INDONESIAN
MATHEMATICAL SOCIETY

JOURNAL ON MATHEMATICS EDUCATION

(IndoMS-JME)

Journal on **MATHEMATICS** Education	Vol.10	No. 1	Page 1-170	Palembang Januari, 2019	ISSN 2087-8885

Journal on Mathematics Education
Doctoral Program on Mathematics Education
Faculty of Teacher Training and Education, Universitas Sriwijaya
Kampus FKIP Bukit Besar
Jl. Srijaya Negara, Bukit Besar
Palembang - 30139
email: jme@unsri.ac.id

p-ISSN: 2087-8885 | e-ISSN: 2407-0610

Table of Contents

Articles

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. v-vi

CELEBRATION OF A DECADE OF JME

Zulkardi

Universitas Sriwijaya, Jalan Srijaya Negara, Bukit Besar, Palembang, Indonesia
Email: zulkardi@unsri.ac.id

Journal on Mathematics Education (JME), formerly called Indonesian Mathematical Society Journal on Mathematics Education (IndoMS-JME), is the first international journal on mathematics education in Indonesia. The primary goal of the journal is to support and facilitate mathematics educators and researchers either from Indonesia or other countries to publish their works in mathematics education.

In 2010, Professor Zulkardi, the vice president of the Indonesian Mathematical Society (IndoMS) as well as a mathematics educator from the Universitas Sriwijaya started the JME. Professor Widodo, the president of IndoMS at that time, launched the first volume of the JME at the opening of National Conference on Mathematics in Universitas Negeri Manado, June 30th, 2010. This editorial focuses not only on the celebration of the achievements and challenges of JME but also the process of the growth of the JME from a local journal, national, and then to be a reputable international journal.

First, as a local journal, JME started with only five articles in the first volume. Luckily, an international expert in mathematics education wrote and published their ideas. Lee Peng Yee, a professor at the National Institute of Education (NIE) Singapore, wrote an article about how to design a mathematics curriculum. Also, Professor Sembiring, from the Institut Teknologi Bandung, wrote about the history of Realistic Mathematics Education in Indonesia. Later on, many authors from various countries published their papers in the next volumes. Some of the well-known researchers in mathematics education have contributed in the JME, such as Professor Berinderjeet Kaur from Singapore; Kaye Stacey, Bardini, and Tom Lowrie from Australia; Christa Kaune and Elmar Cohort from Germany; Edyta Nowinska from Poland; Koeno Gravemeijer, Maarten Dolk, Dolly and Frans van Gallen from the Netherlands. The other authors are from Kingdom of Saudi Arabia, Brunei Darussalam, Indonesia, Turkey, Ghana, United States, and Taiwan.

Second, the JME is nationally accredited by the Ministry of Research, Technology, and Higher Education since September 2015. A year before, the Directory Open Access Journal (DOAJ) and ERIC databases started to index the articles from the JME.

Finally, after waiting for 20 months since the application submitted, the JME is accepted for the Scopus inclusion since September 2018, addressing the JME as one of the reputable international journals from Indonesia. After another three months, Elsevier agreed to cover all articles since volume 9.1 in the Scopus database.

In this celebration moment, as the editor in chief, I would like to say thank you very much to all local staffs, managing editors, reviewers, as well as editorial board members either from abroad or Indonesia who have done excellent work in making JME well known and internationally recognized. Starting from January 2019, JME publishes three editions on each volume.

Finally, in the future, JME has an effort to improve the quality of publication both management and the content of the articles. Also, JME has a task to fulfil the requirements to be indexed in the Web of Science. Of course, this is not an easy task, especially for JME team.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 1-20

INVESTIGATION OF MATHEMATICS TEACHERS' OPINIONS ABOUT PROBLEM POSING

Cengiz Erdik

Near East University, Yakındoğu Blv, Lefkoşa, Turkey
Email: erdikcengiz@gmail.com

Abstract

Problem posing in mathematics education is one of the most important skills. Since mathematics teachers are one of the most important parts of mathematics education and teaching, this research was conducted to evaluate their views on this important skill and the implementation process. The research was carried out by 56 mathematics teachers working at different schools with different seniority times. We evaluated the teachers' opinions by applying content analysis. The importance of problem posing skills in mathematics education has come from knowledge and practice that teachers have.

Keywords: Mathematics education, Problem posing, Teachers' opinion

Abstrak

Problem posing merupakan salah satu keterampilan yang penting dalam pendidikan matematika. Guru matematika juga dipandang sebagai bagian yang penting dalam pendidikan dan pengajaran matematika. Oleh karena itu, penelitian ini dilaksanakan untuk mengevaluasi pandangan para guru matematika terhadap keterampilan *problem posing* dan proses implementasinya. Subjek penelitian ini adalah 56 guru matematika yang mengajar di berbagai sekolah yang berbeda dengan tingkat senioritas yang berbeda pula. Pandangan dari para guru tersebut dievaluasi dengan menerapkan analisis konten. Pentingnya keterampilan *problem posing* dalam pendidikan matematika bersumber dari pengetahuan dan pengalaman yang dimiliki guru.

