back to primitive knowing level	• Having an overview of a concept used in solving mathematical problems.
Property Noticing	• Explaining the similarities/differences in the various descriptions of a topic.	• Explaining the similarities/differences in the various descriptions of a topic. • Conducting folding back to image having a level.
Formalising	• Making abstraction of a mathematical concept based on a mathematical problem • Carrying out folding back to Primitive knowing level	• Making abstraction of a mathematical concept based on a mathematical problem • Carrying out folding back to primitive knowing level
Observing	• Linking mathematical concepts understood with the problem at hand.	• Linking mathematical concepts understood with the problem at hand. • Conducting folding back to image-making level
Structuring	• Linking one concept to another based on logical arguments • Carrying out folding back to observing the level	• Linking one concept to another based on logical arguments

Table 1 shows the differences in the levels of understanding of male and female students in solving mathematical problems properly. This is in addition to the differences associated with the level of image-making, MS and FS understanding, and the ability to develop specific knowledge. However, FS folding back to the primitive knowing level due to the difficulty factor. Furthermore, at the image level, MS and FS have different levels of understanding, with an overview of the concepts used in solving mathematical problems. Similarly, there are differences in the level of understanding between male and female students in solving problems at the property, noticing, formalising, observing, and structuring levels. Male students fold back 3 times at the level of understanding growth, while their female counterparts carried out the process 4 times.

These results indicate differences in students' level of understanding in solving math problems based on gender, as shown in Figures 5 and 9. Male students engage in fewer folding back activities and understand problems than their female counterparts. This result in boys being more likely to correctly answer difficult, unfamiliar, and life-related math problems than girls, as supported by several researchers (Hornburg, Rieber, & McNeil, 2017; Innabi & Dodeen, 2018; Reinhold et al., 2020). The ability of gender differences to affect the way students solve problems associated with learning was also acknowledged by Cvencek, Meltzoff, and Greenwald (2011).

Folding back is the primary key in the growth of Pirie-Kieren understanding of mathematics and essential activity in the building, strengthening, and expanding students' knowledge of mathematics in learning. Students' understanding of mathematics takes place with the help of folding back between levels (Pirie & Martin, 2000; Martin, 2008). Therefore, based on the results above, students do not always go through the problem-solving stages at every step. However, the dynamic growth of mathematical understanding varies between students in problem-solving, according to preliminary studies (Pirie & Kieren, 1994; Pirie & Martin, 2000; Martin, Lacroix, & Fownes, 2005; Martin, 2008; Martin & LaCroix, 2008; Martin & Towers, 2014; Martin & Towers, 2016). Furthermore, this study provides more insight provided by the maps than the original manufactured by Pirie & Kieren (1994).

The Pirie–Kieren theory was used in this study to unpack activities associated with students' mathematical understanding, known as folding back actions. Meanwhile, the use of this theory is not the focus of this study. However, it provided a framework for investigating the role of participants' folding back in the process of mathematical understanding. The prominence of folding back in mathematical problem-solving lends support to the notion that folding back is critical in the process of mathematical understanding, which is in accordance with the Pirie–Kieren theory. This discovery contributes significantly to the solution of mathematical problems by elaborating folding back and proposing a broader framework for its categorization based on its source, form, and outcome. The framework enables the identification of various sources and forms, as well as describe their impact on students' mathematical understanding practices, particularly mathematical problem-solving.

CONCLUSION

This study explored the characteristics of students' level of understanding in solving arithmetic problems, with a focus on folding back based on gender. The results showed differences at the level of image-making, image having, property noticing, observing, and structuring. The understanding activities performed by male students are image having level, students folding back to the level of primitive knowing. Students had a mental picture of the topic, and in formalising level, they utilize the folding back to the level of primitive knowing process. Furthermore, their abstract to mathematical characteristics or properties of the image, create a concept and then write it into a formal definition or algorithm. At the structuring level, folding back is accomplished to the level of observation. Students have the ability to link a theorem to another and demonstrate it based on rational argument. Meanwhile, the level of understanding of female students includes image-making level, students folding back to primitive knowing level. They could imagine the concept with mental and physical action utilizing preliminary information. At property noticing level, students folding back to image level and tend to link the description of a topic to others. In formalising level, they are folding back to primitive knowing level is used to determine abstract mathematical concepts based on their properties. In observing level, students conduct folding back to the level of image-making to combine new knowledge structures with mathematical concepts.

