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COMPARING MODEL-BUILDING PROCESS: A MODEL PROSPECTIVE TEACHERS USED IN INTERPRETING STUDENTS' MATHEMATICAL THINKING

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Abstract

Mathematical thinking is an important aspect of mathematics education and, therefore, also needs to be understood by prospective teachers. Prospective teachers should have the ability to analyze and interpret students' mathematical thinking. Comparing model is one of the interpretation models from Wilson, Lee, and Hollebrands. This article will describe the prospective teacher used the model of the building process in interpretation students' mathematical thinking. Subjects selected by considering them in following the students' strategies in solving the Building Construction Problem. Comparing model is a model of interpretation in which a person interprets student thinking based on student work. There are two types comparing model building process prospective teacher use in interpreting students' mathematical thinking ie. comparing work and comparing knowledge. In comparing works, prospective teachers use an external representation rubric. This is used to analyze student activities in order to provide an interpretation that is comparing the work of students with their own work. In comparing knowledge, prospective teachers use internal representation rubrics to provide interpretation by comparing the students' work with their knowledge or thought.

Keywords: Comparing Model, Interpretation, Students' Mathematical Thinking

Abstrak

Pemikiran matematis merupakan aspek penting dalam pendidikan matematika dan, oleh karena itu, juga perlu dipahami oleh calon guru. Calon guru harus memiliki kemampuan untuk menganalisis dan menafsirkan pemikiran matematika siswa. Model Membandingkan adalah salah satu model interpretasi dari Wilson, Lee, dan Hollebrands. Artikel ini akan menjelaskan bagaimana calon guru menggunakan model proses membangun dalam menafsirkan pemikiran matematis siswa. Subjek dipilih dengan mempertimbangkan bagaimana mereka mengikuti strategi siswa dalam menyelesaikan Masalah Konstruksi Bangunan. Model Membandingkan adalah model interpretasi di mana seseorang menginterpretasikan pemikiran siswa berdasarkan pada pekerjaan siswa. Ada dua jenis proses membangun Model Membandingkan yang digunakan calon guru dalam menafsirkan pemikiran matematis siswa yaitu Membandingkan Karya dan Membandingkan Pengetahuan. Dalam Membandingkan Karya, calon guru menggunakan rubrik representasi eksternal. Ini digunakan untuk menganalisis aktivitas siswa dalam rangka memberikan interpretasi yaitu membandingkan pekerjaan siswa dengan pekerjaan mereka sendiri. Dalam Membandingkan Pengetahuan, calon guru menggunakan rubrik representasi internal untuk menyediakan interpretasi yaitu dengan membandingkan pekerjaan siswa dengan pengetahuan atau pemikiran mereka.

Kata kunci: Model Membandingkan, Interpretasi, Pemikiran Matematis Siswa

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One of the most fundamental goals of mathematics education is being able to use mathematical thinking in solving problems (Stacey, 2006; Shahrill, *et al*. 2018). Therefore, teachers should support the development of students' mathematical thinking. For this purpose, accessing students' thinking when students solve problems is essential to get information about students' potential and also difficulties. Such information is important as a consideration for teachers to choose and prepare appropriate teaching strategies and/or materials. Despite the importance of accessing students' thinking process is not easy because what teachers can directly access are student learning written and oral activities. As mentioned by von Glaserfeld (1995), teachers do not have direct access to students' mathematical thinking. What teachers can do is accessing the evidences of students' mathematical thinking, such as students' works. Student works show students' mathematical activity that might indicate the process of students' thinking. In this respect, teachers can develop hypothesis of students' mathematical understandings by observing students' mathematical activities. Mathematical activities include hypothesizing what students know and understand (Cobb & Steffe, 1983; Steffe & Thompson, 2000). For prospective teachers (PTs), their understanding of students' mathematical thinking is determined much by what they understand or learn about textual theory or knowledge and little about the strategies that students undertake. Understanding students' strategies will lead prospective teachers to understand students' thinking. This relates to how to interpret students' mathematical thinking.

Interpreting student thinking is an important component of high-quality learning and assessment (NCTM, 2014; Teachingworks, 2012; Ahmad, *et al*. 2018). According to Sapti, Purwanto, Mulyati, and Irawan (2016), interpreting students' mathematical thinking is "giving impression, opinion, or a theoretical view towards mathematical information in the form of students' written work in solving problems". A key factor in interpreting students' work is the ability to see essential aspects of students' mathematical thinking (Jacobs, Lamb, & Philipp, 2010; Sherin, Jacobs, & Philipp, 2011; van Es & Sherin, 2002). Paying attention to students' mathematical thinking does not only require attention to children's strategies but also interpretation of mathematical understanding reflected in these strategies. Identifying the extent to which the teacher's evidence shows in interpreting children's understanding is not looking for the best single interpretation but the extent to which participant interpretation is consistent with the details of certain children's strategies and research on children's mathematical development (Jacobs, *et al*. 2010). Teachers can interpret students' mathematical thinking by identifying strategies that students might use in solving a problem. Teachers can also identify why certain problems become difficult and cause problems considering the characteristics of students' thinking (Widodo, Nayazik, & Prahmana, 2019). At the end, students' mathematical thinking can be connected to providing appropriate learning opportunities because different mathematical thinking might require different learning approaches.

Despite the importance of interpreting students' mathematical thinking, a number of studies show prospective teachers' sufficient skill in this respect. Jacobs, Lambs, and Phillip (2010) revealed that prospective teachers and beginner elementary school teachers experience difficulties in identifying and interpreting student strategies. Similarly, Shaughnessy, Boerst, and Ball (2014) and Sleep and Boerst (2012) also found that beginner teachers have difficulties in interpreting students' mathematical thinking and appropriate learning approaches. Due to their lack of teaching experiences, prospective teachers have limited knowledge and experiences about the various strategies of students in solving the problem. Their understanding of student strategies might heavily rely on their theoretical experiences from lecturers and/or their own prior experiences as students. Prospective teachers will compare students' actions with their own actions, either implicitly or explicitly (Wilson, Lee, & Hollebrands, 2011). Such situation is called comparing model analysis. Based on that, to interpret students'mathematical thinking, prospective teachers analyze by comparing, i.e., equating or differentiating student strategy artifacts with their own work or concept. This allows the emergence of two types comparing model: comparing work and comparing knowledge.

Comparing is one of model-building process in interpreting students' mathematical thinking. This relates to the process of building interpretations of students' thinking based on student learning activities. Wilson, *et al*. (2011) depicted a comparing model in the process of constructing interpretations as two separately parallel boxes (see Figure 1). One box contains prospective teachers' (PTs) thinking (TT) and their written work (TW) and the other box contains a student's thinking (ST) and their written work (SW) (Wilson, *et al*. 2011). The arrows were used to represent a PST focusing on his or her attention. We used solid arrow to indicate explicit evidence of a PST's attention and dotted arrow to indicate an implicit of a PST's attention.

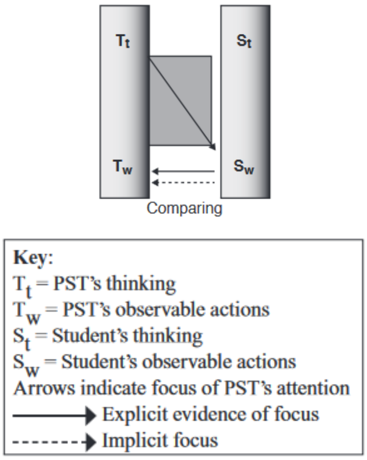

Figure 1. Comparing model to illustrate PTs' analysis of students' work (Wilson, *et al*. 2011)

There are two types of PTs's focus ofattention, those are explicit and implicit attention. The explicitly attention is what PTs attend to the student's written work or what students might think based on student work. Otherwise, implisit attention is inferred based on the PST's work. To modeling how PTs analyze students' work, we follow Wilson, *et al*. (2011) in read the diagram from the upper-left corner and following the arrows indicating their focus of attention.

Considering the importance of developing prospective teachers' skills in interpreting students' mathematical thinking, the present study intend to examine how prospective teachers use the comparing model in interpreting students' mathematical thinking.

METHOD

This qualitative study was conducted on third year graduate students of Mathematics Education Programs in one of a private university in Purworejo, Indonesia. Purworejo is surrounded by favorite public and private colleges in Central Java, Indonesia.

Participant

The participants of this study were 23 prospective teachers (PTs) consisting of 18 female and 7 male graduate students. Participants were asked to follow the strategies of students in completing the the Building Construction Problem (BCP) as shown in the example of student work. There are four fundamental behaviour in attending to student's strategies i.e. making right written solution, making false written solution, not making solution, not making solution but understand the solution. This study selected purposively participant who make correct written solution and participant who did not make written solution but understand the solution and could attend to at least 2 of 4 examples of students' strategies in solving BCP.

Data Collection

We assigned PTs to complete Task of Interpretation of Students' Mathematical Thinking (ToIoSMT). The ToIoSMT provide Building Construction Problem(BCP) and 4 different examples of students' written work about BCP. The following is a BCP that students have done and examples of students' written work.

Building Construction Problem:

> In order to construct a building, the contractor takes 15 months with 120 workers. For a reason, the contractor wants a 3 month accelerated job. If the ability to work for each worker is the same and that the project can be completed on time, how many workers should be added?.

The examples of students' written work in solving BCP represent four characteristics. Student A completed BCP by writing down the method used that was inversed proportion as well as the linkage of information time and many workers. Student A did not write down the steps to get the number 150. He simply wrote that many of the added workers were 150 - 120 = 30 workers, whereas, student D wrote in detail the calculation of inversed proportion. Students B and C both completed the BCP by using direct proportion. Student B divided 12 by 15 and multiplies the result by 120. He multiplied the time ratio by the number of workers, while student C writes in detail the steps of BCP problem solving: known information, completion plans, direct proportion calculations, and conclusions.

We provided these four characteristics of students' written work and presented rewrite of examples of students' written work in solving BCP in the Figure 2.

Inversed Proportion
Time number of worker
15 month → 20 people
15 - 3 = 12 month → x

Number of workers added:
= 150 – 120
= 30 workers

(a)

$$15\overline{)120}\quad 0{,}8$$
120
―――― -
0

$0{,}8 \times 120 = 96$ workers

(b)

15 months → 120 workers
12 months → 120 + x ?

$$15 \times (120 + x) = 12 \times 120$$
$$1800 + 15x = 1440$$
$$1800 - 1440 = 15x$$
$$360 = 15x$$
$$x = \frac{360}{15}$$
$$x = 24$$

So, there are 24 workers that must be added by the contractor.

(c)

Known : * 15 months → 120 workers
* 15 – 3 months = 12 months → x

Asked : how many workers must be added (inversed proportion)

Solution:
Time number of worker
15 months ←→ 120 workers
12 months ←→ x

$$15 \times 120 = 12 \times (120 + x)$$
$$1800 = 1440 + 12x$$
$$1800 - 1440 = 12x$$
$$360 = 12x$$
$$x = \frac{360}{12}$$
$x = 30$ workers

(d)

Figure 2. The rewrite of examples of (a) Student A, (b) Student B, (c) Student C, and (d) Student D written works [*translated and rewritten due to low resolution*]

In The ToIoSMT, PTs assigned to explain what they understand about students' mathematical thinking based on student work. PTs asked to think out loud when completing the task. Furthermore, an interview was conducted to corroborate the PTs' think out loud and written work in interpreting the students' mathematical thinking. The artifact of PTs explanation of students' mathematical thinking named as PTs interpretation.

Data Analysis

Qualitative analysis was used to generate the description of the building process, model of PTs interpretation of students' mathematical thinking based on their written work and think aloud. We conducted data analysis following three stages of qualitative data analysis activities from Miles & Huberman (1999) and six stages of analysis and interpretation of qualitative data from Creswell (2012). This study used triangulation methods by examining the data with different methods: task; think out loud, and interview. Compared PTs written work, think out loud transcript, and interview transcripts resulting data consistency.

By focusing on important themes such as whether PTs make scribbles outside work, convey theories related to problems, this study provide description of how PTs interpret students' mathematical thinking, and characteristics of their interpretations. We used third-order models from Wilson, *et al*. (2011) to describe our desciption of PTs' interpretation of students' mathematical thinking (see Figure 3). This is similar to the point of view of Simon & Tzur (1999) in which they explain “the teacher’s perspective from the researchers’ perspectives” (p. 254).

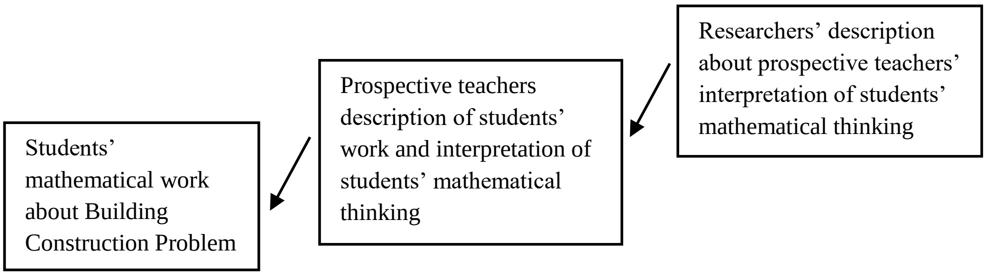

Figure 3. A representation of the process of researchers characterising PTs interpretation of students’ mathematical thinking

RESULT AND DISCUSSION

Looking at how PTs interpret student thinking is done by looking at how they see student work data and how they obtain interpretations. Of the 23 participants, nine participants show the characteristics of comparing models include: three participants doing comparing work, and six participants doing comparing knowledge. This study describes two participant for each comparing model.

These findings trigger the importance to study prospective teachers’ skills in interpreting students’ mathematical thinking. Consideration of what type of reasoning is involved when prospective teachers are asked to analyze student work. This study tends to describe comparing model as building process model prospective teachers used in interpreting student’s mathematical thinking. Comparing model is one of process that PTs used to construct models of students’ thinking(Wilson, *et al*. 2011). In comparing model, PTs compared students’ activity with their own activity, either implicitly or explicitly. The model-building processes in this category include how the PT searched for similarities and differences between theirs and those that the PTs noticed in the students’ work. Comparing between students’ observable action and PTs observable action can lead us to two different characteristic of interpretation. First, comparing work is a characteristic when PTs compare students’s action about the task with their own action. Their own action is their written work about the same task that student’s done. Second, comparing knowledge is the characteristics when PTs compare students’ action with his/her knowledge or theories about given problem. This situation is represented in Figure 4.

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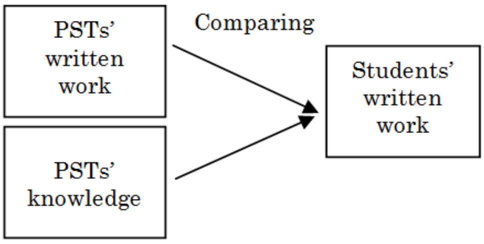

Figure 4. Type of Comparing

Comparing Work Model

The subjects in comparing work group are Ar and Sal. This group begin completing ToIoSMT by reading the BCP and attending to student's strategies. Ar and Sal showed evidence that they used comparing work model in interpreting students' mathematical thinking. Both of them completed the BCP to be compared with the students' written work artifacts. Ar completed the BCP first before attending to the student's work, while Sal completed the BCP after following the student's work. Although Sal solved BCP problems after attending to student work, both used their work explicitly to determine what was true among the four students i.e. belong to A and D.

Ar used his work to determine which of the four students' written work are correct. According to him, the number of workers to be added is 30 people and there are two correct answers. This is evident from the following aloud quotes.

> *If according to my work, e... additional workers will be 30 people. But here are different. Two students answered correctly.*

He said "if according to my work", it shows that he compared the student's work with his own work. At the time of interpreting the mathematical thinking of student A, Ar showed evidence that by completing the BCP he could presume students' unwritten thoughts. Ar's interpretation of student A's mathematical thinking is that student A understands what is known and what to look for eventhough the student does not perform a detailed calculation to obtain 150. Although the calculation process for obtaining 150 was not written, Ar implicitly assumes that students use inversed proportion. It was apparent from what Ar submitted in the passage.

> *... If I modeled the A student's work, A already knows that it used the concept of inversed proportion. .... Na, to get 150, he did not write that. It's quite confusing, how he gets 150. That possibility is 150 from ...could be... it's reversed. It can be reversed or 15 multiplied by 10. Possible as it is.*

This is reinforced by the explanation when asked how she got the number 10.

> *... Students guessed that from the existing number 12, 120, and 15 (refers to student A's work on the part of the relationship between time and many workers), the number 150 is likely to be obtained from 15 multiplied by 10.*

In the beginning, Sal attended to the students' written work according to what the students wrote without giving an argument. Next Sal used gestures while counting. It looks from the following Sal's behavior and think out loud.

> *Student's thinking ... it should be ... (scrutinize and mumble, move hands doing calculations) then 15 per 12 equals x per 120.*

Sal by herself also completed the BCP at the time of interpreting students' mathematical thinking. He did it when completing the second task. Sal used her work to convince herself about the details of student's strategies and mathematical thinking in completing the BCP. Sal saw that the work or the completion of each student is different. Further, Sal used her work to see which students are already understands and which ones are not. This is evident from the following interview excerpt between researcher (R) and subject.

R : *Further you assure which one is right which one is wrong with?*
Sal : *with ... completing the BCP by myself.*
R : *That's means you look back to .. (Sal reply: student work) heeh... then completing the BCP, then?*
Sal : *I conclude which one ... students who already understand, which ones have not.*

This group uses the accuracy of observation as the basis of belief and uses evidence to infer student interpretation (Swartz, 2012). Accurate observation starts from detailing the student's strategy to completion. At the time of writing (following the work of students), the prospective teacher checks whether there are still forgotten or left behind. For example, regarding the operating errors made by student C, they only discovered after reading several times. The characteristics of the subjects in comparing work group were completed the BCP both at the beginning and end of the interpretation process; checked out the right or wrong work of the student based on his/her work; recognized student strategies those match to their strategy. Their interpretations of students' mathematical thinking characterized by : 1) pay attention to the steps used to solve problems in the form of steps to solve the word problems by looking at whether students write the given information, what is asked, detailed steps of completion, as well as withdrawal / writing conclusions, 2) give an assumptions about strategies undertaken by students both written and unwritten, 3) pay attention to the concepts understood or not understood by students in this concept of inversed ratio; and 4) pay attention to the use of variables in completion.

Comparing Knowledge Model

Hap dan Hen analysed student work by comparing student work with their own knowledge while think out loud. They did not complete the BCP to be compared with the students' written work artifacts but compared it with their knowledge. Hap compared the student's work with her knowledge to interpret the students' mathematical thinking in completing the BCP.

Hap starts by reading the questions and proceed with examining the student's work. For example, the Figure 5 is rewriting of A's written work in completing BCP.

Inversed Proportion
Time number of worker
15 month → 20 people
15 - 3 = 12 month → x

Number of workers added:
= 150 – 120
= 30 workers

Figure 5. Rewriting of A's written work

Student A did not write an operation that shows an inversed proportion, A only wrote "inversed proportion". When attended to student A's work, Hap first thinks about the solution. Hap considered her knowledge of inversed proportion, i.e. comparison between variables, i.e. the time ratio is equal to the number of people. Using multiplication will give the number of people needed. While the number of workers added is obtained by reducing the number of workers should be with many existing workers. This is apparent when Hap looks at the work of Student A on the following think out loud excerp.

> *.... Students use the concept: Ta : Tb = na : nb (T denotes time, n states the number of workers, a and b state the first and second events). Later it will produce: the number of worker b (the second event) is equal to the number of worker a (first event) multiplied by Tb and divided by Ta. So the number of workers to be added is obtained by reducing the number of workers in the first time with the number of workers in the second time. That will result in the number of workers that must be added.*

Hap knew the right answer not by doing in writing but rather using his thinking (knowledge). She did it to match students' work with their thinking. In this way Hap becomes aware that the correct answer belongs to student A and student D. This is evident from the following interview excerpt.

R : *Are you scratching or not?*
Hap : *No.*
R : *why?*
Hap : *Because, from the student's work I'm direct ... from this work I was told to give a comment, I look at it from this answer. And I match with my thinking. Yes this is with ... such fondness ... and I get the correct answer between this one (A) and this (D). The point is these ones are true.*

Hen also showed evidence that he was doing analysis with comparing knowledge. When attended to the students' work, Hen did not know the students' answer right or wrong yet. He analyzed by considering the formula that should be used. It was apparent when Hen looks at the student's A work on the following think aloud.

> *The ratio used ... In the ratio formula there are two direct ratio and inversed proportions. The proportion used (while looking at student work) on the problems presented is a inversed proportion.*

In the interview, Hen explained her knowledge of the theory of inversed proportion. In inversed proportion, the formula used is A1 : B2 equal to A2 : B1.

> *Since it is presented that the ratio is reversed then the student should use a formula ... for example, using the ratio A1 : B2 equals A2 : B1 so. Na, this work is appropriate.*

Hen's interpretation of students' mathematical thinking A is that A's students have understood and can determine inversed ratio in BCP.The underlying reason for Hen's assumption that the student understands the inversed ratio is also apparent in the interview about how student A earns 150. According to Hen, students do a inversed ratio considering the longer the time of completion of work the fewer workers. And if work is accelerated then more workers are needed. The Figure 6 is Hen's thinking about the alleged operation A has performed.

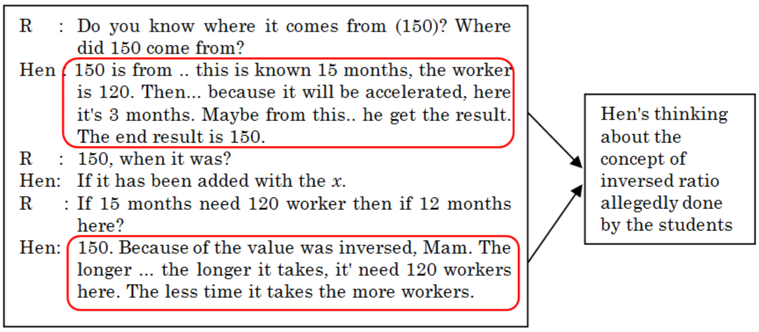

Figure 6. Hen's thinking about the concept of inversed ratio allegedly done by the students

The Comparing Knowledge group also uses accurate observations of the details of student work Swartz (2012) and beliefs about concepts that are understood to ensure their understanding. They based their beliefs on their details about the students' work or strategies they attended. This belief is based on the consideration that during the process of observing, they have considered their thoughts on the concept of proportion. Therefore, they no longer check the work, the order of work, or the concept used.

The characteristics of the subjects in comparing knowledge group are: not make an written solution of the problem, only use their knowledge or thinking about the solution and concept of inversed proportion, checked out the right or wrong of the students'work based on their unwritten solution, and recognized student strategies that match with their strategy.Their interpretation of students' mathematical thinking characterized by : 1) paying attention to the operations performed or suspected of the students and the use of the property of the counting operation on the students' work, 2) paying attention to the concept used is inversed ratio explicitly or implicitly as well as students' mistake, and 3) assessing what students have or have not understood.

The Use of Representation in Interpreting Students' Mathematical Thinking

The interpretation of the characters of different answers is also different. Student A's answer is incomplete, where the students only write what is known, what is asked, and the concept that will be used, namely the inversed proportion, but do not write the operating process. With this type of answer, comparing work and comparing knowledge group makes guesses about the operations performed by students. This is related to the use of the rubric in giving an assessment (interpretation). Prospective teachers need tools to interpret students' mathematical thinking on standard or evaluation criteria. The Comparing group has a rubric that is a set of coherent criteria for student work that includes a description of the level of quality of performance on these criteria (Brookhart, 2013). Both groups use different tools / rubrics to analyze students' work, namely comparing the work of students with their own work or with their thoughts. Comparing work groups use external rubrics to analyze students' mathematical thinking, where they use their work as a tool to assess or interpret. Comparing knowledge groups use their thinking as an internal rubric, where rubrics are not expressed in external representations. Prospective teachers have obtained an idea of how the problem will be resolved, what concepts are involved in it, and how he will provide an external or internal assessment.

Doodles about problem solving or thought of prospective teachers are basically external representations and internal representations of the rubric or the criteria they use to assess. External representations are representations that can be easily communicated to others, while internal representations are images created in mind for mathematical objects and processes (Cuoco, 2001; Hendroanto, *et al*. 2018). Comparing work groups use written solutions as a basis for analyzing or evaluating students' mathematical thinking. This is an external representation of mathematical concepts, acting as a stimulus for the senses (Janvier, Girardon, & Morand, 1993) that helps prospective teachers understand the concepts students use. The group of comparing knowledge uses their thinking as an internal representation of the rubric in solving problems. They describe mathematical ideas about inversed proportion in their thinking (Cuoco, 2001) so that they can be used to interpret students' mathematical thinking.

Representation of a problem is influenced by their past experience (Mason, 2011) where they can usually remember the same situation from their own experience. In contrast to more experienced teachers who can rely on their own teaching experience to understand students' thinking (Zhu, Yu, & Cai, 2018), prospective teachers rely solely on their knowledge and learning experience. Student B's answers immediately use reasoning without writing down what information is known and asked. Student B uses comparison reasoning worth $12/15 \times 120 = 96$. Students' answers end at 96 results without writing conclusions. For the work of student B, all subjects misinterpreted the students' strategies. They suspect that students only carry out operations on known information, namely $120/15 \times 120 = 96$. Even though they were aware of errors or irregularities that 120 divided by 15 were 8 instead of 0.8 but they were not aware of the actual operation performed by student B. Prospective teachers did not recognize the opacity of the students when they used unfamiliar algorithms. They experience challenges in interpreting student understanding or identifying key components of understanding that require attention in line with (Sleep & Boerst, 2012).

Students C and D solve the problems by writing completion steps in detail. The difference is, student C completes using a direct proportion while student D uses inversed proportion. For the work of students C and D, prospective teachers pay attention to the writing of operations in detail including student errors in using a direct proportion (student C) and the use of variable x which is used for t meanings (student D). Beyond that, attention is more to the stages of problem solving in the form of story problems such as writing down information that is known, what is asked, the process of completion, and writing conclusions. Interpretation of prospective teachers towards students' mathematical understanding tends to focus on the sequence of operations performed and the problem solving process. Prospective teachers use descriptive explanation to explain the mathematical procedure and the steps of solving problems such as delivered by Murtafiah, Sa'dijah, Candra, & As'ari (2018). They believe that if students solve problems by carrying out problem solving steps they understand the problem. The problem solving step in question refers to the problem solving phase

(Polya, 1945). The results of this study indicate that prospective teachers have similar interpretative characteristics. Interpreting of students' mathematical thinking are key teaching tasks in which teachers must generate hypotheses about how students' mathematical thinking could be developed (Fernández, Llinares, & Valls, 2012; Norton, McCloskey, & Hudson, 2011).

CONCLUSION

The goal of this study was to gain the description of prospective teacher used model of building process in interpretation students' mathematical thinking. As suggested by literature review, the existing literature in interpreting students' mathematical thinking is relatively sparse. In trying to better understand the model prospective teacher use in interpreting students' mathematical thinking, we find that comparing model-building process is still applicable today. This study has attempted to distinguish detailed comparison models based on representations that are used as a basis for prospective teachers to provide interpretations of students' mathematical thinking.

We find that comparing model-building process in interpreting students' mathematical thinking can be distinguished as two type ie. comparing work and comparing knowledge. The distinction between these two types of comparing model is based on the prospective teacher's implicit or explicit attention to the student's work and the analysis performed. Comparing work does an analysis by considering the external representation rubric by comparing the work of students with the results of their own work in solving the same problem. Whereas Comparing knowledge analyzes by considering the internal representation rubric by comparing students' work with the knowledge they have about the problem.

Prospective teachers use the rubric as a guideline to determine their interpretation of students' mathematical thinking. The rubric is different for both groups of prospective teachers. Comparing work groups use the external representation rubric while the comparing knowledge group uses internal repetition rubrics in the analysis to obtain interpretations of students' mathematical thinking. In the comparing work, prospective teachers completed BCP in written. While in the comparing knowledge, prospective teachers did not. They used their knowledge to analyze students' strategies or students' mathematical thinking but didn't express it in writing. The characteristics of the interpretation of the two groups are relatively same, which emphasizes the concern about the operations performed or allegedly done by the students, students' mistakes, and the assessment of students' understanding. Representation is the basis for determining the model of interpretation of prospective teachers. External representations are used by the compare works group, while internal representations are used by groups of comparing knowledge. The evidence of interpretations of prospective teachers can be used to assess the progress of mathematical understanding and harmonize learning in teacher education programs in a sustainable manner by supporting and developing effective learning.

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REFERENCES

Ahmad, S., Prahmana, R.C.I., Kenedi, A.K., Helsa, Y., Arianil, Y., & Zainil, M. (2018). The instruments of higher order thinking skills. *Journal of Physics: Conference Series, 943*(1), 012053. http://doi.org/ 10.1088/1742-6596/943/1/012053.

Brookhart, S.M. (2013). *How to create and use rubrics for formative assessment and grading*. Alexandria, VA: ASCD.

Cobb, P., & Steffe, L.P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(2), 83–94. https://doi.org/10.1007/978-90-481-9729-3_3.

Cuoco, A.A. (2001). *The Role of Representation in School Mathematics 2001 Yearbook*. Reston, VA: National Council of Teachers of Mathematics.

Fernández, C., Llinares, S., & Valls, J. (2012). Learning to notice students' mathematical thinking through on-line discussions. *ZDM - International Journal on Mathematics Education, 44*(6), 747–759. http://doi.org/10.1007/s11858-012-0425-y.

Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial visualization. *Journal of Physics: Conference Series, 943*(1), 012029. http://doi.org/10.1088/1742-6596/943/1/012029.

Jacobs, V.R., Lamb, L.L.C., & Philipp, R.A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education Journal for Research in Mathematics Education, 41*(2), 169–202. http://doi.org/10.2307/20720130.

Janvier, C., Girardon, C., & Morand, J. (1993). Mathematical symbols and representations. In P. S. Wilson (Ed.), *Research Ideas for the Classroom: High School Mathematics* (p. 81). Reston, VA: NCTM.

Mason, J. (2011). Noticing roots and branches. In M.G. Sherin, V.J. Jacobs, & R. Phillip (Eds.), *Mathematics Teacher Noticing Seeing Through Teachers' Eyes* (pp. 35–50). New York: Routledge. http://doi.org/10.4324/9780203832714.

Murtafiah, W., Sa'dijah, C., Candra, T.D., & As'ari, A.R. (2018). Exploring The Explanation of Pre-service Teacher in Mathematics Teaching Practice. *Journal on Mathematics Education, 9*(2), 259–270.

Norton, A., McCloskey, A., & Hudson, R.A. (2011). Prediction assessments: Using video-based predictions to assess prospective teachers' knowledge of students' mathematical thinking. *Journal of Mathematics Teacher Education, 14*(4), 305–325. http://doi.org/10.1007/s10857-011-9181-0.

Polya, G. (1945). *How To Solve It*. New Jersey: Princeton University Press.

Sapti, M., Purwanto, Mulyati, S., & Irawan, E.B. (2016). Pre-service teacher interpretations of students' mathematical understanding. In *Proceeding of 3rd International Conference on*

Research, Implementation and Education of Mathematics and Science (pp. 435–442). Yogyakarta: Faculty of Mathematics and Natural Science Yogyakarta State University.

Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings, 1952*(1), 020019. https://doi.org/10.1063/1.5031981.

Sherin, M.G., Jacobs, V.R., & Philipp, R.A. (2011). *Mathematics Teacher Noticing Seeing through Teachers' Eyes*. New York: Roudledge-Falmer. http://doi.org/10.4324/9780203832714.

Simon, M.A., & Tzur, R. (1999). Explicating the teacher's perspective from the researchers' perspectives: Generating accounts of mathematics teachers' practice. *Journal for Research in Mathematics Education, 30*(4), 252–264. http://doi.org/10.2307/749835.

Sleep, L., & Boerst, T.A. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education, 28*(7), 1038–1048. http://doi.org/10.1016/j.tate.2012.04.005.

Stacey, K. (2006). What is mathematical thinking and why is it important. *Progress report of the APEC project: collaborative studies on innovations for teaching and learning mathematics in different cultures (II)—Lesson study focusing on mathematical thinking*.

Steffe, L.P., & Thompson, P.W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Research Design in Mathematics and Science Education* (pp. 267–307). Hillsdale, NJ: Erlbaum.

Swartz, R. (2012). Infusing instruction in thinking into content instruction: What do we know about its success? *Sri Lanka Journal of Educational Research*, 1–39.

van Es, E. A., & Sherin, M.G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571–596.

Widodo, S.A., Istiqomah, Leonard, Nayazik, A., & Prahmana, R.C.I. (2019). Formal student thinking in mathematical problem-solving. *Journal of Physics: Conference Series*, *1188*(1), 012087. http://doi.org/10.1088/1742-6596/1188/1/012087.

Wilson, P.H., Lee, H.S., & Hollebrands, K.F. (2011). Understanding prospective mathematics teachers' processes for making sense of students' work with technology. *Journal for Research in Mathematics Education, 42*(1), 39–64. http://doi.org/10.5951/jresematheduc.42.1.0039.

Zhu, Y., Yu, W., & Cai, J. (2018). Understanding students' mathematical thinking for effective teaching: A comparison between expert and nonexpert Chinese elementary mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(1), 213–224. http://doi.org/10.12973/ejmste/78241.

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EXAMINING PROSPECTIVE TEACHERS' BELIEF AND PEDAGOGICAL CONTENT KNOWLEDGE TOWARDS TEACHING PRACTICE IN MATHEMATICS CLASS: A CASE STUDY

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Abstract

Beliefs and pedagogical content knowledge (PCK) are two factors influencing teaching practice in the classroom. This research aims to describe the beliefs and PCK of the prospective mathematics teachers and the relationship between the two factors on the teaching practices in the mathematics classroom. Participant in this research includes a prospective teacher who has taken a micro teaching subject and has good communication skill. Data were collected through interview and video analysis on the teaching practice in the classroom. The data obtained were coded, simplified, presented, and triangulated for the credibility and concluded. The result of the research shows that the prospective teachers who hold a constructivist belief view mathematics as a dynamic knowledge which evolves and is regarded as the space of creation for humans. Their beliefs on the nature of mathematics support the belief in the teaching-learning process in mathematics classrooms. Furthermore, a good understanding of the prospective teachers have on the components of the PCK has been sufficient, which can be identified in every step of practical activities in the classroom. More elaboration on the relationship between the belief and PCK is presented in this research.

Keywords: Beliefs, Pedagogical Content Knowledge, Teaching Practice

Abstrak

Keyakinan dan *pedagogical content knowledge* (PCK) merupakan dua hal yang mempengaruhi praktik pembelajaran di kelas. Penelitian ini bertujuan untuk mendeskripsikan keyakinan dan PCK mahasiswa calon guru matematika, serta hubungan antara keduanya terhadap praktik pembelajaran matematika di kelas. Partisipan penelitian adalah seorang mahasiswa calon guru yang telah menempuh mata kuliah *micro teaching* dan memiliki kemampuan komunikasi yang baik. Pengambilan data dilakukan dengan teknik wawancara dan analisis video praktik pengajaran di kelas. Data yang diperoleh dilakukan pengkodean, penyederhanan, dipaparkan, ditriangulasi untuk mendapatkan data yang kredibel, kemudian ditarik kesimpulan. Hasil penelitian menunjukkan bahwa mahasiswa calon guru yang memiliki keyakinan konstruktivis memandang matematika sebagai suatu ilmu yang dinamis, berkembang, dan merupakan ruang penciptaan manusia. Keyakinannya terhadap sifat matematika mendukung keyakinannya terhadap pengajaran dan belajar matematika. Selanjutnya pemahaman yang baik dari mahasiswa calon guru terhadap komponen PCK khususnya *knowledge of mathematics*, *knowledge of teaching*, dan *knowledge of students* telah mendukung dalam setiap tahap kegiatan praktik pembelajaran yang dilakukan di kelas. Uraian lebih detail tentang hubungan antarkomponen keyakinan dan PCK dibahas dalam penelitian ini.

Kata kunci: Keyakinan, *Pedagogical Content Knowledge*, Praktik Mengajar

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Several factors are contributing to the inconsistency between what is planned and what is practiced in the teaching and learning process at school. A good understanding of mathematical knowledge can influence teaching instruction in the classroom in a positive way. Teachers need the knowledge to transform the content into the representative form, which is believed to be able to help the students to improve their competence (Shulman, 1986; Muhtadi, *et al*. 2018). Research on teacher's knowledge is based on the pedagogical content knowledge (PCK) introduced by Shulman (1986, 1987). Even though PCK facilitates the instructional practices in the classroom, some teachers who are strong in the content knowledge tend to use traditional teaching method (Mewborn, 2001). This is due to the belief that the traditional method is the most effective method for teaching mathematics to the students. Teachers with this belief will tend to use the same method in their classroom teaching practice. Hence, PCK alone is not sufficient in contributing to an effective classroom teaching practice.

Belief and knowledge are two important factors influencing teaching and learning practice (Fennema & Franke, 1992). Empirically, researchers have found that belief consistently affects teaching practice in the classroom (Pajares, 1992; Stipek, *et al*. 2001; Thompson 1992). Instrumentalist belief tends to be associated with the teaching practices that are more traditional (Stipek, *et al*. 2001). A teacher with an instrumentalist belief is also said to be allowing the students to explore and maintain social context, where mistakes are avoided. On the other hand, researchers have found that there is an inconsistency between the belief and practice in the mathematics classroom (Barkatsas & Malone, 2005; Beswick, 2003; Raymond, 1997). Raymond (1997) assert that the teacher's learning process is much more influenced by his/her knowledge than by his/her belief. Barkatsas & Malone (2005) found that teacher belief is not consistent with her teaching and learning practice. This is due to the classroom situation, experience, and social norms.

Research on the relationship between belief, knowledge, and mathematical learning and teaching practice has been conducted by Wilkins (2008) and Belbase (2012). Wilkins (2008) studied 481 math teachers of primary school to investigate the relationship between belief, content knowledge, and attitude on the inquiry-based teaching practice. Data were collected using a questionnaire and were analyzed using structural equation modeling. The result of the research shows that teachers' beliefs have the most significant influence on the teaching practice, while knowledge has a negative correlation with the beliefs in the classroom teaching and learning practice. Belbase (2012) examined and analyzed beliefs, knowledge, and teaching practice of the math teachers through literature review and drew some pedagogical implications. The result of the research shows that belief influences the teaching practice in the context of a supportive learning environment. The teachers' beliefs influence teachers' development. Teachers' knowledge comprises content knowledge, pedagogical knowledge, technological knowledge, and the combination of this knowledge. Both studies provide an initial picture of the relationship between beliefs and content knowledge in PCK. However, they have not described the specification of the beliefs, PCK, and how the belief aspect and PCK can influence the mathematical teaching practice.

Pedagogical Content Knowledge (PCK)

The initial research on pedagogical content knowledge (PCK) was done by Shulman (1986, 1987), and developed by (An, Kulm, & Wu, 2004; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Kilic, 2011). Shulman (1986, 1987) stated that PCK is an integration of content knowledge and pedagogical knowledge realized in the method of presentation and content formulation (learning material) which are comprehensible and can facilitate learning. This definition emphasizes on three aspects: 1) content, 2) pedagogy, and 3) student. PCK does not only emphasize on how to utilize a concept to solve a problem but also on a deep understanding of the concept (Ball, *et al*. 2005). Hence, PCK does not focus merely on how to use an algorithm to solve problems, but also on a deep understanding of the concept.

PCK consists of knowledge of content and student, knowledge of content and teaching, and knowledge of content and curriculum (Ball, *et al*. 2008). However, Hawkins (2012) specifically explained the three components of PCK that one needs to have: (1) knowledge of mathematics (KM), (2) knowledge of teaching (KT), and (3) knowledge of students (KS). Knowledge of mathematics refers to the knowledge and skill of mathematics, which is exclusively used in the teaching practice, such as conceptual knowledge and procedural of the topic of mathematics. Knowledge of teaching is related to the knowledge on how to teach mathematics to students. For example, the knowledge of choosing the examples before starting the lesson, choosing the type of assignment which will help students understand better, the representation that is used, and the questions given during classroom teaching and learning activities. Meanwhile, the knowledge of students refers to the knowledge in understanding students' ideas in solving the problems, diagnosing errors, and making strategies to overcome the errors.

Mathematical content knowledge constitutes a pure knowledge and its organization in the mind of an educator, which includes the ability to explain why one theory needs to be taught, as well as its relation to other theories. Mathematical content knowledge and pedagogical knowledge is one integrated part of the effective mathematical teaching and learning that establishes the concept of mathematics in the student's mind practice (Kahan, Cooper, & Bethea, 2003; Shulman, 1986). Thus, PCK is considered as knowledge of teaching, especially on transforming mathematical knowledge into different kinds of representations and considering obstacles faced by the students, which then enables the comprehension of mathematical content.

Beliefs

Understanding that there might be differences in beliefs is important in developing and ensuring the success of the implementation of mathematics education at schools (Dossey, McCrone, & O'Sullivan, 2006). Beliefs are in the "twilight zone," which means they are between the cognitive domain and the affective domain (Pehkonen & Pietilä, 2003). Belief is the basis for an individual in behaving and in understanding a phenomenon. Belief is a mental condition which is perceived as true and can originate from experiences, either real or imaginative, and influence words and behaviors. Ernest (1989) describes three components of mathematical beliefs: beliefs about the nature of mathematics, teaching, and learning mathematics. Each component of the beliefs consists of three categories of philosophical views, namely: instrumentalist, Platonist, and constructivist. Ernest's views have been adopted and widely used by some scholars (Beswick, 2005, 2012; Buehl & Fives, 2009; Muhtarom, Juniati, & Siswono, 2017a, 2017b, 2018; Muhtarom, *et al*. 2018; Siswono, Kohar, & Hartono, 2017; Siswono, *et al*. 2017; Thompson, 1992). A person who has instrumentalist beliefs sees mathematics as a set of tools consisting of a set of mathematical facts, concepts, rules, and math skills that are not interrelated but useful. Platonist beliefs look at mathematics as being found, not created. It means that mathematics is static, but an integrated field of science where the truth is intertwined by a set of rules, concepts, and theorems. Constructivist view mathematics as dynamic, that is, the space of creation and human discovery that develops continuously where patterns are raised and then filtered into knowledge (Ernest 1989; Thompson, 1992).

There have been many research on the beliefs that focus on students and teacher, but research focusing on prospective teachers are very limited (Bal, 2015; Boz, 2008; Giovanni & Sangcap, 2010). Muhtarom, *et al.* (2017b) assert that from 183 prospective mathematics teacher, most of them do not show consistency beliefs about the nature of mathematics, teaching, and learning mathematics. For example, there are 2.73% prospective teachers who have instrumentalist beliefs about nature of mathematics, but have Platonist beliefs about teaching, and learning mathematics; 7.10% of prospective teachers have Platonist beliefs about nature of mathematics, and teaching, but have constructivist beliefs about learning mathematics. Only 2.19% of prospective teachers consistently have constructivist beliefs about the nature of mathematics, teaching, and mathematics learning.

METHOD

Participants

Before the selection of research subjects, researchers first gave written open questions to 172 prospective mathematics teachers at a private university in Semarang, Indonesia, to get an initial description of his belief in mathematics, teaching and learning mathematics. This data collection is done gradually in each class of the 6^{th} semester. The result is that there are 38 or 22.09% prospective teachers who have instrumentalist beliefs, 10 or 5.81% who consistently have Platonist beliefs and only 3 or 1.74% are consistent with constructivist beliefs, and the rest are not consistent in their beliefs. The research on the consistency of beliefs refers to Beswick (2005).

One of the criteria for choosing a research sample is their willingness to participate and consistently have constructivist beliefs, this is following the current 2013 Curriculum requirements applied in Indonesia in which educators are required to facilitate students to explore information, actively ask questions, and link information is to form a mathematical understanding. Thus, the suitability can be seen between knowledge and beliefs with learning practices conducted in the classroom. A consistent prospective mathematics teacher who has constructivism view was chosen to be the sample of research, who is coded with Fitri (pseudo name). One sample is considered sufficient because the data obtained from all three samples have shown saturation of the data, and there has been sufficient information to replicate the results of research based on repetitive data retrieval (Moleong, 2007). Fitri is a 21-year-old student and Javanese. Fitri was born in Purwokerto Regency, Central Java Province, Indonesia. Before the execution of this research, Fitri did not yet have teaching experience at school.

Instrument and Procedures

Data were collected through semi-structured interviews and observation of classroom teaching practices. Confidential data retrieval was conducted from May to June 2017. The interview on beliefs focused on obtaining data on the beliefs about the nature of mathematics, and beliefs about teaching and learning mathematics, which is adopted from Ernest's (1989) view. The belief in the nature of mathematics is a person's mental state of mathematics as a discipline that is recognized as a truth (Beswick, 2012; Ernest, 1989; Thompson, 1992).

Meanwhile, beliefs about teaching and learning mathematics are a person's mental state on different types of learning approaches, the role of teachers and students in the learning process, activities in motivating students to learn mathematics, and how students learn mathematics. Interviews intended to explore PCK especially related to knowledge of mathematics, knowledge of teaching, and knowledge of students, were conducted in August 2017, when Fitri participated in a three internship program aimed at allowing participants to practice mathematics learning at a state junior high school in Semarang. Interviews on belief and PCK were conducted twice, then triangulation technique was used to obtain credible data. Table 1 illustrates the examples of interview protocols used to explore the beliefs and the PCK of Fitri. All interview protocols are presented so that the readers will have a comprehensive picture of the issues discussed in this research.