Kata kunci: Pandangan guru, Pendidikan matematika, *Problem posing*

How to Cite: Erdik, C. (2019). Investigation of Mathematics Teachers' Opinions about Problem Posing. *Journal on Mathematics Education, 10*(1), 1-20.

Problem posing is seen as an important component of mathematics programs and is at the center of mathematical activities (NCTM, 2000; Mose, Bjork, & Goldenberg, 1990; Silver, 1994). Some mathematics education researchers have stated that problem-solving provides important opportunities for students to improve their mathematics education (Nakano, Murakami, Hirashima, & Takeuchi, 2000). Problems teachers set up to provide opportunities for students and contribute to their understanding of mathematics (Knott, 2010). According to Gonzales (1998), the inclusion of problem posing activities in lectures depends on the guidance that teachers will make in establishing problems for their students. It is important to analyze the pedagogical knowledge of teacher or teacher candidates for problem posing from different dimensions whether the teachers' knowledge influence the process of teaching and students' success (Fennema & Franke, 2006; Kulm, 2008; Rizvi & Lawson, 2007).

It is possible to come across different definitions of problem posing in field writing (Kılıç, 2014). Posing a problem is a necessary process to fulfil a series of mental activities. Some of the definitions of problem posing in the literature are mathematical concepts (English, 1998; English, 2003; Pirie, 2002; Shuk-kwan, 1997) and the transitions between representations to contribute to the establishment of the relationship between processes and everyday life (Dickerson, 1999), as well as the center of mathematics education and mathematical thinking with problem-solving (Silver, 1997). By these, problem posing allows students to form a deep thinking structure. The problem has been proven by the research that has contributed to the thinking organization.

The quality and suitability of mathematical problems have been the subject of many studies in recent years. Many mathematics educators or researchers emphasize that problem-posing processes are important and useful in terms of providing students with knowledge (Cai & Hwang, 2002). We provide some studies conducted in terms of providing students with problem posing in Table 1.

Table 1. Examples from the findings of studies in providing students with problem posing

Problem posing contribution to students	Studies
To establish the relationship between mathematical concepts, operations, and everyday life context	(El Sayed, 2002; Dickerson, 1999; Rizvi, 2007)
Transitions between representations	(English, 1998; Ticha & Hošpesová, 2009; Işık, Kar, Işık, & Guler, 2012)
To your success and attitude	(Silver, 1994)
Critical thinking skills	(Nixon-Ponder, 1995; Nardone & Lee, 2010)
their creativity	(Cankoy & Darbaz, 2010; Silver, 1997)
Problem-solving	(Cankoy & Darbaz, 2010)
Interest and participation	(Brown & Walter, 1993; Nardone & Lee, 2010)
Mathematical reasoning	(Akay, 2006; Crespo & Sinclair, 2008)
To discover mathematical situations	(Akay, 2006; Yuan & Sriraman, 2011)
They can express	(Akay, 2006)
As an evaluation tool	(Lin, 2004; Lin & Leng, 2008; Silver & Cai, 2005)
The relationship between concepts and numbers	(Bonotto, 2006; Nakano *et al.*, 2000)
Reducing dependence on textbooks	(Toluk-Uçar, 2009)
Conceptual meaning	(Silver & Cai, 2005)

In the context of learning and teaching perspectives, problem-posing has an important place. From a learning perspective, students build problem-posing and creativity skills that students contribute to the development of reasoning skills, and strengthen and enrich basic concepts (Brown & Walter, 1983; Richards, 1990; Silver, 1994). From a teaching perspective, problem-posing gives teachers an idea of the skills, attitudes and conceptual learning of a given situation (Lavy & Shriki, 2007).

When we look at recent studies on problem posing, it is seen that studies about teachers (Toluk-Uçar, 2009; Isik *et al.*, 2012) and students (English, 1998; Cankoy & Darbaz, 2010; Barlow & Cates, 2006). Primary school mathematics teachers place the most problem posing activities in the fractional learning area within the numerical learning area. Fractions also found that all of the teachers involved in problem posing activities in the sub-learning field expressed opinions that problem-solving contributed to the conceptual understanding of students and the establishment of the relationship between symbolic expression and everyday life. According to Akay (2006), researches that examine students' cognitive processes in problem posing do not have definite results. At the same time, problem posing has been a subject that has been ignored by teachers and mathematics educators in response to the importance of teaching programs. There were only small number of studies focused on this subject. Therefore, we considered to examine this issue in Turkey versatile. The problem is that teachers are positively involved in thinking about mathematics teaching (Barlow & Cates, 2006), teachers are experiencing problems with problem posing exercises, and self-efficacy perceptions affect teachers' productive passivity. It is necessary to reveal self-efficacy perceptions related to Lin (2004) states that mathematics education should have a strong understanding of problem posing activities in the natural process and teachers should have a strong understanding of problem posing activities. Primary school mathematics teachers included in problem posing activities in the classroom, and that they contributed to the establishment of the relationship between a conceptual understanding of problem-solving and symbolic expression and everyday life.