Subsequently, the results showed that the 2 participants achieved understanding activities by explaining the information obtained from mathematical problems, describing the concept, making reports on a particular topic, identifying similarities and differences in various definitions of a topic, making mathematical abstraction concepts, and linking mathematical ideas to a problem. This study showed that students have not been able to acquire the inventising level. Therefore, further investigation needs to be carried out with qualitative studies at different grade levels, using various topics. The improvement of the understanding map needs to be tested using other issues. Current studies offer new techniques for describing growing students' understanding. The insights observed in this study suggested some implications for students' further development on a broader level of understanding. Some practical considerations were concluded from the results and impact when designing activities to solve arithmetic problems in the preparation program for mathematics teachers. However, this study was limited using data observation, which led to a small-scale investigation, including 2 different gender students, from 33 participants in one public school.

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EXPLORING THE IMPLEMENTATION OF AN INTERVENTION FOR A PUPIL WITH MATHEMATICAL LEARNING DIFFICULTIES: A CASE STUDY

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Abstract

This study presents a single case study of how a remedial mathematics teacher incorporated an instructional intervention into her teaching practices in order to teach counting to a pupil with mathematical learning difficulties. This new theory-driven intervention was developed by the authors of this study. Dyscalculia is a term which refers to a wide range of mathematical learning difficulties or disabilities. Dyscalculic pupils have a specific mathematics learning disorder with a core deficit in representing and processing of numerosity. They might not be able to recognise numerical quantities, performing counting and so on. Early supports such as interventions have a great potential in helping dyscalculic pupils to improve mathematical skills. However, there remains a lack of appropriate instructional scaffolds to help dyscalculic pupils to organise their learning structures by addressing both cognitive deficits and mathematical skills. The present study involves a primary school remedial teacher, Daisy, and an at-risk dyscalculic pupil, David, both pseudonyms. Data were collected through interviews, lesson observations, and reflective journals. The findings revealed that the proposed intervention improved the counting ability of the pupil.

Keywords: Reconnecting Learning, Intervention, Learning Disability, Dyscalculia, Mathematical Learning Difficulties

Abstrak

Penelitian ini menceritakan tentang sebuah studi kasus tunggal tentang bagaimana seorang guru matematika menggunakan intervensi instruksional dalam mengajar berhitung kepada siswa dengan mengalami kesulitan belajar matematika. Intervensi berbasis teori baru ini diciptakan oleh para peneliti dalam penelitian ini. Diskalkulia adalah istilah yang mengacu pada berbagai jenis kesulitan atau ketidakmampuan belajar matematika. Siswa dengan diskalkulia memiliki gangguan belajar matematika yang spesifik yaitu kekurangan dalam permisalan dan pemprosesan bilangan. Mereka mungkin tidak dapat mengenali kuantitas numerik, melakukan perhitungan dan sebagainya. Dukungan awal seperti intervensi memiliki potensi besar untuk membantu siswa dengan diskalkulia untuk meningkatkan keterampilan matematika mereka. Namun, ada kekurangan scaffolding pengajaran yang tepat untuk membantu siswa dengan diskalkulia untuk mengatur struktur pembelajaran mereka dalam mengatasi defisit kognitif dan keterampilan matematika. Penelitian ini melibatkan seorang guru sekolah dasar (Daisy-nama samaran) dan seorang siswa yang berisiko mengalami diskalkulia (David-nama samaran). Pengumpulan data dilakukan melalui wawancara, observasi pembelajaran dan jurnal reflektif. Hasil penelitian menemukan bahwa intervensi yang diusulkan dapat meningkatkan kemampuan berhitung siswa.

Kata kunci: *Reconnecting Learning*, Intervensi, Ketidakmampuan Belajar, Diskalkulia, Kesulitan Belajar Matematik

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Research into learning difficulties and learning disabilities has progressed considerably in recent decades. This indicates growing concern about learning difficulties and learning disabilities, particularly in the context of mathematics. From the perspective of mathematics education, mathematical learning difficulties or disabilities are of paramount importance, as these relate to educational inequalities (Deruaz et al., 2020). Barriers to learning mathematics impede access to and participation in various learning activities; therefore, educators should seek ways to support this group of pupils and provide them with equal opportunities for learning mathematics. The purpose of this paper is to introduce and demonstrate how an intervention may be useful to support pupils with mathematical learning difficulties or disabilities in particular the learning of counting. Pupils must master counting, because it is one of the most foundational skills in mathematics and is a critical prerequisite for the development of basic addition facts (Cotton, 2016).