Table 1. Questions from the Semi-Structured Interview Protocol

Focus of Research	Sub Focus of Research	Questions
Beliefs about the nature of mathematics	Definition of Mathematics	Based on your belief, what is mathematics?
	Relationship between mathematics and daily life	What is the relationship between mathematics and daily life? Elaborate!
	The development of Mathematics science	Elaborate your belief on the development of mathematics science!
Belief about the teaching of mathematics	Teaching Approach	Elaborate your belief on the effective approach implemented in teaching mathematics!
	Role of teachers in the instruction	What is the ideal role of the teacher in mathematics instruction!
	Problem-solving	Elaborate your belief on the effective type of exercises/questions to be given to the students! Who should design them, and what is the source? Give your reasons!
	Motivation on learning mathematics	What method do you apply to motivate the students to learn mathematics? Elaborate! What do you do when students think that mathematics is not relevant to their daily life.
Belief about learning mathematics.	The role of the students	How should students learn math? What should students master to learn math?

Focus of Research	Sub Focus of Research	Questions
Knowledge of mathematics	Conceptual Knowledge	What do you know about linear equation system in two variables concept?
	Procedural Knowledge	What are the steps for setting the substitution, elimination, and combined method
Knowledge of teaching	Learning objective and instructional media used	Elaborate on the learning objective that needs to be achieved! What is the design of instructional media to support learning objective?
	The use of representation	What kind of mathematics representation do you use? Describe the advantages of the media
	Math problems	What example of math problems do you use to start the lesson? Wha math problem do you use to understand the material better?
	Questions	When do you give new questions? When do you use questions that ask for clarification from the students?
Knowledge of students	Knowledge of students' ideas in solving the problems.	Describe the students' idea of solving math problems.
	Knowledge of students' errors	With the aforementioned ideas, Is the student making an error? Elaborate!
	Knowledge in solving students' errors	How do you improve your students' errors?

Two teaching-learning sessions with duration 2 x 40 minute was recorded to obtain data on classroom teaching practices. Fitri teaches in class VIIIA SMP Negeri 6 Semarang, Central Java, Indonesia with the material of linear equation system in two variables. After the series of learning activities have been completed, the interview was conducted to describe Fitri's reflection on what she has stated in the previous interviews so that a relationship between beliefs, PCK, and teaching practices can be made.

Data Analysis

Video recordings and interviews are transcribed, then read and verified again by Fitri to ensure the accuracy of the data. It is then coded and analyzed in an interesting pattern to describe the beliefs and PCKs owned by Fitri. The response patterns associated with beliefs interviews are categorized into instrumentalist, Platonist, and constructivist beliefs for dimensions of mathematical properties and teaching and learning of mathematics. Meanwhile, the pattern of PCK responses describes Fitri's knowledge to teach linear equation system in two variables material by considering the errors made by the students. Overall, the recorded video was played and viewed together with Fitri to explore classroom learning practices.

Data analysis is done through the process of (1) data reduction, which refers to the activity of the election process, centralization of attention, simplification, abstraction, and transformation of raw data. (2) Data presentation, which constitutes organizing data of encoding results through the pairing of first and second data, then subsequently compared to obtaining credible data, and (3) conclusions drawing from data collected and conclusions verification about beliefs, PCK and learning practices undertaken (Moleong, 2007). To demonstrate the relationship between beliefs components, PCK and mathematics learning practices, qualitative data were analyzed using the QSR NVivo 11 software on cluster analysis features (Bazeley, 2007; Bazeley & Jackson, 2013; Muhtarom, Murtianto, & Sutrisno, 2017). Nvivo software is the most effective tool in analyzing qualitative data because it provides completed tools and ideal in analyzing qualitative data (Bazeley, 2007; Bazeley & Jackson, 2013; Hamrouni & Akkari, 2012). Thus, Pearson correlation will be obtained to illustrate the relationship between the components in this research.

RESULT AND DISCUSSION

Fitri's Beliefs about Nature of Mathematics, Teaching and Learning of Mathematics

Fitri tends to consistently have constructivist beliefs about the nature of mathematics, teaching and learning mathematics. Fitri beliefs that mathematics is a dynamic science and mathematical knowledge develops as humans always strive to do continuous research of new problems existing in life has influenced her beliefs in teaching mathematics and learning mathematics. This is in line with the opinions of some experts (Amirali & Halai, 2010; Ernest, 1989; Grigutsch, Raatz, & Törner, 1998; Thompson, 1992; Viholainen, Asikainen, & Hirvonen, 2014). Mathematics, as a dynamic field of human creation, continues to evolve according to discovery patterns and the results remain open to revision (Ernest, 1989). Mathematics is seen as an active construction process (Grigutsch, *et al.* 1998; Shahrill, *et al*. 2018). Ongoing research on problems in life is based on experience and observation of the regularity of the phenomena of social context. Thus, mathematics is always changing and never static (Buehl & Fives, 2009). The relevance of this relationship can be seen when Fitri sees that a suitable approach to be applied in mathematics teaching is those which is student-centered where learning begins with giving the students problems (Barkatsas & Malone, 2005; Ahamad, *et al*. 2018), helping students understand problems, discussing and finding solutions (Felbrich, Kaiser, & Schmotz, 2014; Stipek, *et al*. 2001; Prahmana & Suwasti, 2014), and presenting it. This approach makes learner as a focus rather than the mathematical content (Beswick, 2012).

Fitri believes that students should be given non-routine questions to complete in the learning activities, this is in line with Muhtarom, *et al.* opinion (2018), and students are also allowed to make their problems and solutions independently (Siswono, *et al.* 2017). To facilitate the achievement of the learning goal of mathematics, the teacher acts as a facilitator in the learning activities. This means teachers should facilitate inquiry, allowing students to develop solutions to their problems, and enabling students to play an active role in learning activities. Facilitator can also mean applying classroom activities to help students create new mathematical concepts, as well as to encourage reasoning, creativity and information gathering, learning occurs when there is social interaction involving collaborative dialogue with other students as well as teachers (Stipek, *et al.* 2001), co-operativeconstructivist view (Barkatsas & Malone, 2005), and learner-focused content (Beswick, 2012).

Fitri believes that students need to be given the freedom to construct their knowledge. The teacher only gives direction for further students to dig further understanding. Thus, when students learn mathematics, they need to be able to construct a mathematical concept, develop it, and solve it appropriately. This is in line with the opinion of Ernest (1989) which states that learning is an active understanding construction, perhaps even problem posing and problem-solving. The learning objective constitutes acquiring skills in reasoning and building new things (Felbrich, *et al*. 2014). Beswick (2005, 2012) views mathematical learning as an autonomous exploration of self-interest. Thus, the things students should master in learning mathematics is to construct mathematical concepts, develop them and solve the math problems appropriately through practicing solving varied problems from various sources of references, making summaries, and constructing those mathematical concepts (Ernest, 1989; Thompson, 1992).

Fitri's beliefs that mathematics develops by the problems of daily life, mathematics provides solutions which can be used to solve problems in mathematics itself and everyday life, can be identified Fitri's from the way she started the lesson where she gave students problems to solve. The problem should be made as relatable as possible to the contextual and real problem so that it is close to the students' daily life where mathematics can be used to provide solutions (Muhtarom, *et al.* 2017b; Siswono, *et al.* 2017). Through this method, Fitri has shown her way of motivating students by pointing out the relevance of mathematics to everyday life. The inspirational figures of mathematics also form mathematics based on observations and patterns that exist in life. This is in line with Stipek, *et al*. (2001) which suggest the importance of motivating students during learning activities.

This description has explained the consistency of Fitri's beliefs about how to teach mathematics and how students learn in the classroom as described by some researchers (Boz, 2008; Ernest, 1989; Thompson, 1992). Fitri's belief that mathematics is a dynamic field of human creation, which continues to evolve according to the pattern of discovery has influenced her decision-making to use student-centered learning approach that is more learners-focused than content-focused. This approach suggests that students should practice solving varied problems from various reference sources, making summaries, and constructing concepts and procedures while solving math problems. The conclusion is Fitri's belief that mathematics is a dynamic science, evolves, and is a space of human creation, has supported her belief in her teaching practice.

The above description is supported by the positive relationship between each component of belief using software QSR NVivo 11. The correlation coefficient used in this cluster analysis is the Pearson correlation coefficient. Obviously, Figure 1 shows that there is a strong relationship between beliefs about nature of mathematics and beliefs about teaching mathematics, which is indicated by a correlation coefficient of .651, a fairly strong relationship between beliefs about nature of mathematics and beliefs about learning mathematics, which is indicated by a correlation coefficient of .569, and a strong relationship between beliefs about nature of mathematics teaching and beliefs about learning mathematics, which are indicated by a correlation coefficient of 0.741.

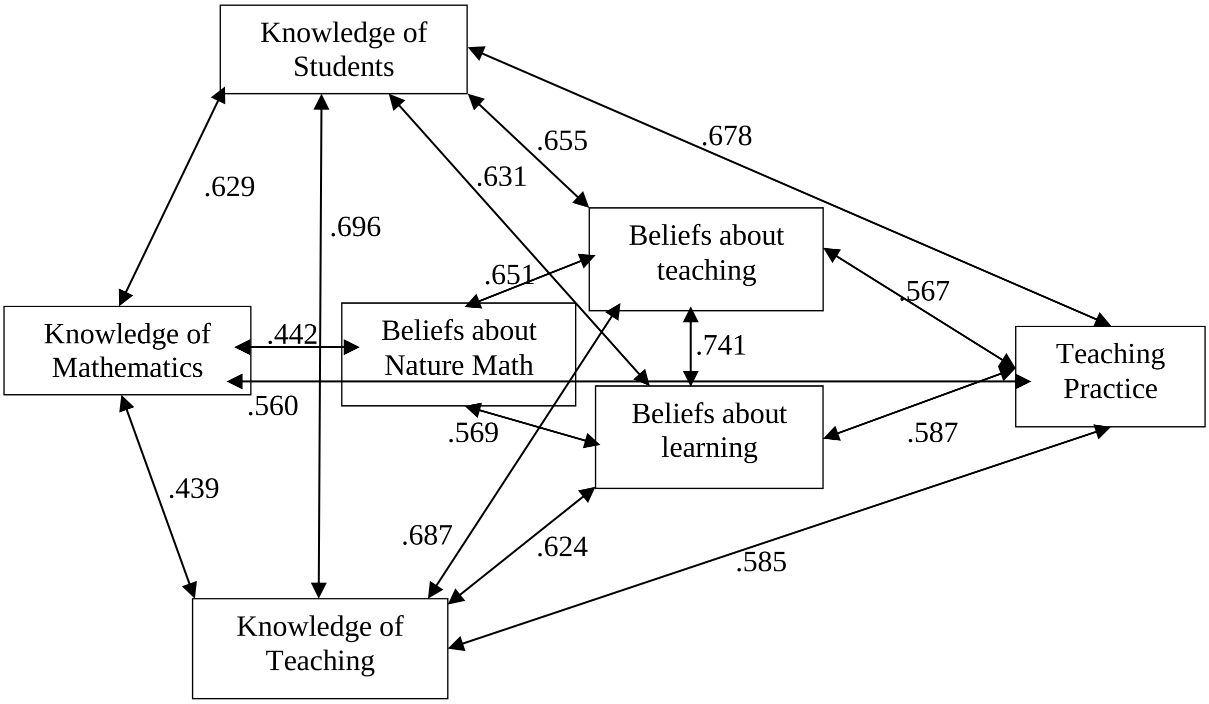


Figure 1. The relation between Beliefs, PCK and Teaching Practice

Fitri's Pedagogical Content Knowledge (PCK)

Fitri has conceptual knowledge and procedural knowledge related to linear equation system in two variables. Conceptual knowledge is demonstrated through understanding the definitions of linear equation system in two variables, the concept of elimination methods and the concept of substitution methods. This can be seen when Fitri understands the concept of linear equation system in two variables that it consists of two linear equations, with every variable to the first power. The concept of elimination is where one variable is being eliminated, while the substitution concept is to replace one variable with another variable by the given equation. Procedural knowledge is identified from an understanding of the steps to solve math problems using methods of elimination or substitution. The understanding of the steps of the elimination method is indicated from the knowledge on determining which variable should be omitted. When the coefficient is different, it must be equalized first by multiplying it with the opposite (look for the smallest multiplier) of the coefficient to be omitted. The second equation should go through the same steps before it can be solved. Understanding the step of the substitution method is indicated from her ability in determining the equation where x or y will be changed into $x = \ldots$ (x in y) or $y = \cdots$ (y in x), then substituting the equation into one of the equations. If the first equation is changed, then it must be substituted to the second equation and vice versa. Next, after the value of x or y is obtained, the next step is to substitute them into one of the equations.

Fitri was able to understand the ideas of students in solving the given problem very well. This is shown from her fluency in describing each answer given by the student and was able to identify the errors made by the students in completing the worksheet or the questions made by the students themselves. Fitri further explains that misconceptions of the student in solving the student worksheet occur in the concept of counting operation, for example the negative number is subtracted by the negative number ($-6-(-3)=-9$), the negative number is subtracting the positive number ($-12-(48)=60$), and another error on the substitution concept when the student cannot change a linear equation of two variables into the variable x in y. While the error procedure in performing algebra operations such as errors in multiplication, addition, subtraction which affects the results. To solve the student's misconception, Fitri gave an example of the problem, emphasized the error, and emphasized the concept of 'sakubeta' on the same variable. If the sign was the same, then it must be subtracted. In contrast, if the sign was different, then it should be added.

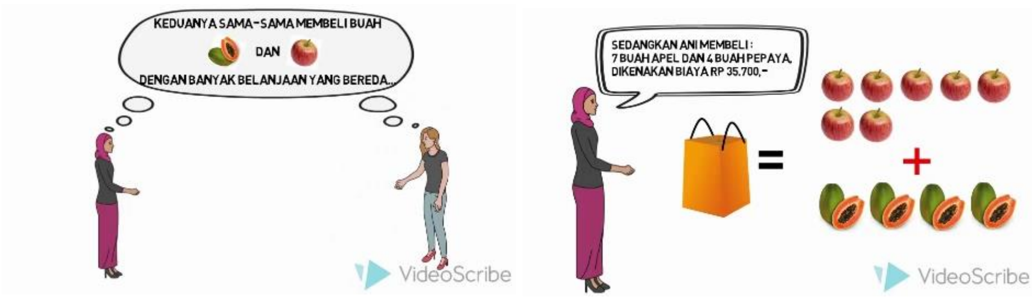
Fitri's understanding of the knowledge of mathematics (KM) has supported her decision in planning classroom activities (KT). For example, Fitri prefers examples at the beginning of the lesson using contextual story problems, while the question to deepen the understanding of the material was done by giving a story problem that has a different level of difficulty with the ones previously used when starting the lesson. Fitri designed the learning activities using the image and symbol representation. The image representation is used to clarify the concept of the given problem so that it is completely contextual to the students. The table representation is used to assist the student in modeling the given problem so that the student can have a clear illustration. Fitri asks new questions when students have understood the lesson and can use linear equation system in two variables method of completion, whereas clarifying questions are given when students have already presented their work and invited other students to respond. A teacher should be able to ask appropriate, meaningful questions to understand students' thinking processes (Muzaini, Juniati, & Siswono, 2019; Turnuklu & Yesildere, 2007). They also need to have the ability to create solutions to overcome students' learning difficulties.

This result is supported by Shulman (1986), who states that understanding of PCK is a strong foundation for effective teacher education. A deep understanding of mathematics and students' knowledge was not enough to teach mathematics (Black, 2008; Turnuklu & Yesildere, 2007). Knowledge of mathematical pedagogy was necessary. Furthermore, teachers should be able to understand students 'difficulties, understand the reasons behind it, and be able to ask appropriate and meaningful questions to understand their thinking processes. Even Kilic (2011) confirms that the content and student knowledge is the determinant of a teacher's success in mastering the class and answering students' questions. The description is supported by cluster analysis with QSR NVivo 11 software, presented in Figure 1. For example, there is a strong relationship between knowledge of mathematics and knowledge of students, which is indicated by a correlation coefficient of 0.629. There is a strong relationship between knowledge of students and teaching practices, which is indicated by a correlation coefficient of 0.678, and the relationship is quite strong between knowledge of teaching and teaching practices, with a correlation coefficient of 0.585.

The Relationship between Beliefs and PCK toward Teaching Practice

Fitri's instructions are constructivist-oriented, which reflect what she believes. Fitri believes that mathematics is a dynamic branch of science. In line with the beliefs about teaching that was mentioned before, the appropriate approach for learning mathematics is student-oriented or student-centered where learning activities start with the provision of contextual problems, teachers help students discuss and find solutions to problems before presenting them. This belief is implemented in the practice of learning through question and answer method and discussion method. Instructional media, which contains contextual illustrations and representations of images and symbols, is an indicator that she has applied what she has planned and what she believes. This can be seen from the teaching practice she underwent. Fitri used image representation and symbols. The image representation is used to clarify the concept of the given problem so that it is completely contextual for the students, while the symbol representation by using certain variables is used to facilitate problem-solving. The following is the record of Fitri's instructions.

PPM.K.06 : Now sit in a group. Please set the chair facing each other (handing out student worksheets) pay attention and watch the video I am showing you. Then please fill in the exercises in your worksheet by the information from the video. Pay attention! (Playing the videos associated with linear equation system in two variables, students are focusing and listening). Please work on and discuss with your friends in your group (Teacher walks around and guides the discussion).



PPM.K.17 : Look at this, here is the linear equation system in two variables table. So the image of the apple and papaya can be presented in a tabular form. This table is used to make it easier for you to create a mathematical model from linear equation system in two variables. The third question, please make the linear equation system in two variables model.

Student : $5x + 5y = 30{,}000$; and $7x + 4y = 35{,}700$; specify the solution for $5x + 10 y = ...$?

PPM.K.18 : I will eliminate y variable, how do you do that?

Student : Through the elimination method, by equating the coefficient of y variable.

PPM.K.19 : Look at this, the y coefficient in equation one is 5, and the y coefficient in equation two is 4. What is the value of x obtained?

Fitri has been able to connect the representations with fundamental ideas and other representations to make students better understand the material being taught (Ball, *et al.* 2008). It should be noted, however, that even though the prospective teachers have used some representations, students are only using symbol representations in solving the math problem. This is possible because the emphasis of conceptual understanding appears to be very limited compared to a procedural understanding of problem-solving. In line with this, Black (2008) has explained that procedural knowledge is very dominant in the practices of classroom instruction. The fact that students' tendency to use symbolic representations related to mathematical problem-solving procedures including in solving the problem of linear equations system of two variables is influenced by the teachers' habits while teaching in the classroom (Nizaruddin, Muhtarom, & Murtianto, 2017). Furthermore, Fitri's content and student knowledge have influenced the organization of her instructional decisions, class explanations, guiding the students, and the way they conduct mathematical discussions.

Fitri's knowledge and beliefs are implemented in her learning practice. Fitri knows that a suitable example to start the instruction is a contextual story problem (see PPM.K.06). She also understands that it will be more effective to give the problem with a different level of difficulty when aiming at deepening the understanding on the content. This is in line with what Fitri believes that teachers should provide non-routine questions for students to complete, as long as learning is made by teachers and students. The following is a quote from Fitri's learning practice.

PPM.K.23 : If you still feel confused, please ask? Now try to do the problem in this worksheet (Distribute the second worksheet). Please work in groups.

Student : (Students discuss problem-solving, and the teacher walks around guiding the students)
In a housing complex residents, there is Mr. Hasyim, Mr. Ikhsan, and Mr. Alwi plans to create a gazebo pole is made of adobe. Mr. Hisham gazebo pole shape is shown in Fig. (1). Mr. Ikhsan has a shape like a gazebo pole Fig. (2). While Mr. Alwi wanted to make a different gazebo pole shape with the shape of the gazebo Mr. Hasyim and Mr. Ikhsan (see Fig. (3)). Determine how high the gazebo pole belongs to Mr. Alwi!

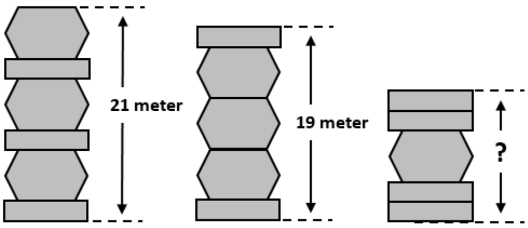


Fig. (1) Fig. (2) Fig. (3)

In accordance with her belief that knowledge of mathematics continues to evolve and is the product of human findings, the student's roles are to construct an active understanding through problem posing and solving (Ernest, 1989), building understanding through reasoning (Felbrich, *et al*. 2014), and building knowledge their own mathematics as facilitated by their teacher (Beswick, 2012). This can be seen from the classroom instruction practiced by Fitri. In her instruction, the use of media is only to motivate students by relating mathematics with everyday life. Fitri does not explain the subject matter (linear equation system in two variables), when the student discusses to solve the problem, she helps the students who were having difficulty solve the problem by giving direction, then the student solve it based on their understanding. Although the understanding of the elimination, substitution, and combination method are still limited; the teacher acted as a facilitator in helping students construct mathematical concepts so that they can solve math problems appropriately. Students were trained to solve problems with different level of difficulty, and are also trained to solve the problems they make themselves. This is consistent with what Fitri has been believed that the teacher acts as a facilitator while the student is an active knowledge constructor.

Focus on the interaction between students and teachers can also be identified in Fitri's classroom teaching practice. During the discussion, teacher-student interactions happened and were established one-on-one with the students in each group. During the presentation, there was an interaction between the teacher and the whole student. The dialogue conducted by Fitri tends to use questions to ask for clarification of student comments or answers. The high frequency indicates that Fitri will approach students to seek information through a series of questions leading to the discussion. Fitri believes that through the provision of questions, students will be actively engaged in the learning process. Through the discussion, students were also trained to be able to construct mathematical concepts, practice solving various problems, and develop the problems. Fitri also used questions as a way to assess the experience and knowledge of her students. In other words, the more the feedback gained from the students, the better the teachers understand the weaknesses and continue to improve them. This fact is consistent with the knowledge of when to ask questions to students and Fitri's belief that the students' involvement in collaborative learning is always beneficial to the learning process. The following is a quote from Fitri's learning practice.

PPM.K.10 : What do you understand about this equation?
Student : Model of linear equation system in two variables, so 5x + 5y = 30,000 and so on
PPM.K.11 : And then?
Student : And then we need to define the solution. Bu I am confused about the question, does it means that the price of 1 apple should be multiplied by this (5 apples) and the same with papaya?
PPM.K.14 : What word indicates what?
Student : The one that asks. So this is the given information only, then we should make an example and then create an equation of a mathematical model, and then solved using the combined method, is that correct?
PPM.K.18 : How to remove y variable?
Student : Eliminate it by equating the coefficient of y variable.
PPM.K.24 : (guiding the discussion) what is the first step to do it?
Students : The information that is known and asked, and then makes an example.

Turnuklu & Yesildere (2007) explain that teachers should be able to ask appropriate, meaningful questions to understand students' thinking processes and should also have the ability to create solutions to student's learning difficulties. The teacher's role is to diagnose student errors and point out student difficulties, guide and facilitate students, rather than providing answers and explanations. Teachers should also recognize students' needs in understanding the lesson. Fitri has done these in the implementation of mathematics learning practice. It is clear that Fitri has used question and answer methods and discussion methods during classroom learning activities. The application of the use of this method of learning is supported by her knowledge of learning and media objectives, the use of examples and tasks is supported by the use of representation, and ideas on when to ask students questions about clarification and when to ask new questions which was done when students have understood the material. This is supported by cluster analysis using QSR NVivo 11 software presented in Figure 1. It is clear that the correlation coefficient between each component of belief and PCK component to the teaching practice is obtained. Overall belief correlates with PCK so as to support the practice of mathematics learning in the classroom.

The above description indicates that this research is consistent with previous research. However, it provides the additional belief that understanding beliefs is an important step in understanding learning practices (Ernest, 1989; Pajares 1992; Stipek, *et al.* 2001, 2001; Thompson 1992). The results of this research are also consistent with the literature research which highlights the positive relationship between content knowledge and teaching practices (Belbase, 2012; Fennema & Franke 1992). As previously described, knowledge of mathematics on linear equation system in two variables material, knowledge of ideas and errors made by students, and knowledge to overcome student errors, have supported prospective teachers in designing learning activities, and then apply them in the classroom.

CONCLUSION

The findings of this research indicate the important role of beliefs and PCK as a factor affecting the learning and teaching practice, as shown by Fitri's experience. The results of this research are consistent with the literature research which highlights the positive relationship between belief and knowledge of teaching practices. Beliefs correlate with PCK and teaching practices as a system. The results of this research can be used as an input for the curriculum of Mathematics Teacher Education program to integrate the philosophy of the belief and to strengthen students' PCK capability which will result in the integration between cognitive ability and mathematical teaching knowledge. It is also important to understand that prospective teachers need to be allowed to have a sustainable learning experience. This is useful for giving them a chance to reflect on their beliefs and PCK so they can improve in the future. The results of this research are hoped to be able to provide a diversity of relationships between beliefs and PCK on learning practices conducted by prospective mathematics teachers.

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REFERENCES

Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series, 943*(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.

Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A Survey of Secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research, 32*(2), 45-61.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the US. *Journal of Mathematics Teacher Education, 7*(2), 145–172. https://doi.org/10.1023/B:JMTE.0000021943.35739.1c.

Bal, A.P. (2015). Examination of the mathematical problem-solving beliefs and success levels of primary school teacher candidates through the variables of mathematical success and gender. *Educational Sciences: Theory and Practice*, *15*(5), 1373–1390. https://doi.org/10.12738/estp.2015.5.2573.

Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29*(1), 14-46.

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554.

Barkatsas, A.T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal, 17*(2), 69–90. https://doi.org/10.1007/BF03217416.

Bazeley, P. & Jackson, K. (2013). *Qualitative data analysis with NVivo*. Los Angeles: Sage Publications.

Bazeley, P. (2007). *Qualitative data analysis with Nvivo qualitative project book*. Thousand Oaks: Sage Publications.

Belbase, S. (2012). *Teacher belief, knowledge, and practice: A trichotomy of mathematics teacher education*. Retrieved from https://files.eric.ed.gov/fulltext/ED530017.pdf.

Beswick, K. (2003). Accounting for the contextual nature of teachers' beliefs in considering their relationship to practice. In Mathematics education research innovation, networking, opportunity: *Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia* (vol. 1, pp. 152–159). Geelong: Pymble N.S.W. MERGA Inc.

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, *17*(2), 39–68. https://doi.org/10.1007/BF03217415.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*, *79*(1), 127–147. https://doi.org/10.1007/s10649-011-9333-2.

Black, J.W. (2008). Content knowledge and pedagogical content knowledge of algebra teachers and changes in both types of knowledge as a result of professional development. *Proceedings of the 5th Annual TEAM-Math Partnership Conference Pre-Session* (pp. 30–40).

Boz, N. (2008). Turkish pre-service mathematics teachers' beliefs about nature of mathematics teaching. *Australian Journal of Teacher Education, 33*(5), 66-80. https://doi.org/10.14221/ajte.2008v33n5.5.

Buehl, M.M., & Fives, H. (2009). Exploring teachers' beliefs about teaching knowledge: Where does it come from? Does it change? *The Journal of Experimental Education*, *77*(4), 367–408. https://doi.org/10.3200/JEXE.77.4.367-408.

Dossey, J. A., McCrone, S. S., & O'Sullivan, C. (2006). Problem solving in the PISA and TIMSS 2003 assessments. *Technical Report NCES 2007-049*. Washington DC: US Department of Education.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. *Science and Technology Education*, *35*, 99-101.

Felbrich A., Kaiser G., Schmotz C. (2014) The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15

Countries. In: Blömeke S., Hsieh FJ., Kaiser G., Schmidt W. (eds) *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*. *Advances in Mathematics Education* (pp. 209-229). Dordrecht: Springer. https://doi.org/10.1007/978-94-007-6437-8_10

Fennema, E., & Franke, M. (1992). Teachers knowledge and its impact. In Grouws D. A. (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan Publishing Company.

Giovanni, P., & Sangcap, A. (2010). Mathematics-related beliefs of Filipino college students: Factors affecting mathematics and problem solving performance. *Procedia Social and Behavioral Sciences*, *8*, 465–475. https://doi.org/10.1016/j.sbspro.2010.12.064.

Grigutsch, S., Raatz, U., & Törner, G. (1998). Einstellungen gegenüber mathematik bei mathematiklehrern. *Journal Für Mathematik-Didaktik*, *19*(1), 3–45. https://doi.org/10.1007/BF03338859.

Hamrouni, A.D., & Akkari, I. 2012. The entrepreneurial failure: exploring links between the main causes of failure and the company life cycle qualitative analysis using NVivo 7 software. *International Journal of Business and Social Science*, *3*(4), 189-205.

Hawkins, W.J. (2012). An investıgatıon of primary teachers' pedagogical content knowledge when teaching measurement to years three and four. *Proceedings of the 12th International Congress on Mathematical Education*. Seoul: Seoul National University.

Kahan, J.A., Cooper, D.A., & Bethea, K.A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of prospective teachers. *Journal of Mathematics Teacher Education*, *6*(3), 223–252. https://doi.org/10.1023/A:1025175812582.

Kilic, H. (2011). The nature of pre-service teachers' pedagogical content knowledge. *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education–CERME7* (pp. 2690–2696). Rzeszów: University of Rzeszów.

Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Teacher Education and Development*, *3*(1), 28–36.

Moleong, L.J. (2007). *Qualitative Research Method* [in Bahasa]. Bandung: Remaja Rosdakarya.

Muhtadi, D., Wahyudin, Kartasasmita, B.G., & Prahmana, R.C.I. (2018). The Integration of technology in teaching mathematics. *Journal of Physics: Conference Series*, *943*(1), 012020. https://doi.org/10.1088/1742-6596/943/1/012020.

Muhtarom, M., Juniati, D., Siswono, T.Y.E., & Rahmatika, I. (2018). Teachers' and students' beliefs in mathematics at State Senior High School 5 Semarang. *Jurnal Riset Pendidikan Matematika*, *5*(1), 64-72. https://doi.org/10.21831/jrpm.v5i1.18734.

Muhtarom, Murtianto, Y.H., & Sutrisno. (2017). Thinking process of students with high-mathematics ability (A study on QSR NVivo 11-assisted data analysis). *International Journal of Applied Engineering Research*, *12*(17), 6934-6940.

Muhtarom., Juniati, D., & Siswono, T.Y.E. (2017a). Exploring beliefs in a problem-solving process of prospective teachers with high mathematical ability. *Global Journal of Engineering Education*, *19*(2), 130-136.

Muhtarom., Juniati, D., & Siswono, T.Y.E. (2017b). *Consistency and inconsistency of prospective teachers' beliefs in mathematics, teaching, learning and problem solving*. AIP Conference Proceedings (Vol. 1868, p. 050014). College Park: AIP Publishing.

Muhtarom., Juniati, D., & Siswono, T.Y.E. (2018). Exploring prospective teachers' beliefs about nature of mathematics. *Journal of Engineering and Applied Sciences, 13*(10), 3547-3554.

Muzaini, M., Juniati, D., & Siswono, T. Y. E. (2019). Exploration of student's quantitative reasoning in solving mathematical problem: case study of field-dependent cognitive style. *Journal of Physics: Conference Series*, *1157*(1), 032093. https://doi.org/10.1088/1742-6596/1157/3/032093.

Nizaruddin, Muhtarom, & Murtianto, Y.H. (2017). Exploring of multi mathematical representation capability in problem solving on senior high school students. *Problems of Education in the 21st Century*, *75*(6), 591-598.

Pajares, M.F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307–332. https://doi.org/10.3102/00346543062003307.

Pehkonen, E., & Pietilä, A. (2003). On relationships between beliefs and knowledge in mathematics education. *Proceedings of the Third Conference of the European Society for Research in Mathematics Education*. Bellaria: University of Pisa and ERME.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The case of rural area's student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Raymond, A.M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching. *Journal for Research in Mathematics Education*, *28*(5), 550–576. https://doi.org/10.2307/749691.

Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings*, *1952*(1), 020019. https://doi.org/10.1063/1.5031981.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14. https://doi.org/10.3102/0013189X015002004.

Shulman, L.S. (1987). Knowledge and teaching: Foundations for the new reform. *Harvard Educational Review*, *57*(1), 1-22. https://doi.org/10.17763/haer.57.1.j463w79r56455411.

Siswono, T.Y.E., Kohar, A.W., & Hartono, S. (2017). Secondary teachers' mathematics-related beliefs and knowledge about mathematical problem-solving. *Journal of Physics: Conference Series*, *812*(1), 012046. https://doi.org/10.1088/1742-6596/812/1/012046.

Siswono, T.Y.E., Kohar, A.W., Rosyidi, A.H., & Hartono, S. (2017). Primary school teachers' beliefs and knowledge about mathematical problem-solving and their performance on problem-solving task. *World Transactions on Engineering and Technology Education*, *15*(2), 126–131.

Stipek, D.J., Givvin, K.B., Salmon, J. M., & MacGyvers, V.L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*(2), 213–226. https://doi.org/10.1016/S0742-051X(00)00052-4.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). Reston: National Council of Teachers of Mathematics.

Turnuklu, E.B., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, *1*, 1-13.

Viholainen, A., Asikainen, M., & Hirvonen, P.E. (2014). Mathematics prospective teachers' epistemological beliefs about the nature of mathematics and the goals of mathematics teaching and learning in the beginning of their studies. *Eurasia Journal of Mathematics, Science & Technology Education*, *10*(2), 159–171. https://doi.org/10.12973/eurasia.2014.1028a.

Wilkins, J.L. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education*, *11*(2), 139–164. https://doi.org/10.1007/s10857-007-9068-2.

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DEVELOPING MATHEMATICS WORKSHEET USING FUTSAL CONTEXT FOR SCHOOL LITERACY MOVEMENT

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Abstract

School Literacy Movement, one of the government's efforts to optimize the ability of students in terms of literacy, needs support from every circle; one of them is from researches in the educational field. This research aims to develop reading texts in a futsal context that will be presented in students' worksheet with valid and practical criteria that will have a potential effect in mathematics learning. This research is using the design research method with a development studies type that consists of three main steps, such as preliminary, prototyping, and assessment with an evaluation plot formative study that used in a prototyping step, including the self-evaluation, expert review and one-to-one, small group, and field test. The research subjects are the students in 7th grade in one of Junior High School in Karawang. There are some steps taken in the data collection process, such as documentation, walkthrough, questionnaire, test, and interview. According to the data analysis, it can be concluded that this research has been producing a product in the form of thirteen reading the text for School Literacy Movement invalid and practical mathematics learning that will have a potential effect for the students' learning result in the mathematics learning process.

Keyword: School Literacy Movements, Futsal, Mathematics Learning

Abstrak

Gerakan literasi sekolah sebagai upaya pemerintah untuk mengoptimalkan kemampuan peserta didik dalam hal literasi membutuhkan dukungan dari berbagai kalangan, salah satunya peneliti dibidang pendidikan. Tujuan dari penelitian ini adalah mengembangkan teks bacaan dengan konteks futsal yang disajikan pada LAS dengan kriteria valid, dan praktis, serta memiliki efek potensial pada pembelajaran matematika. Penelitian ini menggunakan metode *design research* jenis *development studies* yang terdiri atas tiga langkah utama yaitu *preliminary*, *protoyping*, dan *assessment* dengan alur evaluasi yang digunakan pada tahap *prototyping* adalah *formative study*, meliputi tahap *self-evaluation*, *expert review* dan *one-to-one*, *small group*, serta *field test.* Subjek penelitian adalah siswa kelas VII di salahsatu SMP Karawang. Pengumpulan data dilakukan dengan cara dokumentasi, walkthrough, angket, tes, dan wawancara. Berdasarkan hasil analisis data, dapat disimpulkan bahwa penelitian ini telah menghasilkan produk berupa teks bacaan sebanyak 13 teks untuk gerakan literasi sekolah dalam pembelajaran matematika yang valid, dan praktis serta memiliki efek potensial terhadap hasil belajar peserta didik dalam pembelajaran matematika.

Kata kunci: Gerakan Literasi Sekolah, Futsal, Pembelajaran Matematika

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Indonesia has been one of the countries that take action in some international comparative study with an unsatisfied result; one of the examples is the result of the Programme for International Student Assessment (PISA). According to the result of Programme for International Student Assessment (PISA), the average value for Indonesia is still in a low number, compared to the average value of Organization for Economic Cooperation and Development (OECD). The average value was taken from three competitions; science, mathematics, and reading contest. OECD (2015) states that the average value in science is 493. However, Indonesia just got 403, in mathematics, the average value is 490, while Indonesia only got 386, and the last average value is from reading, which is 493, while Indonesia only got 397. This data became one of the reasons why the Indonesian Government organize School Literacy Movement to synergize all of the potentials and expand the public role to grow, develop, and civilize literation in Indonesia (Atmazaki, *et al*. 2017). School Literacy Movement is a movement to grow students' character through a civilization of school literation ecosystem so that they will learn every day (Wiedarti, *et al*. 2016). This activity took 15 minutes to read and consists of three steps, such as habituation, development, and learning step (Kemendikbud, 2015).

The habituation stage of School Literacy Movement has been carried out, starting with students reading their books or the books that they borrow from the library. The reading books are depended on students' favorite such as novels or other non-lesson books. After reading, the students are asked to make a synopsis or retell the text they read. This habituation stage aims to increase love to read outside of school hours, improve reading comprehension, increase self-confidence as a good reader, and develop the use of various reading sources (Retnaningdyah, 2016). Although the habituation stage has been carried out, the development and learning stages have not been implemented due to the difficulty of obtaining non-lesson reading texts related to mathematical material, even though the development stage of School Literacy Movement can help students develop their ability to see the connection of reading material to mathematical material. One of the purposes of the development stage is that students can find the connection between books that they read and their surroundings (Retnaningdyah, 2016). Thus, at the learning stage, students have been able to show the connection of reading texts in the students' worksheet of School Literacy Movement, so the reading text becomes the starting point for learning mathematical material. Through the learning stage of School Literacy Movement, students can understand the text and relate it to personal experience, so the lifelong learners are formed, develop critical thinking skills, process and manage communication skills creatively through responding to textbooks and lesson books (Retnaningdyah, 2016).

School Literacy Movement is expected to develop the ability of students to use the reading results for their life skills rather than just being able to read and write. Therefore, literacy in reading and writing becomes important so that it becomes one of the needs that must be fulfilled and has relevance to the daily lives of students. Mathematical literacy is strongly influenced by realistic mathematics education (RME) because it emphasizes the importance of solving mathematical problems in real-life so that they can be used as a basic theory to develop mathematics-based literacy learning (Zulkardi & Kohar, 2018; Putri & Zulkardi, 2019). The discourse of mathematics concerning its connection with everyday practice (Solomon, 2009; Prahmana & Suwasti, 2014). In the matter of mathematics as a subject that is closely related to the daily lives of students, it is necessary to have a familiar context in their daily lives that is related to mathematical content learned by students so it can be a connector to understand mathematical concepts formally. The context in learning mathematics is offering abstract mathematical concepts that are more easily understood by the students (Fajriyah, Putri, & Zulkardi, 2017; Risdiyanti & Prahmana, 2018; Hendroanto, *et al*. 2018).

Mathematics learning will be more meaningful, interesting, and fun by using the context of sports in the process of learning (Yansen, Putri, & Zulkardi, 2018). Sport is one of the activities that are enjoyed by students, and they usually do this activity at least once a week at school. Sport is a physical activity to maintain health and strengthen the muscles, and in it can be considered as an entertaining, enjoyable activity to improve achievement (Muchlisa, 2017). Things related to sports have become the context used for the starting point in learning mathematics, including the context of sprint on ASEAN games that can help students understand fraction material (Roni, Zulkardi, & Putri, 2017), also, the swimming context can stimulate informal knowledge of students about the meaning of fractions (Putri, Gunawan, & Zulkardi, 2018). Students who learn to use the context of sports will feel that mathematics is more fun because it is different from the repeated math practice questions (Reys, *et al*. 2013). The context of sports helps students to express mathematical ideas that they already have, because they are more comfortable and confident (Sanchal & Sharma, 2017).

Nowadays, futsal is a popular sport among both male and female students. The futsal game is a fun activity for students that can be used as a context for the development of reading texts in School Literacy Movement. Futsal is a ball game that is played by five players that involves speed, and it is played in a room with a smaller size than a soccer field (Sturgess, 2017). From this explanation, it can be concluded that the connection between futsal and mathematics is that in futsal, it requires speed in the movement of players and balls. Speed is one of the mathematical material learned by students in school, so it can be said that there is a connection between mathematics and futsal. Futsal is a game with two teams; each of them consists of five people who have to put the ball into the opponent's goal by manipulating the ball with the foot (Lhaksana, 2011). In futsal, some players already can play futsal. In mathematics, a collection of objects that have properties with definitions are referred to as assets. Set is a collection of objects or objects that can be clearly defined (Walpole, 2010).

METHOD

In this study, researchers used design research method with a development studies type (Akker, *et al*. 2006; Plomp & Nieveen, 2010). This development research is a type of research aimed at producing students' worksheet for School Literacy Movement in learning mathematics which held in 7th grade of junior high school. This study consists of three stages; the first one is the introductory stage, the prototyping stage, and the assessment stage (Plomp & Nieveen, 2010). In the prototyping stage, the evaluation flow using formative evaluation, the phases cover self-evaluation, expert review and one-to-one, and small group, as well as the field test. However, the previous stages are using the stages of assessment (Tessmer, 1993; Zulkardi, 2006; Permatasari, Putri, & Zulkardi, 2018).

Data collection and analysis techniques in this study include documentation, walkthroughs, questionnaires, observation sheets, interviews, and tests. The purpose of the walkthrough is to collect suggestions and comments from expert reviews for the validation of the first prototype. The walkthrough stage was done at the time of validation to the expert review. Documentation was done to collect the results of comments/suggestions from the validator, comments/suggestions from students, the results of the answers from the students, and the photographs of the research. Documentation was done during the one-to-one, small group, and field test stages. Interviews were conducted during one-to-one, small group, and field tests. Questionnaires are given when small groups and field tests were conducted. The observation sheet and test are used in the text field. The results of the walkthrough, documentation, observation sheets, and interviews were analyzed qualitatively.

Meanwhile, questionnaires and tests were conducted to see the potential effects of the prototypes produced on learning outcomes. The potential effects of this study can be seen from the percentage of questionnaires and improvement from student learning outcomes before and after the worksheets are given to *Gerakan Literasi Sekolah* in mathematics learning. This research uses the formula for calculating normalized gain and interpretation from Meltzer (2002).

RESULT AND DISCUSSION

Research has produced a valid and practical worksheet for School Literacy Movement and has a potential effect on student learning outcomes in a complete material. The procedure used in this research is the introductory stage, the prototyping stage, and the assessment stage. In the prototyping stage, the evaluation flow is using formative evaluation; the phase is cover self-evaluation, expert review, one-to-one, and small group, as well as the field test. At the introductory stage, the researcher analyzes the assembling of School Literacy Movement in Junior High School, the ability of students in mathematics learning and the interest of students in futsal games. Also, the researcher is also analyzing the aspects of futsal competition that have relevance to the mathematics material of 7^{th}-grade student in Junior High School based on the curriculum 2013. The results have been published in the 2018 Ed-Humanistic journal.

In the stage of self-evaluation, the researchers conducted their assessment of the design of worksheets for School Literacy Movement, which had been developed in terms of content, constructs, and language. This worksheet contains reading texts with some questions that are by the stages in the School Literacy Movement and especially at the learning stage adjusted to the *Kurikulum 2013* for junior high school. Researchers make the prototype, and expert reviews and students will validate that at the one to one stage.

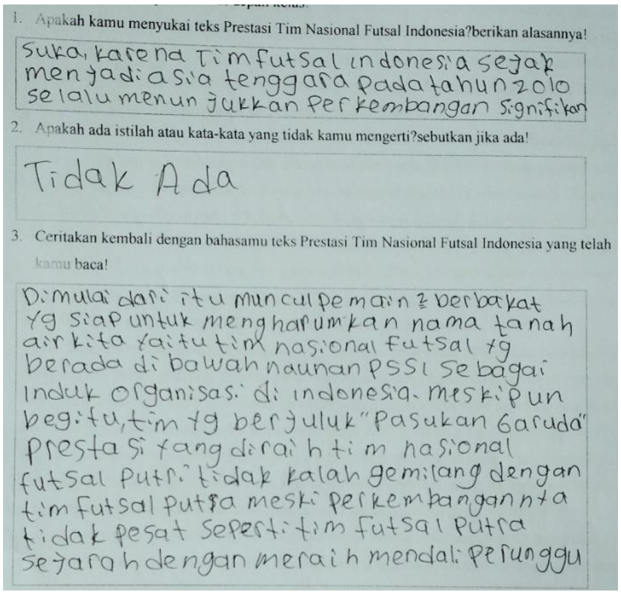
In the prototyping stage, the 1^{st} prototype was validated by the experts and colleagues, then tested at the one to one stage and then to the small group stage to see the validity and practicality of the worksheets that had been developed. The validity of the worksheet can be seen from the expert review comments/suggestions based on the suitability of the reading text and the questions presented on the worksheet according to the content, constructs, and language. In the expert review stage, the researcher asked the opinion of experts and colleagues who had experience in mathematics education, futsal games, and school literacy movement. The expert is the lecturers in the doctoral program in mathematics education at the University of Education in Indonesia, lecturers in sports education as well as futsal coaches at the University of Education in Indonesia, and also teachers of mathematics course who attended School Literacy Movement's training.