This study aims to examine the opinions of mathematics teachers about problem building. Given this goal, we asked the teachers whether they were ready to use the problems they had settled or not, their difficulties in setting up the problem, and their recommendations. We considered that the research teachers' opinions are important thought to contribute to pedagogical field knowledge.

METHOD

Research Pattern

We used phenomenology from qualitative research design to see the teachers' opinions. Events, objects, and experiences have different meanings to different people (Johnson & Christensen, 2008). They see the world as a spectator of the participants, conceives the meaning of their experiences (Lodico, Spaulding, & Voegtle, 2010), and the concept of self (McMillan, 2008). In the study, we used the interview as the data collection technique.

Participants

We reached the mathematics teachers who worked in the urban schools. We did not take the rural schools into the survey. Because the working conditions of the rural schools are different, these schools should be examined separately. The characteristic of participation in the study is voluntary. We reached 56 mathematics teachers in the research. These teachers are 20 female and 36 male. We described the service and age status of the participants in Table 2.

Table 2. Information of the participant group

Age	Year of service				Total
	0-5	6-10	11-15	16 and over	
20-30	13	5	-	-	18
31-40	-	14	13	1	28
41-50	-	-	2	5	7
51 and over	-	-	-	3	3
Total	13	19	15	9	56

Data Collection Technique

We used the interview form developed by the researcher as a result of field survey as the data collection tool. While the interview form was developed, the opinions of two experts working in the field of mathematics teaching were taken and applied to 5 mathematics teachers. In line with the feedback received, the final form was given to the interview form. The ten open-ended questions about mathematics teachers' views on problem-solving, the way they were structured, the criteria they followed, the content of multiple solutions, stages, student level, multiple representations, mathematical and pedagogical situation of problems, and difficulties teachers encountered in problem writing.

Mathematics teachers' opinions about writing problems have been taken in writing. In this context, the interview form was applied by the researcher to the mathematics teachers working in public schools. The reason why the data should be collected in writing is that the participants should be more comfortable writing. Also, the face-to-face interviewer is the weakest point, and the presence of the interviewer can influence the responses of participants (Leeuw, 2008). According to Johnson & Christensen (2008), participation interviews can be conducted face-to-face in participatory research, and experience can be taken in writing from participants.

Validity and Reliability

Long-term interaction, deep-focus data collection, expert analysis and detailed descriptions were used to increase the validity of the study. The teacher's answers were first read by the investigator, and these responses were written down, on separate files of each question. The data were analyzed using Nvivo 8 and the obtained analysis were reviewed several times. During the examination, the raw materials were returned and checked. Besides, there is a direct citation in the finds section without any comment. A scorer association (Creswell, 2009; Miles & Huberman, 1994) was used to confirm the reliability of the study. For this purpose, the level of compliance between the interview forms, which were filled in by the teachers in writing, and the coding, which was coded by another expert experienced in qualitative data analysis, was determined (92%).

Data Analysis

In the analysis of the data, a content analysis method was used. One of the main features of the content analysis method is digitization (Bilgin, 2006). In this study, frequency analysis technique which is one of the content analysis techniques is used. Frequency analysis refers to the frequency with which the unit or objects are viewed numerically. This analysis makes sense of the density and importance of a particular pillar (Bilgin, 2006). Responses to interview questions were read before grouping (Patton, 1987). Responses given by classroom teachers are recorded in the word file for each question. In the analysis of the data, Nvivo's support was obtained. Data were analyzed with Nvivo. Then a table containing the themes, frequencies and the codes in the specified theme was prepared with Nvivo. The data analyzed with raw data were again compared, and inappropriate themes were removed, the themes merged or new themes were created. It is useful to work on the classification of data by multiple evaluators in content analysis (Patton, 1987). Raw data were sent to a specialist at a different university. As a result of the comparison of the opinions of the researcher and the expert, "Interview group", and "Interview group" between the experts were determined by marking. If the researcher and the expert have pointed out the different theme in the relevant question, the "opinion separation" has been accepted, taking the marking that the researcher has done as a reference. Reliability was calculated using Miles & Huberman's (1994) formula as stated in Equation (1).

$$Reliability = \frac{Vision\ Union}{(Vision\ Union + Vision\ Separation)} \quad (1)$$

As a result of applying the formula, a confidence percentage of 94% was obtained.

RESULTS AND DISCUSSION

In this section, the themes of the opinions of the mathematics teachers on the problem posing, the problem-posing criteria, the steps and difficulties are given.

Setting up problems yourself or using ready-made problems

Mathematics teachers in mathematics-geometry courses while describing the problems they have set out by themselves or using ready-made problems are given information. Twenty-five per cent of the teachers said they used ready-made problems, 19% said they had problems, and 26% said they were both ready and they had established themselves. When teacher opinions are examined, it is seen that there are six sub-themes in the ready to use problems category, five sub-themes in their problem posing category, and four sub-themes in both.