Mathematical learning difficulties (MLD) or disabilities are referred to as dyscalculia (Deruaz et al., 2020). Dyscalculia is due to core deficits in representing and processing numerosities (Butterworth, 2019). Numerosity refers to the size of a set and relates to the question ‘How many?’ (Kadosh & Dowker, 2015). Pupils with dyscalculia are known as dyscalculic pupils. They have no number sense, cannot perform subitizing, and are unable to recognise and remember mathematical symbols (Emerson & Babtie, 2014; Wang et al., 2016). Research shows that the prevalence rate of dyscalculia is about 3-6% of the population (Bird, 2017). Westwood (2015) indicated that a critical problem is that very young dyscalculic pupils fail to make a correct one-to-one correspondence between the spoken number and the pointed object in a sequence.

Even though dyscalculia is not rare, there is no standard process in place for dyscalculic pupils to receive sufficient educational support (Morsanyi et al., 2018). Wijaya et al. (2019) indicated that 48% of 28 Indonesian teachers performed re-teaching to overcome students’ learning difficulties, while 31% conducted drills and practices and only 17% of the teachers developed or planned new teaching strategies. This indicates that most teachers had limited strategies to support students with mathematical learning difficulties.

Dyscalculia needs to be assessed separately to evaluate individual’s cognitive strengths and weaknesses, and it has very straightforward implications for designing the appropriate intervention strategy (Butterworth, 2019). He further stated that intervention must strengthen the basic capacity to represent numerosities as sets and understanding basic number concepts (Butterworth, 2019). This would explain the complexities in designing appropriate interventions in remediating dyscalculic pupils. Deruaz et al. (2020) concluded that there is a need for further research with pupils with dyscalculia to develop remedial intervention related to both pedagogy and mathematical content and how a teacher can help at-risk dyscalculic pupils to learn counting effectively. Individuals who have been diagnosed with dyscalculic tendencies via a dyscalculia screener or other standardized instruments are categorized as at-risk dyscalculic pupils. These pupils require effective and efficient instructional intervention so that they do not continue to fall behind their peers (Emerson & Babtie, 2014; Morsanyi et al., 2018). Therefore, this study aims to contribute by introducing a potentially useful instructional intervention, which is known as Reconnecting Learning. This study explores how a remedial teacher has implemented Reconnecting Learning, to teach counting to an at-risk dyscalculic pupil in a remedial mathematics class.

Current Interventions for Teaching At-Risk Dyscalculic Pupils

Research studies have revealed that early interventions show great potential for teaching at-risk dyscalculic pupils (Dowker, 2017; Butterworth, 2019). Researchers have proposed that pedagogical interventions can enhance dyscalculic pupils' mathematics learning, such as the Multisensory Teaching approach (Emerson & Babtie, 2014), the Concrete-Representational-Abstract approach (Bird, 2017), Intervention using an abacus (Lu et al., 2020), intensive intervention (Bryant et al., 2014), integer module intervention (Bryant et al., 2020), intervention combining many evidence-based practices (Powell et al., 2020), intervention based on best instructional practices (Van Garderen et al., 2020), and game-type intervention (Laski & Siegler, 2014). Other studies reported that dyscalculic pupils may gain substantial improvement through technology-based interventions such as computer-based intervention programs (Butterworth & Laurillard, 2010) and Augmented Reality (AR) technology (Miundy et al., 2017).

The aforementioned early mathematics interventions have yielded promising results in contributing to the basic numeracy development of dyscalculic pupils. However, Dowker (2017) argued that these interventions may not focus on cognitive deficits and mathematical skills. Cognitive deficits refer to deficits in representing and processing numerosity. This incomplete evidence may prevent early mathematics interventions from improving numeracy skills of at-risk dyscalculic pupils. The development of mathematical competency begins with the understanding of numerosity and the relationships among numbers (Gebuis & Reynvoet, 2014). Numerosity processing is demonstrated in the performance of numerosity comparison or numerosity judgment tasks (Gebuis & Reynvoet, 2014). Examples include, enumerating small sets (counting) and naming the number of objects in a display are the simple activities for measuring numerosity processing (Butterworth, 2019). Butterworth (2019) further suggested that mathematics educators should design specific interventions and theory-driven assessments for each learner. Appropriate learning strategies should be the focus of these interventions and assessments. In the present study, a theory-driven specific intervention namely Reconnecting Learning is formulated to fill in the aforementioned research gaps. This intervention aims to contribute to the teaching of counting to at-risk dyscalculic pupils by taking into account the cognitive deficits of dyscalculic pupils.