Along with the implementation of the Expert Review stage, one to one stage is also carried out. The 1st Prototype was tested on three students who were not the subject of research, namely three students in 7th grade from one of the junior high schools in Karawang. The purpose of the implementation of this trial is to find out the responses and difficulties faced by students when reading and answering questions in the text. The responses and difficulties observed focused on the readability and clarity of the text, along with the questions in the text. The reading text in this study was compiled for the School Literacy Movement, which consisted of habituation, development, and learning stages. After the trial, the researcher allows students to give comments and suggestions regarding the School Literacy Movement's reading texts. The results from the Expert Review and one to one stages show that the reading text on the worksheet for School Literacy Movement is valid (Effendi, *et al*. 2018).

The 1st Prototype which has been revised in the Expert Review and One to One stages is considered as the 2nd prototype, which will then be tested in the small group stage, consists of 12 students from 7th grade from one of the junior high schools in Karawang which is not the subject of research. The purpose of this small group stage is to see the practicality of the product that has been made. Besides documentation and interview stage, researchers also used a questionnaire to see the practicality of the reading text of the School Literacy Movement in learning mathematics. Practically refers to the extent that user (or other experts) consider the intervention as appealing and usable in normal conditions (Akker, 1999). In the development of worksheets for the School Literacy Movement, practicality is shown by the attraction/liking, usability, and convenience for students. Based on the results of the questionnaire, the percentage aspect of attraction was 79%, the usability aspect was 92%, and the facilitation aspect was 81%. Practicality is seen as the main quality because when something is impractical, it won't last long, no matter how valid, and reliable it is (Jin, 2018). Practical products will make users comfortable when they are using it; that's why the product can last a long time.

The field test stage is the final stage informative evaluation of the development of worksheets for the School Literacy Movement in mathematics learning. After obtaining a valid and practical third prototype, a field test is then carried out. In this field test stage, the research subjects were a 7th-grade student from one of the Junior High Schools in Karawang with a total of 38 students. This field test was divided into four steps, the first stage was using worksheets for habituation phase of School Literacy Movement, the second stage was using worksheets for the development stage of School Literacy Movement, the third stage was using worksheets for learning stage of School Literacy Movement, and the fourth stage is the provision of test results. The purpose of the worksheet test for School Literacy Movement by giving these test questions is to see the potential effects on student learning outcomes.

The worksheets for School Literacy Movement at the habituation stage consist of three texts with the title of *Sejarah Futsal Dunia* (The world history of Futsal), *Sejarah Futsal di Indonesia* (The history of Futsal in Indonesia), and *Prestasi Tim Nasional Futsal Indonesia* (The achievement of the Indonesian Futsal National Team) with questions at the end of the text. At this stage, students are able to understand the reading text and answer questions that are suitable for the purpose of the habituation stage, which is to increase the willingness to read outside of school hours, improve reading comprehension, improve self-confidence as a good reader, develop the use of various reading sources (Retnaningdyah, 2016). In the answer, it can be seen that students like the text their read, there are no difficult words, and students have been able to retell the contents of the text they have read. The results of student's answer are shown in Figure 1.


1. Apakah kamu menyukai teks Prestasi Tim Nasional Futsal Indonesia?berikan alasannya!

Suka, karena Tim futsal indonesia sejak menjadi asia tenggara pada tahun 2010 selalu menunjukkan perkembangan signifikan

2. Apakah ada istilah atau kata-kata yang tidak kamu mengerti?sebutkan jika ada!

Tidak Ada

3. Ceritakan kembali dengan bahasamu teks Prestasi Tim Nasional Futsal Indonesia yang telah kamu baca!

Dimulai dari itu muncul pemain2 berbakat yg siap untuk mengharumkan nama tanah air kita yaitu tim nasional futsal yg berada di bawah naunan PSSI sebagai induk organisasi di indonesia. meskipun begitu, tim yg berjuluk "Pasukan Garuda" prestasi yang diraih tim nasional futsal putri tidak kalah gemilang dengan tim futsal putra meski perkembangannya tidak pesat seperti tim futsal putra sejarah dengan meraih mendali perunggu

Figure 1. Student's Answer

The worksheets for School Literacy Movement at the development stage consist of five texts and the titles are *Lapangan dan Bola Permainan Futsal* (field and ball for Futsal Game), *Teknik Dasar Permainan Futsal* (the fundamental technic in Futsal game), *Strategi Permainan Futsal* (The strategic of Futsal Game), *Sanksi dalam Permainan Futsal* (the punishment of Futsal Game), and *Sistem Pertandingan Futsal* (The system of Futsal). At this stage, students are expected to be able to see the relevance of the text they read with mathematical material. The activities in this phase cover when students read the text and answer the questions in the text compiled based on the objectives of the development stages of School Literacy Movement, one of them is the students find the connection between books that are read by themselves and their surroundings (Retnaningdyah, 2016). In the text entitled the *Sistem Pertandingan Futsal*, the information is provided regarding the match system used by amateur and professional classes, at the district, national and even international levels. The text explained the match system on futsal games that are divided into two systems, such as the knockout system and the competition system starting from understanding, scoring rules to sample charts of match systems. Figure 2 shows that the reading text section is presented regarding scoring for the competition match system.

Penentu kemenangan pada sistem kompetisi adalah jumlah poin yang diperoleh diakhir kompetisi. Poin nol (0) untuk peserta yang kalah poin tiga (3) untuk peserta yang menang, dan poin satu untuk kedua peserta yang seri.

Figure 2. Scoring Information

Furthermore, the example of a competition system match chart shown in Figure 3.

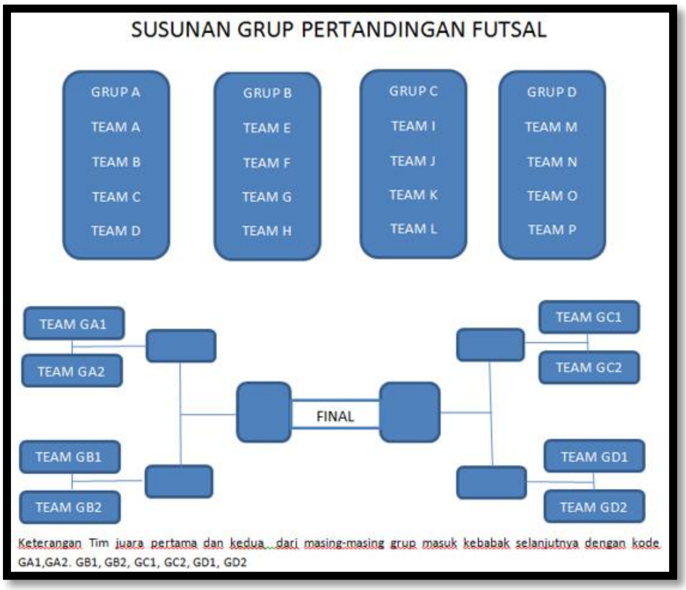


Figure 3. Example of a Competition System Match Chart

From "*Sistem Pertandingan Futsal*" text, students express their idea about benefits that they achieve and the word difficulties that they find on the text, also the connection between mathematics material and the text that can be seen at Figure 4. Students express that the text has a connection with sets and addition in integer's material. Students also write the part of futsal that has a connection with the material that is team division and scoring rules in the futsal competition system.

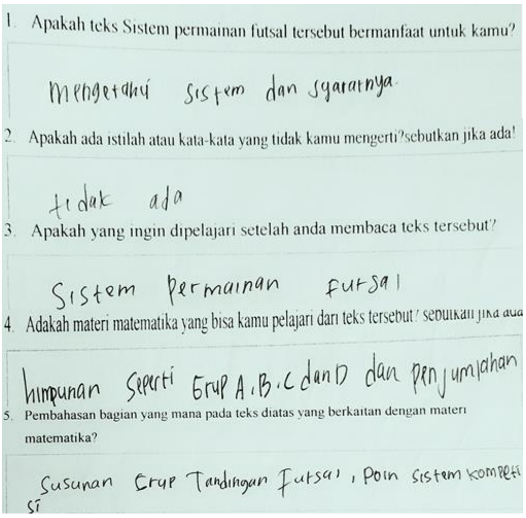
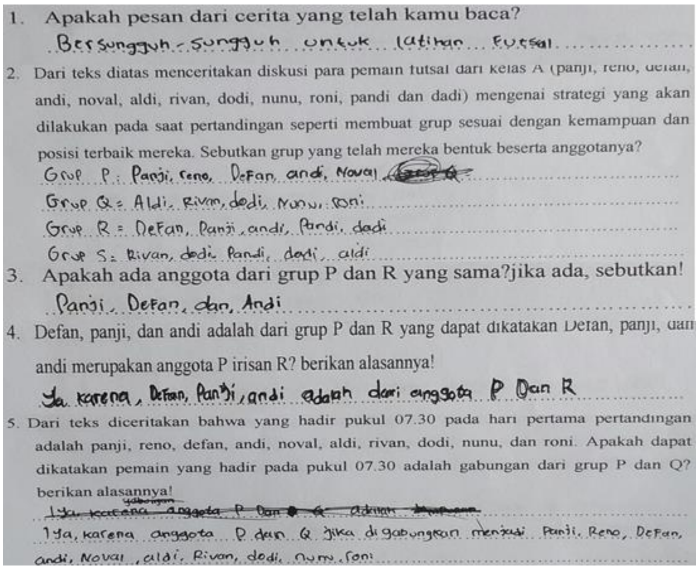


Figure 4. Students' Answer

The worksheets for School Literacy Movement at the learning stage consist of five texts, and the title is *Harumkan Nama Kampus dengan Modal Kebersamaan* (Promote your campus with togetherness). The text on this stage is developed by sets material. The text is divided into five parts, and the questions are adjusted to sets sub-materials, that are set and universal set concept, empty set, and subset, sets on Venn diagram, intersection, and union of sets, difference and complement of sets.

The questions on the text are arranged based on the purpose of the learning stage that is to develop the ability to understand the text and relate it to personal experience so they will become the lifelong learners, develop their critical thinking skills, and process and manage their communication skills creatively through responding to textbooks and learning books (Retnaningdyah, 2016).

Figure 5 shows the students' answers to the text relating to the intersection and the union of sets. From the answers of the students, it was seen that students were able to understand the initial knowledge about the intersection and the union of sets. It is starting from students could write groups formed on futsal team A and its members, determine the same members in group P and group R, and determine the set's intersection from groups P and R accompanied by the reasons. Next, determining the group P and group Q that are present at the same time is a union of sets and accompanied by the reasons. Reading the text in futsal context is one of the students' daily lives and the questions that guide students can make it easier for students to understand mathematical material. Stacey (2011) said that context plays an important role in learning and assessment, because students are prepared to welcome the challenges of the future, so it needs to be introduced to various contexts that cover various aspects of their lives.



1. Apakah pesan dari cerita yang telah kamu baca?
   Bersungguh-sungguh untuk latihan futsal
2. Dari teks diatas menceritakan diskusi para pemain futsal dari kelas A (panji, reno, defan, andi, noval, aldi, rivan, dodi, nunu, roni, pandi dan dadi) mengenai strategi yang akan dilakukan pada saat pertandingan seperti membuat grup sesuai dengan kemampuan dan posisi terbaik mereka. Sebutkan grup yang telah mereka bentuk beserta anggotanya?
   Grup P : Panji, reno, Defan, andi, Noval
   Grup Q = Aldi, Rivan, dodi, Nunu, roni
   Grup R = Defan, Panji, andi, Pandi, dadi
   Grup S = Rivan, dodi, Pandi, dadi, aldi
3. Apakah ada anggota dari grup P dan R yang sama?jika ada, sebutkan!
   Panji, Defan, dan, Andi
4. Defan, panji, dan andi adalah dari grup P dan R yang dapat dikatakan Defan, panji, dan andi merupakan anggota P irisan R? berikan alasannya!
   Ya karena, Defan, Panji, andi adalah dari anggota P Dan R
5. Dari teks diceritakan bahwa yang hadir pukul 07.30 pada hari pertama pertandingan adalah panji, reno, defan, andi, noval, aldi, rivan, dodi, nunu, dan roni. Apakah dapat dikatakan pemain yang hadir pada pukul 07.30 adalah gabungan dari grup P dan Q? berikan alasannya!
   Iya, karena anggota P dan Q jika digabungkan menjadi Panji, Reno, Defan, andi, Noval, aldi, Rivan, dodi, nunu, roni

Figure 5. Students' Answer

After the activities of the literacy movement in the habituation stage, the development, and the learning at the field test stage has been completed; the researcher gives a test on the set material because it is by the material at the learning stage. Tests at the field test stage and giving these test questions are to see the potential effects on student learning outcomes. The test questions consist of three questions with the cognitive level of application (L2) for questions number 1 and number 3 and the cognitive level of reasoning (L3) for question number 2. The level of this question is adjusted to the cognitive level in the guidelines from national standard school exam questions (Kemendikbud, 2018). Tests were given before and after the implementation of the school literacy movement in the learning phase. From the results of the pretest and posttest, it can be seen that there is an improvement in the score of students' tests. Based on the calculation of normalized gain, it shows that the average increase is 0.37, and the interpretation is middle. From the 38 students, there were 38.8% of students who the improvement in test results were low, 55.2% of students had a middle improvement in the test results and 0.08% of students who had a high improvement in the test results. The low of students who have high improvement is because students are still not accustomed to understanding mathematical materials from the verbal language, so students have difficulty writing their arguments in response to the questions given. The students' arguments show their mindset. The important role of mathematical argumentation is one of the goals in developing students' abilities that are by the talents of students (Inglis, Mejia-Ramos, & Simpson, 2007; Soekisno, 2015; Sukirwan, *et al*. 2018).

Based on the results of the questionnaire, it shows that the text and questions in the futsal context presented on the worksheet for the School Literacy Movement have a potential effect on mathematics learning. An important aspect ineffectiveness (potential effects) of an instrument, theory, or model is knowing the level/degree of application of the theory, or model in a particular situation (Reigeluth, 1999). Akker (1999) said that the effectiveness of developing instruments, models, theory in education refers to the consistencies of experience and results of interventions with the intended purpose. The relationship between results and objectives shows potential effects. In this case, the potential effects of the use of reading texts on worksheets for School Literacy Movements can be seen from the results obtained from both the answers to questions in the text and the test questions given at the end of learning activities with expected goals based on the School Literacy Movement program.

Based on the results of the questionnaire, that used to see the potential effects of worksheets for the School Literacy Movement, the 93.8% of the achievement at the habituation stage was shown by students who liked the texts they read, and the texts that were read were not boring. So, they were able to answer questions at the habituation stage because the text presented is clear and easy for them to understand. The 76.3% of the achievement at the development stage was shown by students who stated that they were helped to understand the connection between mathematical material and futsal games. After reading the texts and the questions, it provided could help students to see the connection between the texts presented with mathematics so that students could understand the relevance of the text presented with mathematical material. The 75.4% of the achievement at the learning stage was shown by students who stated that beginning mathematics learning with reading and providing the easily understandable questions could help students to understand the mathematics materials being studied, so it affects students' understanding in mathematics learning. Thus, learning that begins with the activity of reading texts related to mathematical material makes students more interested in learning and the students' literacy skills is better because the presented context is interesting to them and learning becomes more varied.

CONCLUSION

The products are the worksheets that consist of the texts and the questions that are by the stages of School Literacy Movement. There are three texts for the habituation stage regarding the history of futsal games, five texts for the development stage regarding various rules for futsal games, and five texts for the learning stage with the title *Harumkan Nama Kampus dengan Modal Kebersamaan.* The reading texts and questions on worksheets for School Literacy Movement are valid and practical. The validators' assessment shows valid in terms of content, constructs, and language. Practicality is showed by the results of the trials in the small group stage, namely worksheets for the School Literacy Movement that contain all three aspects of practicality, namely easiness, usability, and attraction. The worksheets for the School Literacy Movement have a potential effect on the learning outcomes of the 1st-grade high school students in Karawang. It can be seen from the percentage of the increase in test scores that are 38.8% of students are low, 55.2% of students are middle, and 0.08% of students are high. Also, the results of the questionnaire show that the achievement of the habituation stage is 93.8%, the development stage is 76.3%, and the learning stage is 75.4%. After carrying out this research, the researchers suggest that students used to read various non-learning books directed by the teacher to understand the connection of mathematical material to students' daily lives, so they can understand and resolve contextual problems and provide arguments for resolving these contextual problems, and they also will improve their literacy skills.

REFERENCES

Akker, J. (1999). *Design Approaches and Tools in Education and Training*. Dordrecht: Kluwer Academic Publishers.

Atmazaki, Ali, N.B.V., Muldian, W., Miftahussururi, Hanifah, N., Nento, M.N., & Akbari, Q.S. (2017). *The Guide of the National Literacy Movement [in bahasa]*. Jakarta: Kementerian Pendidikan dan Kebudayaan.

Effendi, K.N.S., Zulkardi, Putri, R.I.I., & Yaniawati, P. (2018). The development of mathematics student worksheet for school literacy movement. *Journal of Physics: Conference Series, 1088*(1), 012033. https://doi.org/10.1088/1742-6596/1088/1/012033.

Fajriyah, M., Putri, R.I.I., & Zulkardi. (2017). Dayung Context in Fraction. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*. Paris: Atlantis Press. https://doi.org/seadric-17.2017.1.

Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial visualization. *Journal of Physics: Conference Series, 943*(1), 012029. http://doi.org/10.1088/1742-6596/943/1/012029.

Inglis, M., Mejia-Ramos, J.P., & Simpson, A. (2007). Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics, 66*(1), 3-21. https://doi.org/10.1007/s10649-006-9059-8.

Jin, Y. (2018). Practicality. In *The TESOL Encyclopedia of English Language Teaching*. Eds J.I. Liontas, T. International Association and M. DelliCarpini. New Jersey: John Wiley & Sons, Inc. https://doi.org/10.1002/9781118784235.eelt0348.

Kemendikbud. (2015). *The regulation of Minister of Education and Culture in Republic of Indonesia Number 23 Year 2015 concerning Growth of Character [in Bahasa]*. Jakarta: Kemendikbud.

Kemendikbud. (2018). *Guide to Preparation of National Standard School Exam Questions [in Bahasa]*. Jakarta: Pusat Penilaian Kemendikbud.

Lhaksana, J. (2011). *Modern Futsal Tactics and Strategies [in Bahasa]*. Jakarta: Be Champion.

Meltzer, D.E. (2002). The relationship between mathematics preparation and conceptual learning gains in physics: A possible “hidden variable” in diagnostic pretest scores. *American Journal of Physics, 70*(12), 1259-1268. https://doi.org/10.1119/1.1514215.

Muchlisa, A.W. (2017). The effect of eye-hand coordination, leg muscle explosive power and achievement motivation on smash skills [in Bahasa]. *Gelanggang Pendidikan Jasmani Indonesia, 1*(2), 213-218.

OECD. (2015). *PISA 2015 results in focus Games*. Accessed from https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf.

Permatasari, R., Putri, R.I.I., & Zulkardi. (2018). Pisa like: Football context. *Journal on Mathematics Education, 9*(2), 271-280. https://doi.org/10.22342/jme.9.2.5251.271-280.

Plomp, T., & Nieveen, N.M. (2010). An introduction to educational design research. *Proceedings of the seminar conducted at the East China Normal University,* Shanghai (PR China), November 23-26, 2007. Stichting Leerplan Ontwikkeling (SLO).

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The case of rural area’s student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Putri, R.I.I., & Gunawan, M.S., Zulkardi. (2018). Addition of fraction in swimming context. *Journal of Physics: Conference Series, 943*(1), 012035. https://doi.org/10.1088/1742-6596/943/1/012035.

Putri, R.I.I., & Zulkardi. (2019). Designing jumping task on percent using PMRI and collaborative learning. *International Journal on Emerging Mathematics Education, 3*(1), 105-116. http://dx.doi.org/10.12928/ijeme.v3i1.12208.

Reigeluth, C.M. (1999). *Instructional Design Theories and Model*. London: Lowrence Earlbown Associates Publishers.

Retnaningdyah, P., Laksono, K., Mujiyem, Setyorini, N.P., Sulastri, & Hidayati, U.S. (2016). *Guide to the Literacy Movement in Junior High School [in Bahasa]*. Jakarta: Direktorat Jenderal Pendidikan Dasar dan Menengah Kementerian Pendidikan dan Kebudayaan.

Reys, R., Reys, R., & Reys, B. (2013). Sport courts and fields: A context for estimation and tessellation. *Mathematics Teaching in the Middle School, 18*(9), 566-570. https://doi.org/10.5951/mathteacmiddscho.18.9.0566.

Risdiyanti, I., & Prahmana, R.C.I. (2018). Ethnomathematics: Exploration in Javanese culture. *Journal of Physics: Conference Series, 943*(1), 012032. https://doi.org/10.1088/1742-6596/943/1/012032.

Roni, A., Zulkardi, Putri, R.I.I. (2017). Sprint Context of Asian Games in the Division of Fractions. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*. Paris: Atlantis Press. https://doi.org/seadric-17.2017.6.

Sanchal, A., & Sharma, S. (2017). Students’ attitudes towards learning mathematics: Impact of teaching in a sporting context. *Teachers and Curriculum, 17*(1), 89-99.

Soekisno, R.B.A. (2015). Problem-based learning to improve students' mathematical argumentation skills [in Bahasa]. *Infinity, 4*(2), 120-139. https://doi.org/10.22460/infinity.v4i2.p120-139.

Solomon, Y. (2009). *Mathematical Literacy: Developing Identities of Inclusion*. New York: Routletge. https://doi.org/10.4324/9780203889275.

Stacey, K. (2011). The PISA view of mathematical literacy in Indonesia. *Journal on Mathematics Education, 2*(2), 95-126. https://doi.org/10.22342/jme.2.2.746.95-126.

Sukirwan, Darhim, Herman, T., & Prahmana, R.C.I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series, 943*(1), 012026. https://doi.org/10.1088/1742-6596/943/1/012026.

Tessmer, M. (1993). *Planning and conducting formative evaluations: Improving the quality of education and training*. London: Kogan Page.

Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Educational Design Research*. London and New York: Routledge.

Walpole, R.E. (2010). *Introduction of Statistics [in Bahasa]*. Jakarta: Gramedia Pustaka Utama.

Wiedarti, P., Laksono, K., Retnaningdyah, P., Dewayani, S., Muldian, W., Sufyadi, S., Roosaria, D.R., Faizah, D.U., Sulastri, Rahmawan, N., Rahayu, E.S., Yusuf, R.A., & Antoro, B. (2016). *Grand Design of the School Literacy Movement [in Bahasa]*. Jakarta: Direktorat Jenderal Pendidikan Dasar dan Menengah Kementerian Pendidikan dan Kebudayaan.

Wijaya, A. (2012). *Realistic Mathematics Education: An alternative mathematics learning approach [in Bahasa]*. Yogyakarta: Graha Ilmu.

Yansen, D., & Putri, R.I.I., & Zulkardi. (2018). Mathematical problems of PISA-like with the 200m swimming contexts in Asian Games. *Journal of Physics: Conference Series, 1088*(1), 012086. https://doi.org/10.1088/1742-6596/1088/1/012086.

Zulkardi, & Kohar A.W. (2018). Designing PISA-Like Mathematics Tasks in Indonesia: Experiences and Challenges. *Journal of Physics: Conference Series, 947*(1), 012015. https://doi.org/10.1088/1742-6596/947/1/012015.

Zulkardi. (2006). *Formative Evaluation: What, Why, When, and How*. Available from http://www.reocities.com/zulkardi/books.html.

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THE INTEGRATION OF A PROBLEM-SOLVING FRAMEWORK FOR BRUNEI HIGH SCHOOL MATHEMATICS CURRICULUM IN INCREASING STUDENT'S AFFECTIVE COMPETENCY

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Abstract

A mathematics framework was developed to integrate problem-solving that incorporated simulation of real-life problems in the classrooms. The framework coined as the *RECCE-MODEL* emphasised understanding and thinking with a view on mathematics embedded in real-life. The *RECCE* which stands for Realistic, Educational, Contextual, Cognitive, and Evaluation encompass the underlying principles of teaching problem solving and guide teachers in planning, designing, developing, and facilitating real-life activity tasks in developing students' problem-solving competencies in mathematics lessons. It also explores students' cognitive competency in their application of abstract mathematical knowledge into real-life problems based on students' developmental status of their thinking and reasoning skills correlating to Meanings, Organise, Develop, Execute and Link (*MODEL*). This study investigated the affective development of the students through activity tasks developed by the sampled teachers using the principles within the framework. In total, 94 students from two high schools in Brunei Darussalam responded to a students' questionnaire constructed to address the *MODEL* aspect of the framework. In particular, the analyses involved the students' affective competencies that corresponded to a 19-item instrument within the questionnaire. The findings showed that Brunei high school students have stimulated beliefs and positive attitudes towards non-routine problem-solving in the learning of mathematics. Meanwhile, meaningful activities developed by the teachers encouraged the development of cognitive-metacognitive and affective competencies of the students. The *RECCE-MODEL* framework paved the way towards understanding the relationships between effective pedagogical approaches and students' learning, and between attitudes and cognitive abilities, and also for teachers to make better-informed decisions in the delivery of the curriculum.

Keywords: Mathematics Framework, Problem-Solving, Curriculum, Affective Competencies

Abstrak

Sebuah kerangka kerja matematika telah dikembangkan untuk mengintegrasikan pemecahan masalah yang menggabungkan simulasi masalah kehidupan nyata ke dalam pengajaran dan pembelajaran di kelas. Kerangka kerja yang diwujudkan sebagai *RECCE-MODEL* menekankan pemahaman dan pemikiran dengan pandangan tentang matematika yang tertanam dalam kehidupan nyata. *RECCE* yang bermakna Realistik, Pendidikan, Kontekstual, Kognitif, dan Penilaian merangkumi prinsip-prinsip asas mengajar pemecahan masalah dan membimbing guru dalam merancang, merekabentuk, membangun, dan memfasilitasi pembuatan tugas aktivitas dari kehidupan nyata dalam membangunkan kompetensi pemecahan masalah siswa dalam pelajaran matematika. Kerangka kerja tersebut juga mengeksplorasi kecekapan kognitif siswa dalam penerapan pengetahuan matematika yang abstrak ke dalam masalah kehidupan nyata berdasarkan status perkembanganpemikiran dan penalaran siswa yang berkaitan dengan Pengertian, Mengorganisasi, Membangun, Melaksana dan Menghubungkan (*MODEL*). Kajian ini menginvestigasi perkembangan afektif siswa melalui tugas-tugas aktivitas yang dikembangkan oleh guru-guru menggunakan prinsip-prinsip dalam kerangka kerja ini. Secara keseluruhan, 94 siswa dari dua sekolah menengah di Brunei Darussalam menanggapi kuesioner siswa yang dibangun untuk membahas aspek MODEL dari kerangka kerja. Secara khusus, analisis melibatkan kompetensi afektif siswa yang sesuai dengan instrumen 19 item dalam kuesioner. Penelitian menemukan bahawa siswa sekolah menengah di Brunei telah menstimulasi keyakinan dan sikap positif terhadap pemecahan masalah yang tidak rutin dalam pembelajaran matematika. Sementara itu, aktivitas-aktivitas yang bermakna yang dikembangkan oleh guru-guru dapat mendorong pengembangan kecekapan kognitif-metakognitif dan afektif siswa. Kerangka kerja *RECCE-MODEL* membuka jalan ke arah pemahaman hubungan antara pendekatan pedagogi yang efektif dan pembelajaran siswa, dan antara sikap dan kemampuan kognitif, dan juga untuk guru membuat keputusan yang lebih bijak dalam penyampaian kurikulum.

Kata kunci: Kerangka Kerja Matematika, Pemecahan Masalah, Kurikulum, Kompetensi Afektif

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Mathematical modelling is one of the applied mathematical tools that support real-life problem solving in mathematics education that has emerged from several perspectives. Blomhøj (2008) identified five main perspectives of research on the teaching and learning of mathematical modelling: 1) The realistic perspective – authenticity of real life modelling in designing problems where students learning is supported by relevant technology, and assess the model and its results against the reality; 2) The epistemological perspective –the development of more general theories and practices in the teaching and learning of mathematics; 3) The contextual perspective – to include research on problem solving and deepening the philosophical role of word problems in its connection to learning theories; 4) The cognitive perspective – students' modelling processes are analysed with the purpose of understanding the cognitive functions and cognitive barriers of the individual going through the modelling process; and 5) The educational perspective–integrating mathematical modelling in the teaching of mathematics, and discuss problems related to assessing students' learning processes using mathematical modelling activities from different types of mathematics curricula.

Barbosa (2012) adopted mainly the education perspective in Brazil where the focus of learning mathematical concepts and the development of 'modelling competencies' are viewed as a way to teach mathematical concepts, in relation to the idea that mathematics education must take part in efforts to educate students be critical, engaged citizens. In the 21st century, it is not sufficient for students to be only competent in applying mathematical knowledge in the context of the framework of the curriculum, which describe the cognitive and educational perspectives. Instead wider perspectives that include embedding real world contexts into the curriculum are needed to support students' cognitive development in engaging new ideas, supporting earlier understandings, and mathematical reasoning from abstraction to solutions. Consequently, it would be appropriate to adapt all five perspectives proposed by Blomhøj (2008) in developing the mathematics framework for Brunei mathematics education. Our teachers need not only teach the curriculum, but continuous support and guidance from relevant stakeholders in educating the future generation is crucial, especially the kind of support and guidance that may elicit confidence and relevance in raising the quality of teaching and learning. Thus, one of the way forward for our mathematics education will be to have our own relevant framework, which guides teachers in preparing their lessons that is realistic, educational, contextually relevant, cognitively challenging for their students.

The Mathematics Framework: RECCE-MODEL

Anthony and Walshaw (2009) identified ten principles of effective mathematics pedagogy, namely an ethic of care, arranging for learning, building on students' thinking, worthwhile mathematical tasks, making connections, assessment for learning, mathematical communication, mathematical language, tools and representations and teacher knowledge, that were found to develop mathematical capability and disposition within an effective learning community. They believed that holistic development of productive students depends highly on effective mathematics pedagogy, which acknowledges the mathematical potentials in *all* students in optimising a range of desirable academic outcomes, and also enhancing a range of social outcomes in classroom. Thus, the ten principles encompass the complex dynamic of a classroom environment within the western education system, where the nature of classroom mathematics teaching focus mainly on students' learning in a safe and supportive environment. This corresponds highly to Brunei's current education system model entitled the National Education System for the 21st Century or *Sistem Pendidikan Negara Abad Ke*-21 or termed as SPN21 (Ministry of Education, 2013). Accordingly, the primary goal of the SPN21 curriculum is based on the principle that each learner is the centre of all teaching and learning through the process of knowledge and understanding, essential skills, and attitudes and values in a well-balanced education system.

In conducting a lesson on problem solving, Lester, Garofalo and Kroll (1989) also proposed that teachers focus on creating a classroom culture of mathematical inquiry through connection and relevant discourse. A design by Lester, Garofalo and Kroll (1989) was also explored to study the effect of instruction on students' cognitive self-regulation of the problem solving processes. This also helps to build the foundation of the current framework. In addition to the ten principles of effective teaching by (Anthony & Walshaw, 2009), the five perspectives proposed by Blomhøj (2008), and the additional five fundamental elements of education by Novak (2013a, 2013b), which are the learner, the teacher, the curriculum, the context, and evaluation, had been incorporated in developing the present mathematics framework. Novak (2013a, 2013b) recognised that in enhancing any successful educational event, each of these five elements must be optimised. Underpinning these principles and perspectives; Pólya's Model (1945), Garofalo and Lester (1985) cognitive and metacognitive framework, Carlson and Bloom (2005) *Mathematical Problem Solving* (*MPS*) framework and modelling cycle by Blum and Leiβ (2007), an emerging mathematics framework representing *Realistic*, *Educational*, *Contextual*, *Cognitive, and Evaluation* - *RECCE* and *Meanings, Organise*, *Develop, Execute*, *Link* - *MODEL* (see Figure 1) was developed for this present study applicable to the mathematics curriculum of Brunei.

The *RECCE-MODEL* is a framework developed to encompass the underlying principles of teaching problem solving by incorporating simulation of real-life problems in classrooms, which emphasized contextually relevance, understanding and expressing thinking with a view on mathematics embedded in real-life. Furthermore, the framework sets direction in learning and assessment of mathematical knowledge and skills in developing students' cognitive, metacognitive and affective competencies. *RECCE* aims to guide teachers in planning and designing their mathematics lessons, developing non-routine activity tasks and evaluate the implementation process of the lesson plans to subsequently make improvement. It is important that the assessment of the learning process in providing information about the progress of students in achieving learning goals are conducted through learning activities between the teacher and the student (Kenedi, *et al*. 2019; Shahrill & Prahmana, 2018; Khoo, *et al*. 2016). The *RECCE-MODEL* framework also echoed similar importance between teaching problem solving and developing competencies through the use of real-life activities and eventually achieving the learning goals. Therefore, this aspect of the framework is focusing on the structuring and development of meaningful lessons to maximize learning in the classroom. Two theoretical perspectives were drawn in developing the conceptual design of the *RECCE-MODEL* framework. Both constructivism and Ausubel's (2000) assimilation of cognitive learning provided the theoretical perspectives in guiding this present study.

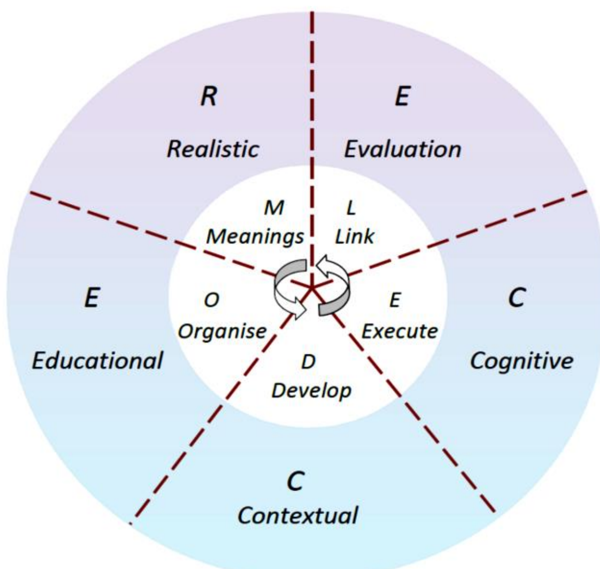

Figure 1. The emerging RECCE-MODEL mathematical problem-solving framework

From Figure 1, the *R*ealistic principle of the framework plays an important role in developing students' cognitive and metacognitive competencies. Teachers are to design lessons focusing on non-routine problems that will develop students' mathematical problem solving skills and thinking skills. This may further improve students' conceptual understanding, application of abstract mathematics and encourage students to be self-aware and regulate own thinking. The *E*ducational principle covers the mathematics curriculum set out by the Ministry of Education. The teachers are to create learning experiences to develop students' understanding of concepts, ideas and applications as an integrated whole process of learning mathematics. Students are encouraged to participate actively in exploring and learning mathematics using worked examples, activities, tasks, and technological aids. The *C*ontextual principle refers to the ability to connect within mathematical concepts and ideas; and also interdisciplinary. This is to help students engage with real problems and make sense of what they are learning, through connecting ideas and regulating thinking precisely, logically and concisely. The *C*ognitive principle is to develop students' thinking skills through seeking solutions, exploring patterns, and formulating conjectures. Students are encouraged to communicate and share their ideas and methods of workings to others as a way of developing their communicating skills. The *E*valuation principle refers to teachers reflecting their teaching approaches and lessons conducted to effectively improve students' competencies in learning and applying mathematics. In addition, the earlier four principles (*R*ealistic, *E*ducational, *C*ontextual and *C*ognitive) must be reviewed to subsequently make improvement in designing lessons that contribute to the success of teaching and learning mathematics.

The *RECCE-MODEL* framework proposes that teachers create a mathematics classroom based on the five guiding principles of *RECCE,* to engage students in mathematical thinking and problem solving through constructivist approach. This is the approach where knowledge is constructed by learners in new experiences from previous learning and propositions of the learning environment, which leads to deeper understanding and flexibility in their mathematical thinking. The key elements of the teacher's role involved planning an overall course of lesson plans; selecting appropriate resources and mathematical problems following the three fundamental requirements for meaningful learning by Novak (2013a, 2013b); monitoring process and progress; and evaluating results.

Therefore, the *RECCE-MODEL* framework aims to create a strong link between teachers' approaches to specifying the mathematical problem solving processes from mathematical content of the curriculum to the mathematical reasoning required in problem solving. Teachers are also expected to foster classroom climate that includes non-routine tasks which, enhances students' beliefs and affects in further contributing to their metacognitive competency towards successful problem solving.

Meanwhile, the *MODEL* framework is used to examine and evaluate students' cognitive-metacognitive competencies in completing a mathematical task. While, students also used *MODEL* in assessing their Level of competencies in completing a task through creating meaning from the real-life problem posed (Level 1); identifying the dependent and independent variables in the problem posed (Level 2); deciding which variables and appropriate mathematical formulae are feasible and possible to use in solving the problem (Level 3); obtaining mathematical solution(s) and contextualise the solution(s) in order to justify for interpretations (Level 4); and finally linking to validate the solution(s) to the problem and reflecting on any error(s) encountered (Level 5). Furthermore, the *MODEL* framework explores students' cognitive competency in six levels, in their application of abstract mathematical knowledge into real-life problems based on students' developmental status of their thinking and reasoning skills correlating to *Meanings*, *Organise*, *Develop*, *Execute* and *Link* (*MODEL*) (shown in Table 1).

Table 1. MODEL cognitive-metacognitive framework categorised in 6 levels in performing a mathematical task

Competency Level & Category	Cognitive-Metacognitive Competency Learners	Key Feature
L0 – No attempt	Did not attempt the problem.	Neither working nor solution is correctly shown.
L1 – *M*eanings	Recall existing propositions; Attempt to make connections; Attempt to make assumptions; Analyse and make meanings of the problem – Understand the problem.	Knowing *about* the problem.
L2 – *O*rganise	Exploring the propositions; Identify strategies; Identify dependent and independent variables; Reflect back to L1.	Knowing *how* to apply.
L3 – *D*evelop	Formulating strategies and variables; Understanding the mathematical concepts needed to solve the problem; Develop a plan; Consolidate L1 and L2.	Knowing *which* to apply.
L4 – *E*xecute	Implement strategies and variables; Monitor progress of the implemented plan; Consolidate L1, L2 and L3 to obtain solution(s) to the mathematical problem.	Knowing *what* and *when* to apply.
L5 – *L*ink	Reflect back solution(s) to the problem; Interpret solution(s) to the problem; Monitor consistency of solution(s); Monitor consistency of plan; Start again if necessary.	Knowing *why* it is applied.

In L1 – *Meanings* (M), students must present some fragments of their abstract knowledge into diagrammatic representation of the problem using concept map, mind map, flowchart, diagrams of all sorts and also any relevant figures. At this Level, students will demonstrate memory recall and reinforced prior knowledge or learning into the real-life problem posed. In L2 – *Organise* (O), students must identify the dependent and independent variables in the real-life problems posed. They will explore and generate ideas, parameters and break down the problem into simpler task by asking questions and linking ideas. In L3 –

Develop (D), students make relevant assumptions based on their ideas and decide which variables are feasible and possible to solve this problem. Students will learn creative decision-making at this Level by choosing the appropriate mathematical formulae to use in solving the problem. In L4 – *Execute* (E), students will obtain mathematical solution(s) at this Level, and will need to contextualize the solution(s) in order to justify for interpretations at the final Level. The learning outcome at this Level is that students will demonstrate their metacognitive competency in reflecting back into the problem. And the fifth Level, L5 – *Link* (L), the metacognitive Level, and students must be able to link and validate their solution(s) to the problem and finally reflecting on any error(s) encountered.

The *MODEL* framework proposes that students to self-scaffolding by following the five levels of problem solving in helping them to become self-aware and self-regulate in their thinking, thus supporting their use of knowledge to help solve a problem. Therefore, with the development of the *RECCE-MODEL* framework, this study aims to investigate the affective development of the students through activity tasks (Chong, *et al*. 2018) developed by the sampled teachers using the principles within the framework. A pilot study was conducted in identifying the affective competencies of Brunei pre-university students (or high school equivalent of Year 12 in the United Kingdom or the 11th Grade in the United States), prior to the development of the *RECCE-MODEL* framework. The pilot study concluded that the affective competencies of Brunei students are stimulated and can be further developed through structured activities in a learning environment (Chong & Shahrill, 2015). Thus, the development of this framework will provide the structure in designing realistic, educational, contextual and cognitive challenging tasks to develop students' affective competencies.

METHOD

A mixed (qualitative and quantitative) research methodology was employed in this study, to engage teachers and students in working with *RECCE-MODEL* in integrating perspectives on problem solving of real-world examples through activity tasks (Chong, *et al*. 2018). The quantitative data were collected using a students' questionnaire, and the qualitative data gathered from semi-structured interviews involving all the participants using open-ended questions and were conducted in groups of four to six students, following the recommendation from Creswell (2013) in relation to focus group interviews. The questionnaire was designed in three sections: the first section consists of questions regarding students' demographic and academic characteristics; the second section consider students' perceptions of the five aspects of the *MODEL* framework; and the last section consider students' affective domain of learning mathematics (beliefs and attitudes).

The students' questionnaire was developed addressing the *MODEL* aspect of the framework and how it interconnects between students' cognitive and metacognitive competencies as they go through the process of problem solving. All the items developed also provided opportunities to critically reflect on individual's attitudes and beliefs of learning mathematics. The development of the questionnaire followed the requirement and criteria set out by Cohen, *et al*. (2011) to obtain as much personal information and academic background of the students as possible and also to assess students' affective competency in learning mathematics. The questions that are designed to capture students' affective competency are in rating scales following Likert scale ranging from never = 1 to always = 5. The design of the questionnaire was concise such that five items that describe the experience of doing and learning mathematics within the context represented each category of the MODEL framework. The questionnaire only required students to read the questions, read the possible responses and mark their responses accordingly. At the start of administering the questionnaire, for ethical considerations, students were informed and assured of the confidentiality, anonymity and non-traceability as all information and data were aggregated into categories. Piloting of the questionnaire was conducted in one of the pre-university institutions prior to implementing the main study.

Meanwhile, the use of activity tasks in this study was to enhance students' cognitive, metacognitive and affective capabilities through communication, self-regulation, and facilitating discovery in enhancing understanding of the problem, and thus supporting students' cognitive, metacognitive and affective development towards non-routine problem solving being part of their learning experiences in mathematics. The subsequent results of the pilot study was also reported in Chong and Shahrill (2016), and the findings showed that Brunei high school students have stimulated beliefs in learning of mathematics and positive attitudes towards non-routine problem solving being part of learning in mathematics.

In reporting the findings in this paper, the students' affective competencies were explored from their responses to a set of 19 questionnaire items that described their beliefs and attitudes towards mathematics and problem solving in general. The 19 items appeared at the last section of the students' questionnaire. In total, the sample size comprised of 94 students from which 42 students were from the first participating high school and the remaining 52 students were from the second high school. There were 33 male students (35.1%) and a total of 61 female students (64.9%). The participating students' ages ranged from 15 to 20 years old.

RESULTS AND DISCUSSION

The reliability score of the 19-items instrument was in the acceptable range of Cronbach's alpha value of 0.76. The results were confirmatory with all 19 items as they fit all the six dimensions in Table 2 below. The questionnaire of this present study was administered after the intervention has been completed. Therefore, the participating students' views of learning mathematics and problem solving in this study was reflective of their attitudes and beliefs after the intervention has been carried out. This was to measure the extent of how *RECCE-MODEL* helps to develop students' affective competency in relation to their cognitive and metacognitive development in solving non-routine problems.

McLeod (1989) viewed emotion as one of the critical factor influencing the process of solving non-routine mathematical problem. The emotion described by McLeod was the feeling of frustration with each unsuccessful attempt; the feeling of anger when a solution cannot be reached; and the feeling of satisfaction and joy when solution is obtained. Therefore, this domain of feelings described by McLeod plays a critical role in influencing the cognitive processes of solving problem, in particular non-routine problems. This is because the extent of the willingness of an individual to solve a problem is greatly dependent on the individual understanding of the problem posed, the kinds of decision-making made during the process and also the working conditions. Schoenfeld (1983) also presented similar views, where he discussed that students manage their cognitive resources through students' belief systems which, included attitudes towards mathematics and confidence about mathematics. Consequently, McLeod (1992) has re-conceptualized beliefs and attitudes towards mathematics as the affective domain in mathematics education and instruction. He categorized beliefs into *beliefs about mathematics* (importance, difficulty, and based on rules), *beliefs about self* (self-concept, confidence and metacognition), *beliefs about mathematics teaching or mathematics classroom instruction*, and *beliefs about the social context* (home environment, parental and peer influences).

Earlier work by Ernest (1988) has distinguished three conceptions of beliefs about mathematics teaching and learning into *the instrumentalist view, Platonist view and the problem-solving view*. The significance of these views is that a learner with *instrumentalist view* will view mathematics as collection of facts, skills and rules with no connection, *Platonist* will view mathematics as a static body of knowledge, and *problem-solving* learner will view mathematics as dynamic with content continually growing (Allen, 2010; Shahrill, *et al*. 2018). In her study, Allen discussed that teachers need to shift their views to one of the problem-solving view in order to be effective teachers of mathematics. Similarly, in the context for a student to be effective learner, one must view mathematics as a process of enquiry and exploration, not just mastery of facts and procedures.