There were 35% of the teachers who use ready-made problems prefer to be informed due to lack of knowledge, 22% due to need, 13% due to the problem, 13% due to student level, 3% insufficient situation and 7 %. They stated that 27% of the teachers who set up the problems themselves were doing at the student level, 18% at the exam format, 18% at the question level, 18% at the original level and 18% at the appropriate level. Teachers who use both expressed that they make 50% of them according to the subject, 21% according to the situation, 21% according to the student level and 8% for the convenience of making meaning. The mapping is illustrated in Figure 1.

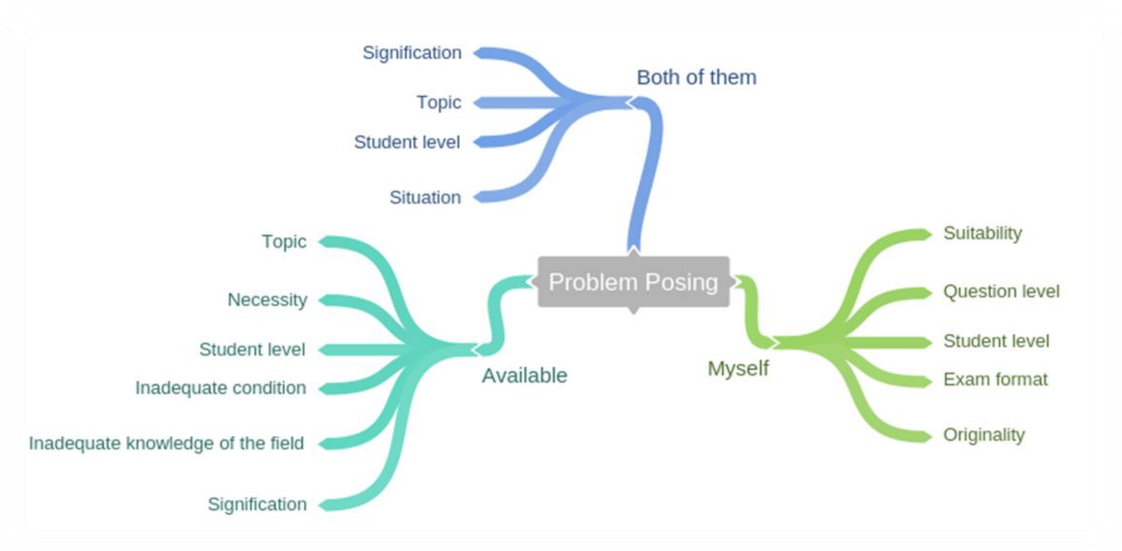


Figure 1. Sub-topics of teachers' views on the causes of problem formation

Some of the teachers' opinions participating in the survey on problem posing were recorded here.

T12 : I am preparing myself according to the success of the class (student-centered).
T36 : I am using prebuilt problems. Because I am not at a level where I can have problems. Setting up problems and generating questions is a serious business. Otherwise, students may lead to misconceptions in the concept.
T14 : I usually build myself. I'm simply adjusting the questions. Some narrative books I advise students.
T3 : I build myself according to the level of the class.
T21 : To make sure that it is understood during and after the narration of the subject, I build myself. I use ready-made problems at the end of the thread.
T48 : I am using ready-made problems. To avoid the problem of incorrect question preparation
T11 : I build myself. Because many ready-made problems are not heuristic-based.

Tracking down problem posing activities

The opinions of the teachers participating in the research on the path they followed in the problems they have established are the themes. They expressed opinions in the themes of using information, understanding, establishing relationships, question level, according to acquisition, daily life, experience, readiness, living by doing, student level, conformity, different representation, quality, attention, multiple solution, The most common ways of thinking are to include acquisition, student level, use of knowledge, daily life, different representation. The mapping is illustrated in Figure 1.

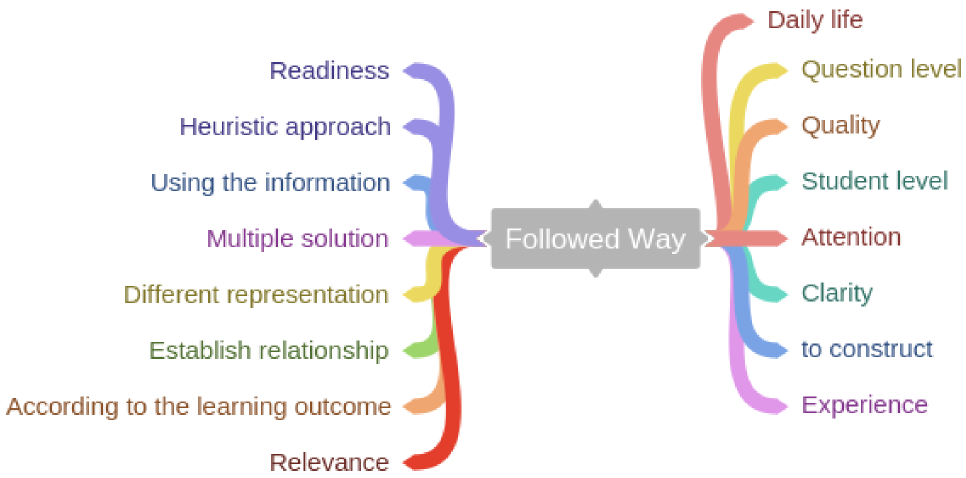


Figure 2. Themes related to the pathways of problem posing activities

Some of the teachers' opinions on the way of monitoring activities were recorded here..