Reconnecting Learning Intervention

Reconnecting Learning is an instructional intervention that has been developed by blending Tall's (2013) theory of mathematical thinking and Feuerstein's (2015) structural cognitive modifiability theory, because remediation is essential to explore teaching at-risk dyscalculic pupils from different perspectives. Reconnecting Learning aims to help at-risk dyscalculic pupils to learn counting skill. The intervention lasted a week and comprised two 30-minute sessions. Counting may be a difficult process for young learners (Villarroel et al., 2011). This is because counting can become an increasingly abstract process (Voutsina, 2016). Counting is initially a highly complicated combination of speaking number words and pointing at objects in succession, stopping with the number at the last object pointed at (Tall, 2013). This invites the question of what abilities are needed in order to comprehend counting.

Humans are born with three abilities, namely recognition, repetition, and language (Tall, 2013). Humans can recognise patterns, repeat an action, and use language to name and refine concepts. All of these underpin the development of Reconnecting Learning. There are three steps in Reconnecting Learning, namely Demonstration, Mediation, and Active Learning. As an illustration, a pupil recognises that the last number in a counting sequence is the total number of objects in that collection. The pupil can visualise the counting process in a demonstration, then can count repeatedly to strengthen one-to-one correspondence counting in the mediation step. The pupil uses language to reason the way to get the answer in the active learning step.

Tall's (2013) theory explains how children learn counting naturally. However, it may not sufficiently address dyscalculic pupils' difficulties. As mentioned previously, the core problem of at-risk dyscalculic pupils is cognitive deficits in representing and processing of numerosity (Butterworth, 2019). Feuerstein et al. (2015) confidently speculate, pending further confirmation that structural cognitive modifiability theory may modify an individual's structure of neuroplasticity. Hence, it is possible to unlock individual's potential by modifying pupils' thinking both cognitively and motivationally, to help at-risk dyscalculic pupils to learn counting by blending Feuerstein's (2015) structural cognitive modifiability theory with Tall's (2013) theory.

The theory of structural cognitive modifiability emerges naturally through Mediated Learning Experience in educational context (Feuerstein et al., 2015). There are three vital parameters of Mediated Learning Experience, namely (1) mediation of intentionality and reciprocity; (2) mediation of meaning; and (3) mediation of transcendence. In our research context, mediation of intentionality may mean that a teacher demonstrates how to count to an at-risk dyscalculic pupil and make sure the pupil is giving positive feedback, so that the demonstration can be continued. Then the teacher mediates the meaning of counting to the pupil. Finally, the teacher bridges the idea of counting using language with the at-risk dyscalculic pupil (Feuerstein et al., 2015).

METHOD

This study employed a case study research design to collect the relevant qualitative data. It was carried out at one of the national primary schools in Malaysia. A two-tier purposive sampling technique is chosen. Merriam and Tisdell (2016) stated that a two-tier purposive sampling is necessary in qualitative case studies when there is a general question that an in-depth study of a particular occasion will elucidate that interest. In the first tier, we screened 15 primary year one pupils by using the Malaysia Dyscalculia Instrument MDI (Wong et al., 2016). One at-risk dyscalculic pupil, David (pseudonym) was identified. In the second tier, the remedial teacher, Daisy (pseudonym) was chosen because she was David’s remedial teacher and had the responsibility to help David to master basic numeracy skills. She had 24 years of teaching experience. Daisy completed a two-day Reconnecting Learning training course prior to implementing it in her lessons. Daisy and David participated this study voluntarily. Data were collected through an in-depth interview with Daisy, classroom observations, and analyses of Daisy's reflection journals and David's worksheet. One of the aims of this study is to explore how does Daisy implement Reconnecting Learning to teach counting to the at-risk dyscalculic pupil, David. This study also explores the impacts of Reconnecting Learning, seeking to answer two specific research questions as follows:

1. How does the teacher implement Reconnecting Learning to teach counting to the at-risk dyscalculic pupil?
2. To what extent does the implementation of Reconnecting Learning support the achievement of the intended learning outcome, which is being able to count 1- 10?