Table 2. The six dimensions of the students' affective competency in learning mathematics and problem solving

Items	Value of Factor matrix	Dimensions (No. of Items Related to the six Dimensions)
1. I seek help from a mathematics tutor.	.745	Attitudes towards social context (3)
2. I seek help from peers (discussion to seek mathematical solutions).	.836	
3. I work in a group to solve mathematics problems.	.655	
4. I think mathematics is useful in everyday life.	.755	Beliefs about mathematics (4)
5. I think that mathematics is used in everyday life.	.749	
6. I use mathematics in everyday life.	.767	

7. I think mathematics will help in my future career path.	.386	
8. I am curious about the mathematical solutions obtained.	.867	Attitudes towards learning mathematics (2)
9. After completing a mathematics question, I try to interpret the solution(s).	.674	
10. I look forward to a mathematics lesson.	.758	Positive beliefs (4)
11. I think mathematics is fun to learn.	.839	
12. I am very keen to learn new ideas and theories in mathematics.	.661	
13. I usually do well in mathematics.	.519	
14. I work individually to solve mathematics questions.	.759	Self-beliefs (3)
15. I finished all assigned mathematics assignments.	.588	
16. I learn mathematics through understanding and problem-solving strategies.	.709	
17. I learn mathematics through memorising of formulae and procedures.	.688	Instrumentalist beliefs (3)
18. I think mathematics is all about solving equations (numerical computation).	.870	
19. I think mathematics solution is just a numerical number.	.709	

Presented in Table 3 are the descriptive statistics of the six dimensions of the students' affective competency in learning mathematics and problem solving. Entries from Table 3 were evident that students have strong beliefs about mathematics and also positive beliefs. These two dimensions recorded the highest mean values in comparison to its total maximum score.

Table 3. Numerical variables between the six dimensions of the students' affective competency

Six dimensions of students' perceptions	Total Minimum score	Total Maximum score	Mean (SD)
1. Attitudes towards social context	3	15	9.9 (1.96)
2. Beliefs about mathematics	4	20	16.4 (2.90)
3. Attitudes towards learning mathematics	2	10	6.5 (1.58)
4. Positive beliefs	4	20	14.7 (2.83)
5. Self-beliefs	3	15	11.3 (2.12)
6. Instrumentalist beliefs	3	15	10.6 (2.33)

These findings were further supported by students' comments from the interviews when asked these questions: why study mathematics and what use of mathematics is important for you to learn? The following are excerpts from the interviews that were relevant to support the findings:

*T1 For me, it is my best subject. I like it and it also gives **a lot of help in my other subjects**. Physics, there's all these Maths, also in Computer Science, there's all these calculations where we're converting numbers in a system to another system. It's very (cradles his head in his hands). Maths is definitely helping in all my other subjects and also one of my goals.*
*H1 Because one, it's **easy** and-second, it's **most job requirements**.*
*X1 I take Maths because it's **essential for life**.*
*Z1 I take Maths because I like Maths and also we get **more job opportunities.***
*L1 Because you need Maths to get a… Because many **university subjects require Maths.***
*A1 It's **fun**.*
*A1 Hehe. Because we… we need- we **need Maths** in every- **in our everyday life.***
*I1 Because it can **help in my career** in the future.*
*N1 It would be **useful for my Economics** because I plan to take Economics degree*
*C1 I take Maths because it is **important.** Because it is **related to Physics**.*
*F1 Hmmm. I find that it is **interesting** and sometimes I can **release my stress** by just doing the past year questions.*
*B1 **I love maths**, and I think I'm good in maths and that's why I'm doing Maths.*
*F1 When I ask my friend, they say that maths is really important when you want to get a job. Nowadays, I think it is the **most important subject**.*
*D1 Because **I like mathematics** and doing calculations*
*G1 Because my father said... Like, **maths is important for all**. Like, any course you want to take. Maths is important.*
*V1 Because it might be **helpful in the future**.*
*O1 I have the **interest** to study Maths in A-Level. It's actually because of **my career**. I have two career basically either become engineering or the doctor. So to be engineering, engineer, so I need to take Maths.*
*K1 Basically we **use mathematics everyday** either we do realize or not so if we **don't have knowledge in Maths we will be lost** in such a ways…*
*P1 Uhh because **I love Maths**.*
*Q1 Uhh the reason why I take Maths is because I want to **pursue law**, for my University course and I did a lot of research about requirements what I should take for my A' Level subjects to pursue Law and most of it says that it is better for me to take Maths, History and English Lit and it's also a bonus point that I enjoy all of my subjects, I enjoy Maths so pretty much why I'm taking it.*
*U1 I think Maths is basically **useful in our everyday life** and I'm also interested in becoming a chemical engineer, so that needs Maths.*
*P1 Because **Maths is important for life**.*

Further analysis of the interview excerpts showed that the participating high school students have very strong perception of the purpose and importance of learning mathematics. They believed that mathematics will be able to support their future career paths and is essential for life. During the intervention, the mathematics pedagogical approaches developed by the sampled teachers using *RECCE-MODEL* framework have shaped the students' beliefs and their behaviour in learning problem solving. In particular, the teachers' actions in scaffolding students' learning during the interactions using the activity tasks (Chong, *et al*. 2018), the technology, the resources and their peers, were crucial to the success of solving the tasks. This seemingly simple findings have important implications on how students learn and apply the metacognitive processes and strategies during the activity tasks. For example, a task on designing a school car park was viewed as the most challenging task for majority of the students, but it enriched their metacognitive experience as the students continually check the appropriateness of their solutions and justifying the final solution. The task was designed to give students the opportunities to reflect on their strategies following their engagement in the problem-solving task with their group members. Furthermore, they had to test, redesign if necessary and review their solutions repeatedly during the problem solving process guided by the *MODEL* framework. Consequently, all groups persevered and managed to complete this task through good discussion and strategic collaboration. This was attributed by the teachers' influences on changing the culture of the classroom by bringing the *realistic* experiences of learning mathematics through non-routine problem solving in the classroom.

CONCLUSION

The sampled teachers in this study provided meaningful tasks that encouraged the development of cognitive-metacognitive and affective competencies of the students. The progress of the *RECCE-MODEL* framework has paved the way towards understanding the relationships between effective pedagogical approaches and students' learning, and between attitudes and cognitive abilities, and also for teachers to make better informed decisions in the delivery of the curriculum. Goos, *et al*. (2017) identified mathematical knowledge base, heuristics, self-awareness, self-regulation, beliefs, affects and classroom environment are the factors that contribute to successful problem solving. And these factors are inter-connected to one another. Evidently, a teacher plays a critical role in shaping students' beliefs and attitudes towards a learning environment. Therefore, a simple change in teachers' classroom practice in this study appeared to influence and articulate students' beliefs and dispositions in deepening their mathematical engagement. Through synthesis of researches, Lesh and Zawojewski (2007) pointed out that developing a productive problem-solving persona involves complex, flexible, and manipulatable profile of affect. Therefore, co-developing affective and metacognitive competency can contribute to how cognition develops in learning mathematics. Sari and Mutmainah (2018) also highlighted similar significance of teacher's role in delivering the subject matter to motivate learning of mathematics for students through creative, open and joyful learning. It can be suggested that with high cognitive demand tasks, students may be more engaged and become active in the exploration stage, and may be able to use strategies that were meaningfully connected to concepts. To conclude, this present study marked the beginning of integrating a mathematics framework called the *RECCE-MODEL* into the Brunei school curriculum in developing students' affective competencies in the learning of mathematics.

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REFERENCES

Allen, B. (2010). The primary mathematics specialists – What do they think about teaching and learning mathematics? In M. Joubert & P. Andrews (Eds.), *Proceedings of the British Congress for Mathematics Education April 2010* (pp. 9-16).

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. *Journal of Mathematics Education*, *2*(2), 147-164.

Ausubel, D.P. (2000). *The Acquisition and Retention of Knowledge: A Cognitive View*. Dordrecht, Netherlands: Springer Science+Business Media.

Barbosa, J.C. (2012). Mathematical modelling in classroom: A socio-critical and discursive perspective. *ZDM: The International Journal on Mathematics Education*, *38*(3), 293-301. https://doi.org/10.1007/BF02652812.

Blomhøj, M. (2008). Different perspectives on mathematical modelling in educational research-Categorising the TSG21 papers. In ICME 11 International Congress on Mathematics Education (pp. 1-13).

Blum, W., & LeiB, D. (2007). How do students and teachers deal with modelling problems? In C. Chaines, P. Galbraith, W. Blum & S. Khan (Eds.): *Mathematical Modelling: Education, Engineering and Economics* International Perspectives on the Teaching and Learning of Mathematical Modelling, ICTMA 12 (pp. 222-231). Chichester: Horwood Publishing.

Carlson, M.P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, *58*(1), 45-75. https://doi.org/10.1007/s10649-005-0808-x.

Chong, M.S.F. & Shahrill, M. (2015). An exploratory study of Brunei Darussalam pre-university students' affective mathematical modeling competencies. In C. Vistro-Yu (Ed.), *In Pursuit of Quality Mathematics Education for All: Proceedings of the 7th ICMI-East Asia Regional Conference on Mathematics Education* (pp. 417-423). Quezon City: Philippine Council of Mathematics Teacher Educators (MATHTED) Inc.

Chong, M.S.F., & Shahrill, M. (2016). The use of an emerging framework to explore students' cognitive competency. *Indian Journal of Science Technology*, *9*(16), 1-12. http://dx.doi.org/10.17485/ijst%2F2016%2Fv9i16%2F78812.

Chong, M.S.F., Shahrill, M., Putri, R.I.I., & Zulkardi (2018). Teaching problem solving using non-routine tasks. *AIP Conference Proceedings, 1952*(1), 020020. https://doi.org/10.1063/1.5031982.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research Methods in Education* (7th Ed.). London: Routledge Falmer.

Creswell, J.W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Thousand Oaks, CA: SAGE Publications.

Ernest, P. (1988). The problem solving approach to mathematics. *Teaching Mathematics and its Applications*, *7*(2), 82–92. https://doi.org/10.1093/teamat/7.2.82.

Garofalo, J., & Lester Jr, F.K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, *16*(3), 163-176. http://dx.doi.org/10.2307/748391.

Goos, M., Vale, C., & Stillman, G. (2017). *Teaching secondary school mathematics: Research and practice for the 21st century* (2nd ed.). Crows Nest, NSW: Allen & Unwin.

Kenedi, A.K., Ariani, Y., Zainil, M., Helsa, Y., & Hendri, S. (2019). Mathematical connection of elementary school student to solve mathematical problems. *Journal on Mathematics Education*, *10*(1), 69-80. https://doi.org/10.22342/jme.10.1.5416.69-80.

Khoo, J.S., Shahrill, M., Yusof, N., Chua, G.L.L., & Roslan, R. (2016). Graphic organizer in action: Solving secondary mathematics problems. *Journal on Mathematics Education*, *7*(2), 83-90. https://doi.org/10.22342/jme.7.2.3546.83-90.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 763-804). Charlotte, NC: Information Age.

Lester, F.K., Garofalo, J., & Kroll, D. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven classes.* Final report to the National Science Foundation of NSF project MDR 85-50346. Retrieved from http://files.eric.ed.gov/fulltext/ED314255.pdf

McLeod, D.B. (1989). The role of affect in mathematical problem solving. In D. B. McLeod, & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 20-36). New York, NY: Springer.

McLeod, D.B. (1992). Research on affect in mathematics education: a reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). New York, USA: MacMillan.

Ministry of Education. (2013). *Sistem Pendidikan Negara Abad ke-21 SPN21 (Revised Ed.)*. Ministry of Education, Brunei Darussalam.

Novak, J.D. (2013a). Empowering learners and educators. *Journal for Educators, Teachers and Trainers*, *4*(1), 14-24.

Novak, J.D. (2013b). Meaningful learning is the foundation for creativity. *Qurriculum: Revista de teoría, investigación y práctica educative (Revista Curriculum)*, (26), 27-38.

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Reys, R.E., Suydam, M.N., & Lindquist, M.M. (1992). *Helping children learn mathematics.* (3rd ed). Hoboken, NJ: John Wiley & Sons.

Sari, W.R., & Mutmainah, S. (2018). Increased motivation and results of student math learning through realistic mathematics education. *Journal of Mathematics Education*, *3*(1), 7-13.

Schoenfeld, A.H. (1983). Beyond the purely cognitive: Belief systems, social cognitions, and metacognitions as driving forces in intellectual performance. *Cognitive Science*, *7*(4), 329-363. https://doi.org/10.1016/S0364-0213(83)80003-2.

Shahrill, M., & Prahmana, R.C.I. (2018). The connection commonalities in the mathematical content of lesson sequences. *Journal of Physics: Conference Series*, *1097*(1), 012136. https://doi.org/10.1088/1742-6596/1097/1/012136.

Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. AIP Conference Proceedings, 1952(1), 020019. https://doi.org/10.1063/1.5031981.

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ELEMENTARY SCHOOL TEACHER'S OBSTACLES IN THE IMPLEMENTATION OF PROBLEM-BASED LEARNING MODEL IN MATHEMATICS LEARNING

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Abstract

This study aims to describe the teachers' obstacles in applying problem-based learning model in Mathematics learning of elementary schools. This method of this study was a qualitative descriptive. The data sources were three third-grade teachers of the elementary schools. The data analysis consisted of stages of data collection, data reduction, data presentation, and data verification. The results of the study show: in the planning stage, obstacles occur when teachers need careful preparation in making learning plans and determining problems at the beginning of learning. In the implementation phase, the obstacle that occurs is a lack of time in maximizing activities in all phases. The teacher finds it difficult to direct students to problems that require solutions. Teachers need enough time to organize students in group activities. Also, they have difficulty dividing time when guiding groups because students are still waiting for the teacher to explain to the group without doing it themselves first. Another difficulty faced is making students actively ask questions or respond to learning activities, and feedback from problem-solving is less profound.

Keywords: Elementary school, Problem-based learning, Teachers' obstacles.

Abstrak

Penelitian ini bertujuan untuk mendeskripsikan hambatan guru dalam menerapkan model pembelajaran *problem-based learning* pada pembelajaran matematika di sekolah dasar. Penelitian ini merupakan penelitian deskriptif kualitatif. Sumber data adalah tiga guru sekolah dasar kelas tiga di kecamatan banjarsari kota Surakarta. Analisis data yang dilakukan mulai dari tahap pengumpulan data, pereduksian data, penyajian data, dan verifikasi data. Hasil penelitian menunjukkan: pada tahap perencanaan, kendala terjadi ketika para guru membutuhkan persiapan yang cermat dalam membuat perencanaan pembelajaran dan menentukan masalah pada awal pembelajaran. Pada tahap implementasi, kendala yang terjadi adalah kurangnya waktu dalam memaksimalkan kegiatan di semua fase. Guru merasa kesulitan untuk mengarahkan siswa ke masalah yang membutuhkan solusi. Guru membutuhkan waktu yang cukup untuk mengatur siswa dalam kegiatan kelompok. Selain itu mereka mengalami kesulitan untuk membagi waktu pada saat membimbing kelompok karena siswa masih menunggu guru untuk menjelaskan kepada kelompok tanpa melakukannya sendiri terlebih dahulu. Kesulitan lain yang dihadapi adalah membuat siswa secara aktif bertanya atau merespons kegiatan pembelajaran, dan umpan balik dari penyelesaian masalah kurang mendalam.

Kata kunci: Hambatan guru, *Problem-based learning*, Sekolah dasar.

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Mathematics is one of the subjects that has an important role in education (Eviyanti, *et al.* 2017; Muhtadi, *et al*. 2017). Mathematics takes the role as a tool to organize our daily lives in society (Risdiyanti & Prahmana, 2018). We use Mathematics as a tool in many jobs and also in making calculated decisions (Ali, 2013). The ability to solve mathematical problems is very important. Many efforts have been implemented intensively and sustainably by the Indonesian government to improve education (Darma, 2018). Therefore, mathematical problem solving is the necessary ability for students and becomes important to learn.

The Regulation of the Indonesian Ministry of Education and Culture Number 22 of 2006 concerning the Content Standard for Primary and Secondary Education states that Mathematics learning is provided to promote learners using logical, analytical, systematic, critical, and creative thinking ability as well as the ability to work together. Thus, the learners can have the ability to obtain, manage, and utilize the information to survive in a situation that always changes, uncertain, and competitive (Ahamad, *et al*. 2018). The achievement of those competencies is in the form of increased knowledge, skills, and attitude development through the learning process.

Based on the regulation, problem-solving is a focus on Mathematics learning. In increasing the problem-solving ability, students need to develop several skills, namely the skill to comprehend the problem, creating Mathematical models, solving the problems, and interpreting the Solutions (Widodo, *et al*. 2019). In any chances, Mathematical learning should begin with the introduction of problems that are appropriate to the students' real situation (contextual problem).

NCTM (2000) asserts the importance of mathematical problem-solving, which states that problem-solving is an integral part of Mathematics learning. Therefore, it should not be separated from mathematics learning. Also, problem-solving skills are the goals of mathematics learning. Mathematical problems are tools that are used not only to help the students develop their thinking skills but also help them to develop their basic skills in solving problems, especially problems in their daily life (Pimta, Tayraukham, & Nuangchalerm, 2009; Muhtadi, *et al*. 2017; Shahrill, *et al*. 2018).

Based on the research of PISA (Program for International Student Assessment) in 2015, Indonesia is ranked 62 out of 70 countries participating in PISA (OECD, 2018). The average score of mathematics achievement of all countries is 490; meanwhile, the average score of mathematics students in Indonesia is to 386. Based on that description, Indonesia's ranking is still far below the international average. It means that the mathematics ability of Indonesian students is still low. Therefore, the mathematics learning process in elementary schools needs improvement. Mathematics learning should be designed according to the goals that are listed in the curriculum.

Low mathematics learning achievement is caused by several factors that influence the learning process. Based on interview results, the learning is still dominated by material explanations using lectures method. The teacher has not implemented the meaningful learning that was done by starting the learning through the presentation of problems related to daily life so that the learners were easy to remember the material. The teacher has not allowed the students to think freely to look for the concepts and solve the problems that were related to the presented material. The presentation of problems that are not by the learners' daily life makes it difficult for the students to apply their knowledge in the real world. The students are less skilled to ask during the learning process. The students have not been brought into real situations during the learning process. The students tend to memorize the formulas to solve the problems so that they have the difficulties in facing the developed problem. And moreover, the teachers tend not to enable the students to solve the problems.

The problems can be overcome using the innovative learning model. The innovative and fun learning models can support learning to be more meaningful for the learners. Not all of the learning models are appropriate and suitable to be applied in solving mathematical problems. Problem-based learning is a suitable learning model that is used to overcome mathematics problems. Problem-Based learning requires the teachers to apply meaningful learning by presenting the problems related to the learners' daily life. The teachers also have to fully give the students a chance to think freely to find the concept as well as solve the problem. Padmavathy & Mareesh (2013) describes problem-based learning that is believed to create the learning environment where the problems can encourage learning. In sum up, learning begins with a problem to be solved, and the posed problem can be the way for the students to acquire new knowledge before they can solve the existing problems. The students prefer interpreting the problem, gathering the required information, identifying the possible solutions, evaluating options, and presenting conclusions instead of looking for the one that correctly answers the question.

In applying the problem-based learning model, it is necessary to know the existing obstacles to make it easier in finding the solution for learning. It is required to conduct the learning optimally. If the implementation of the current learning model experiences several obstacles, the teachers need to analyze and find the right solutions. Therefore, student achievement can be attained optimally according to the purpose of learning. Wijayanti, Roemintoyo, & Murwaningsih (2017) explains that teachers can choose a learning model that can support teaching and learning activities that can be effectively managed. Based on Allen, Donham, & Bernhardt (2011), teachers should project different roles of students in group work; these analyzes can more accurately indicate which areas of study are possible for them. This study aims to describe the teachers' obstacles in applying problem-based learning model in mathematics learning of elementary schools.

METHOD

Research design

The type of this study was qualitative descriptive that was aimed to describe the teachers' obstacles in applying the Problem-Based Learning model in Mathematics learning of the elementary school. This study was conducted at the public elementary school in Banjarsari sub-district of Surakarta.

Research sample

The sampling technique was purposive random sampling, which represented the elementary school population in Banjarsari sub-district, which was taken from three categories, namely high, medium, and low quality. The Elementary Schools that were used in this study were SDN Madyotaman, SDN Bromantakan, and SDN Bibisluhur I. The subject of study was the third-grade teacher in the current elementary school.

The data validity test used triangulation technique. The methods used were observation, data documentation, and interview. The researcher examined the validity of the data by examining the observation data, data document, and the interview result about the teachers' obstacles in applying the Problem-Based Learning model in Mathematics learning of the third-grade of elementary school. Observation is a method done by the researcher to observe the learning process of social science of natural and artificial environment by applying Problem-Based learning model. An interview is a method done by the researcher to obtain the data by conducting the oral question and answer process between the researcher as an interviewer and the third-grade teachers as the informants. The interview was conducted at the last meeting after the teacher applied the problem-based learning model. Meanwhile, the interviews were conducted individually in which each of the third-grade teachers in each school was interviewed directly by the researcher through in-depth, free, and clear interviews without any interference from others. The observation, data document, and interview instruments were adapted using the syntax of problem-based learning model, which consists of five phases.

Data analysis

The data analysis techniques were interactive analysis model of Miles & Huberman (1994) through data collection, data reduction, data presentation, and data verification. The results of data analysis of the research were presented using qualitative descriptive with narrative text to depict teachers' obstacles in applying problem-based learning model in mathematics learning of the elementary school.

RESULT AND DISCUSSION

The results of the study are grouped into two sub-chapter, namely the planning stage and implementation stage in the implementation of problem-based learning.

The planning stage of the problem-based learning model

The learning process will be well-conducted if the teacher prepares the lesson carefully by making the lesson plan. It is in line with the statement of the teacher of SDN Bibisluhur I stating that the problem-based learning model can work well when the teachers prepare all the documents well. However, preparing the learning document also takes a long time. It is proven by the following transcript:

> *"The implementation of the problem-based learning model will be maximized if all of the learning document are well-prepared and well-planned, whereas making a good learning tool takes a long time."*

Mutholib, Sujadi, & Subanti (2017) mentions that teachers are an important factor in the implementation of the curriculum. Therefore, how the teachers teach and how they use the learning models in Mathematics will affect the students' understanding. Thus, the teachers need to make the lesson plan to make the maximal learning to achieve the learning objectives.

Teachers were reluctant to use the problem-based learning model because the preparation was quite complicated and time-consuming. In making lesson plans, the teachers needed to create the problems that will be solved by the students. The obstacles experienced by teachers is in determining the right problem for Mathematics learning using the method of problem-based learning. The teachers preferred to use the problem on the subject matter of the textbook. It was supported by the statement of the teachers of SDN Bromantakan that said:

> *"Usually, the problem for learning is only taken from the book. It is due to the difficulty in determining the problems in problem-based learning that needs to pay attention to certain criteria".*

Yusof *et al.* (2012) explains that learning with a problem-based learning model begins with an unstructured problem that has more than one answer. Learning using problem-based learning model is an effective learning method to face the challenges of the 21st century. Napitupulu, Suryadi, & Kusumah (2016) explains that problem-based learning is an instructional approach which uses problems to trigger the learning. The students collaboratively work in groups to solve the problems. Teachers play their part to facilitate the learning using scaffolding techniques by providing indirect directions or posing questions to stimulate the students to use their reasoning and experience to explore the possible ways to get the temporary or final solutions.

By implementing the problem-based learning model, the students are trained to think through the learning process stage. Therefore, the learning process must be given precedence. The learning objectives that are not achieved are also considered as the teachers' general obstacles in implementing the problem-based learning model.

The implementation stage of problem-based learning

According to Hosnan (2014), five phases are conducted during the learning process of the students' orientation phase on the problem in the Problem-Based Learning model, namely organizing the students to learn, guiding individual and group investigations, developing and presenting the work, and analyzing and evaluating the problem-solving process. Learning activities through problem-based learning model began with student activities to solve the real problems that have been made previously. The process of problem-solving has the implications for the formation of student skills in solving problems, stimulating critical thinking, and forming new knowledge. However, there were some obstacles experienced by the teachers in the implementation of the problem-based learning model. It is presented in Table 1 based on observations and interviews.

Table 1. Obstacles in the application of problem-based learning model

Phases in PBL	SDN Madyotaman	SDN Bromantakan	SDN Bibisluhur I
Orientation the students upon the problem	Delivering the students to the problems	Involving the students in problem-solving activities	Giving the understanding related to the problems
Organizing the students to learn	Dividing the students into groups, requiring the time	Ask the students to make a group	The teacher needed the time to organize the students within the groups

Guiding both individual and group investigation	Prioritizing all of the groups to be guided	Dividing the time to guide the groups	There was insufficient time to guide the investigation
Developing and presenting the work	The students are not allowed to give responses	The students were not eager to deliver the result of the discussion	No students asked the question
Analyzing and evaluating the process of problem-solving	Limited time allotment	The feedback is not well-explored	The time allotment was insufficient

Based on Table 1, in the student's orientation phase onto the problem, the teacher posed a problem related to the material that will be learned. The students provided responses or questions individually about the information they may get. It was aimed to stimulate the students' curiosity about the solution to the problem. The obstacles occurred in this phase were because several causes, namely the teacher found it difficult to direct the students to the problem that needed the solution and the students were not accustomed to receiving the problems without the direction. Thus, the students were not trained to identify and understand the problem. Moreover, the students also had weak prior knowledge, and they were not accustomed to understanding the contextual problems because the Problem-Based Learning model has never been previously applied.

In the learning process using the ideal problem-based learning, the students began to identify the nature of the problem. They should expand their knowledge and try to find effective solutions. This process requires a structured and systematic approach in which the students are motivated to present the problem positively and systematically (Alrahlah, 2016). The results of Padmavathy & Mareesh (2013) show that the problem-based learning model is more effective in teaching mathematics. By adopting a problem-based learning model in teaching, the mathematics teacher can stimulate the students to think creatively, make important decisions, and solve the urgent problems in the world's competition. Also, the problem-based learning strategies affect the student's knowledge that provides greater opportunities for students to learn. They also have a chance to be more involved and to increase their active participation, motivation, and interest among them. It makes the students have a positive attitude towards mathematics and help them to improve their achievements that will result in long-term memory. It also provides new and desirable experiences for the students.

In the organizing phase of mathematics learning, the teacher gave the students the problems that have been written down in the student worksheet. They also got the instructions on what they should do in the group discussion. Based on the results of observations and interviews contained in table 1, it can be seen that the obstacles occurred when the teachers needed more time to organize the students into groups. The research was conducted in the third grade of elementary school; therefore, it took patience to arrange the students in their groups. The teachers had difficulty in directing the students to form groups as the classroom became rowdy. In this phase, the teacher had to organize the learning tasks related to the problem. The students did not listen to the teacher's instructions carefully. It is only the smartest students that were willing to pay attention to the teacher. It was explained by the teacher of SDN Madyotaman as follows:

> *"What often happens is the clever students are more dominant in solving the problems. They should help other friends. It also happened to the less smart students that tend to ignore other friends and not try to participate".*

Harun, *et al.* (2012) mentions that in the implementation of problem-based learning, the learners are trained to learn independently. It was proven that this method is effective to solve real problems existing in the students' lives. Hariadi & Wurijanto (2016) describes a learning process that has the purpose of having an understanding of new information in the social principles of learning. The students in groups consisting of various abilities also can undertake the learning activities to understand new information.

In the phase of guiding both the individual and group investigations, the students were not accustomed to constructing the knowledge based on the given problem. Meanwhile, in the problem-based learning model, the students required to find out their knowledge. However, the concept error arose since the students were not accustomed to doing such a thing. Thus, teachers need to guide the process of group discussion. The obstacles occurred when the teacher found it difficult to divide the time to guide each group because each group asked at the same time. The classroom became very rowdy when the certain group asked the teacher; meanwhile, another group that also needed the teacher's explanation chose to wait for the teacher to approach their group. However, they did not do the worksheet so that the class atmosphere became noisy. This situation made the students lazy to solve problems that can be done in the group because the group did not get a chance to be guided first.

All groups want to be prioritized when teachers guide or pose the questions. These obstacles occur very often in the learning process. The students sometimes find that their teachers are unfair because teachers frequently do not meet their requests to ask questions in group activities. Also, the obstacles in this phase were largely caused by the lack of time allotment while working on the worksheet. Due to the lack of discussion time, many students did not maximally master the material. It will affect the next material understanding.

Etherington (2011) explains that there may be a diversity of solutions to a given problem. The students should be encouraged to investigate the basic principles underlying the system thoroughly. It means that the teacher makes constant reference to the importance of the observed patternCertain principles must be mastered in advance before it becomes possible to learn other things, such as solutions.

In the phase of developing and presenting the work, the students were directed to make a report of the discussion result to be presented in front of the class. Each member of the group should be involved in the presentation of the report because the creativity of each group is required to make the systematic report. Each group presented their work in front of the class. Communicating the results of the discussion is an important step in learning. During the group presentation, the other group was asked to provide responses or questions so that concept errors could be clarified. However, none of the students was eager to respond or ask the question to the group who has the presentation in front of the class. Widiana & Jampel (2016) mentions that student interactions can work well if they are given the opportunities to work and discuss.

The phase of analyzing and evaluating the problem-solving process is a very important phase because the students will be given the feedback in a problem-solving process to make the learning meaningful. This phase is also used to emphasize the correct steps of the expected solution, as there may be many solutions for a certain problem. The obstacles faced in this phase was the limited time to explain the deep material of the problem-solving process done by the students. Therefore, the implementation of this phase was not maximal because the time was insufficient.

The results of Al-Bashir, Kabir, & Rahman (2016) study show that the teachers have an important role in improving their own students' ability to understand the process of self-regulation. They are also an essential source of external feedback. Good feedback not only can provide useful information in improving their learning, but it can also give the teachers proper information that ultimately enhances the learning experience for the students.

CONCLUSION

The obstacles experienced by teachers in the application of problem-based learning model occur at the planning stage and the implementation stage of each phase of the learning model. During the planning stage, the obstacles that occur are the teachers that require careful preparation in making the lesson plan. Also, in determining the problem, the teachers are reluctant to determine their problems, and they prefer to use the problem in textbooks. Furthermore, in the implementation stage, the obstacles experienced by the teachers are the lack of time allotment to maximize the activities in all phases. The implementation of problem-based learning required good time management to make the learning process run optimally. In the orientation phase of the problem, the obstacles that occur in this phase are the teacher difficulties in directing the students onto the problem that needs the solution; the students are not accustomed to experience the problems without guidance. Thus, the students are not trained to identify and understand the problem the phase of organizing the students to learn, the obstacles that occur is the teachers require sufficient time to organize the students in the group activities. In the phase of guiding both individual and group investigations, the teachers have difficulty in dividing the time to guide one group with another since each group asks at the same time.

Moreover, the students still wait for the teacher without doing it themselves in advance. In the phase of developing and presenting the work, the obstacle is to make the students actively ask or respond to the group that is doing the presentations. In the phase of analyzing and evaluating the process of problem-solving, the material explanation or feedback from problem-solving is not profound as the time is up. To face the obstacles that have been exposed, the teacher should prepare the lesson plan in advance because it takes time and preparation. Furthermore, in determining the problem appropriately, the teachers should consider the problem criteria in the problem-based learning model. In the implementation of the learning, the teacher should consider the available time allotment and perform the classroom management well.

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REFERENCES

Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, *943*(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.

Al-Bashir, M.M., Kabir, M.R., & Rahman, I. (2016). The value and effectiveness of feedback in improving students' learning and professionalizing teaching in higher education. *Journal of Education and Practice*, *7*(16), 38-41.

Ali, S.A.H. (2013). The motivation of lifelong mathematics learning. *AIP Conference Proceedings*, *1522*(1), 762-765. https://doi.org/10.1063/1.4801203.

Allen, D.E., Donham, R.S., & Bernhardt, S.A. (2011). Problem-based learning. *New Directions for Teaching and Learning*, *2011*(128), 21-29. https://doi.org/10.1002/tl.465.

Alrahlah, A. (2016). How effective the problem-based learning (PBL) in dental education. A critical review. *The Saudi Dental Journal*, *28*(4), 155-161. https://doi.org/10.1016/j.sdentj.2016.08.003.

Darma, I.K. (2018). Improving mathematical problem solving ability through problem-based learning and authentic assessment for the students of Bali State Polytechnic. *Journal of Physics: Conference Series*, *953*(1), 012099. https://doi.org/10.1088/1742-6596/953/1/012099.

Etherington, M.B. (2011). Investigative primary science: A problem-based learning approach. *Australian Journal of Teacher Education*, *36*(9), 53-74. http://dx.doi.org/10.14221/ajte.2011v36n9.2.

Eviyanti, C.Y., Surya, E., Syahputra, E., & Simbolon, M. (2017). Improving the students' mathematical problem solving ability by applying problem based learning model in VII grade at SMPN 1 Banda Aceh Indonesia. *International Journal of Novel Research in Education and Learning*, *4*(2), 138-144.

Hariadi, B., & Wurijanto, T. (2016). Influence of web based cooperative learning strategy and achiever motivation on student study outcome. *International Journal of Evaluation and Research in Education*, *5*(3), 189-199. http://doi.org/10.11591/ijere.v5i3.4538.

Harun, N.F., Yusof, K.M., Jamaludin, M.Z., & Hassan, S.A.H.S. (2012). Motivation in problem-based learning implementation. *Procedia-Social and Behavioral Sciences*, *56*, 233-242. https://doi.org/10.1016/j.sbspro.2012.09.650.

Hosnan, M. (2014). *Scientific and contextual approach in 21st century learning [in Bahasa]*. Bogor: Ghalia Indonesia.

Miles, M.B., & Hubermen, A.M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks: SAGE Publications.

Muhtadi, D., Sukirwan, Warsito, & Prahmana, R.C.I. (2017). Sundanese Ethnomathematics: Mathematical Activities in Estimating, Measuring, and Making Patterns. *Journal on Mathematics Education*, *8*(2), 185-198. https://doi.org/10.22342/jme.8.2.4055.185-198.

Mutholib, A.A., Sujadi, I., & Subanti, S. (2017). Mathematics teachers' beliefs about scientific approach (SA) and implementation in mathematics learning. *AIP Conference Proceedings*, *1868*(1), 050036. https://doi.org/10.1063/1.4995163.

Napitupulu, E.E., Suryadi, D., & Kusumah, Y.S. (2016). Cultivating upper secondary students' mathematical reasoning-ability and attitude towards mathematics through problem-based learning. *Journal on Mathematics Education*, *7*(2), 117-128. http://dx.doi.org/10.22342/jme.7.2.3542.117-128.

NCTM. (2000). *Principles and Standards for School Mathematics*. Reston: National Council of Teacher of Mathematics.

OECD. (2018). *PISA 2015 Result in Focus*. Paris: Organisation for Economic Co-operation and Development.

Padmavathy, R.D., & Mareesh, K. (2013). Effectiveness of problem based learning in mathematics. *International Multidisciplinary e-Journal*, *2*(1), 45-51.

Pimta, S., Tayraukham, S., & Nuangchalerm, P. (2009). Factors influencing mathematic problem-solving ability of sixth grade students. *Journal of Social Sciences*, *5*(4), 381-385.

Risdiyanti, I., & Prahmana, R.C.I. (2018). Ethnomathematics: Exploration in Javanese culture. *Journal of Physics: Conference Series*, *943*(1), 012032. https://doi.org/10.1088/1742-6596/943/1/012032.

Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings*, *1952*(1), 020019. https://doi.org/10.1063/1.5031981.

Widiana, I.W., & Jampel, I.N. (2016). Improving students' creative thinking and achievement through the implementation of multiple intelligence approach with mind mapping. *International Journal of Evaluation and Research in Education*, *5*(3), 246-254. http://doi.org/10.11591/ijere.v5i3.4546.

Widodo, S.A., Istiqomah, Leonard, Nayazik, A., & Prahmana, R.C.I. (2019). Formal student thinking in mathematical problem-solving. *Journal of Physics: Conference Series, 1188*(1), 012087. http://doi.org/10.1088/1742-6596/1188/1/012087.

Wijayanti, N.W., Roemintoyo, R., & Murwaningsih, T. (2017). The impact of Numbered Heads Together model on the learning outcomes of science viewed from students' self regulated learning. Journal of Education and Learning, *11*(3), 257-261. http://dx.doi.org/10.11591/edulearn.v11i3.5838.

Yusof, K.M., Hassan, S.A.H.S., Jamaludin, M.Z., & Harun, N.F. (2012). Cooperative problem-based learning (CPBL): Framework for integrating cooperative learning and problem-based learning. *Procedia-Social and Behavioral Sciences*, *56*, 223-232. https://doi.org/10.1016/j.sbspro.2012.09.649.

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EFFECTIVENESS OF GUIDED DISCOVERY-BASED MODULE: A CASE STUDY IN PADANG CITY, INDONESIA

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Abstract

One of the research objectives was formulated to improve the quality of education, namely by understanding the problems of students and teachers. There were three main groups of quality characteristics of education highlighted in scientific and academic literature in Russia, namely the quality of educational objectives, the quality of the educational process, and the quality of educational outcomes. The problem of education in Indonesia is the quality of the education process. The development of teaching materials is an alternative to improve the quality of the education process. Research so far has only focused on the implementation of learning models by teachers in the classroom, but the model is not integrated into the teaching materials used. This paper examines how the effectiveness of a product that has been developed for two years on 27 Indonesian students. The product produced is a guided discovery-based module in the Complex Analysis subject. This product was developed after the preliminary analysis (defining) process. Students are given 16 modules during lectures; then students are given a final test containing all the competencies that must be achieved. Test results are scored, and statistical analyses are conducted to compare them with student score before using the module. The test used is t-test. The design used is one of the experimental research designs, One Shot Case Study. The results of the study showed that students who were taught with modules developed effectively to improve student learning outcomes. Further research can be done by implementing different learning models in teaching materials.

Keywords: Module, Guided Discovery, Effectiveness, Learning Outcome

Abstrak

Salah satu tujuan penelitian dirumuskan untuk meningkatkan kualitas pendidikan,yaitu dengan memahami masalah siswa dan guru.Ada tiga kelompok utama karakteristik kualitas pendidikan yang disorot dalam literatur ilmiah dan akademik di Rusia, yaitu kualitas potensi tujuan pendidikan, kualitas proses pendidikan, dan kualitas hasil pendidikan.Masalah Pendidikan di Indonesia adalah kualitas proses pendidikan. Pengembangan bahan ajar adalah alternatif untuk meningkatkan kualitas proses pendidikan. Penelitian selama ini hanya berfokus pada implementasi model pembelajaran oleh guru di kelas, namun model tersebut tidak terintegrasi pada bahan ajar yang digunakan.Tulisan ini meneliti bagaimana efektivitas suatu produk yang telah dikembangkan selama dua tahun terhadap 27 pelajar Indonesia. Produk yang dikembangkan adalah modul berbasis penemuan terbimbing pada matakuliah Analisis Kompleks. Produk ini dikembangkan setelah proses analisis awal (pendefinisian). Siswa diberikan modul selama perkuliahan sebanyak 16 pertemuan, kemudian siswa diberikan tes akhir yang berisi semua kompetensi yang harus dicapai.Hasil tes diberi skor dan dilakukan uji statistik untuk membandingkannya dengan skor hasil siswa sebelum menggunakan modul. Tes yang digunakan adalah t test. Desain yang digunakan adalah salah satu desain penelitian eksperimen, yaitu One Shot Case Study. Hasil penelitian menunjukkan bahwa siswa yang diajar dengan modul yang dikembangkan efektif untuk meningkatkan hasil belajar siswa. Penelitian lanjutan dapat dilakukan dengan mengimplementasikan model pembelajaran yang berbeda pada bahan ajar.

Kata kunci: Modul, Penemuan Terbimbing, Efektivitas, Hasil Belajar

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One of the problems in education is the incompatibility between the implementation and the education system, as well as the insufficient readiness of lecturers to teach (Fominykh, *et al*. 2016; Hanapi &

Nordin, 2014). Several researcher already research to improve the quality and readiness of teachers in education (Prahmana & Suwasti, 2014; Shahrill, *et al*. 2018; Muhtadi, *et al*. 2018; Ahamad, *et al*. 2018). These studies were carried out to improve the quality of learning process that provide appropriate solutions to help students' and teachers' weakness in learning mathematics.

There are various research to diagnose student weaknesses, such as studies about the difficulties and characteristics of students in solving a problem (Shahrill, *et al*. 2018; Harisman, *et al*. 2017; Sukirwan, *et al*. 2018). The students' difficulties can be seen from various aspects ranging from gesture outward up to their high level of thinking. One of the studies that look at the difficulties from the student's gesture is the research conducted by Harisman, *et al*. (2017) who saw how consistency gesture two junior high students in solving geometry problems. The conclusion of this study is there is a significant difference from the gesture shown by both boys and girls. The results were used as a diagnostic tool at the time of giving the material, whether girls or boys familiar with the material given or the material needs to be repeated.

Research conducted by Muir, *et al*. (2008), which examines the consistency of the behavior of 20 elementary school students in solving mathematical problems is research to diagnose a high level of thinking students. The Results of Muir's study is obtained information about the problem-solving behavior of students in grade 6. A detailed description of the type of strategies they used and how this strategy implemented has been discussed in the paper. The impact of Muir's research by reviewing research from Lowrie (1999) is recommended to use the problem posing in the learning process; he discovered that the problem-posing could help children to think about the problem-solving process with a sophisticated manner. It can be concluded that one way to improve the quality of learning is to choose a strategy that fits with the student's difficulties. This issue is also supported by Kaur & Blane (1994), using the same problem with Muir, *et al*. (2008); they diagnosed a variety of different strategies used by students at various grade levels in Singapore. The research result detects student weaknesses and provide appropriate solutions to overcome these problems.

Based on these studies, many ways made by teachers (including lecturers) in diagnosing the weaknesses of students. This study was also conducted to diagnose student weaknesses and then find a solution. The previous studies provide a solution with the teaching strategies or models (Sovia, 2015a; Sovia, 2015b; Sovia, *et al*. 2016). Furthermore, this study examines how a developed book can contribute to improve the student achievement. In the first year has seen the score of learning outcomes from Complex Analysis courses, students who receive grades of less than 65 (categories C, D, and E) of 44.91%. The fact that it is still far from the expectations due to students not understanding the material presented in teaching materials. During this time, the learning process of Complex Analysis use the lecture method and teaching material, namely textbook. Based on the observations of researchers, the use of conventional method cause learning becomes monotonous. Teaching material used in the learning process has not been able to construct and guide students to develop their knowledge. It is because learning model is not integrated into teaching material. Students do not understand the material presented in teaching material (Sovia, 2015b). Therefore, the guided discovery module is suitable for teaching material.

This study is important because one way to improve the quality of learning is by doing research development. There has been a report of a survey of 1,722 teachers in 31 schools in the United States on effective ways to help teachers learn to change and develop the confidence, knowledge, potential, and change the way they teach that the desired instructional goals can be achieved (Kisa & Correnti, 2015). Their research form is survey research to improve the professionalism of teachers in teaching. In addition to teaching, according to Fishman, *et al*. (2013) curriculum is a core component of teaching, and learning context means that the curriculum can be developed to improve the quality of learning. The curriculum is closely associated with learning devices ranging from the syllabus, planning for learning, and teaching materials. Teaching materials are not by the characteristics of students will affect student learning outcomes. In addition to the development of teaching material, development of assessment tools can also be performed. A total of 24 teachers develops strategies for formative assessment; this was due to the pressure on schools to improve the results achieved by students in various tests (Wiliam, *et al*. 2004).

Hill, *et al*. (2004) conducted a diagnosis of the way teachers teach with designing measures to teach basic math. It is one form of researches to develop ways to teach basic math. Furthermore, one study in the field of mathematics studies on the development of teaching materials is research by Gomez, *et al*. (2015). Three college faculty of engineering science is used as a subject to design, develop, and improve contextual mathematics lessons where we concluded that language and literacy pedagogy gives a prominent influence in teaching materials. In this paper will be seen how the influence of the use of teaching materials developed by teachers (lecturers) to students mathematics learning outcomes. In the other hand, this paper also contains the final stage of the research that has been done continuously and periodically since 2015. This paper discusses whether there is an influence or effect of the use of modules to the 27 students who took a course Complex Analysis. For one semester, students learn using the guided discovery-based module, which has been valid and practical. At the end of the semester, students are given problems, then score outcomes after using the modules compared with a score of student results before using the module. This study aims to answer the question of whether there is an effect of the use of the module to the achievement of learning outcomes.

METHOD

Description of study

This study was a continuation and the last part of the research that has been conducted for two years. The guided discovery-based module, in the course of Complex Analysis, developed with 4D (Define, Design, Develop, and Disseminate) development methods. The first phase analyzing the needs of the students obtained the results that they need instructional materials capable of guiding to find the concepts. After going through a defining phase, that is the analysis of the syllabus, analysis of textbook, interviews, analysis of student needs, and analysis of the literature, can be concluded that students require module-based guided discovery learning in Complex Analysis, at STKIP PGRI Sumatera Barat. The second phase of research and development (4D model) is the design stage (Sovia, 2015a). The module is designed of several learning activities where each activity contains learning materials, examples, exercises, feedback, follow-up, and an answer key. The next phase is the validation. The results of the validity of the product obtained that guided discovery-based module developed is very valid. Therefore, the module is eligible to be tested (Kariman, *et al*. 2015).

This research continued in 2016 to know the practicality of the modules. The module is tested on students at a large scale and small scale. On a small scale, six students were asked to fill questionnaires to understand the practicalities of modules that have been designed and tailored to the aspect of practicality; it can be concluded that the guided discovery-based module is very practical (Kariman, *et al*. 2016). In large-scale trial also showed that the modules are practically in every aspect (Sovia, *et al*. 2016).