T4 : If I have two problems, one is simple. On the other, I want additional information. For example, I am asking complex number information in the logarithm of the questions.

T32 : Understanding mathematical knowledge and creating a relationship between this knowledge. Creative and reflective the problems they will be able to use the skills of thinking analysis and synthesis.

T13 : The problem is firstly updated so that the student can understand that the acquisition is appropriate. I make sure that the daily life is appropriate.

T16 : We take care to prepare the questions understandably. According to the level of the students, I have questions ready.

T24 : By embodying, we can judge visually; I try to solve problems by taking advantage of it.

T9 : I am having problems with everyday life and with examples that students can take pictures of them.

T31 : According to the way it works, I am following a difficult path from my hand.

T16 : First, problems with the single acquisition (simpler level) and visual problems, here we go. At the level of analysis, after the problems are not understood, I want the children to set up their problems.

When Figure 2 and the opinions of some teachers were examined, they expressed the criteria that the mathematics teachers should be careful to understand the problems of the ways they see, to prepare the problems from the easy, current and daily life, students' preliminary knowledge and multiple solutions.

Criteria to establish the problem

Teachers who participated in the research stated that they consider some criticisms in the subject expressions and in the problems they have established at the end of the subject. Some teachers have specified more than one criterion. The themes created according to the views taken from the teachers are given in Figure 3.

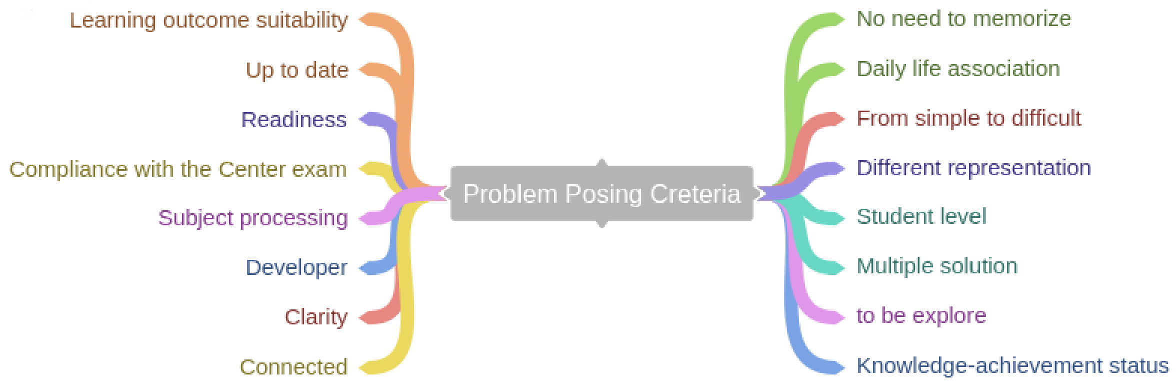


Figure 3. The criteria teachers use to solve problems

T5 : I do not solve problems that lead to learning based on memorization. I use canonical problems.
T15 : I take care to be up to date. I wish daily life is appropriate. Consider events that students may understand.
T23 : To be suitable for the level of the learner, to learn by exploring.
T19 : We take care to prepare the questions in a way that improves the students' minds from simple to complex.
T2 : I take into account the profitability. I pay attention to he questions that will enable them to establish relations between them are clear dictionaries and examples of daily life.
T33 : I pay attention to the fact that there are problems with the points that will direct the student to the goal. I'm careful not to be confused if it's clear and clear.

The most important aspect of the problem posing criteria

When the opinions of the teachers who participated in the research are examined regarding the most important in the problem building criteria, the sub-themes of the interviews are given in Figure 4 below. The importance of these criteria is mostly related to daily life, acquisition suitability, student level and the up to date problem.

Figure 4. Situations where teachers are most important in problem posing criteria.

The opinions of some teachers participating in the research on the issues that are most important in problem posing criteria were recorded here.

T4 : I am setting the problems with incarnation principle (straight from the arm). According to the bell, last questions it's getting harder, and it's asking for additional information.
T17 : Class level and university exam questions
T32 : Students will be able to collect, analyze and reason data. Different thought problems they can solve.
T23 : I would like to be the situation that most students can meet or meet in their life. I make sure that there are as many current situations as possible.
T38 : Student level because if the student is (chalanged) at the above level, then the student cannot do the task; then mathematics becomes difficult, and the students will be bored.
T26 : Whether it is open to problem development. In case of trouble, can be created. Regarding how to solve problems by turning into a game frame and see whether all students participate. The place of this problem in my life is my priority.
T48 : Students can understand without difficulty. I attach importance to the difficulty of describing mathematics. So we pay attention to the fact that it is understandable when setting up the problems.