Background of the Research Participants

Reconnecting Learning was conducted in a one-on-one setting. A remedial mathematics teacher and an at-risk dyscalculic pupil were involved in the study. David was a Year One pupil, and his chronological age was seven years old. Malaysia Numeracy Remedial Assessment (MNRA) is a numeracy screening instrument that aims to identify at-risk pupils, and it helps remedial teachers to prepare and design remedial teaching and learning activities (MOE, 2012). Based on his past MNRA result, he has been categorized into a group which indicates that he did not master pre-numbers and number concepts and that he could not recognise numbers.

Daisy is a female teacher with 24 years of teaching experience. Daisy followed the guidelines provided in the training course to prepare her daily lesson plans and teaching resources. She always recorded her reflections immediately after the lessons. Daisy had two mathematics lessons (30 minutes in each lesson) with her year one numeracy remedial pupils on every Monday and Friday.

Data Collection and Analysis

Data were collected through multiple sources during the one-week intervention. We examined Daisy's journal reflections, David's worksheets, triangulate with the lesson observations data and the 30 minutes in-depth interview to ensure trustworthiness (O'Leary, 2014). These journal reflections serve as the primary data of this study (Yin, 2018). Daisy always wrote her journal reflections after her lessons. Two formal classroom observations were conducted by the researcher. Anecdotal notes and pictures were taken during the observations. Rich data sources are crucial to capturing Daisy's multifaceted patterns of thoughts, beliefs, and values that underlie her teaching experiences (Yin, 2018). After the classroom observations, a 30-minute in-depth interview was conducted to allow Daisy to reflect on her experience of implementing the intervention and to explore her perceptions regarding Reconnecting Learning.

Data from journal reflections, interview transcripts, and anecdotal notes were analysed using Clarke and Braun's (2013) thematic analysis. The analysis involved four steps. In the first step, categories were created based on indicators that emerged from the journal entries which are considered relevant to Reconnecting Learning and the research questions. In the second step, the interview data were transcribed from the audio recordings. In the third step, data from Daisy's reflective journals, interview transcripts, and classroom observation anecdotal notes were compared respectively, and new categories of data will be created if necessary. In the last step, we evaluated the reliability of the themes by re-reading and re-analysing the coding concepts and ensured no new themes were uncovered, in order to promote the trustworthiness and rigor of the findings.

RESULTS AND DISCUSSION

Two main themes emerged in the present study: (1) Reconnecting Learning is an instructional-based intervention; and (2) Reconnecting Learning appears to be a potentially useful intervention for teaching counting to the dyscalculic pupil. This section provides a case profile of Daisy's teaching process. The case description clarifies how she implemented Reconnecting Learning to teach counting to an at-risk dyscalculic pupil in her remedial lessons.

Theme 1: Reconnecting Learning as A Structured Approach Intervention

The results presented in this section are used to answer the first research question. Reconnecting Learning is a structured approach intervention which focuses on teaching counting for at-risk dyscalculic pupils. Daisy organised each lesson into three distinct phases, namely Demonstration, Mediation and Active Learning.

Demonstration

Daisy demonstrated how to count the pips on domino cards to David.

Daisy: Good Morning, David. Are you ready for today's lesson?

David: Morning… (He nods his head.)

Daisy: Do you notice what is this? (She displays the domino cards and numbers in front of David.)

David: Number… card (He urges to touch the learning materials)

Daisy: These are domino cards and numbers.

David: (He nods his head.)

Daisy: Do you like to play counting game? We are going to learn counting today.

David: Yes…

Daisy: Good. Let's start our game. Our mission is to find out how many pips altogether in domino number cards.

Daisy demonstrated how to count the pips on each mathematics domino cards by using point to count strategy (see Figure 1). Daisy spoke the number words aloud and pointed to the pips in correspondence then told David that the final spoken number word represented the total number of pips.

David: (He looks at the ground)
Daisy: Look here (pointing at the pips), do not look at the ground.
David: Eight. (He stared at the mathematics domino cards.)
Daisy: How do you know?
David: I saw it.

Figure 1. Daisy demonstrated point to count strategy

In the interview, Daisy elaborated that:
"Firstly, I demonstrated how to count using mathematics domino cards, he (i.e., David) was urged to touch the learning materials."