Lastly, this study reveals how the effectiveness of the modules that have been developed. The effectiveness of the module to be reviewed on the aspects of learning outcomes 27 students who participate in Complex Analysis courses. This study aims to prove that there is influence the use of teaching materials developed to student results. One way of presenting the impact of development programs or products are designed closely related to the effectiveness of the program (Kennedy, 2016).

Participant

This research has been conducted for two years toward 27 students who took courses Complex Analysis in STKIP PGRI Sumatera Barat, Indonesia. One of the relevance of the subject matter that can be chosen for the study is conditioned by the difference between (a) the educational orientation pedagogical modern at future teachers and training based on the subject matter and content, and (b) the need to develop pedagogically. Based on these opinions, if the orientation of the teacher (or lecturer) is different from the orientation of the students, then improvement needs to be done. They have a characteristic, such as low learning achievement indicated by the data of lecturers, students who receive grades of less than 65 (categories C, D, and E) of 44, or 91%. It is, of course, different from the orientation of the lecturers, who expect students have good point.

Task

The task given to measure student learning outcomes at the end of the course for a semester after the use of the module is as follows.

1. Let $u(x,y) = y^3 - 3x^2y$, specify a function $v(x,y)$ so$f(z) = u + iv$ analytic. Then stated $f(z)$ in the term of z.
2. Specify
   a) $\frac{d}{dz}\left(2z\cos^{-1}(\ln z)\right)$
   b) $\lim_{z\to m\pi i}(z - m\pi i)\left(\frac{e^z}{\sin z}\right)$

c) $\int_0^{\pi i} sin^5 z\, dz$

3. Note $z(t) = 3\cos t + i \sin 2t$ with $0 \leq t \leq 2\pi$. Investigate whether $z(t)$ is Jordan curve.
4. Count $\oint_c (\bar{z}^2 + 1)dz$ around the closed curve bounded by $y = x$ and $x = 2$.

Four issues have been selected because it contains the entire basic competence in the subject of Complex Analysis. Each lecturer of the course has discussed this task. Task or problem has also been validated by the team of quality groups from mathematics education courses STKIP PGRI Sumatera Barat. The validation matter relating to the language problem, content, compliance with the material, level of difficulty, distinguishing features, and things that need to be considered in the selection of matter. The problem also considers the phases of mental development of students. It is providing the social situation of the development of age-appropriate. If all task is fulfilled, the students would show more activity and high creativity.

Procedure

A total of 27 students in teaching with Complex Analysis module for one semester were given the test with questions that are on the task at the end of the lecture. Having obtained a student answer sheet, the value is compared with the student before using the module. This research can be classified to experimental research. The design used is one shot case study that can be seen in Table 1.

Table 1. Research design

Class	Treatment	Post-test
Experiment	X	O

Description:

X: Treatment of a sample class, which is teaching and learning activities using complex analysis module based on guided discovery

O: Students test at the end of treatment

Learning outcomes were analyzed, and it aims to test the hypothesis. The hypothesis is:

H_0: There is no effect of Complex Analysis module based on guided discovery, to the learning outcomes

H_1: There is an effect of Complex Analysis module based on guided discovery, to the learning outcomes

This design was chosen because it was considered appropriate to resolve the problems faced by the students of 2013. The research looked at whether there is influence the use of the module to the achievement of student learning outcomes. Systematic teaching corrected later seen its effects on students. Lu, *et al*. (2014) did the same with the aim of the research was to evaluate the effectiveness of a synergistic approach to educational research. This research was conducted for graduate students in Hispanic-Serving Institution (HSI). The interdisciplinary group of faculty members to develop a series of workshops. Researchers conducted a pre-posttest and survey methods to evaluate all of the workshops. The result shows that student learning outcomes are increased. A synergistic approach, effective on a group of students.

Data Analysis

Data were analyzed by correcting the entire answer sheet, 27 students. Once the data was obtained in the form of a score of student results, scores were compared with scores of students before being given treatment (before using the module). The data is processed by using the t test to answer the research hypothesis. Trials or experimental research is really to see the cause-effect relationships of a phenomenon (Kadir, 2019).

RESULT AND DISCUSSION

Effective or not a product, method, or learning model can be seen by the increase or changes in motivation, learning outcomes, behavior, and so forth in a better direction. In this paper will be seen, the effectiveness of the product in the form of guided discovery-based modules, the lectures Complex Analysis in STKIP PGRI Sumatera Barat, Indonesia. Effectiveness highlighted here is on aspects of learning outcomes of students who attend the lecture. The effectiveness of student learning in mathematics, being TIMMS or PISA assessment is determined by several factors (Sugilar, 2016; Stacey, 2011). The Indonesian education system is often said that the students' achievement in mathematics is a major factor for teachers (Shadiq, 2013; Fitri & Prahmana, 2019). One indicator of success in learning is good learning achievement by students after going through the learning process.

Class begins by giving modules that have been developed over two years through the research design development of teaching materials with the type of 4D. Students are asked to read the material on the module. If there are difficulties in understanding the material, the student can ask the lecturer. Since reading and understanding the material, students are asked to do exercises in the module. The example of the student answers after learning by using the module can be seen in Figure 1.

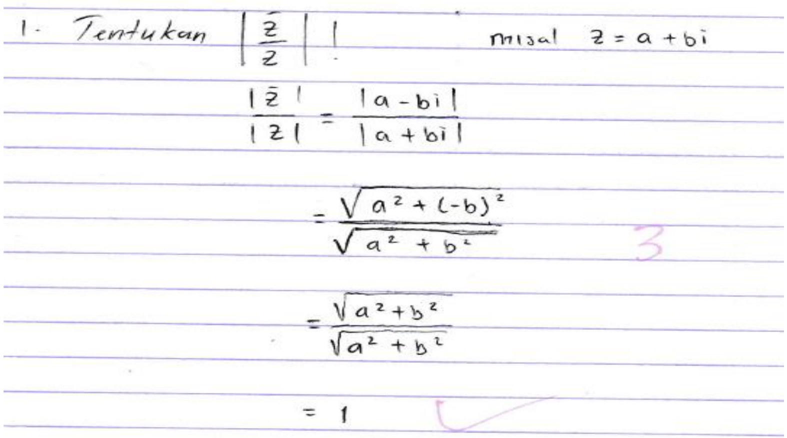

Figure 1. Student's answer in solving exercise 2 no. 1 in the module

Figure 1 shows that students can resolve the problem of the absolute values perfectly. It means that the module can make the students managed to learn independently, with the guidance of the steps on the description of the material without the guidance of a lecturer. After the students do the exercise, the lecturer gives reinforcement to the students about the parts that are considered important. Furthermore, the training modules also presented a practical exercise to do at home. Homework is given for the student to repeat the subject matter at home. The characteristics of human memory has a relationship with the construction of personality (Dudina, *et al*. 2016). As a result, sometimes some students have a short memory, so it must often repeat the lessons at home. One form of answers from students in completing the tasks assigned for homework can be seen in Figure 2.

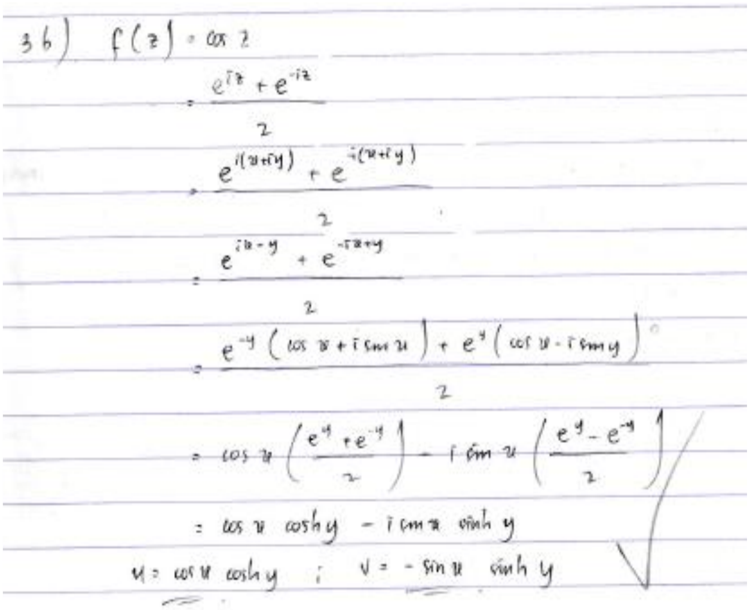

Figure 2. Students form of response to the Task (exercise 1) number 3b

Figure 2 shows the student's answer who also had no problems in answering the question of trigonometric functions. It is due to the module has been guided to construct a detailed knowledge through the guidance of the material that is needed to resolve the problems.

To determine whether the hypothesis is accepted or rejected, then the score of the learning outcomes before and after using the modules was analyzed using t test two-way. The student learning outcomes data before and after using the modules that are calculated through average, standard deviation, the highest value and the lowest value of 27 students were given treatment and were not given the treatment can be seen in Table 2.

Table 2. Calculation of average ($\bar{X}$), standard deviation (S), the highest value (X_{maks}), the lowest value (X_{min}) final test sample class

Sample Class	$\bar{X}$	S	X_{maks}	X_{min}
Before treatment	32,11	17,41	63	8
After treatment	41,70	13,52	69	15

Table 2 revels an average score of student results after given treatment is higher than the average score of student's learning outcomes before treatment. It indicates that the value is better after treatment than before treatment. Furthermore, performed statistical tests to see if there is the influence of the use of the modules to students. Hypothesis test results on the real level $\alpha = 0{,}05$ obtained $t_{count} = -2{,}46$

and $t_{table} = 2{,}05$, because $t_{count} < -t_{table}$ then reject $H_0$and accept H_1. Thus, it can be concluded that the use of Complex Analysis module in lectures effect on learning outcomes. How students can complete all the tests were given. The answer to the task No. 1 at the final test of ability levels of students (high, medium, and low) can be seen in Figure 3.

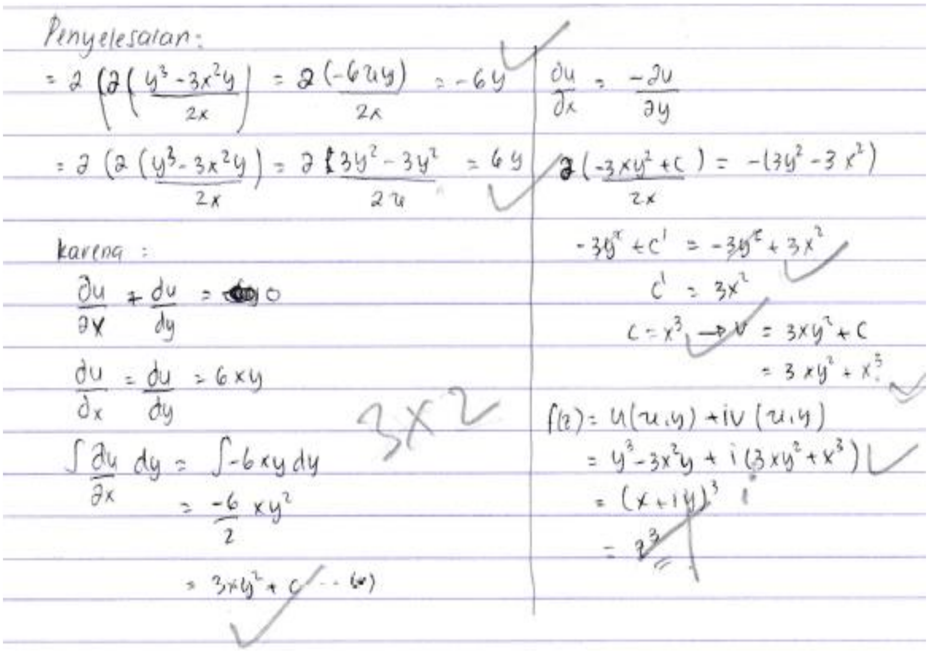

Figure 3. Forms of student's answer (with low ability) for Task 1 in final test

Based on the student's answer in Figure 3, the low-ability student has been able to solve problems on harmonic functions and Cauchy-Riemann equations, just at the end there are deficiencies in the states $f(z)$ in the term of z.

Figure 4. Forms of Student's Answer with Medium-Ability for task 1 in the final test

Figure 4 shows that students who have the medium-ability able to answer the question and almost perfect. But, just as low-ability students, there is still a bit of a mistake at the end of the answer, which is expressed $f(z)$ in the term of z. Furthermore, for high-ability students, the form of representation of one of their answers can be seen in Figure 5.

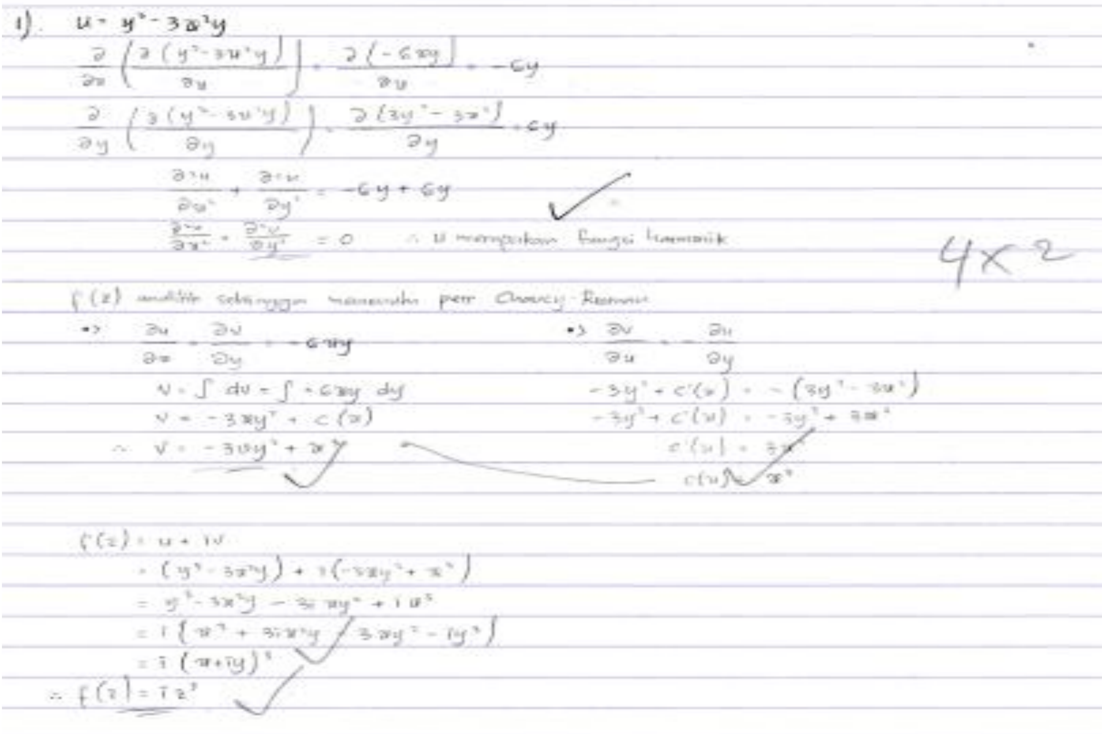

Figure 5. Forms of Student's Answer with High-Ability for task 1 in the final test

Figure 5 appears that high-ability students can solve problems correctly. It shows that the students already understand the concept of harmonic functions, Cauchy-Riemann equations, and understand how to express $f(z)$ in the term of z. If the answer of the three students who have different abilities compared, then the student with low and medium ability will usually have a naive way or routine in solving a given problem. It is problem-solving for beginners, usually by manipulating the existing numbers on the problem (Muir, *et al*. 2008; Ahamad, *et al*. 2018; Shahrill, *et al*. 2018). It is not visible in answer to both the student level; they tend to demonstrate the ability to think that not only manipulate numbers or symbols on the matter but is already showing the way or the chosen strategy towards more sophisticated. It shows the effectiveness of using the guided discovery-based modules that have been developed. The results of this study are similar to the research conducted by Yurniwati & Hanum (2017)**,** which conclude that guided discovery learning improves students' mathematics learning outcomes.

CONCLUSION

The study was conducted over two years, starting from the significance of module definition until the module development stage. In 2015, the module was developed based on guided discovery to lectures of Complex Analysis in STKIP PGRI Sumatera Barat. This article is the final part of the study, which aims to see whether there is an effect of the use of modules on the learning outcomes of students. A total of 27 students is given a final test in the form of four questions that have been validated by education experts. Then, the data is processed and performed statistical tests (comparing the results with a score of student results before using the module). Therefore, it can be concluded that the use of Complex Analysis module in lectures effect on learning outcomes. For future research, we can use guided discovery-based module to increase the behavior of students in problem-solving and understanding the concept. The subject for the next research prefers to use a big sample. Directions of research that is being promoted by every country in the world are to improve the system and the quality of learning. The learning system can be improved by analyzing the weaknesses of educators and students then find a solution.

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REFERENCES

Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, *943*(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.

Dudina, M.M., Khamatnurov, F.T., & Trubetskaya, O.V. (2016). Experience of modeling skill of memorizing short stories as a part of the development of neuro-linguistic programming techniques. *IEJME-Mathematics Education Mathematics Education*, *11*(8), 2914-2924.

Fishman, B., Konstantopoulos, S., Kubitskey, B.W., Vath, R., Park, G., Johnson, H., & Edelson, D.C. (2013). Comparing the impact of online and face-to-face professional development in the context of curriculum implementation. *Journal of Teacher Education*, *64*(5), 426-438. https://doi.org/10.1177/0022487113494413.

Fitri, N.L., & Prahmana, R.C.I. (2019). Misconception in fraction for seventh-grade students. *Journal of Physics: Conference Series*, *1188*(1), 012031. https://doi.org/10.1088/1742-6596/1188/1/012031.

Fominykh, M.V., Uskova, B.A., Mantulenko, V.V., Kuzmina, O.N., & Shuravina, E.N. (2016). A model for the education of a student of a vocational pedagogical educational institution through the gaming simulation. *IEJME-Mathematics Education*, *11*(8), 2014–2840.

Gomez, K., Gomez, L.M., Rodela, K.C., Horton, E.S., Cunningham, J., & Ambrocio, R. (2015). Embedding language support in developmental mathematics lessons: Exploring the value of design as professional development for community college mathematics instructors. *Journal of Teacher Education*, *66*(5), 450–465. https://doi.org/10.1177/0022487115602127.

Hanapi, Z., & Nordin, M.S. (2014). Unemployment among Malaysia graduates: Graduates' attributes, lecturers' competency and quality of education. *Procedia-Social and Behavioral Sciences, 112,* 1056-1063. https://doi.org/10.1016/j.sbspro.2014.01.1269.

Harisman, Y., Noto, M.S., Bakar, M.T., & Amam, A. (2017). The different patterns of gesture between genders in mathematical problem solving of geometry. *Journal of Physics: Conf. Series, 812*(1), 1–6. https://doi.org/10.1088/1742-6596/755/1/011001.

Hill, H.C., Schilling, S.G., & Ball, D.L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11–30. https://doi.org/10.1086/428763.

Kadir. (2009). *The Collection of Questions and Material Summary Comprehensive for Mathematics Exam Preparation [in Bahasa]*. Bandung: Sekolah Pasca Sarjana Universitas Pendidikan Indonesia.

Kariman, D., Sovia, A., & Harisman, Y. (2015). Developing guided discovery-based module in complex analysis lectures [in Bahasa]. *Proceeding of ACER-N Conference 2015*. Malaysia: Universitas Kebangsaan Malaysia.

Kariman, D., Sovia, A., & Harisman, Y. (2016). The practicality of complex analysis module based on guided discovery [in Bahasa]. *Prosiding Seminar Nasional MIPA 2016* (pp. 14–17). Bali: UNDIKSHA Press.

Kaur, B., & Blane, D. (1994). Probing Children's Strategies In Mathematical Problem Solving. In *The AARE Conference*. University of Newcastle.

Kennedy, M.M. (2016). How does professional development improve teaching? *Review of Educational Research, 86*(4), 945-980. https://doi.org/10.3102/0034654315626800.

Kisa, Z., & Correnti, R. (2015). Examining implementation fidelity in America's choice schools. *Educational Evaluation and Policy Analysis, 37*(4), 437-457. https://doi.org/10.3102/0162373714557519.

Lowrie, T. (1999). Free problem-posing: Year 3/4 students constructing problems for friends to solve. *Proceedings of the Twenty-second Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 328–335). Adelaide: MERGA.

Lu, M.T.P., Ward, H.C., Overton, T., & Shin, Y. (2014). The synergetic approach to effective teachers' research education: An innovative initiative for building educational research capacity in a hispanic-serving institution. *Journal of Hispanic Higher Education*, *13*(4), 269-284. https://doi.org/10.1177/1538192714536188.

Muhtadi, D., Wahyudin, Kartasasmita, B.G., & Prahmana, R.C.I. (2018). The Integration of technology in teaching mathematics. *Journal of Physics: Conference Series*, *943*(1), 012020. https://doi.org/10.1088/1742-6596/943/1/012020.

Muir, T., Beswick, K., & Williamson, J. (2008). "I'm not very good at solving problems": An exploration of students' problem solving behaviours. *Journal of Mathematical Behavior*, *27*(3), 228-241. https://doi.org/10.1016/j.jmathb.2008.04.003.

Prahmana, R.C.I., & Suwasti, P. (2014). Local Instruction Theory on Division in Mathematics Gasing: The Case of Rural Area's Student in Indonesia. *Journal on Mathematics Education*, *5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Shadiq, F. (2013). *The Important Role of Mathematics Teachers in Educating Students [in Bahasa]*. Jakarta: P4TK Kemendikbud Indonesia.

Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings, 1952*(1), 020019. https://doi.org/10.1063/1.5031981.

Sovia, A. (2015a). Designing the prototype of guided-based module in complex analysis lectures [in Bahasa]. *Prosdiding Seminar Nasional Matematika dan Pendidikan Matematika Universitas Negeri Surabaya* (pp. 60-67). Surabaya: Universitas Negeri Surabaya.

Sovia, A. (2015b). The define stage in developing guided discovery-based module in PGRI STKIP Sumatera Barat [in Bahasa]. *Prosiding Seminar Nasional Pendidikan Matematika STKIP PGRI Sumatera Barat* (pp. 54-59). Padang: STKIP PGRI Sumatera Barat.

Sovia, A., Kariman, D., & Harisman, Y. (2016). Practicality of (big scale) complex analysis module based on guided research. *Proceeding of 6 th International Conference on Language, Education, and Innovation* (pp. 54–62). Singapore.

Stacey, K. (2011). The PISA view of mathematical literacy in Indonesia. *Journal on Mathematics Education, 2*(2), 95-126. https://doi.org/10.22342/jme.2.2.746.95-126.

Sugilar. (2016). Identification of horizon mathematical knowledge for teaching fraction division at elementary schools. *IEJME - Mathematics Education*, *11*(8), 3160-3175.

Sukirwan, Darhim, Herman, T., & Prahmana, R.C.I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series*, *943*(1), 012026. https://doi.org/10.1088/1742-6596/943/1/012026.

Wiliam, D., Lee, C., Harrison, C., & Black, P. (2004). Teachers developing assessment for learning: impact on student achievement. *Assessment in Education: Principles, Policy & Practice*, *11*(1), 49–65. https://doi.org/10.1080/0969594042000208994.

Yurniwati, & Hanum, L. (2017). Improving mathematics achievement of Indonesian 5^{th} grade students through guided discovery learning. *Journal on Mathematics Education*, *8*(1), 77–84. http://dx.doi.org/10.22342/jme.8.1.3209.77-84.

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PRIMARY SCHOOL STUDENTS' ABSTRACTION LEVELS OF WHOLE-HALF-QUARTER CONCEPTS ACCORDING TO RBC THEORY

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Abstract

Whole-half-quarter are important mathematical concepts that form the basis of fractions and should be well understood for advancing mathematical topics. The aim of this study is to determine the primary school students' abstraction levels of whole-half-quarter concepts according to RBC theory. The participants of the study are six students (8 age group) from the second grade of primary school. The data of the research which is a case study were collected through worksheets and semi-structured interviews. The data obtained from interviews were analyzed by qualitative data analysis steps. The abstraction levels of students were evaluated according to RBC theory. As a result of the study, it was seen that many of the students could not abstract the whole, half and quarter concepts. It was determined that difficulties of students to abstract the whole-half-quarter concepts resulted from reasons such as not understanding the half and quarter concepts, not being able to divide the whole into two equal parts, not being able to divide one dimensional shapes into half and quarter, generalizing dividing into quarter as putting a "+", not being able to divide into four equal parts for quarter.

Keywords: Abstraction, Fractions, Mathematics Education, RBC Theory, Whole-Half-Quarter Concepts

Abstrak

Konsep *whole-half-quarter* adalah konsep penting matematika yang membentuk dasar pecahan dan harus dipahami dengan baik untuk menguasai materi matematika. Tujuan dari penelitian ini adalah untuk menentukan tingkat abstraksi siswa sekolah dasar dari konsep *whole-half-quarter* menurut teori RBC. Subjek penelitian pada penelitian ini adalah enam siswa (usia 8 tahun) kelas dua sekolah dasar. Data untuk penelitian studi kasus ini dikumpulkan melalui lembar kerja siswa dan wawancara semi-terstruktur. Data yang diperoleh dari wawancara dianalisis menggunakan analisis data kualitatif. Tingkat abstraksi siswa dievaluasi sesuai dengan teori RBC. Hasil penelitian ini menunjukkan bahwa banyak dari siswa tidak dapat memahami bentuk abstrak dari konsep keseluruha, setengah, dan seperempat. Hasil ini menunjukkan bahwa kesulitan siswa untuk mengabstraksi konsep *whole-half-quarter* yang dihasilkan dari sejumlah sebab, diantaranya tidak memahami konsep setengah dan seperempat, tidak mampu membagi keseluruhan menjadi dua bagian yang sama, tidak mampu membagi satu bentuk bangun datar menjadi setengah dan seperempat, menggeneralisasi pembagian menjadi seperempat sebagai menempatkan "+", tidak dapat membagi menjadi empat bagian yang sama untuk seperempat.

Kata kunci: Abstraksi, Pecahan, Pendidikan Matematika, Teori RBC, Konsep *Whole-Half-Quarter*

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Mathematics is a system consisting of ideas (structures) and relations developed as successive abstraction and generalization processes (Baykul, 2011). Abstraction is defined as a cultural activity that leads to the creation of new meanings in the process of reorganizing and restructuring known mathematical knowledge as a new structure (Bikner-Ahsbahs, 2004). Thus, understanding mathematical concepts for primary school students can often be difficult.

Abstraction is the process of vertically reorganizing previous constructs that lead to the emergence of a new mathematical construct (Dreyfus, 2007). Vertical mathematization is the process of building a new mathematical structure that incorporates mathematics itself and its mathematical meanings and usually involves reorganizing previous structures, establishing relationships and connections between them, using a single mathematical thinking process to achieve a new mathematical structure (Dreyfus, 2007). The abstraction process consists of three observable epistemic actions: Recognizing – Building with – Constructing (RBC). According to the RBC model, the most important actions in the abstraction processes are mental actions and they cannot be directly observed. The idea of epistemic actions helps us to overcome this obstacle. Epistemic actions are the visualization of mental actions through verbal expressions or physical actions of students (Dreyfus, 2007). In this model of abstraction, which is discussed in terms of sociocultural viewpoint, it is the prevailing thought that the abstraction process has come to life with abstract thought, that the experiential thinking for scientific concepts does not lead to the formation of abstract knowledge, and therefore the dialectic rationale is necessary for the process of abstracting scientific concepts (Dreyfus, 2007; Dreyfus & Tsamir, 2004; Hershkowitz, Schwarz & Dreyfus, 2001).

The three epistemic actions in the RBC model are subjective, that is, a student can recognize a structure he has constructed in the previous activity. For this reason, the student's subjective knowledge determines which structures can be recognized and whether a particular task will lead to building-with or constructing. A structure recognized by a student may cause another student to construct a new structure (Dreyfus & Tsamir, 2004). These actions are not linear but intrinsic, in other words, the creative process does not follow the recognizing and building-with, but it simultaneously requires these epistemic actions (Dooley, 2007). The student behaviors required by the three actions of this theory in the abstraction process can be summarized as follows:

“Recognizing” action is recognizing the properties of a previously solved problem (Hershkowitz et al. 2001), explaining the results of past activities related to the subject (Schwarz, Dreyfus, Hadas & Hershkowitz, 2004), recognizing the existence of a familiar mathematical structure (Bikner-Ahsbahs, 2004). It contains recognizing a familiar mathematical notion, process or idea occurs when a student realizes that the given mathematical situation is inherent in (Dreyfus & Tsamir, 2004; Monaghan & Özmantar, 2006).

"Building with" action is using of the mathematical structure that was created before, to achieve the desired goal (Schwarz et al. 2004) and the process of combining familiar pieces of knowledge into a new context (Bikner-Ahsbahs, 2004). This action consists of combining existing artifacts in order to meet a goal such as solving a problem or justifying a statement (Dreyfus & Tsamir, 2004). It emerges not when the situation is enriched with more complex structural information, but when the students use their existing structural knowledge to deal with the problem at hand (Dooley, 2007).

“Constructing” can be expressed as the process of gathering information structures that allow information to be reorganized vertically (Schwarz et al. 2004). It is the process of reorganizing and restructuring what is recognized and known to construct new meanings (Bikner-Ahsbahs, 2004). In other words, it means bringing together elements of information to produce a new structure (Dreyfus, 2007). In the abstraction process, constructing is more important and rarer than other actions, and it involves gathering the information to create a new information structure that is familiar to the learner (Monaghan & Özmantar, 2006). Students construct more complex structures from simpler ones, involving the reorganization of mathematical elements, resulting in a more refined structure (Dooley, 2007).

"Constructing incorporates the other two epistemic actions in such a way that building-with actions are nested in constructing actions and recognizing actions are nested in building-with actions and in constructing actions" (Dreyfus & Tsamir, 2004, p. 273). Because of nested epistemic actions of abstracting these three actions, the model is called "the dynamically nested RBC model of abstraction". According to the model, abstraction emerges through the following three steps; the need for a new structure, the construction of a new abstract structure, and the consolidation of the abstract structure through the repeated recognition and use of the new structure (Dreyfus & Tsamir, 2004). In this respect Dreyfus, Hadas, Hershkowitz and Schwarz (2006) stated that the stages of construction and consolidation are mostly intertwined and in order to support this, they studied the students' abstraction process of the subject of probability. They analyzed the mechanisms that students used to consolidate new knowledge structures and found three mechanisms that showed epistemic movements; consolidation during building-with process with constructing, consolidation during projection of constructing, consolidation and in order to recognize the structure with regard to constructing more structures during a new abstraction process. Dreyfus (2007) has added consolidation into the RBC model because of the centralization of consolidation in learning processes in such research results, and RBC + C (consolidation) model has emerged. According to this model, the constructing phase of abstraction does not mean consolidation. An unconsolidated abstract mental structure can be fragile. Consolidated knowledge has become an integral part of the student's current knowledge. The constructed knowledge is that the student uses a specific problem only under specific circumstances in a specific context. Consolidation allows the student to use abstract thinking in a fluid and safe way in a variety of situations. In other words, consolidation gives students a sense of flexibility, trust and clarity (Dreyfus & Tsamir, 2004).

The whole-half-quarter concept is a matter of critical subject that must be comprehended at a young age because it forms the basis of fractions. Fraction is also required in many advanced mathematical studies and students should experience many concepts including parts, ratio and division to fully understand fractions (Van de Walle, Karp & Bay Williams, 2012). The students come across expressions with part-whole relations such as "whole, half, quarter" in the preschool period and they interpret these phrases as different sizes (Olkun & Toluk, 2003). In addition, the researches show that the students have difficulty in learning fractions, ratio and proportion and that their comprehension level is insufficient (Alacacı, 2010; Brown & Quinn, 2006; Kaplan, İşleyen & Öztürk, 2011; Sowder & Wearne, 2006; Stafylidou & Vosniadou, 2004). The concept of fractions can be constructed on the bases of abstracting different meanings of fractions in the student (Temur, 2015). The use of multiple physical representations of translations between pictorial, manipulative, verbal, real-world and symbolic representations in the initial comprehension of fractions significantly enhances the achievement of students in terms of fractions (Cramer, Post & del Mas, 2002). Different methods can be applied in the comprehension of the whole-half-quarter subject to attract younger students. Also, many types of research about different subjects on RBC abstraction theory have been made (Bikner Ahsbahs, 2019; Dooley, 2007; Dreyfus, 2007; Dreyfus, Hershkowitz & Schwarz, 2001; Gilboa, Kidron & Dreyfus, 2019; Guler & Gurbuz, 2018; Halverscheid, 2008; Kidron & Dreyfus, 2008; Monaghan & Özmantar, 2006; Özmantar & Monaghan, 2007). However, there is no study which analyzes the abstracting process of the whole-half-quarter concepts of students according to the RBC theory. Therefore, in this study, the primary school students' abstraction levels of whole-half-quarter subjects were examined.

METHOD

Research design

The case study of qualitative research methods was used in the study. The case study is an in-depth description and analysis of a limited system. The most important characteristic feature of the case study is the limitation of the object, that is the case of the study (Merriam, 2013). In some studies, researchers choose more than one case to analyze and compare, while in other studies a single case is analyzed (Creswell, 2013). In this study, students' abstraction levels of whole-half-quarter concepts were analyzed.

Participants

Participants of the study consisted of six students (8 age group) training in the second grade of a state primary school in Turkey. According to the primary education program in Turkey, the whole-half relation is the subject of the first grade of primary school. The aim of the program in this regard is *"The student shows the whole and half with appropriate models, and explains the relationship between the whole and half"* (National Ministry of Education [NME], 2018, p. 28). In the second grade, quarters are taught. The aim in the second-grade education program is *"The student shows the whole, half and quarter with appropriate models, and explains the relationship between the whole, half and quarter"* (NME, 2018, p. 34). Thus, the second grade was selected. Participants were selected with purposive sampling method from among the most successful and unsuccessful students in the class in accordance with the views of the classroom teacher. Three successful students (Ceyda, Yaren, Berkay) and three low achievers (Kadirhan, Yunus Emre, Kenan) in mathematics course participated in the study. Two of the participants were female students (Ceyda, Yaren) while four were males (Kadirhan, Yunus Emre, Kenan, Berkay). For ethical purposes, the students' pseudonyms names were used.

Data collection tools

To determine students' abstraction levels of the whole-half-quarter concepts, two worksheets were developed consisting of open-ended questions. The first worksheet contained questions about the half concept and the second one was about quarter. The questions on the worksheet were prepared in accordance with the RBC theory on the basis of the opinions of the classroom teacher. Especially the examples about two-dimensional forms (such as square, triangle, rectangle) and one-dimensional shapes (straight line) were included in the worksheets. Since the teacher told that she taught the subject with examples of two-dimensional forms (such as square, triangle, rectangle), such shapes were included in the worksheet. The teacher stated that he did not teach how to divide a straight line. For this reason, the straight line (one-dimensional shape) is also included in the worksheet as a shape that students have never split half and quarter before. An interview form was also developed to determine the students' abstraction levels of these concepts based on the worksheets. The worksheets and interview form were reorganized through the expert opinions working in the field of mathematics. The students solved the questions on the worksheets and they were interviewed about their solutions.

Data analysis

The data obtained from interviews were analyzed using qualitative data analysis steps. Merriam (2013) states that data analysis has two steps in case study; case analysis and cross-case analysis. In the case analysis, each situation is first seen as a comprehensive situation within itself, the data are collected, and the researcher can learn as much as possible about the contextual variables. Cross-case analysis begins when case analysis is complete for each case. Cross-case analysis can result in a sample description, cause the theme or categories that conceptualize the data in all cases or result in a fixed theory that provides an integrated framework covering many cases. The RBC abstraction theory was used in determining the level of abstraction of the students. Interviews were analyzed by considering the epistemic mental actions required by the recognizing, building-with, and constructing stages of the RBC theory. The data were analyzed by two different researchers, the results of both analyses were compared, and the consensus of them was determined.

Reliability and validity arise from the presence of the investigator, the mutual communication of the investigator and the participants, the interpretation of data perceptions and the rich explanatory triangles, as opposed to the experimental designs described before the research (Merriam, 2013). In this study, the data collected from the students were analyzed according to the steps of the qualitative data analysis and explained in detail in the findings section. In addition, findings were supported by direct quotations from the student interviews.

RESULTS AND DISCUSSION

When the interviews were analyzed, it was observed that students abstracted whole-half-quarter concepts at different levels. The first student interviewed was Ceyda. Ceyda correctly answered the questions about the half and quarter on worksheets and showed in the interviews that she fully understood these concepts. Ceyda recognized the information required by the questions and she could divide the shapes into half and quarter by using her existing knowledge. The fact that she could even divide the one-dimensional straight line, which she did not know before, accurately shows that she reached the right conclusion, that is, she constructed half and quarter concepts by interpreting and reorganizing her existing knowledge on the subject with new examples. The answers of Ceyda to the questions were as follows:

Researcher: What is half, Ceyda?
Ceyda: Half of a whole.
Researcher: So, how much of the whole?
Ceyda: We divide the whole into two pieces. And, it becomes half. The equal parts of the whole are called half.
Researcher: What do you call a quarter, Ceyda?
Ceyda: We divide the half into equal parts and it becomes quarter.
Researcher: How many parts do we divide the whole into?
Ceyda: Four.
Researcher: Can we divide the whole randomly?
Ceyda: No. They must all be equal.

Kadirhan divided into half all the shapes in the first worksheet. He divided most of the shapes in the second worksheet into quarters. But he could not divide the straight line into quarters. Figure 1 shows the second worksheet of Kadirhan. After analyzing Kadirhan's answers about quarters in Figure 1, it was seen that he divided all shapes into quarters by adding a "+" (plus) sign. However, the way he tried to divide a straight line by adding a plus sign showed that he could not comprehend the quarter concept. In fact, Kadirhan did not correctly answer any of the questions about what the whole, half, and quarters are, although he could divide all shapes into half and quarters (except for the straight line). His answers to the questions about the half and quarter showed that Kadirhan could not grasp the half and quarter and that he could divide them correctly by adding + rotely. He has not figured out how the half and quarters are formed and the whole-half-quarter relationship and memorized the subject of dividing shapes into half and quarters in a formal way. Kadirhan is able to think based on the visual characteristics of shapes and cannot make any conclusion about dividing in half and quarter independently. It is possible to understand this from his expression *"it becomes a quarter when we put a +"*. In this case, it is understood that half and quarter knowledge was not obtained by Kadirhan and he could not abstract these concepts at all.

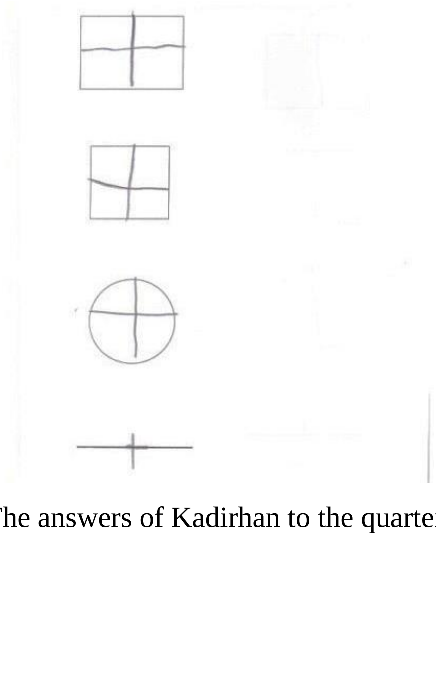

Figure 1. The answers of Kadirhan to the quarter worksheet

The interview between researcher and Kadirhan was as follows:

Researcher: What do you call a whole, Kadirhan?
Kadirhan: Quarter
Researcher: Can you divide this into half?
[After the student divides correctly]
Researcher: What is this called?
Kadirhan: A circle.
Researcher: What do you call a quarter?
Kadirhan: A line.
Researcher: What do you mean? Can you divide this into quarters?
[After the student correctly divides the two-dimensional shapes]
Researcher: What are these parts of the whole?
Kadirhan: Rectangular.
Researcher: Can you divide this straight line into quarters?
[After student puts a "+" on the straight line]
Researcher: Is this a quarter?
Kadirhan: Yes.
Researcher: How? What do you call a quarter, Kadirhan?
Kadirhan: Rectangular.
Researcher: How did you make the quarter?
Kadirhan: By putting a +. When we put a +, it becomes a quarter.

Figure 2 shows Kenan's answers to the questions in the quarter worksheet. Kenan could divide two-dimensional shapes into halves and quarters. But it was seen that he could not divide the straight line into halves and divided into quarters by putting three "+". The interviews showed that he had no understanding of what half and quarter were. He explained how he could divide the shape into quarters by saying *"We put + to divide into quarters"*. This shows that Kenan only memorized how to divide the shape into halves and quarters instead of comprehending the whole-half-quarter relation. The student adheres visually to formal characteristics of half and quarter in the questions and as a result, cannot make any conclusions about these concepts. It was understood that Kenan could not construct whole-half-quarter concepts.

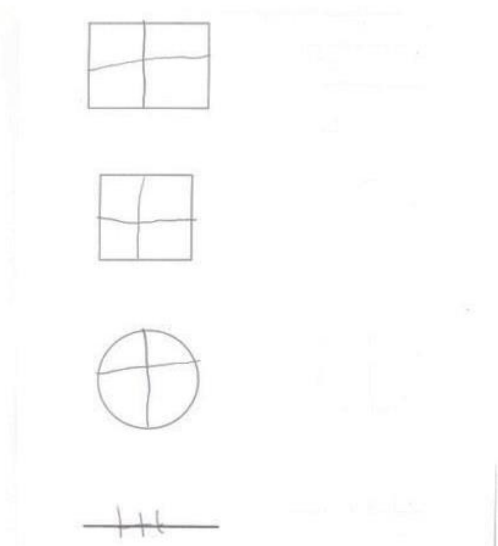

Figure 2. Kenan's worksheet about quarters

Berkay's second worksheet about quarter and his answers in the interviews can be seen in Figure 3. Berkay divided all the shapes into halves, but could not divide anything into quarters. His expressions in the interviews showed that he fully comprehended and constructed the half concept.

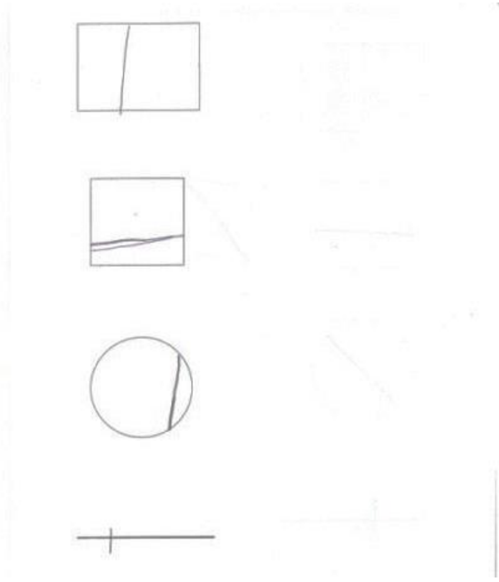

Figure 3. Berkay's quarter worksheet

The student correctly divided all shapes into half using his knowledge of the subject, moreover, he showed that he abstracted this subject by even dividing the straight line into half which was a shape he had not learned before. However, Berkay described the quarter as *"quarter is a part of the whole"*, so he divided the shapes false on the worksheet (Figure 3). It was understood that Berkay could not construct the quarter concept. The interview between researcher and Berkay was as follows:

Researcher: Berkay, what do we call a whole?
Berkay: The entire part of a shape.
Researcher: What do you call a half, then?
Berkay: If you cut a plank of wood in the middle, you get two halves.
Researcher: Can you divide these shapes into half?
[After the student divides]
Researcher: So, you divided them into what?
Berkay: Into half.
Researcher: How many pieces did you get? And, what do we call each one of them?
Berkay: Two halves.
Researcher: What is this?
Bannon: A line.
Researcher: Can you divide this into half?
Berkay: Yes.
[After the student divides]
Berkay: I couldn't divide exactly.
Researcher: How should it be?
Berkay: Right in the middle.
Researcher: What do you call a quarter?
Berkay: This, a little shorter than half.
Researcher: How do we divide into the quarter? If I get this piece, can I call it a quarter?
Berkay: Yes.

Researcher: Can you divide this into quarters?
[After he divides]
Researcher: How did you divide it?
Berkay: I divided it a little towards the front of the middle.
Researcher: Berkay, how do you divide into the quarter?
Berkay: Small, a little smaller than a half.

Yunus Emre divided the two-dimensional shapes correctly into half in the questions posed to him, but could not divide the one-dimensional shape into the half, and did not answer any questions about quarters. When asked what the half was, he answered correctly. His answer *"If we divide it into two right in the middle, it will be half"* showed that he comprehended the whole-half relation. The answers given to the first worksheet showed that Yunus Emre recognized and used the half concept in the questions. However, the fact that he could not divide the straight line, which was a new kind of question that he had not encountered before, showed that he could not construct the half concept and therefore could not completely abstract this concept. Also, the student did not answer any questions about the quarters. Failure to questions about quarters in the interview also indicates that the student could not obtain the quarter knowledge. Yaren's answers to the questions about the half and quarter were as follows:

Researcher: Yaren, what is a whole?
Yaren: The whole is like a cube. If we cut the cube, it becomes a half, if we do not cut it, it becomes whole.
Researcher: How should we cut it so that it becomes a half?
Yaren: We should cut it right in the middle.
Researcher: Can you divide those shapes into half?
[After the student divides]
Researcher: What is this now?
Yaren: A half.
Researcher: How many halves do a whole consist of?
Yaren: Two.
Researcher: Can you divide this? [Showing the straight line] Can you divide this shape into half?
Yaren: It cannot be divided.
Researcher: Why not?
Yaren: Because it is a line.
Researcher: So, how should it be so that it can be divided into half?
Yaren: It needs to be a shape like a cube or a triangle.
Researcher: What do you call a quarter?
Yaren: .....