İncluding multiple solutions in the problem-posing process

The opinions of the teachers who participated in the research on the thinking of having multiple solutions in problems in problem posing process were examined. Teachers stated that they think and apply more than 85%, 1% do not think and do not use it, 7% sometimes use it partially.

Considering the explanations of the teachers who think that they have multiple solutions in their problems, it was seen that they were collected in 10 main headings. Reasons for using the multiple solutions most frequently are as follows: student perspective, conceptual learning, ensuring the development of creative ideas, attracting attention, linking issues, increasing differences in individual differences, increasing interpreting skills, choosing the right solution and achieving success in central exams. The reasons for those who do not reflect the problems with which they have created multiple solutions are that they are confused by the students' minds and the difficulty of preparing the problems. The sub-themes of these views are given in Figure 5.

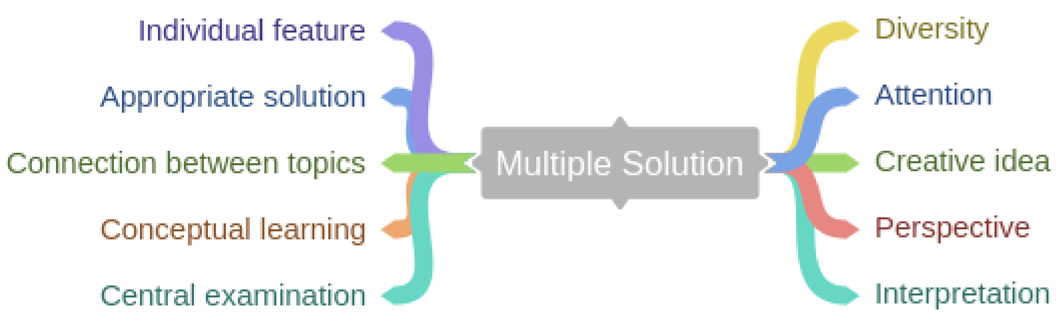


Figure 5. Sub-themes for multi-solution in problem-solving

Some of the views of teachers participating in the survey on whether they have multiple solutions in the problems they have established are given below.

T8 : In the problem-posing process, the inclusion of more than one solution, the student's ability to solve the question negatively. Already the student does not adopt the solution much by the method of establishing the equation. When he finds the solution correct, he applies this solution to more than one question, and if he gets the same question, he finds his solution and adopts it.
T19 : Not always. Students are confused.
T54 : Partially finding the solutions when establishing problems according to the topic.
T50 : Yes, the student should understand the problem and make the appropriate solution (the solution that the student knows one of the ways)
T24 : Problems bring different solutions when students are given opportunities; opportunity should be given. They can interpret it in the problem they have not encountered before.
T33 : I think that finding different solutions will create creative ideas in the problem-solving.
T45 : I specifically use more than one solution. I think you have improved the perspective of the learner.
T5 : I definitely think. What kind of provision you make in comparing the solution of the behave with consciousness. Discuss the different solutions of the students. I identify with examples how some solutions are used in other problems.
T22 : Yes, absolutely. Especially due to the way of thinking at the end of the university exam, the solution is sensible.

Consider the problem-solving stages in the problem posing process

Whether or not the teachers involved in the research take into account the problem-solving stages and their reasons for the reason are examined. It has been seen that teachers take into account the problem-solving stages. Also, when the explanations were evaluated, five sub-themes emerged. The most important reason for considering the problem phases is that students in the sense of the problem point out that the convenience of students is useful in establishing relationships, sorting between given and desired, improving interpreting skills and facilitating central examinations. The sub-themes of the teachers' views are given in Figure 6.

Figure 6. Sub-themes to consider opinions on problem-solving phases

The teachers's opinions on the problem-solving stages are recorded here.

T3 : Understanding and interpreting the problem. Establish the relationship among information.
T19 : We take it mandatory when it is directed to the examination.
T38 : The solution should be appropriate and understandable during the establishment of the problem. The data are given regarding information and logical order.
T9 : I buy it. I want a summary of what's given. I expect to establish a relationship between the given information and the desired information.
T53 : I buy it. A good reading comprehension and narrative can be expressed in their own words, the ability to express correctly in the words they say, the solution of the solution in the mind by rolling the solution, and the comparison of different solutions to compare different solutions to consider the possibility of preparing an appropriate activity and dancing problems.

The most difficult situation in establishing the problem

The opinions of the mathematics teachers participating in the research on the difficult situations or situations in the problem-posing process have been examined. From the information obtained, the existence of the situation, the situation in which it exists and the solution proposal for these situations have emerged. Two teachers said that they do not have any difficulty in problem posing process. Other teachers expressed their suggestions describing the difficulties they experienced. The challenges and solution proposals have been put together, and the views have been combined under the themes as follows; Student level (problem posing, teacher manual, repeat, current example, information refresh, level classes), class status (level determination), problem diversity- quality (question variety, number change, teacher manual, (use of materials, daily life and reality), concretization (drama), measurement and evaluation (measurement evaluation criteria), and feedback (individual education, peer review)), levels of experience-social experiences (taken into account) and abstraction (material use). The mapping is given in Figure 7.