She further explained:
"At first, he felt so shy and nervous in my class. Then, when I took out the mathematics domino cards…oh…He realised that the teacher wanted to play a game with him. Then he felt more relaxed, then he was excited to touch the domino cards."

Demonstration can capture pupils' attention and provide opportunities for teachers to introduce and explain concepts in a highly visual and auditory way (Ware & Johnson, 2013). In Reconnecting Learning, Intentionality and Reciprocity were embedded in the demonstration. Intentionality means the mediator's effort to change a child's attention and perception (Feuerstein et al., 2015). This was achieved by telling David the learning objectives of the lesson and demonstrating the counting process so that

David can focus on the recognition of quantity. To ensure that David was paying attention to the lesson, Daisy gave explicit instructions by saying "Look here (pointing at the pips), do not look at the ground."

Intentionality cannot stand alone without the pupil's reciprocity (Feuerstein et al., 2015). Reciprocity refers to the learner responds vocally, verbally, or non-verbally to the teacher's behaviour (Feuerstein et al., 2015). David responded by nodding his head and said, "I saw it." Daisy realised there was a positive change in David's attitude and was motivated to learn and urged to touch the domino cards during the demonstration.

Mediation

David was engaged in the lesson, and he was motivated to speak aloud the number when he was counting. Daisy led David to do the counting and pointed at the corresponding pips concurrently. David can verbally count "one, two, three …nine " in Malay.

When Daisy asked David: "How many pips are there on the domino card?" David looked around and paused for a while, then he started to count verbally: "one, two, three, four, five" and answered "five" in Malay. Daisy explained to David, "Just now you counted the pips from the right lattice, how about you try to count the pips from the left lattice?" David counted and gave the same answer "five." Mediation of meaning happened when Daisy grabbed David's dominant hand (i.e., right hand) to count the pips on the domino cards by using point to count strategy (see Figure 2). Daisy explained in her interview, "I mediated him by grabbing his hand to mediate the meaning of counting…. erm….one-to-one correspondence to him…" In Daisy's reflective journals, she remarked her teaching and learning went on smoothly and David could focus his attention all the time.

Figure 2. Daisy grabbed David's right hand to count the pips on the domino cards using point to count strategy

Mediation of meaning happens when a teacher conveys the main point and the purpose of an activity to a learner in both cognitive and affective levels during the interaction (Feuerstein et al., 2015).

Using Reconnecting Learning, Daisy aimed to convey the meaning of how to count to David. As Gelman and Gallistel (1978) pointed out, there are three 'how-to-count' principles namely the one-to-one principle, the stable-order principle, and the cardinal principle. In this case, the one-to-one correspondence principle means only one numeral can be given to each item in a set. The stable-order principle denotes that the numerals used to count must be used in the same order in any one count. The cardinal principle means the numeral, or the last word spoken in counting a set of items represents the total number of items in the set (Gelman & Gallistel, 1978).

This mediation phase aimed to teach the one-to-one correspondence principle and stable order rule to David. Daisy interpreted the mediation phase as grabbing David's dominant hand to do the counting together and pointing at the corresponding pips concurrently. From the observation data, David could verbally count "one, two, three …nine " in Malay. This data show that he could speak one numeral for each item in a set and followed the same order in any one count. This is supported by Feuerstein et al. (2015), who state, "Do it through me, and with me. Don't do it for me". Additionally, Daisy tried to mediate the meaning of the order-irrelevant principle to David. Per the observation data, Daisy challenged David whether he could get the same answer as he started to count the pips from right to left and vice versa. Then, David tried to count the pips start from the right lattice, then he continued to count the same number of pips from the left lattice using the point to count strategy. Finally, David got the same number of five pips. From this activity, Daisy tried to mediate the meaning that the order in which objects are counted is not important to David. As a consequence, David could count the pips in all instances. David could count correctly because he could apply the one-to-one correspondence principle using the count all strategy and he stated the last number as the amount of the set. From the affective perspective, David was motivated to learn during the mediation phase as he was willing to speak aloud the number when he was counting and completed the small tasks assigned by Daisy. For example, Daisy asked David to count the pips from the left lattice. Daisy’s reflection journal stated that David could concentrate on his learning during the mediation process. The mediation of meaning helped David to learn how to count cognitively.