Yaren's worksheet about half was presented in Figure 4. Yaren says that two-dimensional shapes such as square and cube can be divided into half, but the straight line cannot be divided. Yaren responded correctly to the question of what the half was, she actually knows that the half should be equal two halves of the whole, but she cannot grasp how a straight line is divided. When she was asked to divide a straight line, she replied: *"this shape cannot be divided"*. Yaren recognized the half concept and used her knowledge about the halves, used it to divide two-dimensional shapes. However, she stated that one-dimensional straight line, which was a new shape for her, could not be divided into half. Not being able to use her existing knowledge on a newly encountered question shows that she could not construct and abstract the knowledge. Yaren did not answer any questions about quarters expressing that she did not know how to divide into quarters. Not having any familiar knowledge indicates that the concept of quarters was not constructed, too.

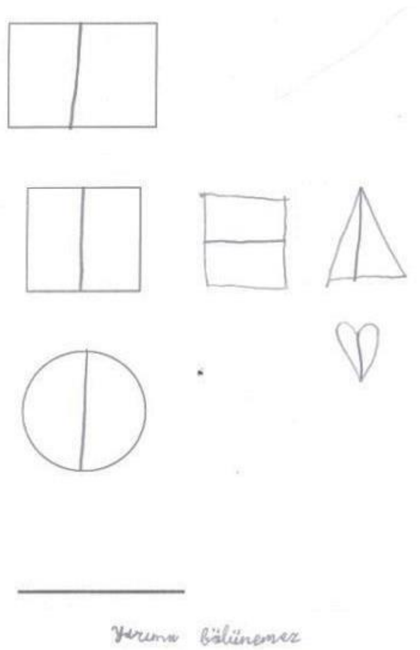

Figure 4. Yaren’s first worksheet about half

The abstraction levels of the students are revealed as in Table 1. As it can be seen in Table 1, it is understood from Ceyda's responses that she constructed and completely abstracted the half and quarter concepts. While only Ceyda and Berkay abstracted the concepts of the half, nobody abstracted quarter concept except Ceyda. However, it was determined that students had difficulty in understanding these subjects, they could not comprehend and abstract the whole-half-quarter concepts in the interviews even though they could divide them correctly on the worksheets.

Table 1. Students' levels of abstraction for the whole-half-quarter concepts

	Abstraction of Concepts	
	Half	**Quarter**
Ceyda	+	+
Yaren	-	-
Berkay	+	-
Kadirhan	-	-
Yunus Emre	-	-
Kenan	-	-

This study was conducted to examine the abstraction levels of whole-half and quarter concepts of primary school students. According to RBC theory, the abstraction steps are Recognizing – Building with – Constructing and there are properties of these steps like “recognizing the existence of a familiar mathematical structure” for recognizing; “combining familiar pieces of knowledge into a new context” for building with and “reorganizing what is recognized and known to construct new meanings” (Bikner-Ahsbahs, 2004) as mentioned above. In the interviews, the students’ abstraction levels were examined according to abstraction steps and as a result of the study, it has been seen that most of the students could not abstract whole-half-quarter concepts.

When students’ abstraction levels were analyzed, it can be seen that most of them failed. Only two of them were able to answer the questions and abstract the “half” concept accurately. However, the interviews showed that only Ceyda could construct and comprehend the whole-half-quarter concepts correctly. It can be concluded that while a student abstracted all the concepts, many of them memorized the definitions of these concepts, but they did not understand the subject at all, they adhere to the definitions and could not abstract the concepts.

They have difficulties in learning and understanding these concepts. It was understood that difficulties of students to abstract the whole-half-quarter concepts resulted from reasons such as not understanding the half and quarter concepts, not being able to divide the whole into two equal parts, not being able to divide one dimensional shapes into half and quarter, generalizing dividing into quarter as putting a "+", not being able to divide into four equal parts for quarter. It was concluded that the reasons for these learning difficulties were mostly caused by rote learning.

Some students memorized half-and-quarter concepts as shapes (putting + to divide into quartiles), while others have not been able to make any definitions and answering correctly the questions, some of them could not generate any information even at the level of recognition. It can be assumed that these students could not abstract these concepts because of memorizing the definitions or mentally adhering to the shapes that the classroom teacher had shown in the courses before. Because students could not make any conclusions when they were asked to divide half and quarter of a different shape (one-dimensional straight line) that has not been taught by their teacher before, and they could not divide it into half and quarter. Moreover, not only unsuccessful but also successful students experienced these mistakes.

The results of the study are consistent with the Van Hiele Geometric thinking levels. This is because students can make inferences appropriate for the zero and one level they belong to according to this model. It has been understood that they could not comprehend the division into half and quarter independently of the definition of the shape being taught. As a matter of fact, in the literature, it has been found that the misconceptions about fractions are caused due to not being divided equally (Alacacı, 2010; Stafylidou & Vosniadou, 2004) and that these misconceptions can be avoided by making meaningful activities in the first, second and third classes in which the concept of fraction is introduced (Erbilgin, Şahin & Arıkan, 2017).

CONCLUSION

For the correct learning of the concept of fractions, points to note were stated in the literature. In order for the fractions to be grasped correctly, it is especially important for students to understand that the whole is divided, the pieces must be of equal size, and that a region can be divided equally into different groups (Charalambos & Pitta-Pantazi, 2007; Temur, 2015). It is aimed to accomplish effective fraction teaching by using rules of real-life situations and concrete tools instead of teaching the rules of fractions (Temur, 2015). It can be claimed that different teaching methods or computer programs can be beneficial in this regard and will enable students to correctly structure their future learning. However, there are points that need attention. The method used by the teacher should not prevent students from thinking and structuring their forms of understanding, so the teacher needs to be careful (Van de Walle et al. 2012). For this reason, the abstraction levels of students and the success of mathematics education depends on the characteristics of teacher. The teachers need to be careful to achieve the goals of the mathematics topics.

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REFERENCES

Alacacı, C. (2010). *Öğrencilerin kesirler konusundaki kavram yanılgıları.* In E. Bingölbali ve M. F. Özmantar (Eds). İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri, (p. 63-94), Ankara: Pegem Akademi.

Baykul, Y. (2011). *İlköğretimde matematik öğretimi (10. Baskı).* Ankara: Pegem Akademi.

Bikner-Ahsbahs, A. (2004). *Towards the emergence of constructing mathematical meanings.* Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2, 119-126.

Bikner-Ahsbahs, A. (2019). *The research pentagon: A diagram with which to think about research.* In Compendium for Early Career Researchers in Mathematics Education (pp. 153-180). Springer, Cham.

Brown, G., & Quinn, R.J. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher, 62*(4), 28-40.

Charalambous, C.Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational studies in mathematics, 64*(3), 293-316. https://doi.org/10.1007/s10649-006-9036-2.

Cramer, K.A., Post, T.R., & Delmas, R.C. (2002). Initial fraction learning by fourth-and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education, 33*(2), 111-144. https://doi.org/10.2307/749646.

Creswell, J.W. (2013). *Nitel araştırma yöntemleri, beş yaklaşıma göre nitel araştırma ve araştırma* *deseni (3. baskıdan çeviri).* Çev. Ed: M. Bütün, S. B. Demir. Ankara: Siyasal Kitabevi.

Dooley, T. (2007). *Construction of knowledge by primary pupils: The role of whole-class interaction.* In Proceedings of CERME, 5, 1658-1668.

Dreyfus, T. (2007). Processes of abstraction in context the nested epistemic actions model. Retrieved from https://pdfs.semanticscholar.org/d190/0be9d6a043ac815c81344caa8c2713dcc329.pdf.

Dreyfus, T., Hadas, N., Hershkowitz, R. & Schwarz, B. (2006). *Mechanisms for consolidating knowledge constructs.* Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education. Prague, Czech Republic.

Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2001). Abstraction in context II: The case of peer interaction. *Cognitive Science Quarterly, 1*(3/4), 307-368.

Dreyfus, T., & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *The Journal of Mathematical Behavior, 23*(3), 271-300. https://doi.org/10.1016/j.jmathb.2004.06.002.

Erbilgin, E., Şahin, B., & Arıkan, S. (2017). Pasta dilimlerinin fiyatı ne kadar?. *Journal of Inquiry Based Activities, 5*(1), 34-47.

Gilboa, N., Kidron, I., & Dreyfus, T. (2019). Constructing a mathematical definition: the case of the tangent. *International Journal of Mathematical Education in Science and Technology, 50*(3), 421-446. https://doi.org/10.1080/0020739X.2018.1516824.

Guler, H.K., & Gurbuz, M.C. (2018). Construction process of the length of [cube root of 2] by paper folding. *International Journal of Research in Education and Science, 4*(1), 121-135.

Halverscheid, S. (2008). Building a local conceptual framework for epistemic actions in a modelling environment with experiments. *ZDM Mathematics Education, 40*(2), 225-234. https://doi.org/10.1007/s11858-008-0088-x.

Hershkowitz, R., Schwarz, B., & Dreyfus, T., (2001). Abstraction in context: epistemic actions. *Journal for Research in Mathematics Education, 32*(2), 195-222. https://doi.org/10.2307/749673.

Kaplan, A., İşleyen, T., & Öztürk, M. (2011). The misconceptions in ratio and proportion concept among 6th grade students. *Kastamonu Education Journal, 19*(3), 953-968.

Kidron, I., & Dreyfus, T. (2009). *Justification, enlightenment and the explanatory nature of proof.* In Proceedings of the ICMI Study 19 Conference: Proof and proving in mathematics education, 1, 244-249.

Merriam, S.B. (2013). *Nitel araştırma desen ve uygulama için bir rehber (3. baskıdan çeviri).* Çev. Ed: S. Turan. Ankara: Nobel Yayıncılık.

Monaghan, J., & Ozmantar, M.F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics, 62*(3), 233-258. https://doi.org/10.1007/s10649-006-8753-x.

National Ministry of Education. (2018). Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar). Retrieved from http://mufredat.meb.gov.tr/ProgramDetay.aspx?PID=329.

Olkun, S. & Toluk, Z. (2003). *Matematik öğretimi*. Ankara: Anı Yayıncılık.

Ozmantar, M.F., & Monaghan, J. (2007). A dialectical approach to the formation of mathematical abstractions. *Mathematics Education Research Journal, 19*(2), 89-112. https://doi.org/10.1007/BF03217457.

Schwarz, B., Dreyfus, T., Hadas, N. & Hershkowitz, R. (2004). *Teacher guidance of knowledge construction.* Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education. Bergen, Norway.

Sowder, J., & Wearne, D. (2006). What do we know about eighth – grade achievement? *Mathematics Teaching in the Middle School, 11*(6), 285-293.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*(5), 503-518. https://doi.org/10.1016/j.learninstruc.2004.06.015.

Temur, Ö.D. (2015). Opinions of teachers of fourth and fifth grade about teaching fractions: A phenomenograhic research. *Dumlupınar Üniversitesi Sosyal Bilimler Dergisi*, *29*.

Van de Walle, J.A., Karp, K.S. & Bay Williams, J.M. (2012). *İlkokul ve ortaokul matematiği gelişimsel yaklaşımla öğretim* (Yedinci baskıdan çeviri, S. Durmuş Çev. Ed.). Ankara: Nobel Yayıncılık.

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SAILING CONTEXT IN PISA-LIKE MATHEMATICS PROBLEMS

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Abstract

Developing PISA like mathematics problems using daily life context helps to improve the quality of learning. This study aimed to generate a valid, practical, and having potential effects on mathematics literacy ability PISA like mathematics problems with the context of sailing in the 2018 Asian Games. This research involved three expert reviews and 32 fifteen-years-old tenth-grade students of SMA N 10 Palembang, a public senior high school in Palembang Indonesia, as the research subjects. This study used the design research method of development studies type. The data were collected through documentation, walkthrough, tests, and interviews. The problem developed in this study is related to the length of wood required to span the sail. The results show that the problems are valid. The validity was viewed from its compatibility with the PISA framework, in which it had related the problems with daily life context of sailing in the 2018 Asian Games and space and shape content. The problems are also practical, viewed from students' understanding of the problem. And, the problems have potential effects when tested in learning issue of mathematical literacy ability. The dominant ability is reasoning and representation ability, while the communication ability is still low because the students do not give detailed answers.

Keywords: Design Research, Mathematics Literacy, PISA, Sailing Context

Abstrak

Mengembangkan soal matematika tipe PISA menggunakan konteks yang berhubungan dengan kehidupan sehari-hari dapat membantu meningkatkan kualitas pembelajaran. Penelitian ini bertujuan untuk menghasilkan soal matematika tipe PISA dengan konteks cabang olahraga layar pada Asian Games 2018 yang valid, praktis dan memiliki efek potensial terhadap kemampuan literasi matematika. Penelitian ini melibatkan 3 *expert review* dan 32 siswa yang berusia 15 tahun kelas X SMA N 10 Palembang sebagai subjek penelitian. Metode penelitian yang digunakan adalah *design research* tipe *development study*. Teknik pengumpulan data terdiri dari dokumentasi, *walkthrough*, tes dan wawancara. Soal yang dikembangkan berhubungan dengan panjang kayu yang dibutuhkan untuk membentangkan layar. Hasil penelitian menyatakan bahwa soal yang dikembangkan telah (1) valid dilihat dari kesesuaiannya dengan *framework* PISA yaitu telah menghubungkan permasalahan dengan kehidupan sehari-hari dengan menggunakan konteks cabang olahraga layar pada Asian Games 2018 dan konten *space and shape*, (2) praktis dilihat dari pemahaman siswa terhadap soal yang dikembangkan, (3) memiliki efek potensial ketika diujicobakan dalam pembelajaran memunculkan kemampuan literasi matematis. Kemampuan dominan yang muncul dari soal yang dikembangkan adalah kemampuan penalaran dan representasi sedangkan kemampuan komunikasi masih rendah disebabkan oleh jawaban yang dituliskan siswa tidak dituliskan secara rinci.

Kata kunci: Desain Research, Literasi Matematika, PISA, Konteks Layar

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Programme for International Student Assessment (PISA) is an international study which is conducted every three years to measure literacy ability of 15 years old students (OECD, 2016). Since 2000, Indonesian students have participated in the PISA test, but they did not get good results. The score obtained by Indonesian students, especially in mathematical literacy is only around 350 - 400, which is still far from 500 as the average international PISA score. It occurs because Indonesian students are only able to answer the problems in the low category, and almost no student can answer the problems that need high levels of thinking (Stacey, 2011; Stacey, 2012). Therefore, the unavailability of problems that measure mathematical literacy ability causes the students not be able to answer the PISA problems (Lutfianto, *et al*. 2013, Pulungan, 2014).

Mathematical literacy is needed by everyone in facing the problems in life because it is mostly used to help people to understand the role and usefulness of mathematics in daily life (Stacey, 2012). Furthermore, literacy ability also presents concepts that can add the mindset of students and teachers (Sturgeon, 2018). The mindset is trained through the process of literacy, especially mathematical literacy by formulating, applying, and interpreting the problems. The mindset is also consistent with the definition of mathematical literacy that "*Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts ....*". Based on this definition, literacy also uses context in its problems. The context used is adapted to the development of the 21st century (OECD, 2015).

Moreover, the NCTM (2014) states that using the context of the real world is expected to be a stepping stone in understanding the concept of mathematics. However, the context used in the PISA problem is still strange to the students because it is still related to the foreign context (Sasongko, 2016). Also, students are also difficult in turning problems into mathematical problems (Wijaya, *et al.* 2014). Contextual learning provides a mindset to the students that mathematics is not only about counting, but what is learned in mathematics can also be useful in life (Widiati, 2015).

The use of context is one of the characteristics of the *Pendekatan Matematika Realistik Indonesia* (PMRI) adopted from the Realistic Mathematics Education (RME) which emphasizes the meaningfulness of mathematical concepts in the form of problem (Prahmana, *et al*. 2012; Zulkardi, 2002; Prahmana & Suwasti, 2014). Context can be applied in the form of a problem that is seen as an important aspect of mathematics learning (Chapman, 2006; Prahmana, *et al*. 2012). The problems can be in the form of the development of a PISA like problem designed by the teacher so that it can be applied in classroom learning (Zulkardi, 2010). Based on the study from Putri (2015), giving problem in the form of test with PMRI approach can make students happy and challenged to answer them so that an optimal learning result of mathematics can be achieved. Clair (2018) and Putri (2015) suggest that the context applied in classroom learning should use the context which is close to the student. Therefore, one of the contexts that can be used is the context of sailing in the 2018 Asian Games.

The 2018 Asian Games is a sports competition held every four years which aims to enhance and strengthen the friendship between Asian countries. The Asian Games is also referred to as the energy of Asia because it shows the spirit of unity and sports achievement in Asia. Jakarta and Palembang will be the host of the 2018 Asian Games. One of the sports is sailing. Sailing is a part of the aquatic sports that contribute a lot of medals so that it becomes a mainstay sports at the 2018 Asian Games (INASGOC, 2018). By using sports as a context of the developed problem will make students prefer mathematics and also remember the event of Asian Games 2018. Many studies developed the design of learning mathematics using sports context in the 2018 Asian Games, using this context can help students understand mathematics (Gunawan, *et al*. 2017; Fajriyah, *et al*. 2017; Rahayu, *et al*. 2017; Roni, *et al*.

2017; Efriani, *et al*. 2018; Nizar, *et al*. 2018). Therefore, this article will discuss the developed problem using the context of sailing in the 2018 Asian Games.

METHOD

This study was design research of development study. The stages conducted in this study were preliminary and prototyping (formative evaluation) which included self-evaluation, expert reviews, one to one, small group, and field test (Tessemer, 1993; Zulkardi, 2002).

In the preliminary stage, the writers determined the subject and the place of the study and analyzed the subject matter, including the curriculum as well as the PISA problems that would be developed to produce the draft. The next stage was the formative evaluation started from the self-evaluation. A draft that had been developed to produce Prototype I was reviewed; then, piloted at the expert reviews and one to one stages.

In the expert review stage, Prototype 1 was validated by experts. The validation process was conducted in three ways, namely face to face review, mails review, and focus group (Tessemer, 1993). Along with the expert review, one to one stage was done with three students having various capabilities. Students were asked to read, examine, and give feedback about the legibility and clarity of the problem. Suggestions and comments from experts and students were taken into consideration to revise Prototype 1 to produce Prototype 2. Then, Prototype 2 was tested in the small group stage.

In the small-group stage, the problem was tested to six students having various abilities. They were asked for their opinions and comments about the problem they had been working. This stage focused on knowing the practicality of the problem. Students' suggestions and comments in the small group stage were taken into consideration to revise the Prototype 2 to produce Prototype 3. Next, Prototype 3 was tested in the field test stage.

In the field test stage, the problem was tested to 32 tenth grade students of class X MIA 1 of SMA N 10 Palembang to see its potential effects on the mathematical literacy ability. The data collection techniques used in this study was: (1) documentation, (2) walkthrough, (3) test, and (4) interview. The data from the expert review, one to one comments, student answers and interviews were analyzed descriptively.

RESULTS AND DISCUSSION

The results of this research are 12 questions of PISA like mathematics problems with the context of sport in Asian Games. This article only discusses one of developed PISA like problems with the context of sailing. In the preliminary stage, the writers determined the 15-year-old students of SMA N 10 Palembang as the research subjects and the content material about space and shape for the problem. The content was based on the 2013 curriculum, and it can be related to the context of sailing in Asian Games. Then, the PISA problems from 2000 to 2012 were collected. The problem was made concerning the existing PISA problems and the developed one.

The reference problem used in this study is the 2012 PISA problem with the context of a sailing ship. The problem asked to determine the length of the rope required when the angle and height of the sail are known. Then, the writers paraphrased the problem by changing this adopted problem from the Bairac method (2005) into determining the height of the wood required to tie the sail in the sailing ship. The purpose of this problem is to make students understand that to span the sail to catch the wind. The sail must be attached to the wooden pole; therefore, they will know how long the wood is needed to tie the sail.

The next stage is the preliminary stage that began with the self-evaluation stage. At this stage, the writers evaluated and examined the draft of the PISA-like problem. The developed problem was related to the context of sailing as a sport. The type of problem was a closed-constructed response. It was predicted to be in Level 4 because it corresponded to the characteristics of the PISA problem shown by making the model implicit in concrete but complex situations, selecting and combining different representations including symbolizing and relating to real situations, as well as using good skill development and presenting flexible reason and perspective according to the context. At this stage, the problem was by the PISA framework, so there was no revision. The problem at this stage was called Prototype 1.

The Prototype 1 validation process was done via mail review. Experts who acted as the validators were (1) Ross Turner (E1), the director of the Australian Council for Educational Research (ACER) and a MEG PISA staff, (2) Kaye Stacey (E2), the Chairman of MEG PISA from University of Melbourne, Australia and (3) Ahmad Fauzan (E3), a lecturer in mathematics at the State University of Padang. Along with the expert review stage, one to one stage was done with students who had various abilities (high (O1), medium (O2), and low (O3)). The comments and suggestions from the experts and the students in Prototype 1 are presented in Table 1.

Table 1. Comments and Suggestions from Experts and Students

Code	Comments and Suggestions
E1	- Describe the situation clearly, maintain the prerequisites for students to interpret and transform information into a mathematical form - Use rubric from PISA assessment by using a range of answers - Add the abilities of mathematizing, using language, formal and technical operational and symbolic language, as well as problem-solving.
E2	- Reasoning and argumentation abilities in the problem are good, in which it has been analyzed carefully - Less information about the length of the base of the ship to the base of the sail
E3	- Revise the spacing in the problems - The problems may raise students' different perceptions related to the sail size
O2	- The student had correctly put the information about the picture, but he was still confused with the phrase “the length side cloth under the pole.” - Students predict the length of the pole from the base only based on the image without thinking about the wood that is plugged in because the information about the problem is not given

These comments were taken into consideration to revise prototype 1. Based on the validation process at the expert review and one to one stages, the problem was maintained with some revisions, namely (1) adding information "to span the sail, a wood is put in the ship pole" which is useful for guidelines for determining the answer, (2) replacing the question "the height of the wood" into "the length of wood needed to span the sail". This question gave a benefit for students so that they did not only calculate the length of the wood, but they also knew that the sail could not be unfolded if it did not have the crossing wood, (3) changing the scoring rubric based on the PISA assessment rubric by adding a range of answers. Based on experts' suggestion which says that many students would use the right method but would not get the right results indicated the need of the range for various answers, and (4) revising the mathematical literacy ability indicators based on the steps of work. The result of this revision was called Prototype 2.

Furthermore, the problem in Prototype 2 was tested in a small group stage on November 7, 2017. First, students were given the problem to be solved individually. During this process, the students with high, medium, and low abilities had understood the purpose of the problem by trying to find the solution by writing down what was known from the problem. Some students got the answer by using only one step, while others used some steps to get the answers. Next, the students formed small groups to discuss the problem. In the small groups, they determined the solutions based on the answers they got individually. The students got the same answers in various ways. Because the problem can be answered by all students without any obstacles, the writers consulted it with the advisor. The advisor suggested that the information of the problem be changed. The previous problem asked the student to answer the length of the wood needed to span the sail if the height of the wood and the area of sail were known. Meanwhile, the revised problem asked students to answer the length of the wood needed to span the sail if the three sides of the sail were known, as shown in Figure 1.

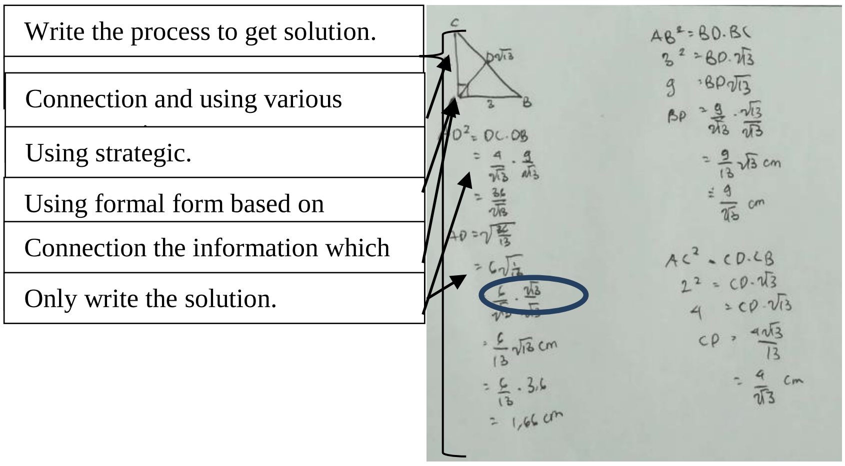
PERAHU LAYAR

Gambar 1. Perahu Layar
(Sumber : http://www.google.com)

Perahu layar adalah alat transportasi tradisional yang menggunakan layar sebagai penangkap angin untuk berlayar. Perahu di atas dipasang sebuah layar berbentuk segitiga. Sisi layar berturut-turut adalah 2m, 3m dan $\sqrt{13}$m. Untuk membentangkan layar, dipasanglah sebuah kayu pada tiang perahu. Tentukanlah panjang kayu yang dibutuhkan untuk membentangkan layar tersebut !

Figure 1. Prototype 3.

The next stage was the field test held on November 9, 2017. After the students finished with the problem, the writers interviewed them to explore the potential effects of the problem. One of the students' answers is correct, as shown in Figure 2.



P: Why did you use this formula?
S: Because, to find the length of this wood I must know the length both of this (student pointed the picture).
P: Where did you get this formula?
S: From the course.
P: Do you know the origin of this formula?
S: I don't know.

Figure 2. Correct student's answer.

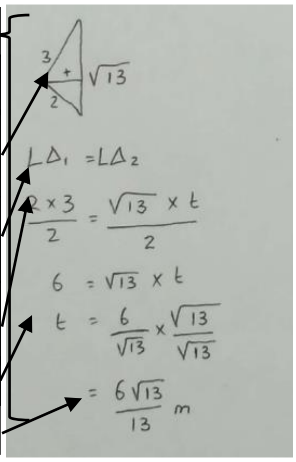
From the answers shown in Figure 2, the student wrote the problem into a mathematical statement by imagining the representation from the information given. Using the figure she had made, she symbolized the figure herself to make it easier to find the solutions. As shown in Figure 2, she gave symbols A, B, C, and D. Then, she chose a strategy by relating to the various equations that had been made. Finally, she got the last equation of $AD^2 = DC.DB$, in which the symbol AD was the length of the wood needed to span the sail asked in the problem. Although the students' answer was correct, the writers also interviewed her related to how she got the answer using the formula.

Based on the above interview, the student said she found the solutions using the formula which she got from her course outside the school. However, she did not know the origin of the formula that she used. The literacy ability involved by the student was that she could write the process of achieving the solution completely. She used the image representation by making the figure by herself to make it easier for her to find the solution. Next, she related the previous information about the length of the three sides of the sail consecutively, which is 2, 3, and $\sqrt{13}$ by writing them on the figure she made before. Then, she used the strategy based on the definition and the existing rule of the equation $AD^2 = DC.DB$ to find the solution. However, before getting the length of AD, she found the length of DC and DB from the information that she got before. Once the solution was obtained, she should have been able to conclude the solution, but she only wrote down the solution without any conclusion. The answer was correct with different algorithms, as shown in Figure 3.

Figure 3 shows that the student responded by using the area of the right triangle. The image representation form was divided into two triangles; then, he used the equation of the area of triangle 1 equals the area of triangle 2. However, the writers investigated the reason why the student used the area of right-triangle to find the answer.

Furthermore, Figure 3 also shows the interview results that the student knew that the triangle was a right triangle so that it could be calculated by using the area of a triangle. The triangle was viewed from two different corners so that by using the comparison between the two triangular areas, he got the length of wood needed to span the sail.

Write the process to get solution.
Using understanding context.
Connecting and using various representation.
Using formal form based on definition and rule.
Connecting information which got before.
Using strategic.
Only write the result.



P: Why did you use the right-triangle area formula? While the figure and information have not shown that the right triangle!
S: It easy. That's right, there is no information about right triangle but I can prove that if it is right angle using the information (student proof the right triangle).
P: Why does it have two triangle area while in the figure just has one triangle?
S: Of course, in the figure only has one triangle but if I look from different corners, it has two triangles.
P: That's right.

Figure 3. Correct student's answer with different algorithm.

From the mathematical literacy ability involved, students could write down the process of getting a complete solution by relating the previous information about the side of the sail which was 2, 3, and √13 and use various representations namely the image representation by making the figure of the sail and symbol representation by using triangle symbol. Furthermore, he used a strategy based on the existing definitions and rules stated that calculating the area of a triangle can be done with two different corners, but the reason was only found out from the interview. Then, the student got the answer of $\frac{6\sqrt{13}}{13}$. Student only wrote the result without any explanation. This strategy is in line with Hasratuddin's opinion (2010) that students solve a problem only by finding solutions, then operating it, but they do not provide any explanation of the solution. The process often occurs because the learning of mathematics pays less attention to students' mathematical communication ability (Saragih & Rahmiyana, 2013). The student had the potential to explain the reasons why he used the right triangle formula. Moreover, his incorrect answer is shown in Figure 4.

Not exactly using representation.
Not exactly using strategic.
not exactly connecting the information.
Not exactly understanding the context.
Not exactly using formal form based on definition and rule.
Uncorrect to conclude the solution.

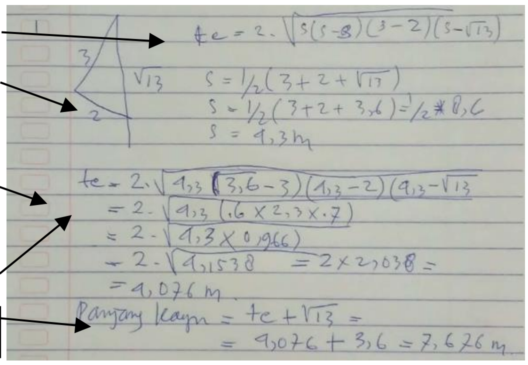


Figure 4. Incorrect student's answer.

From the answer shown in Figure 4, the student used the area of a triangle when the three sides were known. However, this answer did not match with what the question wanted since it asked for the length of wood needed to span the sail, while the student determined the area of the sail. Therefore, his answer did not get any score (no credit). From the answer that he made, the writers interviewed him to find out the reasons why he determined the length of wood needed by using the area formula. The interview between the student and the writers is shown as follows.

P: What is the problem that is asked?
S: The length of wood.
P: Why did you use the area of triangle formula?
S: Because based on the formula area of the triangle, if it is known three sides of the sail, it can use this formula.
P: Is it the length of wood same with the area of the triangle?
S: It's different.
S: Why do you still use this formula.
P: Because I just know the formula of the area if three sides of the sail are known.

Based on the interview above, the student understood what was known in the problem by translating the problem into the mathematical statement using the image representation. However, he was wrong in the use of the information provided. He used the formula of triangle area to calculate the length of the sail due to the lack of prerequisite knowledge about the use of the area formula. From the mathematical literacy ability involved, he had tried to write the process of getting the solution, but the representation of the contextual understanding was not related to the desired question. The strategy used was also not appropriate for the problem which asked to determine the length of wood needed to span the sail instead of determining the area of the triangle. Therefore, he could not relate the information and conclude the solution. The mistake happened when he decided on the strategy to determine the solution. The ability to settle the implementation plan is the core of the problem-solving stage (Hapizah, 2016). Therefore, if the student could not determine the right strategy in planning the settlement, it would cause an error in solving the problem. The field test results showed that out of 32 students, only ten students obtained full credit, 13 students obtained partial credit, and nine students obtained no credit. Furthermore, we could also capture the mathematical literacy ability involving the 32 students.

Communication ability with the indicator to write the process in achieving the solution showed ten students writing it completely and correctly, and five students writing it incorrectly. Whereas, the indicator concludes the mathematical result obtained that none of the students concluded correctly and completely, and nine students did not conclude. The mistake happened because the students did not read the data from the information in the problem carefully, so they were wrong in transforming the information into the calculation. The students placed the order of the sail side size in reverse. Also, the students did not use to writing down the stages, along with the explanations about the process of finding the solutions because they only focused on getting the answer.

The mathematical ability with the indicators of using context comprehension to solve mathematical problems showed ten students could use the understanding of the process to problem-solving appropriately, and nine students could not. This ability is a basic ability that students must have because they must be able to understand the context first before trying to find a solution to get the right solution. This phenomenon is in line with the opinion of Sudarman (2010), who states that students can be said to understand the problem if they can disclose data that are known and asked from the problem given.

Representation ability with the indicator to use and connect various representations in problem-solving showed ten students could use the representation completely and correctly, and five students did not use it correctly. The students' mistake occurred when they represented the figure and wrote the information on the figure. They made a mistake in putting the existing information, which later caused a mistake in the calculation process.

The reasoning and argument ability with the indicator of relating the previously available information to find the solution showed ten students could use reasoning correctly, and nine students did not use reasoning in finding the solution. To get the right solutions, students must be good at relating the previously available information. This phenomenon happened because the problems with the sail context was a high-level problem which needed more reasoning (Sulastri, *et al*. 2014). Also, with good reasoning, students can also solve problems well (Hapizah, 2014).

The ability to choose strategies to solve problems with the indicator of using strategies through various procedures that lead to solutions and conclusions showed nine students could choose the strategy appropriately and nine students did not choose the strategy in problem-solving. There were many strategies to be used for this problem; however, it would only depend on the skills of students to anticipate the relationship. Also, before students determine the suitable strategy, they must first understand the question of the problem, which is the length of the wood needed to span the sail in which the wood is located between the wooden buffers of the sail.

The ability to use language as well as symbolic, formal and technical operations with the indicator of using formal form based on the definition and mathematical rules showed that eight students could use it completely and nine students did not use it correctly.

Based on the above description, the dominant ability arose from the developed problem was the reasoning and representation ability because the students already understood the meaning of the problem well so that students could connect the information of the problem by making the figure repeatedly (Hendroanto, *et al*. 2018; Sukirwan, *et al*. 2018). However, the communication ability was still low because the students only focused on finding the answers, so they were not accustomed to writing answers in detail. Students had potential in developing mathematical literacy ability. However, this ability was not fully appointed but only as a compliment. This ability was needed to train students in solving problems properly (Ahamad, *et al*. 2018). It is also expressed by Sapitri & Hartono (2015) that every student has the potential for the ability of mathematical literacy. However, the students' ability in understanding mathematics is different from one another, so the teacher must train students to optimize their ability. Therefore, mathematical literacy ability of students can be extracted by giving the routine problem for them and asking them to solve it by writing down solutions in detail.

CONCLUSION

The problem was declared to be valid, practical, and had a potential effect on mathematical literacy ability based on the problem development process that had been generated. Valid was assessed from the validator results at the expert review and one to one stage, which stated that the problem was good in terms of content, constructs, and language. Valid was viewed from its compatibility with the PISA framework that had related it to daily life using the context of sailing in the 2018 Asian Games and the content of space and shape. The problem was also considered practical from the small group stage. Practicality was viewed from the students' understanding of the problem. The problem with the context of sailing in the 2018 Asian Games could help students understand the mathematics problems in daily-life contexts. From the problem, students knew that the length of the wood which was used in the sail had the size which could be counted in mathematics. Also, the problem had a potential effect when piloted in the learning of mathematical literacy ability. The dominant ability was reasoning and representation ability because they could understand it well so that they could connect its information by making a figure repeatedly. Whereas, the communication ability was still low because the students only focused on finding the final answer so that they were not accustomed to writing the answers in detail. Therefore, it is suggested to develop a PISA mathematics problem with sailing context on other content and can also use the equipment and rules in the sport. Also, PISA like mathematics problems that have been developed can be given to students regularly to train their mathematical literacy ability.

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REFERENCES

Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series, 943*(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.

Bairac, R. (2005). Some methods for composing mathematical problems. *International Journal for Mathematics Teaching and Learning*, *1*(3), 1-7. Retrieved from: http://www.cimt.org.uk/journal/bairac.pdf.

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics*, *62*, 211–230. https://doi.org/10.1007/s10649-006-7834-1.

Clair, J.S. (2018) Using cartoons to make connections and enrich mathematics. *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference* (vol. 2, article 12). Georgia: Georgia Southern University. https://doi.org/10.20429/stem.2018.020112.

Efriani, A., Putri, R.I.I., & Hapizah. (2018). PISA like mathematics problem: a case of soccer context. *Journal of Physics: Conference Series, 1097*(1), 012108. https://dx.doi.org/10.1088/1742-6596/1097/1/012108.

Fajriyah, M., Putri, R.I.I., Zulkardi. (2017). Dayung context in fraction. *Advances in Social Science, Education and Humanities Research* (vol. 100). Banjarmasin: Atlantis Press. https://dx.doi.org/seadric-17.2017.1.

Gunawan, M.S., Putri, R.I.I., Zulkardi. (2017). Learning fractions through swimming context for elementary school students. *Advances in Social Science, Education and Humanities Research* (vol. 100). Banjarmasin: Atlantis Press. https://dx.doi.org/seadric-17.2017.14.

Hapizah. (2014). Developing the instruments of students' mathematical reasoning abilities in the course of differential equations [in Bahasa]. *Kreano, Jurnal Matematika Kreatif-Inovatif*, *5*(1), 73-81. https://doi.org/10.15294/kreano.v5i1.3280.

Hapizah. (2016). Student's ability to solve problem-solving problems in mathematics [in Bahasa]. *Jurnal Pendidikan dan Pembelajaran*, *23*(2), 124-131.

Hasratuddin. (2010). Improving the critical thinking skills of junior high school students through realistic mathematics approach [in Bahasa]. *Jurnal Pendidikan Matematika*, *4*(2), 19-33. https://doi.org/10.22342/jpm.4.2.317.

Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial visualization. *Journal of Physics: Conference Series, 943*(1), 012029. http://doi.org/10.1088/1742-6596/943/1/012029.

INASGOC. (2018). *Asian Games 2018*. Retrieved from: http://www.asiangames2018.id.

Jannah, R.D., Putri, R.I.I., & Zulkardi. (2018). PISA-like mathematics problem: The context of basketball in Asian Games. *Journal of Physics: Conference Series, 1088*(1), 012019. http://doi.org/10.1088/1742-6596/1088/1/012019.

Lutfianto, M., Zulkardi & Hartono, Y. (2013). Unfinished student answer in PISA mathematics contextual problem. *Journal on Mathematics Education*, *4*(2), 188-193. https://doi.org/10.22342/jme.4.2.552.188-193.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston: National Council of Teachers of Mathematics.

Nizar, H., Putri, R.I.I., & Zulkardi. (2018). Developing PISA-like mathematics problem using the 2018 Asian Games football and table tennis context. *Journal in Mathematics Education, 9*(2), 183-194. https://doi.org/10.22342/jme.9.2.5246.183-194.

OECD. (2015). *Assessment and Analytical Framework*. Paris: OECD. http://dx.doi.org/10.1787/9789264281820-en.

OECD. (2016). *PISA 2018 Draft Analytical Framework*. Paris: OECD.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The case of rural area's student in Indonesia. *Journal on Mathematics Education*, *5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Prahmana, R.C.I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education, 3*(2), 115-132. https://doi.org/10.22342/jme.3.2.1931.115-132.

Pulungan, D.A. (2014). Developing the instrument of PISA model mathematical literacy test [in Bahasa]. *Journal of Educational Research and Evaluation*, *3*(2). 74-78.

Putri, R.I.I. (2015). The interaction effect of learning approach in the formative test form of the mathematics learning outcomes [in Bahasa]. *Jurnal Pendidikan dan Pembelajaran*, *22*(1), 69-75.

Rahayu, C., Putri. R.I.I., Zulkardi. (2017). Multiplication of fraction with natural number by using hurdles. *Advances in Social Science, Education and Humanities Research*, (vol. 100). Banjarmasin: Atlantis Press. https://dx.doi.org/seadric-17.2017.10.

Roni, A., Zulkardi, Putri, R.I.I. (2017). Learning divisions of fractions through sprint running pictures. *Journal of Education and Learning*, *11*(4) 381-393. http://dx.doi.org/10.11591/edulearn.v11i4.5982.

Sapitri, S., & Hartono, H. (2015). The effectiveness of STAD and GI cooperative learning in terms of critical thinking skills and mathematical communication [in Bahasa]. *Jurnal Riset Pendidikan Matematika*, *2*(2), 273-283. http://dx.doi.org/10.21831/jrpm.v2i2.7346.

Saragih, S., & Rahmiyana. (2013). Increasing mathematical communication skills of senior high school / MA students in Simpang Ulim District through the STAD type cooperative learning model [in Bahasa]. *Jurnal Pendidikan dan Kebudayaan*, *19*(2), 174-188.

Sasongko, T.P.M. (2016). Developing of a package of PISA model space content and shape to determine the mathematics literacy of junior high school students [in Bahasa]. *Jurnal Edukasi*, *3*(1), 27-32.

Stacey, K. (2011). The PISA view of mathematical literacy in Indonesia. *Journal on Mathematics Education*, *2*(2), 95-126. https://doi.org/10.22342/jme.2.2.746.95-126.

Stacey, K. (2012). The international assessment of mathematical literacy: PISA 2012 framework and items. *Proceedings of the 12th International Congress on Mathematical Education Programme*. Seoul: COEX.

Sturgeon, A. (2018). Why literacy should be included in an effective elementary math curriculum. *EURASIA Journal of Mathematics, Science and Technology Education*, *14*(2), 557-560. https://doi.org/10.12973/ejmste/76957.

Sudarman. (2010). Thinking process of junior high school students based on Adversity Quotient (AQ) in solving mathematical problems [in Bahasa]. *Unpublished Dissertation*. Surabaya: Universitas Negeri Surabaya.

Sukirwan, Darhim, Herman, T., & Prahmana, R.C.I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series*, *943*(1), 012026. https://doi.org/10.1088/1742-6596/943/1/012026.

Sulastri, R., Johar, R., & Munzir, R. (2014). The ability of mathematics education department students at FKIP Unsyiah to solve the PISA questions in most difficult level [in Bahasa]. *Jurnal Didaktik Matematik*, *1*(2), 13-21.

Tessemer, M. (1993). *Planning and Conducting Formative Evaluations: Improving the Quality of Education and Training*. Philadelphia: Kogan Page.

Widiati, I. (2015). Developing mathematical representation ability of junior high school students through contextual learning [in Bahasa]. *Jurnal Pengajaran MIPA*, *20*(2), 106-111. http://dx.doi.org/10.18269/jpmipa.v20i2.571.

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast*, *11*(3), 555-584. Available at: https://scholarworks.umt.edu/tme/vol11/iss3/8.

Zulkardi. (2002). Developing a learning environment on realistic mathematics education for Indonesian student teachers. *Dissertation*. Enschede: University of Twente.

Zulkardi. (2010). PISA, KTSP and UN. *Prosiding Konferensi Nasional Matematika XV*, 53-54. Manado: UNIMA.

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PISA-LIKE MATHEMATICS PROBLEMS: USING TAEKWONDO CONTEXT OF ASIAN GAMES

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Abstract

This research aims to produce a valid, practical, and having potential effects PISA-like mathematics problems using taekwondo context in Asian Games. The subjects were MIA 3 student of SMA 10 Palembang. This study was design research of development study in which had two stages: the preliminary and formative evaluation. The formative evaluation includes self-evaluation, one-to-one and expert review, small group, and field test. The context is used to have the students estimate maximum numbers of exercising athletes in a hall with a specific size. The result of the analysis shows that the problems which were reviewed by three expert reviews are valid qualitatively based on the PISA framework; it is also practical and easy to understand the problem. Based on the analysis of students' answer, the developed problems display potential effects on student's diverse basic mathematical abilities on the various process of answering the problems. The basic mathematics abilities emerging among which are reasoning and argument ability. It appears that students can develop and solve the problem by modeling using their assumptions. Also, the other ability is designing strategies to solve problems in which students use various procedures in solving problems leading the conclusion.

Keywords: Asian Games, Design Research, PISA, Taekwondo

Abstrak

Tujuan dari penelitian ini adalah menghasilkan soal matematika tipe PISA menggunakan konteks olahraga taekwondo pada Asian Games yang valid, praktis serta mengetahui efek potensial. Subjek penelitian ini adalah siswa kelas MIA 3 SMA 10 Palembang. Penelitian ini merupakan *design research* tipe *development study* yang terdiri dari dua tahap: *preliminary* dan *formative evaluation*. *Formative evaluation* meliputi: *self-evaluation*, *one-to-one* dan *expert review*, *small group*, dan *field test*. Penggunaan konteks ini untuk meminta siswa mengestimasi jumlah maksimal atlet yang dapat latihan dalam aula dengan ukuran yang ditentukan. Hasil penelitian ini menghasilkan soal yang telah dinyatakan valid, sesuai berdasarkan *framework* PISA yang divalidasi 3 *expert review* secara kualitatif; praktis juga dan mudah dalam memahami soal. Berdasarkan analisis hasil jawaban siswa, soal yang dikembangkan ini memiliki efek potensial terhadap kemampuan dasar matematis siswa yang beragam pada proses penyelesainnya. Kemampuan dasar matematis yang muncul pada permasalahan ini diantaranya kemampuan penalaran dan argumen. Terlihat siswa dapat mengembangkan dan menyelesaikan masalah dengan *modelling* menggunakan asumsi sendiri. Selain itu, kemampuan lainnya adalah merancang strategi untuk memecahkan masalah, terlihat siswa menggunakan berbagai prosedur dalam memecahkan masalah dengan menggiring pada satu penarikan kesimpulan.