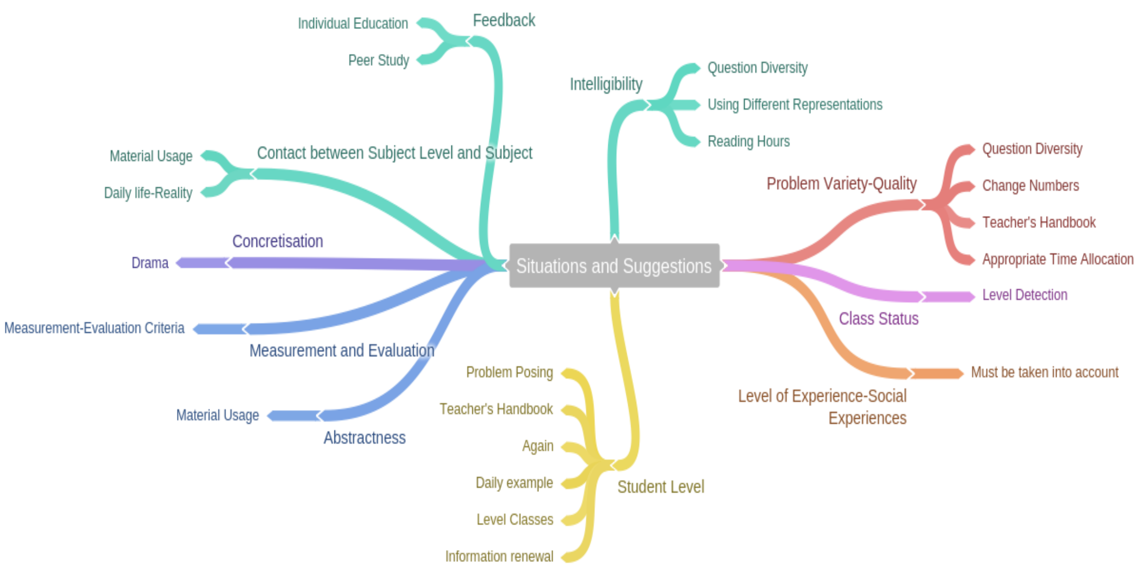


Figure 7. Challenges faced by teachers during the problem posing process and the proposed solution

Some of the opinions of mathematics teachers about the situations they have experienced in the problem posing process and the solutions they have expressed about these situations are recorded here.

T9 : Know the measurement evaluation criteria of the people who set the problem and know which question type is advantageous according to the situation. The purpose of some problems is not explicitly questioning the information, but problems that require special solutions.

T41 : Sometimes the problem we are having is not fully understood by the student. I try to make this situation more clear by using some explanation picture shape for this.

T13 : The level of confrontation is the most difficult situation for the level of the teacher. I'm trying to tell you this until you understand it.

T32 : I can face difficulties in determining the method of expression appropriate to the level of the learners. I use the method of generalization with simple examples in the case of Yada when the student falls into concept confusion in the process of evaluation and equations.

T50 : The shortcomings in the basic mathematical knowledge (students) make it difficult to understand the abstractness of mathematics, and using more material may perhaps make sense easier.

Preparing the problem according to the level of the student's level

Mathematics teachers who participated in the research investigated the appropriateness of the problems in the process of problem posing and the opinions about how they made this appropriateness. They all stated that the problem was taken into account when setting up the student. Then, it concluded that this conformity is based on what they have done and they have done as follows: behaviour, observation, readiness, curiosity-willingness, pre and post evaluation, comprehension, performance, solving ability, perceiving ability, understanding ability.

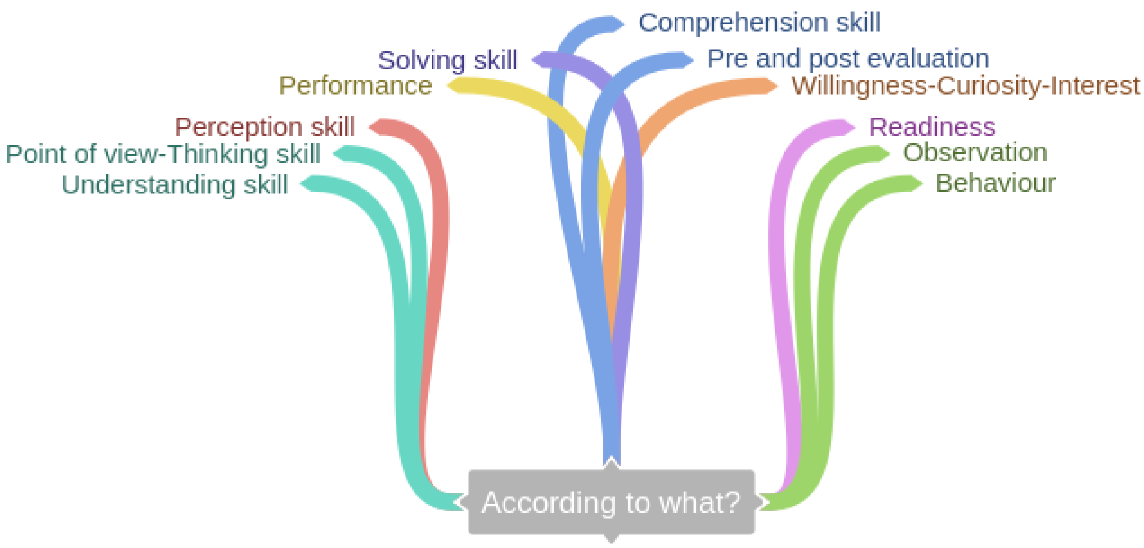


Figure 8. The theme of how teachers determine problems according to student level

The problems of the teachers in the process of forming the problem are recorded here.