Active Learning

Active learning means pupils are intensely engaged in both mental and physical exercises during the learning process (Green & Casale-Giannola, 2017). David was encouraged to choose his favourite domino card and perform point to count during the active learning phase (see Figure 3). In the reflective journals, Daisy remarked: "During the active learning phase, David was encouraged to choose his favourite domino card and perform counting. I will give him minimal guidance if he makes mistakes…" During the lesson, it was observed that Daisy facilitated David carefully and motivated him to choose his favourite domino card for counting. Daisy's intention was to allow David to comprehend and make sense of counting by himself with minimal guidance. Mediation for transcendence was embedded in active learning. Mediation for transcendence is characterized by going beyond the pupil's concrete context or the knowledge (Feuerstein et al., 2015). In this case, mediation for transcendence is evidenced by the fact that David could match the numeral with the correct quantity of pips but not just merely being able to repeat the process of counting the pips. Daisy mediated David to go beyond the how to count knowledge, David was able to recognise the numbers and match them with the correct quantity of pips. At last, David learned how to count through Reconnecting Learning, consisting of three systematic learning processes: demonstration, mediation, and active learning.

Figure 3. David performed point to count by using his index finger

Theme 2: Reconnecting Learning Appears to be A Potentially useful Intervention for Teaching Counting to the Dyscalculic Pupil

The result presented in this subsection is used to answer the second research question. In the interview, Daisy explained that David did not know how to express quantity prior to implementing Reconnecting Learning:

> "*He didn't know how to count using his fingers. Erm… I asked him to show me seven using his fingers, he failed to do that, I don't know why…*"

After the lessons, Daisy said, "It was great that David mastered counting after the remedial session." The results show that David was able to count 1-10 using domino cards and match the quantities of pips with the correct numerals during the lessons. In addition, David was able to count and match the quantities of sandwiches with the numbers in the worksheet correctly (see Figure 4). He was able to represent the quantities using numbers in two different instances.

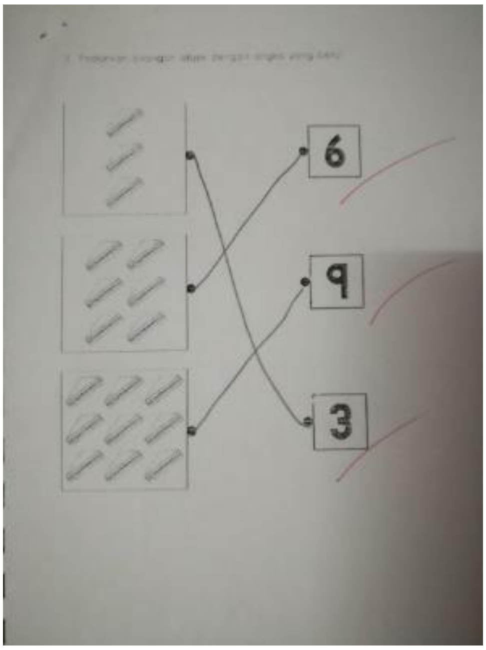

Figure 4. David counts and matches the quantities with the numbers correctly in the worksheet

Daisy' reflective journals indicate that Reconnecting Learning is a practical intervention, as David can pay attention and learn counting easily in a one-to-one setting. She remarked:

"After carried out the remedial activity, I felt that one-to-one setting in Reconnecting learning is practical. My pupil can pay attention more easily."
"I use mathematics domino cards in my teaching and learning activity. My pupil can learn more easily using the learning material."

Moreover, Daisy also illustrated:

> *"After the training, I learned how to teach at-risk dyscalculic pupils…Erm… Reconnecting Learning can motivate my pupils to learn mathematics, it is different from other teaching strategies that I had learned before…"*

She further elaborated that Reconnecting Learning includes some significant elements of social aspects that is valued and loved:

> *"He loves teacher to touch his hand, he felt he is valued and being loved. I think this part is the most different from other teaching strategies. I learned and felt this step is so motivated."*

The affective responses of David coupled with the involved cognitive processes had supported the achievement of the intended learning outcome. In short, the implementation of Reconnecting Learning has supported the achievement of the intended learning outcome cognitively and affectively.

Our purpose was to describe Reconnecting Learning for teaching counting to the at-risk dyscalculic pupil and to highlight that the intervention shows promising results for supporting the pupil to learn counting. Dyscalculic pupils do not stagnate in their learning development if they are given sufficient time and appropriate intervention (Emerson & Babtie, 2014). David did not know how to express quantity prior to implementing Reconnecting Learning. Building on these circumstances, David was able to recognise the presented quantity of pips and he was able to repeat the demonstrated counting action. He used language to speak aloud the number words while counting and reasoning his experiences after this remediation.