Kata Kunci: Asian Games, *Design Research*, PISA, Taekwondo

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PISA (Programme for International Student Assesment) is an international study organized by OECD (Organization for Economic Cooperation and Development) which assessment students' literacy skills (Edo, *et al*. 2013). PISA held every three years to see academic ability both in literacy reading, mathematics literacy, science literacy, and financial literacy (OECD, 2016). Based on Indonesia's participation in PISA 2015, as shown that the average for Indonesia students was 386 and ranked 63 out of the 70 participating countries (OECD, 2016). The involvement of Indonesia in PISA is an effort to see the position of literacy ability of students in Indonesia compared to the student in other countries and the problem affecting it (Khairuddin, 2017).

The low result of Indonesian students was caused by students' failure to work on the PISA problem which lay on their difficulty in formulating everyday problems into formal mathematical forms, understanding mathematical structures and evaluating mathematical results to real-world contexts (Edo, *et al.* 2013; Jupri & Drijvers, 2016). Moreover, when they have found the mathematical solution of the problem, then it was not followed by interpreting the solution back to the given context/situation of the problem (Jupri, *et al*. 2014; Lutfianto, *et al*. 2013; Hendroanto, *et al*. 2018). Putri (2012) states in using a context, students would not learn directly using the formula, such as culture (Risdiyanti & Prahmana, 2018; Maryati & Prahmana, 2019), games (Prahmana, *et al*. 2012; Kaune, *et al*. 2013), or several sports in Asian Games (Nasution, *et al*. 2018; Gunawan, *et al*. 2017; Rahayu, *et al*. 2017; Roni, *et al*. 2017). Classroom activities designed by the teachers also have engaged students in group and classroom discussions. One effort to familiarize students with the PISA model problem is to provide the problem as early as possible or at an early phase of middle school (Barczi, 2008). PISA results show that students who were able to answer the problem correctly on geometry were 47.5%, on statistics were 61.96%, and on number were 53.7% (Wardani, 2011). The result is one of the reasons for the revision of the 2006 curriculum, which results in the development and implementation of the 2013 curriculum (MOEC, 2014). Putri (2013) reveals that one approach in line with the 2013 curriculum is PMRI. PMRI is one approach using context (Zulkardi & Putri, 2006; Prahmana, *et al*. 2012; Ginting, *et al*. 2018). Aminuddin (2012) on his study which developed a mathematical problem of PISA model on space and shape content aiming at finding out the ability of mathematical connection of junior high school students stated that less than 50% of students could solve math problems of PISA model level 4,5 and 6. Furthermore, Purnomo, *et al*. (2015) indicates a poor ability of students in solving PISA problem on space and shape content based on Rasch model analysis is still lacking.

Moreover, Charmila, *et al*. (2016) concludes that it is important to integrate the context in the surrounding environment. Mathematics learning will be more meaningful, interesting, and fun by using the context of sports in the process of learning (Yansen, *et al*. 2019). Various researches on the development of mathematical PISA-like problems for High School students with various focus both on content and ability have been conducted (Jannah, *et al*. 2019). In 2018, Indonesia hosted the XVIII Asian Games. It is a sporting competition conducted every four years with participating countries of members of the Olympic Council of Asia. A lot of applicable contexts related to space and shape are available in the event either from the equipment used, the situation of events, and place of sport. Several studies developed PISA-like problem in Asian Games, such as Rahayu, *et al*. (2017) states context of athletics hurdles which is an Asian Games sport is used as the starting point used as a helpful media to solve the problems associated with fractional multiplication operations with natural numbers. Roni, *et al*. (2017) states used the context of sprint sport at the Asian Games give the impression of something new and different. Meanwhile, Gunawan, *et al*. (2017) used the swimming context is chosen because it can represent fractions using measurements. The shape of the pool is one model that allows one to represent parts of the whole.

Many studies developed the design of learning mathematics using sports context in the 2018 Asian Games, using the context can help students understand mathematics (Gunawan, *et al*. 2017; Fajriyah, *et al*. 2017; Rahayu, *et a*l. 2017). Pratiwi, *et al*. (2019) states use of PISA-like mathematics problems with a long jump in Asian Games context made students more interested and active during discussions in the learning process. Different with those studies, the researcher developed PISA like problems that using taekwondo context is chosen because the context is used to have the students estimate the maximum number of exercising athletes in the hall with the specified size with their assumptions. Therefore, this research aims to produce a valid and practical PISA math problem with the taekwondo context at Asian Games 2018 and to know the potential effect of the problem on students' mathematical ability.

METHOD

This research method used in the study is design research with the type of development studies which had two stages: the preliminary and formative evaluation. It includes self-evaluation, one-to-one and expert review, small group, and field test (Tessemer, 1993; Zulkardi, 2002). By the provisions of the PISA framework, subjects in this study are 15-year-old students in MIA 3 class of SMA 10 Palembang.

Initially, researchers evaluated and reviewed the prototype draft. The researcher also designed several instruments (lesson plan, question cards, scoring rubrics, and PISA questions on Space and Shape content based on PISA questions criteria). This research was started by describing the validity of the problem. Therefore, the subjects involved in this stage were three students with various levels of ability, high, fair, and low. They were the students in the one-to-one stage who given the Prototype 1. It was what the researcher had developed previously. This prototype was also given to experts at the expert review stage. It was reviewed by PISA experts Kaye Stacey, Ross Turner and Ahmad Fauzan who focused on three characteristics: content, constructs, and languages. The revision of Prototype 1 is called Prototype 2. This prototype was given to a small group of six students who were not the research subject with low, fair, and high-ability. At this stage, the appearance and use of questions were also evaluated to see the responses, assessments, and practicality and the results as input to revise the design for the next stage. The revision result of the small-group stage was called prototype III. It was, then tested with the subject of research by analyzing the results of student answers. It aims to see the potential effects emerging on students' mathematical abilities.

The data were collected using walk-through. It was used in compliance with the PISA framework. Furthermore, documentation was used as physical data of related documents. The test was registered to see comments from students on clarity, legibility. The results of the student answer showed basic mathematical skills. Moreover, interviews served to gather information about what students think after completing the test.

RESULT AND DISCUSSION

This study has produced nine problems of PISA type using taekwondo (2 problems) and game (7 problems, 6 of small balls and 1 of large balls) contexts. From the problems, two were level 3, level 4, and level 5 for each level and one were level 1, level 2, and level 6 also for each level. In this paper, researchers discussed one problem of taekwondo. It was chosen since there were various ways of completion students could use and many resulting assumptions that students use in solving problems given.

Preliminary

At this stage, the researcher determines the place and subject of the research, analyzes and designed Prototype 1, created a test table of specification containing appropriate indicators from the curriculum, designs question cards and scoring rubrics in compliance with PISA framework. Also, the researcher contacted the subject teachers whose working place was used as research locations and prepares other needs such as scheduling and working procedures with classroom teachers.

Formative evaluation

Self-evaluation

In the self-evaluation step, the researcher reviewed the prototype design by examining the compliance of the problem design with the PISA 2015 framework in terms of content, context, language, and level prediction. It aimed at checking the error in the process of resolving the problem before the prototype is used in the next stage. The prototype was given to experts at the expert review stage and one-to-one. Furthermore, we also designed instruments such as test table of specification, question cards, scoring rubrics and PISA problems based on the PISA framework. No revision was needed at this stage for its compliance to PISA framework and it was called Prototype 1.

Researchers were motivated to develop problems from the PISA problem with a rock concert. In this case, researchers change the context into taekwondo. Researchers wanted to estimate the maximum number of taekwondo players who can practice in a hall with a given size. Researchers also want to know whether or not students who solve these problems have determined reasonable, relevant, and accountable assumptions given their assumptions. The content used in these problems were space and shape. Prediction level is at level 5. The comparison between the original PISA problem with the developed problem can be seen in Table 1.

Table 1. The comparison between the original PISA problem with the developed problem

PISA problem	Developed problem
M552: Rock Concert **Question 1: ROCK CONCERT** M552Q01 For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert? A 2 000 B 5 000 C 20 000 D 50 000 E 100 000 **(PISA, 2009)**	 Taekwondo training hall Taekwondo National Training Camp will hold a series of warm-up training before participating in the 2017 Asian Games in Kuala Lumpur. It will be held in a rectangular hall measuring 150 meters in length and 75 meters wide. What is the maximum number of taekwondo athletes who can exercise in the hall?

Expert review and one to one

Experts then validated it at the expert review stage and by students at the one-to-one stage. Both stages were carried out simultaneously. This stage tried to see the validity of the instrument using the context of taekwondo. The expert review is a qualitative stage of validation, which was done through emails. The experts were Kaye Stacey, Ross Turner, and Ahmad Fauzan. While experts at the review panel item were Zulkardi (Sriwijaya University Lecturer), Somakim (Sriwijaya University Lecturer) and Ika Pratiwi (Sriwijaya University graduate students).

The three experts provided the following inputs, Ross Turner states that the problem was categorized as a modeling problem, so the assessment rubric will include whether or not students can determine reasonable, relevant or accountable assumptions by each student. Kaye Stacey adds that the matter can match the actual number of athletes at the Asian Games 2018, so the estimated number of athletes does not exceed the actual capacity in the Asian Games 2018. While Ahmad Fauzan has no comments on this problem so that this matter can be forwarded. While from the panel item, it was suggested to add a goal or reason why the context was used in developing the problem to give students more motivation in solving the problem.

In the one-to-one stage, Prototype 1 was also tested to three students whose abilities were of high, fair, and poor. The focus was to get students' comments on the clarity of the purpose of the question, to propose changes or alternatives, investigate why students are confused or have difficulty or even other interesting things from some aspect of the problem. The three students were CAR, MFR, and NA. NA finds the word maximum confusing to interpret while CAR was confused because he did not know the number of movements done in the exercise.

Based on the comments and suggestions from the validator and students at the one-to-one stage, the researcher revised Prototype 1 by adjusting the length and width of the hall by the actual number of taekwondo athletes in the Asian Games 2018 so the estimate does not exceed the capacity of the actual number of an athlete. Then the problem-solving strategy was added to the scoring rubric. Based on the revision of suggestions and comments from expert review and one to one conducted in parallel was Prototype 2.

Small group

On the small group stage, it was tested to 6 students whose ability of every two students were of high, fair, and poor. They were given Prototype 2 simultaneously and were given time to work on the problem individually; then after a few minutes, they were asked to discuss with their group members to solve the problems.

The researcher's focus at this stage was to see whether or not information such as tables, figures, numbers can be seen and well understood. While they were doing it, the researcher went around to see whether or not there are any problems the students encountered in the process of solving the given problem. After they had completed the given problem, one of the group representatives was interviewed to ask how the problem was solved. On this stage, all students could solve the problem without difficulty. So no revision was needed. The result of this stage was called Prototype 3. It is shown in Figure 1.

Taekwondo training hall

Taekwondo National Training Camp will hold a series of warm-up training before participating in the 2017 Asian Games in Kuala Lumpur. It will be held in a rectangular hall measuring 16 meters in length and 5 meters wide. What is the maximum number of taekwondo athletes who can exercise in the hall?

Figure 1. Prototype 3

Field test

In the field test phase is tested on the subject of which was students of grade X MIA 3 SMA Negeri 10 Palembang, 33 students worked on prototype 3. Researchers also observed students in working on the problem to find out the difficulties they faced. One of the goals of the field test stage is to know the potential effect of the problem on the students' mathematical literacy ability, which was seen from the result of the student's answer. The discussion of the results of student answers in solving problems with their respective strategies will be presented in Figure 2.

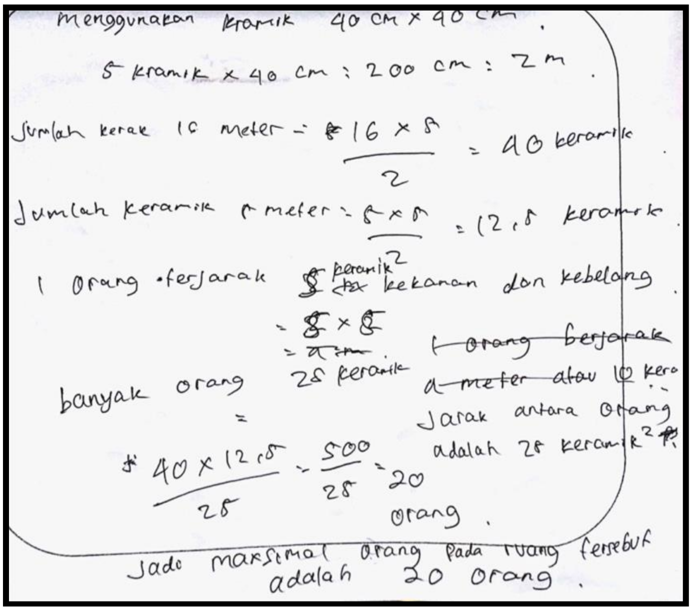
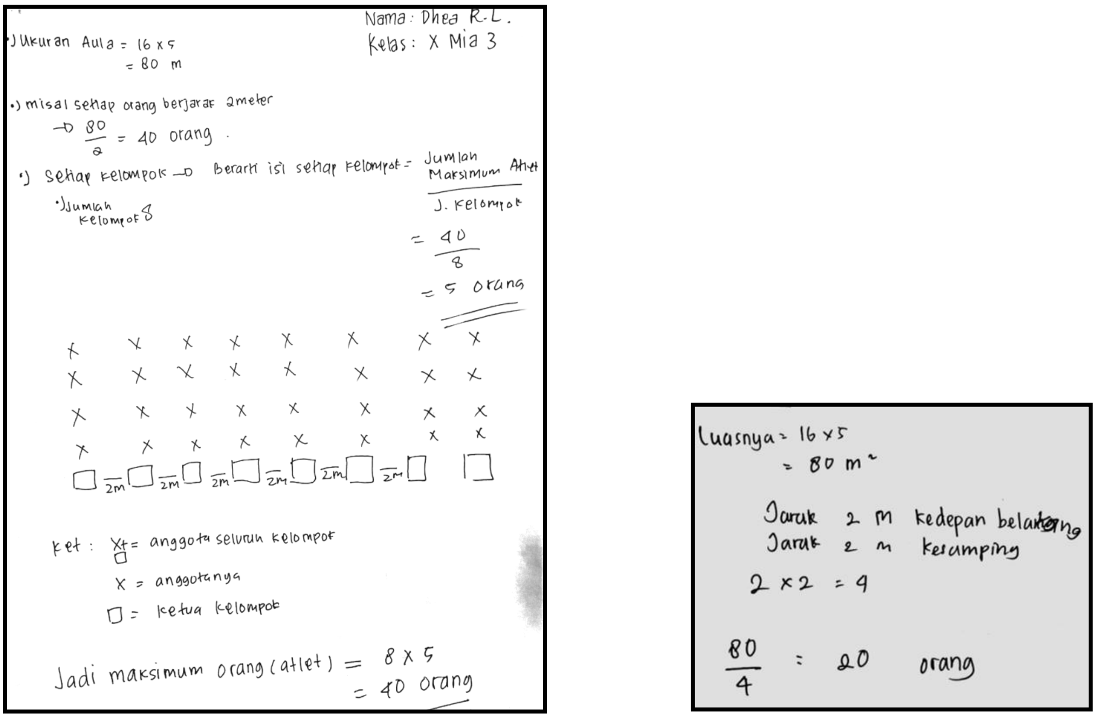


Figure 2. MDF‘s answer

From the analysis of students’ answer, it appeared that the MDF estimates the maximum number of taekwondo athletes who can exercise in the hall using ceramic tiles measuring 40 cm x 40 cm as a benchmark of the distance between athletes. Although as it was seen in the picture that the hall's floor was made of parquet wood, he tried using ceramic tiles as a strategy to answer the problem. In this case, students were able to use representational skills. It was also visible students made explanations and reasoning supporting in qualifying a mathematical solution to a contextual problem. In this case, he was able to use reasoning and argument ability.

Initially, MDF estimates the distance required for one athlete. By applying for 5 ceramics x 40 cm = 200 cm = 2 m. Here, MDF estimates the space required for athletes to avoid touching each other. Furthermore, MDF calculates the area of the hall by adjusting to the width and length of the hall using ceramics tiles. After that, MDF concluded that one athlete requires 25 tiles, five tiles' extending sideways, and five backward. So, MDF concluded that the maximum number of athletes who could exercise in the hall was 20 people. Nasution (2017) stated each activity in the learning used the problems of rowing context provided in a series of activities showed that most students had to understand and know the rowing context so that it can be integrated into the learning. The students answer by using other strategies to prove their answers (see Figure 3).



(a) (b)

Figure 3. DRL's answer (a) and N's answer (b)

It appears that a student with the initials DRL answered with a strategy that simultaneously shows the sketch assumed himself. DRL assumes that everyone has space 2 meters for eight groups. So with 80 m^2 hall and 2 meters for every athlete, it could be concluded 40 athletes could exercise in the hall with each group having five athletes. While the student initials N estimates the maximum number of athletes who can practice in the hall was 20 people. She used a similar strategy taken by MDF and DRL, N also estimated that there was a distance between athletes. N estimates the required distance was 2 meters to the front and 2 meters to the side (right and left). So one person had a $4m^2$ area. He concluded that the estimated maximum number of athletes in the hall was 20 people.

Based on field test results, 17 out of 33 students can use reasoning and arguments ability. It appears that students can make the explanation and reasoning from the mathematical solution to the contextual problem completely. It could be seen when students argued that situations, they were assumed to be reasonable, such as the assumption of distance size between students. There are also 6 out of 33 students who could define the complete range of mathematical solutions. They stated the reasons for limiting the distance required from each athlete. So, in this case, students were able to use the ability of mathematization.

Also, there are 14 out of 33 who could communicate their explanations and arguments in the context of the problem completely. In this case, students were able to use communication ability. There were also 15 out of 33 who could use a representational ability. It appeared that students could interpret mathematical results in the form of representation completely. Based on students' answers, the potential basic mathematical ability emerging in this problem includes communication ability, as seen from the ability to read and to interpret questions from the images given. It also showed that students were capable of reasoning and argumentation, mathematical ability, and representational ability. It can be seen from students' ability to represent real-world situation problems into math problems. Hapizah (2014) states the ability of reasoning is the ability to direct the mind to produce a statement in reaching conclusions when solving a problem.

In general, the achievement of students in the field test stage in working on the problem in Prototype 3 had shown a potential effect. The problem has the potential to bring up various basic mathematical skills in the process of completion. Based on the results of the interview, the students stated that the test had provided a new experience for them since the questions given were interesting because they use various sports contexts and varied according to their level. As a result, they could imagine more in answering the problem and make use of assumption and logic. The context was the main point for students in developing mathematics ability, and the context itself should be meaningful for them and real for their mind (Putri, *et al*. 2015; Prahmana & Suwasti, 2014).

Furthermore, Nasution, *et al*. (2018) also states the use of the rowing sport can be a bridge of students' thinking and help students in understanding the operation of addition and subtraction of fractions. Gunawan, *et al*. (2017) added with the support of context and learning media, students learning will be more enthusiastic. Moreover, many students revealed that solving the problem requires sufficient reasoning and problems solving ability. This argument is in line with Putri, *et al.* (2015) saying that learning mathematics through sports can make students like it because they will adapt faster since it concerns daily activities. The concept of learning will be effective and minimize the level of difficulty. As stated by Jurnaidi & Zulkardi (2013) in their research which concluded that in the results of interviews with 5 students in the field test, that it was illustrated that in general the problems of mathematical reasoning PISA model can trigger to reason in solving the problem, even though some students still have problems in understanding and resolving it. This condition means that the PISA model with sports content is capable of exploring students' mathematical ability and give positive effects. It is concluded that the problem has potential effects for the students.

CONCLUSION

This research has produced PISA mathematical problem with Taekwondo context in Asian Games 2018. In the preliminary stage, the researchers developed about 9 item using sports context in Asian Games 2018. All the item, at the expert review stage and one-to-one, had some revision. In a small group, all the problems have fulfilled the needed characteristics. As a result, in prototype 3, it was produced nine items. This prototype generated is a valid and practical PISA type using the context of taekwondo. Student achievement in the field test stage in working on the problem of prototype 3 showed that it could develop students' mathematical literacy skills and explore students' potentials. Based on the analysis of the results on 33 students, it displayed potential effects on basic mathematical abilities, including the ability of reasoning and argument. It also exhibited that students can develop and solve problems with modeling using their assumptions. In addition, the ability to design problem-solving strategies emerged in using various procedures in solving problems that led to the conclusion.

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REFERENCES

Aminudin. (2012). Developing the mathematical problems in the PISA model on shape and space content to know the mathematical connection ability in junior high school students [in Bahasa]. *Unpublished Thesis*. Palembang: Sriwijaya University.

Barczi, K. (2008). A Study on how Hungarian Students solve problems that are unusual for them. In S. Turnau (ed.), *Handbook of Mathematics Teaching Improvement: Professional Practices that address PISA* (pp. 119-130). Krakow: University of Rzeszów.

Charmila, N., Zulkardi, & Darmawijoyo. (2016). Developing the mathematical problems in the PISA model using the Jambi context [in Bahasa]. *Jurnal Penelitian dan Evaluasi Pendidikan*, *20*(2), 198-207. http://dx.doi.org/10.21831/pep.v20i2.7444.

Edo, S.I., Putri, R.I.I., & Hartono, Y. (2013). Investigating secondary school students' difficulties in modeling problems PISA-Model Level 5 and 6. *Journal on Mathematics Education*, *4*(1), 41-58. https://doi.org/10.22342/jme.4.1.561.41-58.

Fajriyah, M., Putri, R.I.I., & Zulkardi. (2017). Dayung context in fraction. *Advances in Sosial Science, Education and Humanities Research* (Vol.100) Banjarmasin: Atlantis Press. https://dx.doi.org/seadric-17.2017.1.

Ginting, M.S., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the reasoning ability of elementary school student through the indonesian realistic mathematics education. *Journal on Mathematics Education*, *9*(1), 41-54. https://doi.org/10.22342/jme.9.1.5049.41-54.

Gunawan, M. S., Putri, R. I. I., Zulkardi. (2017). Learning fractions through swimming context for elementary school students. *Advances in Social Science, Education and Humanities Research* (vol. 100). Banjarmasin: Atlantis Press. https://dx.doi.org/seadric-17.2017.14.

Hapizah. (2014). Developing the instruments for students' mathematical reasoning abilities in the course of differential equations [in Bahasa]. *Kreano, Jurnal Matematika Kreatif-Inovatif, 5*(1), 73-81. https://doi.org/10.15294/kreano.v5i1.3280.

Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial visualization. *Journal of Physics: Conference Series, 943*(1), 012029. http://doi.org/10.1088/1742-6596/943/1/012029.

Jannah, R.D., Putri, R.I.I & Zulkardi. (2019). Soft tennis and vollyball context in ASIAN GAMES for PISA-like mathematics problems. *Journal on Mathematics Education*, *10*(1), 157-170. https://doi.org/10.22342/jme.10.1.5248.157-170.

Jupri, A., & Drijvers, P.H.M. (2016). Student difficulties in mathematizing word problems in algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(9), 2481-2502. https://doi.org/10.12973/eurasia.2016.1299a.

Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683-710. https://doi.org/10.1007/s13394-013-0097-0.

Jurnaidi, J., & Zulkardi. (2014). Developing the PISA model problems on the content of change and relationship to determine the mathematical reasoning abilities of junior high school students [in Bahasa]. *Jurnal Pendidikan Matematika*, *8*(1), 25-42. https://doi.org/10.22342/jpm.8.1.1860.25-42.

Kaune, C., Nowinska, E., Paetau, A., & Griep, M. (2013). Games for enhancing sustainability of year 7 maths classes in Indonesia: Theory-driven development, testing and analyses of lessons, and of students' outcomes. *Journal on Mathematics Education*, *4*(2), 129-150. https://doi.org/10.22342/jme.4.2.412.129-150.

Khairuddin. (2017). PISA, try out the PISA problems and the students' strategies to answer the questions [in Bahasa]. *Proceedings of Seminar Nasional Matematika SEMNASTIKA 2017* (vol. 9). Medan: Universitas Negeri Medan.

Lutfianto, M., Zulkardi & Hartono, Y. (2013). Unfinished student answer in PISA mathematics contextual problem. *Journal on Mathematics Education*, *4*(2), 188-193. https://doi.org/10.22342/jme.4.2.552.188-193.

Maryati, & Prahmana, R.C.I. (2019). Ethnomathematics: Exploring the activities of culture festival. *Journal of Physics: Conference Series*, *1188*(1), 012024. https://doi.org/10.1088/1742-6596/1188/1/012024.

MOEC. (2014). *The Regulation of the Minister of Education and Culture of the Republic of Indonesia number 69 year 2013 about the Basic Framework and Structure Curriculum of the Senior High School [in Bahasa]*. Jakarta: Indonesian Ministry of Education and Culture.

Nasution, M.F., Putri, R.I.I., & Zulkardi. (2018). Rowing sport in learning fractions of the fourth grade students. *Journal on Mathematics Education*, *9*(1), 69-80. https://doi.org/10.22342/jme.9.1.4270.69-80.

OECD. (2016). *PISA 2015 Assessment and Analytical Framework: Science, Reading, Mathematic and Financial Literacy*. Paris: OECD Publishing. http://dx.doi.org/10.1787/9789264255425-en.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The Case of Rural Area's Student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Prahmana, R.C.I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education, 3*(2), 115-132. https://doi.org/10.22342/jme.3.2.1931.115-132.

Pratiwi, I., Putri, R.I.I & Zulkardi. (2019). Long jump in ASIAN GAMES: Context of PISA-like mathematics problems. *Journal on Mathematics Education*, *10*(1), 81-92. https://doi.org/10.22342/jme.10.1.5250.81-92.

Purnomo, S., Dafik, & Kusno. (2015). Analysis of student responses to PISA problems with the content of space and shape using the Rasch model [in Bahasa]. *Prosiding Seminar Nasional UNEJ*. Jember: Universitas Jember.

Putri, R.I.I. (2012). Developing learning trajectory using traditional games in supporting students learning greatest common divisor in Indonesian primary school. *Proceedings of the 12th ICME* (vol. 1, p. 7721). Seoul: COEX.

Putri, R.I.I. (2013). Developing the PISA type questions for junior high school students and their implementation in the 2011 Mathematics Literacy Contest [in Bahasa]. *Prosiding Seminar Nasional Matematika dan Terapan*. Banda Aceh: Indonesian Mathematical Society of Aceh and North Sumatera.

Putri, R.I.I., Dolk, M., & Zulkardi. (2015). Professional development of PMRI teachers for introducing social norms. *Journal on Mathematics Education*, *6*(1), 11-19. https://doi.org/10.22342/jme.6.1.1900.11-19.

Rahayu, C., Putri. R.I.I., & Zulkardi. (2017). Multiplication of fraction with natural number by using hurdles. *Advances in Social Science, Education and Humanities Research*, (vol. 100). Banjarmasin: Atlantis Press. https://dx.doi.org/seadric-17.2017.10.

Risdiyanti, I., & Prahmana, R.C.I. (2018). Ethnomathematics: Exploration in Javanese culture. *Journal of Physics: Conference Series*, *943*(1), 012032. https://doi.org/10.1088/1742-6596/943/1/012032.

Roni, A., Zulkardi, & Putri, R.I.I. (2017). Learning divisions of fractions through sprint running pictures. *Journal of Education and Learning*, *11*(4) 381-393. http://dx.doi.org/10.11591/edulearn.v11i4.5982.

Tessemer, M. (1993). *Planning and Conducting Formative Evaluations: Improving the Quality of Education and Training*. Philadelphia: Kogan Page.

Wardani, K. (2011). Developing the mathematical questions on the PISA model for seventh grade student in enrichment programs [in Bahasa]. *Unpublished Thesis*. Palembang: Sriwijaya University.

Yansen, D., & Putri, R.I.I., & Zulkardi. (2019). Mathematical problems of PISA-like with the 200m swimming context in Asian Games. *Journal of Physics: Conference Series*, *1088*(1), 012086. https://doi.org/10.1088/1742-6596/1088/1/012086.

Zulkardi, & Putri, R.I.I. (2006). Designing your own contextual math problems [in Bahasa]. *Proceedings of Konferensi Nasional Matematika XIII*. Semarang: Universitas Islam Sultan Agung.

Zulkardi. (2002). Developing a learning environment on realistic mathematics education for Indonesian student teachers. *Dissertation*. Enschede: University of Twente.

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TRANSFORMATIVE PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHERS

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Abstract

This paper was an attempt to redesign the current professional development training for Mathematics teachers in the Philippines. Mathematics teachers claimed that most training and seminars they attended ignored their local work context; was routinely and was hardly applicable to their classroom milieu. By utilizing the transformative professional development training, the teachers identified the classroom issues that had confronted them; restructured their useful pedagogical ideas and instructional plans and materials; implemented these in their classrooms; and shared their reflections on the new teaching experiences. Qualitative data were gathered from focus group discussions and key informant interviews. University researchers (3), secondary Mathematics teachers (28), and students (250) from four rural public schools in the Hamiguitan Range participated. The teachers and students revealed that their lack of self and environmental understanding were the prevalent issues that led to critical behavior in Mathematical cognition and learning. By applying the transformative education in the classroom, promising results like better teacher performance, improved students' interest, and maximized student participation were evident. This transformative professional development training adequately responded to the teachers' work needs and was recommended to other areas of learning.

Keywords: Pedagogical Ideas, Student Interest, Teacher Professional Development, Transformative Learning, Transformative Mathematics Education.

Abstrak

Artikel ini bertujuan untuk mendesain ulang pelatihan pengembangan profesional yang saat ini ada bagi guru matematika di Filipina. Para guru matematika mengklaim bahwa sebagian besar pelatihan dan seminar yang mereka hadiri mengabaikan konteks pekerjaan lokal mereka, sekadar menjadi kegiatan rutin, dan hampir tidak dapat diterapkan di lingkungan kelas mereka. Dengan memanfaatkan pelatihan pengembangan profesional transformatif, para guru mengidentifikasi masalah kelas yang telah mereka hadapi; merestrukturisasi gagasan pedagogis mereka yang bermanfaat, rencana, serta bahan pengajaran; menerapkannya di kelas; dan membagikan hasil refleksi mereka terhadap pengalaman mengajar yang baru. Data kualitatif dikumpulkan dari diskusi kelompok terpumpun dan wawancara informan kunci. Subjek yang terlibat dalam penelitian ini adalah tiga peneliti universitas, 28 guru matematika sekolah menengah, dan 250 siswa dari empat sekolah negeri di daerah Hamiguitan. Guru dan siswa mengungkapkan bahwa pemahaman terhadap diri dan lingkungan yang rendah menjadi penyebab utama rendahnya perilaku kognitif dalam pembelajaran matematika. Penerapan pendidikan transformatif di kelas menjanjikan hasil seperti kinerja guru yang lebih baik serta peningkatan minat siswa dan partisipasi siswa yang maksimal. Pelatihan pengembangan profesional transformatif ini menjadi jawaban yang memadai untuk kebutuhan pekerjaan guru dan direkomendasikan untuk bidang pembelajaran lainnya.

Kata kunci: Gagasan pedagogis, Minat siswa, Pembelajaran transformatif, Pendidikan matematika transformatif, Pengembangan profesional guru.

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The declining performance of the National Achievement Test (NAT) in mathematics in the Philippines (50.70% in 2005, 47.82% in 2006, and 46.37% in 2012) showed some inadequacy of mathematics education. This phenomenon was also supported with the results in the Trends in International Mathematics and Science Society 2003 (TIMMS) (Mullis, *et al.* 2004), wherein the

Philippines students scored below average in all areas of Mathematics Achievement Test and ranked fifth from the last out of 45 participating countries. This happened in spite of the country's series of large-scale reform efforts (Lomibao, 2016) on curriculum and instruction. These indeed suggest the need to improve mathematics education in the country.

One crucial aspect that needs improvement is the design and delivery of the Teacher Professional Development (Ganzer, 2000; Crowther, *et al.* 2002; Brookfield, 2005; Wenglinsky, 2001; Mizell, 2010; Tahir & Thien, 2013; Ekawati & Lin, 2014; Putri, *et al.* 2015). As teacher competencies are inadequate (Wenglinsky, 2001), there is also a need to restructure the current teacher professional development model in the Philippines and consider an alternative model that is potentially more inclusive and meaningful to teachers and eventually to students. Many teachers said that the training they attended ignored their local work context, and it tends to be merely a routine and hardly applicable to the teachers' classroom milieu. Attributed to such concerns is the traditional model of teacher professional development that is delivered ready-made to a mass of teachers (Putri, *et al*. 2015; Lowrie & Patahuddin, 2015). The model privileges only ‘expert knowledge' that teachers are required to receive during the training, while it ignores their own work needs and the opportunity to reflect and act on their values and beliefs critically. These reduce their role to be empowered, critical, and active recipient of instructions from seemingly authoritative speakers. So the teachers often find the training and workshops routine and irrelevant.

The teachers, together with the researchers as mentors, went through the transformative professional development actions and sought answers to the following objectives. First, to examine/identify the classroom issues that have usually confronted the Mathematics teachers as they examined their values and beliefs relevant to these issues. Second, to restructure pedagogical ideas, developed instructional plans and materials that they perceived would help address the issues. Third, to implement these plans in their classrooms. And fourth, to share reflections on their teaching experience before and after the training as manifested by their students.

METHOD

Research design

This unique transformative educational research utilized the qualitative inquiry by employing both the criticality and the interpretivism as paradigms (Taylor & Medina, 2013; Taylor, *et al*. 2012). This research is a qualitative inquiry because it gives qualitative answers to questions that this research hoped to obtain. The questions given to the teachers were triangulated by the students for verification purposes. For this reason, focus group discussions and key informant interviews were done.

Criticalism is a critical inquiry that was done by elevating the conscious awareness of teachers about established values and beliefs that underpin their seemingly natural teacher-centered classroom roles (Taylor, 2008). Practicing this process helps introduce critical theory (e.g., critical pedagogy, cultural inclusiveness, social justice). Similarly, it helps stimulate teachers' creative thinking about designing curricula and assessment that are more student-centered, inquiry-oriented, culturally sensitive, community-oriented, and socially responsible (Taylor & Medina, 2013). Also, the interpretive inquiry makes teachers become reflective practitioners in developing and understanding the life-worlds of their students (Palmer, 1998). It involves a broader focal point on the social, political, historical and economic forces influencing the methods of teaching and learning, curriculum rules and schooling structures in which teachers are engrossed (Taylor & Medina, 2013).

The combination of these methods gives new literary genres, modes of thinking, and quality standards and becomes a compelling means for transformative professional development (Taylor, *et al.* 2012). The goal of our narrative inquiry in this research project is to engage others on critical self-awareness and critical understanding for them to develop a sense of urgency to act towards mathematics education hence writing for pedagogical thoughtfulness (Creswell & Garrett, 2008; Van Manen, 1990).

Perspectives and issues that emerged about mathematics teaching and learning, school ethos, and the professional development approach were described in Peter Charles S. Taylor's five ways of knowing in transformative learning: cultural-self knowing, relational knowing, critical knowing, visionary and ethical knowing, and knowing in action. Transformative learning is a theory of adult learning (Baldwin, 1996; Boyd & Meyers, 1998) that utilizes disorienting dilemmas to challenge students' thinking. Students are then encouraged to use critical thinking and questioning to consider if their underlying assumptions and beliefs about the world are accurate.

We focused on how the transformative education can be connected with the training on pedagogical ideas that we had and other relevant learning from workshops we attended before the project. We also theorized that in developing the mathematics education to become more relevant to the students and the teachers, the application of transformative learning and instruction to pedagogical ideas, different classroom activities and other related experiences be considered.

Research participants

We had university researchers (3), secondary mathematics teachers (28), and a good number of students (250) from four rural public schools in the Hamiguitan Range participated in a context-driven, action-oriented, inquiry-based teacher training and classroom-level applications. We had selected the region because of the availability of the environment that transformative learning would want to utilize. Moreover, we hoped to contribute to the preservation of one of the UNESCO Heritage sites utilizing Mathematics learning as a tool. The teachers (16) in the implementation of the program through classroom observations and assessments had volunteered, and they all agreed in the process.

Data collection, analysis, and tools

To ensure the credibility of this research, we spent three – day training workshop and three to six months of immersion in the field (school) by observing classroom instructions, mentoring teachers, and interviewing students in the aspects of transformative education. By using narrative writing, we provided a rich and detailed account of what the participants did in the classroom. Similarly, the researchers prepared daily data logs to ensure that everything that transpired in the process was documented.

The whole process allowed the participants to learn deeply about their contexts by enabling them to write reflective journals on their past learning activities and their short life histories related to Mathematics learning as a proof of an educative and catalytic endeavor. We employed Key Informant Interviews (KII) by using both the vernacular and English and Focus Group Discussion (FGD) as means in acquiring additional information from the participants. We also utilized classroom observations. Similarly, student interviews were used to triangulate the teachers' responses to the questions given to them.

We also reflected on how their experiences resonate with our experiences as mathematics teachers. This allowed both the researchers and the participating teachers be engaged in understanding subjective realities (Willis, *et al.* 2006). Through following the experiences of our participating teachers, we came up with universal themes as to why mathematics education in the country did not intentionally educate our students towards lifelong learning (knowing in action).

RESULT AND DISCUSSION

Our conduct of the plenary (three days) via leveling the expectation as well as the articulation of aspirations and challenges (a meta-card group activity) led to identifying the teachers' visions/aspirations for their students, school, and for themselves through their stories. The plans of action of teachers were built logically from their own experiences. The participants were empowered to apply the concepts learned inside their classrooms, hoping that the delivery of transformative mathematics education could contribute to educational change.

The classroom issues

The mathematics teachers attested that the most prevalent issue in a mathematics classroom was the students' interest in learning. Teachers said that student interests were manifested in the classroom discussions where most students found difficulty in participating and sharing their ideas or opinions in mathematics. Teachers also connected students' interest with problems in understanding the relevance of mathematics in the real world, so when questions related to the lessons were asked, they found difficulty in answering. Moreover, students' interest arose when their students became lazy in making and submitting their assignments, and other paper works assigned to them. They all agreed that these classroom issues had caused poor learning retention.

The teachers also said that in the new K to 12 curriculum, the topics were difficult to explain and made students afraid even more of Mathematics. Teachers also made mention of their difficulty in establishing good relationships with their students. Finally, the teachers cited some school and environmental set-ups, the lack of facilities and pedagogical materials to help facilitate the discussion in mathematics more effectively and in return may accelerate student interest.

These manifestations of behaviors were verified when the researchers did random interviews with students. A good number of them said that in the classroom discussion, they experienced difficulty in sharing the concepts that they did not understand. In return, they couldn't interact with their teachers and classmates, felt bored, and developed negative feelings about the subject. They also said that when their teachers gave them assignments, they just copied their answers from their classmates. Moreover, they said that they just studied their lessons during examination times and forgot the lessons after.

The students said that their teachers were not making them realize the importance of mathematics in everyday living. They answered that their teachers discussed topics in Mathematics less appealing. However, they cited the role of the teacher in learning regarding attitude and behavior but noted that some mathematics teachers had problems with their personality. They also said that some teachers in mathematics had problems in imparting knowledge to the students. They also said and felt that some teachers couldn't create and develop an excellent critical discourse. The students also said that their teachers tried their best so hard in providing improvised materials and facilities and giving lively discussions. They said that their parents also showed some indifference towards the subject but most importantly, they narrated that poverty also made the subject less-interesting, costly, and difficult.

These classroom issues that had usually confronted the teachers and the students about learning mathematics was on student interest and learning retention – an indication of critical behaviors in mathematical cognition (Azmidar, *et al*. 2017; Khayati & Payan, 2014; Heinze, *et al*. 2005; Nyman, 2017). It had shown that the teachers failed to examine their values and beliefs critically and identifying the very nature of students' abilities attributed to low learning retention. Because of these, the students developed negative feelings on Mathematical cognition like indifference, lack of self-respect, neglect, and inattentiveness (Colomeischi & Colomeischi, 2015; Trezise & Reeve, 2018; Mata, *et al*. 2012). The results also highlighted some of the characteristics of teachers in Mathematics that were fully aware of their roles to their students inside and outside of the classrooms (Azmidar, *et al.* 2017; Khayati & Payan, 2014; Frenzel, *et al.* 2010; Lazarides & Ittel, 2013). Teacher's expectations, styles, and sufficient support may create a positive influence on learning mathematics (Cornelius-White, 2007; Prahmana & Suwasti, 2014). Also, the values and beliefs of the teachers and students to these issues were hoped to be addressed so that the teachers can improve their capabilities in imparting Mathematical knowledge (Brophy, 2000; Wentzel, 2002) as these were essential tools in the delivery of learning. Finally, the results revealed that effective communication was a factor in learning.

In transformative learning, there are two knowing that were essential in this discourse; the critical self-knowing and the relational knowing. The self-knowing or self-realization recognizes the culturally positioned selves, in particular. This is how these premises underpin the worldview – the shared values, beliefs, ideals, emotionality, and spirituality (Taylor, 2008). The researchers believed that the majority of the teachers had failed to identify the nature of their students. This was primarily because of the student to teacher ratio in the public schools and the number of teaching preparations. Relational knowing involves learning to connect empathically and compassionately with the true (non-selfish) selves, the local community, the culturally different other, and the natural world (Taylor, 2008). The classroom issues confirmed that one of the many factors of student interest in Mathematics includes the social set-up, the family, and the school environments (Khayati & Payan, 2014). Also, the classroom environment helped influence both the emotional and cognitive dimensions of students in Mathematics (Watt, *et al*. 2017; Huetti, 2016). The transformative professional development training is a tool that starts with the assessment of the issues that confront the teachers and students in the academic exercise so that it can directly address the issues and suggest a solution.

Addressing the issues

The training had established an avenue for the participants to develop instructional materials in the classroom. The objective of the exercise was for the Mathematics teachers to enhance these instructional materials without distorting the DepEd K-12 curriculum competencies. The criteria given on the generated instructional materials and other pedagogical inputs were accuracy, appropriateness, identification of learning competency, as well as the alignment of learning competency and the activity. Upon completion, the researchers found out that most of the teachers had difficulty in connecting their teaching materials with the realities inside and outside the classroom, as these are factors that affect students' interest. Thus, the facilitators taught the teachers the transformative processes in making instructional materials through accuracy in drawing and making diagrams, appropriateness in giving definitions about angles and other shapes, aligning the learning competency (ultimate versus enabling) with the lessons presented, and the classroom activities (teaching-learning activity) that connects Mathematics to real-life situations. Then, we introduced transformative teaching strategies.

Most teachers admitted that they also lack new teaching strategies and that their pedagogical ideas inside the classroom were not updated. They found difficulty in applying Mathematics in the environment. The reason why students failed to be interested in Mathematics was that they did not understand its relevance in society. The pedagogical ideas like the "5E's and Metaphors of Learning", "Realistic Mathematics," and "Journaling" were shared, explained and demonstrated by the facilitators so that the teachers can be guided on its utilization inside the classroom. The five E's of learning talks about engaging, exploring, explaining, elaborating, and evaluating. The metaphors of learning by Sfard (2008) focus intensely on learning by acquisition, then learning by participation. It also embraces the commognitive (communication and cognition) framework of learning. Realistic Mathematics education makes the subject applied to the environment while journaling has something to do with communicating the solutions of mathematical problems using journal writing.

Moreover, aspirations and success stories gave flavor to the session. We also shared the "algebra walk," the "recipe Mathematics," and the "Mathematics jingle," which supplied energy to the tiring day. This sharing also addressed and articulated the common issues and concerns in the classroom setting.

Most researchers agreed that standardizing the classroom experience via dominant instructional approach with a tremendous amount of innovation is needed in the conduct of teaching (Garcia-Santillan, *et al.* 2016; Foley & Reveles, 2014; Mumu, *et al*. 2018). Specifically, they said that connecting the pedagogical ideas with the classroom experience is needed to gain student interest in Mathematics and help reduce anxiety in teaching and learning (Garcia-Santillan, *et al.* 2016). As students listen to teachers' aspirations and inspirational stories, it helps improve the Mathematics achievements of the students (Khattab, 2015) and teachers' beliefs, judgments, and a positive outlook in life bring a livelier classroom experience (Biesta, 2015).

In the realms of transformative learning, there is visionary and ethical knowing. Ethical knowing is an approach about views on teaching as a form of service. It goes beyond the understanding of the self and the environment and considers all actions that are deliberate and involve a decision of correct and incorrect (Taylor, 2008). The transformative professional development training is also a tool that when the issues are already addressed, a particular solution is offered and must be implemented.

Classroom implementation of transformative learning

There was a consensus among the Mathematics teachers as to who will try out the process of transformative learning in the classroom. Sixteen teachers accepted the invitation, seven from Compostela Valley, and nine from Davao del Norte. These teachers were teaching Mathematics in Grades 7, 8, and 9 in which the pedagogical ideas were embedded during the fourth grading period. Careful planning was made by the teachers and the researchers to make sure that students would benefit from these exercises. The mathematics classes adopted the transformative teaching and learning pedagogies employing the newly developed Instructional Materials.

We discussed among the volunteer teachers the procedures of classroom implementation, and they accepted the challenging points. We made critiquing about the teaching methodology, the pedagogical inputs, the students' response and reactions to these new inputs. As expected, the volunteer teachers were engaged in several activities in the classrooms that made Mathematics learning even more enjoyable. There was a lively discussion involving the teachers and the students.