This study provides a significant window into the way that Daisy demonstrated how to implement Reconnecting Learning that involved three main steps. She began with Demonstration and intentionally provided David with the first input regarding counting with one-to-one correspondence using mathematics domino cards. In the Mediation step, Daisy and David counted the pips on the mathematics domino cards together by grabbing David's dominant hand to point at the pips. Daisy explicitly delivered the counting principles: one-to-one correspondence principle and cardinal principle to David. In the Active Learning step, David actively engaged in the activity, and he was able to count the pips on the mathematics domino cards in a correct correspondence. Finding reveals that the blending of Tall's theory (Tall, 2013) and the theory of structural cognitive modifiability (Feuerstein et al., 2015) to perform instructional intervention to help at-risk dyscalculic pupils to learn counting skill was successful. The findings are consistent with the study of Aunio et al. (2021) that investigated if early numeracy skills of 267 at-risk of mathematical learning difficulties children in South Africa can be improved with an intervention program. The main result shows that the intervention group had improved more in numerical relational skills as compared to the control group. Aunio et al. (2021) implemented explicit mathematics instruction intervention program and included the cognitive measures in their intervention study. They focused on language skills and measured executive function through recording the children's reaction times and accuracy of their answers per item whereas the present study focused on how an at-risk dyscalculic pupil processed and represented numerosities by collecting qualitative evidence. The present study fills the gap in the literature by focusing on the abilities of processing and representing numerosities of dyscalculic pupils.

In addition, the present study has exemplified the gradual release of responsibility model of instruction which was proposed by Fisher and Frey (2013). The model is about an instructional teaching design for a continuous shift of the cognitive load across time. Learners' cognitive load should gradually and purposefully shift from instructor modelling to joint accountability between teachers and pupils, and then to independent practice and application (Pearson & Gallagher, 1983). The gradual release of responsibility model of instruction was exemplified through the present study by focusing on teaching counting to an at-risk dyscalculic pupil in a one-on-one setting.

The present study is supported by Vygotsky's sociocultural theory which claims that cognitive development needs to happen within a social context (Gauvain, 2008). For example, Daisy acted as a mediator to convey mathematical concepts to David using language and David reacted by giving positive feedback to Daisy when learning counting. David can engage in more complex cognitive activities, such as counting with the assistance of Daisy through the sociocultural approach.

At the end of the intervention, David was able to count the sandwiches and match the quantities with the numbers correctly in his worksheet. This shows that David was able to process and represent the quantities using numbers in two different instances. Activities such as counting and naming the number of objects in a display are the measurements for numerosity processing (Butterworth, 2019). Reconnecting Learning was able to tackle the cognitive deficits in processing and representing numerosities and develop counting skill for the at-risk dyscalculic pupil. Consistent with Butterworth (2019), in order to assist dyscalculic pupils in making progress, interventions should emphasise on developing pupils' basic capacity to represent numerosities as sets and link them to number words and numerals. Teachers may guide pupils carefully from concrete work to abstract work. In this context, Daisy used mathematics domino cards to illustrate quantities and adapted her teaching to fit David's current level of understanding of the subject.

CONCLUSION

Daisy implemented Reconnecting Learning by organising her each lesson into three distinct phases, namely Demonstration, Mediation and Active Learning. The empirical evidence indicates that Reconnecting Learning did support the achievement of the intended learning outcome which is being able to count 1-10.

The study clearly has limitations. Even considering the limitations in terms of the sample size and the relatively narrow focus of the study, there are some implications that are evident. Reconnecting Learning was successful in helping Daisy to teach counting to the at-risk dyscalculic pupil, David. As numerous previous studies have discussed remedial interventions focused on teaching strategies (devices, tools, interactions) designed to help students progress, rather than students' cognitive abilities (Deruaz et al., 2020), the main contribution of this study is the introduction of Reconnecting Learning which is a potentially useful intervention to help at risk dyscalculic pupils to learn counting cognitively. Further research should be undertaken with a larger sample size and a broader content focus to investigate the effectiveness of Reconnecting Learning in teaching mathematics to at-risk dyscalculic pupils.

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