Volunteer teachers realized that the application of transformative learning in the pedagogical inputs to the students accelerated, even more, their interest in the subject. The transformative learning incorporated in the discussion was about the teaching of Mathematics integrating the real-life situations and other related experiences. Some teachers had to incorporate dramatics in discussing functions specifically in the aspects of relationships were the millennials are very acquainted with. The teacher had to convert the lecture on direct, inverse and negligible relationships by allowing the students' real – life situations to become part and parcel of the drama. By doing these, the students can involve emotions in the discussion of functions which will become a lifelong learning. Some teachers also had to utilize kite flying while discussing angles and trigonometry. The students, as happier as they were, measured the angles made by the thread and their fingers then discuss in the classroom the importance of angles to kite flying. The students' eagerness to comprehend the learning out the activity (specifically on slopes) was worth emulating. Some teachers also utilized games while discussing probability. Though the activities distorted time and class schedules, the teachers promised that mastery on the implementation will be studied and applied.

We had seen that the levels of students' interest were harnessed. It was not difficult for both the teachers and students to recall the processes of learning. Though the students' responses to the 'transformations of teachers' were varied. Some said that the classroom now was even enjoyable because of the diverse activities and different application to real-life situations presented by the teacher. The students' noticed that their participation was maximized, and they became empowered to perform the task given to them. Every time the teacher introduced a topic, they became interested in how this topic related to real-life situations. They enjoyed group dynamics and lively discussions. A good number of students said, "Before the discussion in Mathematics was very boring, the classroom was just an ordinary avenue for learning, the students were not so engaged in the activities but today, the learning was fun, and it was memorable and did not deviate from accuracy. The teaching was fun, the learning was fun, and it was just very fantastic." There were two weeks of classroom observations.

In each activity recorded, assessments were done. The students still had difficulty in the mastery of the concepts given but in terms of the narrative evaluation, they were able to explain the learning process without them knowing those. The researchers had to give credit to the metaphors of learning by Sfard (2008). This metaphor suggested that students must be able to first utilize the different senses in learning Mathematics. It is in the utilization of their senses that students developed understanding to certain concepts without their awareness. We had uncovered that several students cannot compute for the values of the unknowns in functions but was able to write the processes on how to solve those. In transformative learning, this is very critical especially on the practice of learning by acquisition. This practice involved those activities presented by the volunteer teachers, where the students tasted (cooking with Math), felt (dramatics in Math), touched (games in Math), smelled (cooking with Math), seen (kite flying) and heard (essay writing in Math) Mathematics. We had seen that when the students became familiar with the concepts presented, they undergo learning by participation. These metaphors of learning helped the students acquire knowledge through the development of skills and dexterities, as well as abilities. The transformative professional development training is also a tool that when solutions are offered, a reflection on the application is made.

Reflections on their teaching experience

Aside from assessing their capability to develop instructional materials and guides, the teacher participants were told to write a reflective journal about their experiences before and after the training. These were assured to become part of the assessment of the training, that is, the realization of the teachers to become critical on their journey of learning and be able to interpret the positive impact of this training to their students. Similarly, their reflective journals was for the Mathematics teachers to explain the relevance of applying the transformative teaching and learning Mathematics without distorting the competencies featured in the K-12 curriculum.

A good number of teachers wrote, "This was the only training, in which we are actively participating. In many seminars we attended, we merely listened to the speakers. We, the participants, were often instructed immediately to do certain activities. We did not like to participate in those activities. We did not like to speak, either. We never expected that this training turned out to be distinctive. Our voice and expectations were recognized and heard. We are grateful to our trainers for choosing our school and ourselves for the conduct the training." Some teachers also wrote, “If only that we were allowed to participate in this type of training and application before; we would not have encountered difficulty in teaching Mathematics. The students appreciated our efforts, and we became empowered. What happened was an eye opener for us, allowing us to grow more.”

Most teachers agreed that the application of transformative education to teaching and learning was one of the relevant lessons that they learned from those exercises. Moreover, they claimed that this training and seminar was within the radius of their local work contexts and was considered relevant in the classroom setting.

The researchers were amazed and fascinated by the responses of both the teachers and the students in the application of transformative learning. According to Sagor's (2005) research framework, the trainers are just "sage on the stage and guide on the side." This was what we wanted this training to give us – to empower teachers to become effective agents of change. We learned that once they were authorized, their creativity and resourcefulness became skills in carrying out their missions. The teachers requested to propagate this training and workshop in other schools, especially to those in the far-flung areas where human resources and facilities are limited. The short closing impressions about the sessions (informal group interview with the teacher-participants) made us aware of the critical points.

The changes that occurred in the implementation did not deviate from the existing K-12 teaching/learning guide of DepEd. The goal to increase the level of interest of students as well as to increase student learning retention was the banner of this paper. Conforming to the words of Mahatma Gandhi, ‘we must become the change we want to see in the world,' the trainers believed that empowering Mathematics teachers would lead to producing empowered students to engage in lifelong learning. This training had applied the teachers’ local work context in their classroom milieu. In the aspects of transformative education, these exercises knew in action. The transformative professional development training takes out all the positive effects of the solution to issues that affect the academic exercise as part and parcel of the learning process.

CONCLUSION

The most prevalent classroom issue that had usually confronted the Mathematics teachers as they examined their values and beliefs relevant to the issue was about low student interest. This level of student interest results to difficulty in understanding mathematics, participating in the classroom discussion, making and submitting assignments, teacher and student relationship, environmental set-up as well as sharing their ideas in the classroom. In the long run, the level of retention of students' knowledge of the subject is deteriorating. Self-knowing and environmental knowing are important aspects of the practice of teaching and learning.

The restructuring the pedagogical ideas developed instructional plans and materials that they perceived would help address the issues had to be aligned with transformative learning. This was needed because of the difficulty of the Mathematics teachers to connect their teaching materials with the reality inside the classroom. The teachers admitted that they lack teaching strategies and that their pedagogical ideas inside the classroom are not updated. Thus, Mathematics turned out to be less appealing to the students.

The implementation of transformative learning in Mathematics to their classrooms was very promising. Most volunteer teachers said that the application of transformative learning in the pedagogical inputs to the students accelerated, even more, their interest in the subject. The transformative learning incorporated in the discussion was about the teaching of Mathematics integrating the real-life situations and other related experiences. This helped in addressing the students' interest and retention as it helped harnessed what they wanted to learn and do. It made the classroom learning even more challenging and enjoyable because of the diverse activities presented by the teacher. Student participation is maximized, and they become empowered to perform the task given to them.

The sharing of reflections on their teaching experience before and after the training proved the importance of transformative learning. The Mathematics teachers realized the relevance of the five aspects of knowing in transformative learning. The teachers were empowered, became authorized, creative, and resourceful. They wanted to propagate this training and workshop in other schools, especially to those in the far-flung areas where human resources and facilities are limited. Their willingness to be adaptive to change is the key. The benefits of transformative teacher development are enriching because it captures the systematic growth not only of a teacher but the students as well. Most importantly, it was proven that the five aspects of transformative learning develop and make both the teachers and the students become critical agents.

REFERENCES

Azmidar, A., Darhim, D., & Dahlan, J.A. (2017). Enhancing students' interest through mathematics learning. *Journal of Physics: Conference Series*, *895*(1), 012072. https://dx.doi.org/10.1088/1742-6596/895/1/012072.

Baldwin, R.G. (1996). Faculty career stages and implications for professional development. In D. Finnegan, D. Webster, & Z. F. Gamson (Eds.), *Faculty and faculty issues in colleges and universities* (2nd ed.). Boston, MA: Pearson Custom Publishing.

Biesta, G. (2015). What is education for? On good education, teacher judgement, and educational professionalism. *European Journal of Education*, *50*(1), 75-87. https://dx.doi.org/10.1111/ejed.12109.

Boyd, R.D., & Meyers, J.G. (1988). Transformative education. *International Journal of Lifelong Education, 7*(4), 261–284. https://doi.org/10.1080/0260137880070403.

Brookfield, S. (2005). *Becoming a critically reflective teacher*. San Francisco: Jossey Bass.

Brophy, J. (2000). *Educational Practices Series 1*. Retrieved from: https://eric.ed.gov/?id=ED440066 .

Colomeischi, A.A., & Colomeischi, T. (2015). The students 'emotional life and their attitude toward mathematics learning. *Procedia-Social and Behavioral Sciences*, *180*, 744-750. https://doi.org/10.1016/j.sbspro.2015.02.192.

Cornelius-White, J. (2007). Learner-centered teacher-student relationships are effective: A meta-analysis. *Review of Educational Research*, *77*(1), 113-143. https://doi.org/10.3102/003465430298563.

Creswell, J.W., & Garrett, A.L. (2008). The "movement" of mixed methods research and the role of educators. *South African Journal of Education*, *28*(3), 321-333.

Crowther, F., Kaagen, S.S., Ferguson, M., & Hann, L. (2002). *Developing teacher leaders: How teacher leadership enhances school success*. Thousand Oaks: Corwin Press.

Ekawati, R., & Lin, F.L. (2014). Designing teacher professional development for mathematics teaching with variation theory. *Journal on Mathematics Education*, *5*(2), 127-137. https://doi.org/10.22342/jme.5.2.1497.127-137.

Foley, B.J., & Reveles, J.M. (2014). Pedagogy for the connected science classroom: Computer supported collaborative science and the next generation science standards. *Contemporary Issues in Technology and Teacher Education, 14*(4), 401-418.

Frenzel, A.C., Goetz, T., Pekrun, R., & Watt, H.M. (2010). Development of mathematics interest in adolescence: Influences of gender, family, and school context. *Journal of Research on Adolescence*, *20*(2), 507-537. https://doi.org/10.1111/j.1532-7795.2010.00645.x.

Ganzer, T. (2000). Ambitious visions of professional development for teachers. *National Association for Secondary School Principals, 84*(618), 6-12. https://doi.org/10.1177/019263650008461802.

García-Santillán, A., Escalera-Chávez, M.E., Moreno-García, E., Santana-Villegas, J.D.C. (2016). Factors that explain student anxiety toward mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(2), 361-372. https://doi.org/10.12973/eurasia.2016.1216a.

Heinze, A., Reiss, K., & Franziska, R. (2005). Mathematics achievement and interest in mathematics from a differential perspective. *Zentralblatt füur Didaktik der Mathematik*, *37*(3), 212-220. https://doi.org/10.1007/s11858-005-0011-7.

Huetti, K.J. (2016). The relationship between poverty and student achievement. *Culminating Projects in Teacher Development*, Paper 16. Minnesota: St. Cloud University.

Khattab, N. (2015). Students' aspirations, expectations and school achievement: what really matters?. *British Educational Research Journal*, *41*(5), 731-748. https://doi.org/10.1002/berj.3171.

Khayati, S., & Payan, A. (2014). Effective factors increasing the students' interest in mathematics in the opinion of mathematic teachers of Zahedan. *World Academy of Science, Engineering and*

Technology, International Journal of Social, Behavioral, Educational, Economic, Business and Industrial Engineering, 8(9), 3069-3077.

Lazarides, R., & Ittel, A. (2013). Mathematics interest and achievement: What role do perceived parent and teacher support play? A longitudinal analysis. *International Journal of Gender, Science and Technology, 5*(3), 207-231.

Lomibao, L.S. (2016). Enhancing mathematics teachers' quality through Lesson Study. *SpringerPlus, 5*(1), 1590. https://doi.org/10.1186/s40064-016-3215-0.

Lowrie, T., & Patahuddin, S.M. (2015). ELPSA as a lesson design framework. *Journal on Mathematics Education, 6*(2), 77-92. https://doi.org/10.22342/jme.6.2.2166.77-92.

Mata, M.D.L., Monteiro, V., & Peixoto, F. (2012). Attitudes towards mathematics: Effects of individual, motivational, and social support factors. *Child Development Research, 2012,* 876028. http://dx.doi.org/10.1155/2012/876028.

Mizell, H. (2010). *Why professional development matters*. Oxford: Learning Forward.

Mullis, I.V., Martin, M.O., Gonzalez, E.J., & Chrostowski, S.J. (2004). TIMSS 2003 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades. Chestnut Hill: TIMSS & PIRLS International Study Center, Boston College.

Mumu, J., Prahmana, R.C.I., & Tanujaya, B. (2018). Construction and reconstruction concept in mathematics instruction. *Journal of Physics: Conference Series, 943*(1), 012011. https://doi.org/10.1088/1742-6596/943/1/012011.

Nyman, R. (2017). Interest and engagement: Perspectives on mathematics in the classroom. *Dissertation*. Gothenburg: University of Gothenburg. Available at http://hdl.handle.net/2077/51917.

Palmer, P.L. (1998). *The courage to teach: Exploring the inner landscape of a teacher's life*. San Francisco: Jossey-Bass.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The case of rural area's student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Putri, R.I.I., Dolk, M., & Zulkardi. (2015). Professional Development of PMRI Teachers for Introducing Social Norms. *Journal on Mathematics Education, 6*(1), 11-19. https://doi.org/10.22342/jme.6.1.1900.11-19.

Sagor, R. (2005). *The action research guidebook: A four-step process for educators and school teams*. Thousand Oaks: Corwin Press.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. New York: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511499944.

Tahir, S., & Thien, L.M. (2013). *Southeast Asia regional standards for mathematics teachers (SEARS-MT): Setting the bar for the teachers*. Penang: SEAMEO RECSAM.

Taylor, P.C. (2008). Multi-paradigmatic research design spaces for cultural studies researchers embodying postcolonial theorizing. *Cultural Studies in Science Education, 4*(3), 881-889. http://dx.doi.org/10.1007/s11422-008-9140-y.

Taylor, P.C., & Medina, M.N.D. (2013). Educational research paradigms: From positivism to multiparadigmatic. *Journal for Meaning-Centered Education*, *1*(1). Available at http://hdl.handle.net/20.500.11937/40413.

Taylor, P.C., Taylor, E.L., & Luitel, B.C. (2012). Multi-paradigmatic transformative research as/for teacher education: An integral perspective. In *Second international handbook of science education* (pp. 373-387). Dordrecht: Springer.

Trezise, K., & Reeve, R.A. (2018). Patterns of anxiety in algebraic problem solving: A three-step latent variable analysis. *Learning and Individual Differences*, *66*, 78-91. https://doi.org/10.1016/B978-0-12-802218-4.00005-4.

Van Manen, M. (2016). *Researching lived experience: Human science for an action sensitive pedagogy*. Abingdon-on-Thames: Routledge.

Watt, H.M., Carmichael, C., & Callingham, R. (2017). Students' engagement profiles in mathematics according to learning environment dimensions: Developing an evidence base for best practice in mathematics education. *School Psychology International*, *38*(2), 166-183. https://doi.org/10.1177/0143034316688373.

Wenglinsky, H. (2001). *Teacher classroom practices and student performance: How schools can make a difference*. Princeton: Educational Testing Services, Statistics and Research Division.

Wentzel, K.R. (2002). Are effective teachers like good parents? Teaching styles and student adjustment in early adolescence. *Child Development*, *73*(1), 287-301. https://doi.org/10.1111/1467-8624.00406.

Willis, S.L., Tennstedt, S.L., Marsiske, M., Ball, K., Elias, J., Koepke, K. M., ... & Wright, E. (2006). Long-term effects of cognitive training on everyday functional outcomes in older adults. *JAMA*, *296*(23), 2805-2814. http://dx.doi.org/10.1001/jama.296.23.2805.

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DEVELOPING MATHEMATICS QUESTIONS OF PISA TYPE USING BANGKA CONTEXT

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Abstract

This study aims to develop a math problem type PISA to familiarize students using problems with PISA standard and produces a valid, practical, PISA context of Bangka (Tanjung Kalian Lighthouse) type and see the Basic Math Skills (BMS) seen from the context of Tanjung Kalian lighthouse. The research method used is design research with the type of research development or development studies. The result of the research is a valid PISA math problem at the expert review stage and one to one, while small group stages do the practicality. The ability that is found in the form of communication, representation, mathematical, reasoning, and argument, and formulates a strategy to solve the problem.

Keywords: PISA, Mathematical Literacy, Bangka Context, BMS

Abstrak

Penelitian ini bertujuan untuk mengembangkan soal matematika tipe PISA untuk membiasakan siswa menggunakan soal-soal berstandar PISA dan menghasilkan soal tipe PISA konteks Bangka (Mercusuar Tanjung Kalian) yang valid, praktis dan melihat kemampuan dasar matematika (KDM) yang terlihat dari konteks mercusuar tanjung kalian. Metode penelitian yang digunakan adalah *design research* dengan jenis penelitian pengembangan atau development studies. Hasil penelitian berupa soal matematika tipe PISA yang valid pada tahapan *expert review* dan *one to one*, sedangkan kepraktisan dilakakukan tahapan small group. Kemampuan yang ditemukan berupa komunikasi, representasi, matematisasi, penalaran dan argument, serta merumuskan strategi untuk memecahkan masalah.

Kata kunci: PISA,Literasi Matematika, Konteks Bangka, KDM

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Based on the results of PISA in 2012 and 2015, Indonesia is still in a position under dependent neighboring countries; this result is still not good in the eyes of the world. The ability of Indonesian students in solving problems that require the ability to review, giving reasons and communicating effectively, and solve and interpret problems in various situations is still lacking (Kamaliyah, *et al*. 2013). It is based on students 'weak mathematical literacy about Indonesian students' mathematical literacy skills. Mathematical literacy emphasizes the ability of students to analyze, reason, and communicate ideas effectively in fractions of mathematical problems they encounter (OECD, 2009).

Mathematical abilities require disposition knowledge to think and act systematically in applying certain mathematical principles to everyday problems and involve critical aspects and related to life (Wijaya, 2016; Permatasari, *et al*. 2018). Mathematical Literacy is a person's ability to formulate, implement, and interpret mathematics in various contexts of everyday life problems efficiently (Sari, 2015). So that learning in the classroom must be with a variety of situations related to everyday life.

Also, students are accustomed to obtaining formal mathematical knowledge in the classroom so that it causes students' weak ability to work on problem-based questions such as PISA questions (Novita, *et al*. 2012). Similar to what was said by Novita and Putra (2016) that students always get non-routine problems and formal knowledge in their classrooms, so that students lack mathematical literacy skills.

In connection with the OECD results and previous researchers, Indonesian students are relatively low in answering high-level questions, and the teaching process in schools only learn mathematics conventionally without using the context of daily life or cultural context. This result has become the foundation of researchers to improve the quality of education in Indonesia by creating and developing mathematical questions in the context of daily life or cultural contexts. By previous researchers, Indonesian students should change their habits and try learning-based PISA questions that include basic math skills. One way to help teachers implement PISA-based learning to familiarize students is to provide banks with the model PISA (Zulkardi & Kohar, 2018). It is by what is obtained from the score of PISA in 2012, Indonesia was scoring 371, while in 2015 Indonesia obtained a score of 375.

Indonesian students should be familiarized in the learning process using the problem of PISA to be able to improve students' literacy skills, so that will impact on the results of PISA in the next year. Learning uses the context of making students find meaningful relationships between abstract ideas and practical applications in real contexts (The Cornerstone of Tech Prep, 1999). Whereas, the use of local contexts can help students understand the phenomenon of mathematics from the perspective of their own life experiences (Charmila, *et al*. 2016). Furthermore, OECD (2013) stated that mathematical literacy is the ability of individuals to formulate, apply, and interpret mathematics in various contexts, not only about the context in everyday life. Making a question is not only reasonable that can be solved, but also the problem must be authentic that relates to the real world and is modeled mathematically (Zulkardi & Kohar, 2018). On the other hands, the use of context in learning is very important because context can present mathematical problems in abstract form to representations that are easily understood by students (Fajriyah, *et al*. 2017).

Researchers use the context in Indonesia to get students to familiarize themselves with the standard PISA problems and improve math scores at the OECD level. Several researchers already develop the PISA-like problems by using the context in Indonesia, such as Indonesia natural and cultural heritage (Oktiningrum, *et al*. 2016), rice fields context in Karawang (Aini, *et al*. 2019), and some games in the Asian Games in Indonesia (Nizar, *et al*. 2018; Permatasari, *et al*. 2018; Jannah, *et al*. 2019; Yansen, *et al*. 2019). Therefore, the researcher developed a PISA type math problem using a valid and practical Bangka context.

METHODS

This research method is design research with the type of research development or development studies. This development research aims to generate a PISA mathematical problem using a valid and practical Bangka context as well as to see how the potential effects of the problems developed on the mathematics ability of junior high school students. This math research development study consists of two stages of preliminary or formative evaluation. (Zulkardi, 2002). Stages of formative evaluation consist of self-evaluation, expert reviews, and one-to-one, and small group and field test (Tessmer, 1993).

At the preliminary stage, the researcher performs student analysis, curriculum analysis, context analysis of Bangka, and PISA analysis. Furthermore, the researcher designed the problem of lattice problem, problem rubric, and question card. The process of this stage is called an early prototype, then performed at the formative evaluation stage. The first stage of the formative evaluation is self-evaluation.

This stage is in the self-evaluation by the researchers, after the researchers asked colleagues to see the problem developed whether there is a mistake on reading, and then evaluated by the supervisor and testers at the first defense of this process is called prototype 1. The next step the researchers validate to experts about PISA is called the stage expert review; simultaneously, the researchers perform one to one stages to the students. After doing the process, the researchers revised the pass by the validator. The results of the expert review stage and one to one is to look into the validity of a prototype. This prototype is called prototype 2. The next stage, the researchers tested into small groups or small groups while viewed in this process to see the practicality of prototype 2. Results revision at this stage, then researchers get prototype 3. The last step is the field test using prototype three, where the field test stage, the researcher sees a potential effect on basic mathematical skills that arise.

The data collection techniques used in the form of documentation in the form of PISA design, walkthrough, test in the form of a third prototype tested to students who have been determined. The results of this field test are used to obtain data about the potential effects of PISA-type mathematics that are developed to the mathematical communication ability of junior high school students of class IX, interviews, and Questionnaire.

RESULTS AND DISCUSSION

This development uses design research with the type of development studies. The development of a PISA mathematical problem using the Bangka context, namely *Tanjung Kalian Mercusuar,* produces valid and practical questions and has a potential effect on students' basic math skills.

The test subjects include students of class IX SMP Srijaya State Palembang city. In the preliminary stage the researchers conducted an analysis of students, curriculum, and PISA problems that will be developed this is called the initial prototype, subsequently carried out formative evaluation in the form of self-evaluation, this stage uses the initial prototype for the re-check by colleagues, supervisors, and lecturers testers who did when the proposal seminar.

This research produces prototype three consists of 11 Mathematical Problem Type PISA using Bangka context. As a detail that of the 11 questions, there are 3 questions with the context of *Tanjung Kalian Mercusuar* (the lighthouse of Tanjung Kalian), such as 1 question with the context of the lake of kaolin, 1 question with the context of the three pyramid, 2 questions with the context of the population in Bangka, 2 questions with the context of the Bangka Gold bridge, and 2 problem with the context of Bangka Botanical Garden. This study produced 11 PISA type Mathematics questions from various contexts, and various levels used. Mathematical problems in context Bangka has valid and practical criteria and has a potential effect seen in students' mathematical literacy abilities.

Product validation was developed based on the comments of the validators. The validation process in terms of content by PISA, the construct adapts to student characteristics and the level of PISA questions, and the language used does not interpret clear and understandable double spelling and language spelling. The potential effects are generated in the analysis of student answers carried out in the stages of the field test, analysis of student answers aims to see seven basic mathematical abilities.

At the next stage, the product developed was evaluated so that it became prototype 1, then one-to-one and expert stages were carried out. One of the problems in the context by using the context of Tanjung Kalian Lighthouse can be seen in Figure 1.

Sumber : http://idantravel.com/wp-content/uploads/2016/11/mercusuar-tanjung-kelian.jpg

Mercusuar ini dibagun pada tahun 1862, mercusuar tanjung kalian di atas memiliki 117 anak tangga, setiap 10 anak tangga terdapat ruang dengan sebuah jendela untuk melihat keadaan di luar mercusuar, dan mercusuar tanjung kalian hingga saat ini masih beroprasi dengan tenaga lampu sebesar 1.000 watt dengan jangkauan 40 mil. (sumber : kompasiana.com)

Pertanyaan :

1. Berapakah ketinggian dari mercusuar tanjung kalian dengan informasi yang sudah diberikan?

Figure 1. The problem of Tanjung Kalian Lighthouse

Along with the stage of the expert review conducted one-to-one stage with students of class IX SMP Srijaya Negara, used as many as three students who have high ability, medium, and low and aged 15 years. Prototype 1 is given to the validator at the expert review stage. As for the validator at this stage is Prof. Ahmad Fauzan, and Prof. Hasratuddin. The validation process is done via email delivery. The results of the good validator from Prof. Ahmad Fauzan and Prof Hasratuddin that the questions in the context are good, but improve the level of question number 3.

The validators validate the problem, the rubric of the problem, the question card, the lattice grid about views of content, constructs, and language. As stated by Zulkardi (2006), the validity of the problems in terms of content was by the domain of mathematics literacy in PISA, such as content, context, and mathematical process. The construct was by the characteristics of PISA problems level and capabilities of the tenth-grade students; language was the problem of the use of enhancing spelling, could be understood, and did not have a variety of meanings. And Then, the challenges include the difficulties experienced by the designers when developing PISA like tasks, namely creating more authentic, more accessible regarding the use of language structure, and more cognitively-demanding tasks (Zulkardi & Kohar, 2018).

Furthermore, small group stages are done using students from the same school of SMP Srijaya Negara. Students are in use as many as six students or 2 groups that each group has high-ability students, moderate, and low. In the small group process to see the practicality of a problem that has been developed. According to Zulkardi (2006), the practicality of the problem was illustrated from the result of the small group where the problems could be understood, easy to use, could be administrated, and interpreted well by the students. After this process, the researcher revises to be continued in the field test in grade IX students of SMP Srijaya Negara.

In the last stage or field test, the problems used in the form of prototype 3. Prototype 3 is tested to students of class IX D SMP Srijaya Negara with a total of 29 students. The problem is given as many as 11 problems done for 80 minutes. Students are asked to do all the problems and solve the PISA problem with the students' creativity strategy to answer the question. This activity is to see what basic mathematical skills emerge from student answers.

The question shown is in the form of a matter of context of Tanjung Kalian Tower with three questions. On the question, it looks easy, but the results of the answers of students working on various kinds of perceptions so that it is quite interesting to the question. When interviewing students about the question, they were happy because they got some answers to the various processes carried out, this was very good in terms of student development when finding similar questions.

On the matter of number 1, it discusses the height of the lighthouse. In completing the PISA problem creativity, broad intelligence is needed in interpreting a given problem. Hoerr (2000) stated that comprehensive intelligence consists of linguistic intelligence, logical-mathematical, musical, spatial, kinesthetic, interpersonal, intrapersonal, naturalist, and existential intelligence. In this problem, students can be creative in answering and using logical reasoning to what has been provided in the picture and information on the matter In this case, students can be creative in answering and using logical reasoning for what has been provided in the image and information about the issue that can be seen in Figure 2.

This context is chosen because of the lighthouse function. The content contained in this context is a comparable comparison that is learned in junior high school, so it is by the object of junior high school students. On this issue raises some basic mathematical skills other than students using reasoning and argument, the basic mathematical ability that arises is the mathematics in which it performs. As described by Anisah, *et al*. (2011), mathematical reasoning is a thought process that is carried out by drawing conclusions and reasoning abilities that can make students solve problems in life, inside and outside of school.

Sumber : http://ikdantravel.com/wp-content/uploads/2016/11/mercusuar-tanjung-kelian.jpg

Mercusuar ini dibagun pada tahun 1862, mercusuar tanjung kalian di atas memiliki 117 anak tangga, setiap 10 anak tangga terdapat ruang dengan sebuah jendela untuk melihat keadaan di luar mercusuar, dan mercusuar tanjung kalian hingga saat ini masih beroprasi dengan tenaga lampu sebesar 1.000 watt dengan jangkauan 40 mil. (sumber : kompasiana.com)

Pertanyaan :

1. Berapakah ketinggian dari mercusuar tanjung kalian dengan informasi yang sudah diberikan?

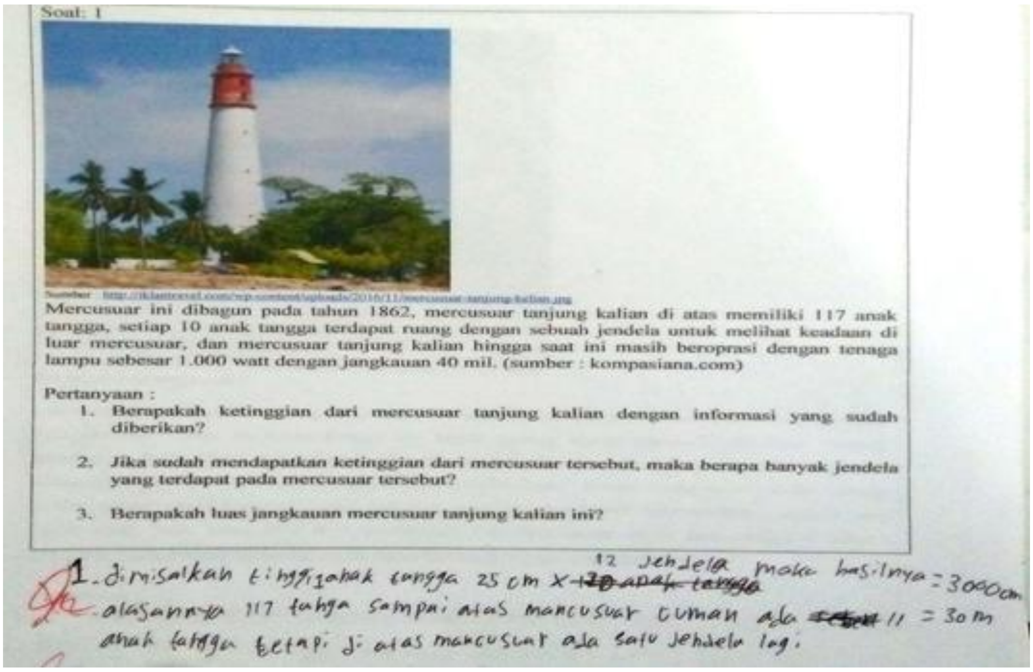
Figure 2. Problem number 1 context of Tanjung Kalian Lighthouse

OECD (2003) stated that mathematical PISA content is related to students' ability to analyze, reason, and communicate effective ideas because they describe, formulate, solve, and interpret mathematical questions in various situations problem. Here, there are two answers from different students that can be seen in Figure 3 and Figure 4.

In Figure 3, the researcher assumes that the student solves the problem with mathematical ability and the representation of what has been informed on the question. Besides this student uses his strategy in solving the problem with information on the problem or known in the matter (Prahmana & Suwasti, 2014). This student's answer has been predicted by the researchers the possibility of students using the strategy, only that there is a difference in determining the height of each ladder. This answer is not a problem because a ladder across different places has a difference. And the student understands that the height of each ladder is the same as that which the researcher justifies what the student is answering. Therefore the matter of the lighthouse of your cape can make students out the ability mathematization, communication, representation, reasoning and arguments and the ability of students in using strategies to solve the problem. The students with reasoning abilities can understand, formulate, and solve problems properly and correctly (Ahyan, *et al*. 2014; Saleh, *et al*. 2018).

The second answer sheet for this student is correct (Figure 4). On the results of this student answer using his strategy is different from other friends. The student solves the height of the lighthouse by comparing something in the picture. These students use coconut trunks to compare with the height of the lighthouse. This context is justified according to the researchers.

As Stacey (2014) pointed out that in PISA, all logical methods can be used in solving the given problem given full value. The result of this student's answer has a mathematical ability, a communication in which the student can explain again what he has answered, the reasons and arguments are seen from the student's answer using the reason that the height of the lighthouse can be calculated by comparing with how many coconut trunks, the PISA problem.


Soal: 1

Mercusuar ini dibagun pada tahun 1862, mercusuar tanjung kalian di atas memiliki 117 anak tangga, setiap 10 anak tangga terdapat ruang dengan sebuah jendela untuk melihat keadaan di luar mercusuar, dan mercusuar tanjung kalian hingga saat ini masih beroprasi dengan tenaga lampu sebesar 1.000 watt dengan jangkauan 40 mil. (sumber : kompasiana.com)

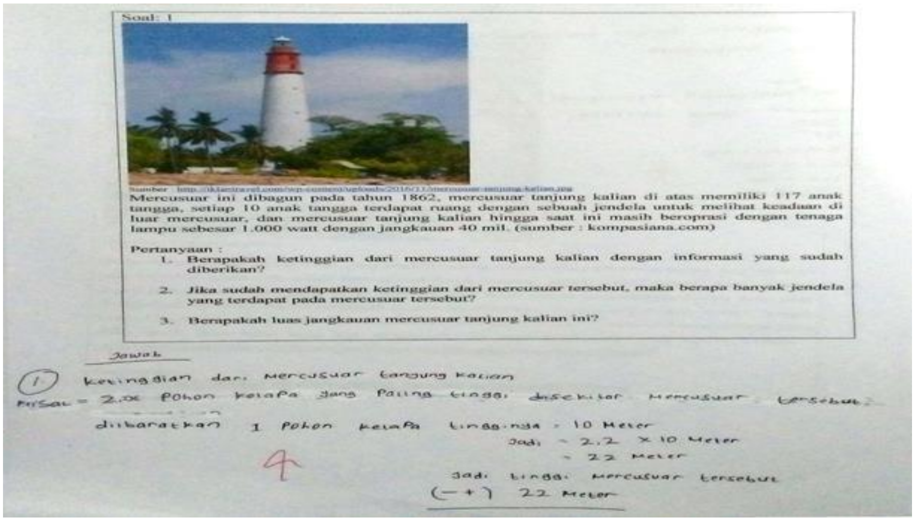
Pertanyaan :

1. Berapakah ketinggian dari mercusuar tanjung kalian dengan informasi yang sudah diberikan?
2. Jika sudah mendapatkan ketinggian dari mercusuar tersebut, maka berapa banyak jendela yang terdapat pada mercusuar tersebut?
3. Berapakah luas jangkauan mercusuar tanjung kalian ini?

1. dimisalkan tinggi 1 anak tangga 25 cm x 12 jendela maka hasilnya = 3000 cm alasannya 117 tangga sampai atas mancusuar cuman ada 11 = 30 m anak tangga tetapi di atas mancusuar ada satu jendela lagi.

Figure 3. Student answer number 1.

The difference between these two answers the researcher does not blame for what has been said to be valid and practical. But this question is already well proved by different student answers and can lead to basic mathematical skills.


Soal: 1

Mercusuar ini dibagun pada tahun 1862, mercusuar tanjung kalian di atas memiliki 117 anak tangga, setiap 10 anak tangga terdapat ruang dengan sebuah jendela untuk melihat keadaan di luar mercusuar, dan mercusuar tanjung kalian hingga saat ini masih beroprasi dengan tenaga lampu sebesar 1.000 watt dengan jangkauan 40 mil. (sumber : kompasiana.com)

Pertanyaan :

1. Berapakah ketinggian dari mercusuar tanjung kalian dengan informasi yang sudah diberikan?
2. Jika sudah mendapatkan ketinggian dari mercusuar tersebut, maka berapa banyak jendela yang terdapat pada mercusuar tersebut?
3. Berapakah luas jangkauan mercusuar tanjung kalian ini?

Jawab

(1) ketinggian dari mercusuar tanjung kalian
misal = 2.2x pohon kelapa yang paling tinggi disekitar mercusuar tersebut.
diibaratkan 1 pohon kelapa tingginya = 10 Meter
Jadi = 2.2 x 10 Meter
= 22 Meter
Jadi tinggi mercusuar tersebut (−+) 22 Meter

4

Figure 4. Answer sheet of a student to 2

In Figure 5, question number 3 students are required to imagine the function of the lighthouse. Students must always associate their mathematical knowledge with practical situations or problems encountered in everyday life (Hayat dan Yusuf, 2010; Hendroanto, et al. 2018; Risdiyanti & Prahmana, 2018). So students can see broadly about the context. When students can know the function of the lighthouse and what is formed if the lighthouse is operating.

Sumber : http://idiantravel.com/wp-content/uploads/2016/11/mercusuar-tanjung-kelian.jpg

Mercusuar ini dibagun pada tahun 1862, mercusuar tanjung kalian di atas memiliki 117 anak tangga, setiap 10 anak tangga terdapat ruang dengan sebuah jendela untuk melihat keadaan di luar mercusuar, dan mercusuar tanjung kalian hingga saat ini masih beroprasi dengan tenaga lampu sebesar 1.000 watt dengan jangkauan 40 mil. (sumber : kompasiana.com)

Pertanyaan :

3. Berapakah luas jangkauan mercusuar tanjung kalian ini?

Figure 5. About PISA Lighthouse Tanjung Kalian Bangka

Students are required to use their strategy how to solve question number three, students are required to understand the function of the lighthouse, after which the students implement the function of the lighthouse into a question on the question, so students will be able to solve the problem. Here the researchers present one of the answers found when the field test stage (Figure 6).

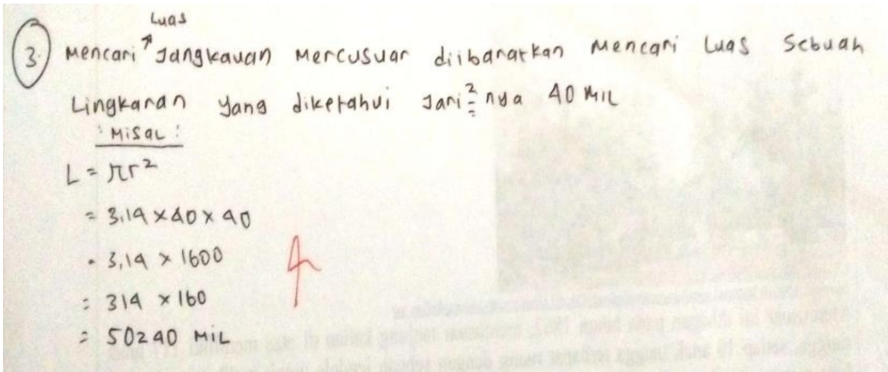


Figure 6. Answer the student

Figure 6 shows one of the students' answers found in the field test stage. In the above answers, the researcher assumes that the students do an understanding of the circle-shaped lighthouse function it resembles the reasoning capability and meaningful argument that students reason logically to connect problems so that students can find solutions to problems. Furthermore, students perform mathematical calculations obtained on the ability of reasoning and visible student representation of the images given in the above problems. The students 'mathematical ability is seen in the students' answers which assume the widest range of the lighthouse, as well as modeling the mathematics in the context of the context problem, and in the student's answer seen the communication where the students solve the problem and present the answers according to the justification that the students do. So, in this problem can bring up some basic math skills in the form of reasoning ability and argument, mathematization, representation, and communication.

In general, the results of the learning process show students can solve the context problem involving seven basic mathematical abilities or mathematical literacy abilities. However, there are still students who have difficulty in answering these questions, because they are still not accustomed to students working on problems that are contextual and creative thinking in which if ordinary students are only given formal lessons such as those in textbooks. The results of this research process researchers find pleasure in students by conceptualizing students' answers to student creativity without any limitations, and students can think broadly when listening to answers that are different from other friends.

CONCLUSION

This study resulted in 11 mathematical questions of type PISA using a Bangka context that was valid and practical and had potential effects. Validity is carried out at the stage of expert review and one-to-one, in this case, in terms of content, constructs, and discussion. While in practicality carried out at the small group stage, the interpretation of practicality is administratively when used in learning and said by the students. The results of the analysis of the Tanjung Lighthouse Tower unit show that 15 of the 29 students were able to engage in communication skills, mathematical abilities, and reasoning abilities. While ten students were able to involve mathematical abilities, only four students had difficulty in answering the question. Based on interviews with 15 students regarding the context of your cape lighthouse tower that this context can improve students' abilities in reasoning, mathematical and communication. while for 4 students who had difficulty just saying they did not understand the questions and images given, this the researcher assessed that these students were not familiar with questions that needed creativity and reason.

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REFERENCES

Ahyan, S., Zulkardi., & Darmawijoyo. (2014). Developing mathematics problems based on PISA level. *Journal on Mathematics Education, 5*(1), 47-56. https://doi.org/10.22342/jme.5.1.1448.47-56.

Aini, I.N., Zulkardi, Putri, R.I.I., & Yaniawati, P. (2019). PISA-like mathematics problems using rice fields context in Karawang. *Journal of Physics: Conference Series, 1188*(1), 012073. https://doi.org/10.1088/1742-6596/1188/1/012073.

Anisah, Zulkardi, & Darmawijoyo. (2011). Developing the mathematical problem in the PISA model on content quantity to measure the mathematical reasoning ability of junior high school students [in Bahasa]. *Jurnal Pendidikan Matematika, 5*(1), 1-15. https://doi.org/10.22342/jpm.5.1.333.

Charmila, N., Zulkardi., Darmowijoyo. (2016). Developing the mathematical problems in the PISA model using the Jambi context [in Bahasa]. *Jurnal Penelitian dan Evaluasi Pendidikan. 20*(2), 198-207. https://doi.org/10.21831/pep.v20i2.7444

Fajriyah, M., Putri, R.I.I., & Zulkardi (2017). Dayung Context in Fraction. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017* (pp. 1-6). Banjarmasin: Universitas Lambung Mangkurat. https://doi.org/seadric-17.2017.1

Hayat, B., & Yusuf, S. (2010). *The Quality of Education [in Bahasa]*. Jakarta: Bumi Aksara.

Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial visualization. *Journal of Physics: Conference Series, 943*(1), 012029. http://doi.org/10.1088/1742-6596/943/1/012029.

Hoerr, T.R. (2000). *Becoming A Multiple Intelligences School*. Virginia: ASCD.

Jannah, R.D., Putri, R.I.I., & Zulkardi. (2019). Soft tennis and volleyball contexts in Asian games for PISA-like mathematics problems. *Journal on Mathematics Education, 10*(1), 157-170. https://doi.org/10.22342/jme.10.1.5248.157-170.

Kamaliyah, Zulkardi, & Darmawijoyo. (2013). Developing the sixth level of PISA-like mathematics problems for secondary school students. *Journal on Mathematics Education, 4*(1), 9-28. https://doi.org/10.22342/jme.4.1.559.9-28.

Nizar, H., Putri, R.I.I., & Zulkardi. (2018). Developing PISA-like mathematics problem using the 2018 Asian games football and table tennis context. *Journal on Mathematics Education*, *9*(2), 183-194. https://doi.org/10.22342/jme.9.2.5246.183-194.

Novita, R., & Putra., M. (2016). Using task like PISA's problem to support student's creativity in mathematics. *Journal on Mathematics Education, 7*(1), 31-42. https://doi.org/10.22342/jme.7.1.2815.31-42.

Novita, R., Zulkardi., Hartono, Y. (2012). Exploring primary student's problem solving ability. *Journal on Mathematics Education, 3*(2), 133-150. https://doi.org/10.22342/jme.3.2.571.133-150.

OECD. (2003). *The PISA 2003 Assessment Framework: Mathematics, Reading, Science, and Skills*. Paris: OECD Publishing.

OECD. (2009). *Learning Mathematics for Life: A Perspective from PISA.* Paris: OECD Publishing.

OECD. (2013). *PISA 2012 Results: What Students Know and Can Do: Student Performance in Mathematics, Reading, and Science. (Volume I)*. Paris: PISA-OECD Publishing.

Oktiningrum, W., Zulkardi, & Hartono, Y. (2016). Developing Pisa-like mathematics task with indonesia natural and cultural heritage as context to assess students mathematical literacy. *Journal on Mathematics Education, 7*(1), 1-8. https://doi.org/10.22342/jme.7.1.2812.1-8.

Permatasari, R., & Putri, R.I.I., & Zulkardi. (2018). PISA-like: Football context in Asian games. *Journal on Mathematics Education, 9*(2), 271-280. https://doi.org/10.22342/jme.9.2.5251.271-280.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The case of rural area's student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Risdiyanti, I., & Prahmana, R.C.I. (2018). Ethnomathematics: Exploration in Javanese culture. *Journal of Physics: Conference Series, 943*(1), 012032. https://doi.org/10.1088/1742-6596/943/1/012032.

Saleh, M., Prahmana, R.C.I., Isa, M., & Murni. (2018). Improving the Reasoning Ability of Elementary School Student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education, 9*(1), 41-54. https://doi.org/10.22342/jme.9.1.5049.41-54.

Stacey, K. (2014). The PISA view of mathematical literacy in Indonesia. *Journal on Mathematics Education, 2*(2), 95-126. https://doi.org/10.22342/jme.2.2.746.95-126.

Tessmer, M. (1993). *Planning and Conducting Formative Evaluations: Improving the Quality of Education and Training*. London: Kogan Page.

The Cornerstone Tech Prep. (1999). *Teaching Mathematics Contextually.* Texas: CORD.

Wijaya, A. (2016). Students' information literacy: A perspective from mathematical literacy. *Journal on Mathematics Education, 7*(2), 73-82. https://doi.org/10.22342/jme.7.2.3532.73-82.

Yansen, D., Putri, R.I.I., Zulkardi, & Fatimah, S. (2019). Developing PISA-like mathematics problems on uncertainty and data using Asian games football context. *Journal on Mathematics Education, 10*(1), 37-46. https://doi.org/10.22342/jme.10.1.5249.37-46.

Zulkardi, & Kohar, A.W. (2018). Designing PISA-like mathematics tasks in Indonesia: Experiences and challanges. *Journal of Physics: Conference Series, 947*(1), 012015. https://doi.org/10.1088/1742-6596/947/1/012015.

Zulkardi, (2006). *Formative Evaluation: What, Why, When, and How*, (Online). Available at http://www.reocities.com/zulkardi/books"html.

Zulkardi. (2002). Developing a learning on realistic mathematics education for Indonesian students teachers. *Doctoral Dissertation*. Enschede: University of Twente.