T32 : I try to pay attention to the student level. In a non-homogeneous class, I choose appropriate problems according to each student's level. Try to determine according to students' understanding and solving capacities.

T19 : If you do not prepare the problem by observing the level of a group of students who do not have a basic mathematical thinking system like you are speaking English to people who do not speak English ... like taking oranges from your pharmacy. It should be appropriate for the student level.

T23 : It is preparing for students and class level. There are students I know over time in the classroom, probing students, different perspectives and sometimes with fewer questions.

T45 : I prepare according to the student's level. The student's level is determined by the questions themselves. Their levels come from simple to difficult, right or different ways of thinking and solving.

T54 : Yes, I am preparing for the level of the student. I also determine the level of the learners according to the positive or negative reaction given by the student.

T35 : I can observe the student's level with the leveling exam I applied at the beginning of the year and the topic examination at the end of each topic. Partly because it is difficult for me to deal with the problems.

T40 : In the process of problem posing, the majority of times I prepare the problem according to the level of the student. It is necessary to prepare the problem according to the present situation of the learner, perception. Otherwise, the class disappears (in terms of teaching).

T6 : I prepare at the student level. I make use of in-class screening exams and in-class performances that I did before the actual exams.

Expressing the problem with multiple representations

The opinions of mathematics teachers participating in the research on whether the problems established in problem posing process are paying attention to the expressiveness of multiple representations have been examined. It has been seen that 11% (3 teachers) of the teachers do not pay attention to multiple representations and 89% of them pay attention. When the views of those who care are examined, the explanation they make is the theme. The opinions of teachers about why they pay attention to multiple solutions are as follows: student level, concrete-thinking, understanding-understandability, information use-transfer-activation, interest-motivation, connection-association. The mapping is given in Figure 9.

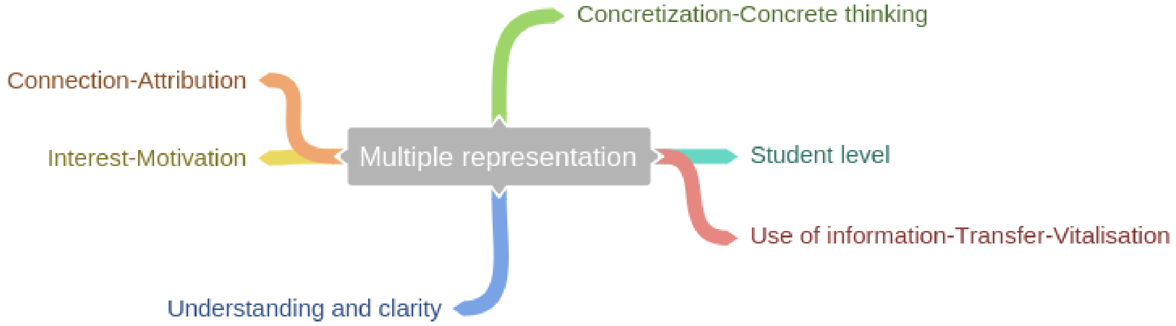


Figure 9. The ability of teachers to express the problem in multiple representations

Some of the opinions expressed by teachers about expressing problems with multiple representatives in the problem posing process are recorded here.

T2 : Yes, by writing intangible data with an abstract problem, we must ensure that the student has a good and complete understanding of the question.

T44 : Yes, I believe that the problem is very important for the rise of the student body.

T19 : Yes, I pay attention. I use multiple representations by linking to past issues.

T33 : In general, Osym uses this in 1-2 questions, but helping publishing houses do not give much to such questions. So I rarely use this type of question.

T24 : I pay attention. Too many variables can cause too many problems. The use of the least number of representations is important for the sake of legibility of the solution.

T39 : It depends on the topic. I support the visuals on topics that are available for table-graphics (drawing).

Do you think that the problems you set are mathematically and pedagogically satisfactory?

When the opinions of the mathematical teachers participating in the research are examined regarding their mathematical and pedagogical sufficiency, 70% of the teachers are not sufficient. Teachers believed to be competent have stated that mathematical competence is more than pedagogical competence. Some of the teachers' comments on the interview are recorded here.

T9	:	It may be mathematically sufficient, but I do not think it is enough for pedagogical students. Because he does not work in a student with a memorizing education system, he goes to the way of solving it as an equation instead of thinking and researching thinking.
T47	:	I see that it is difficult for the student to grasp because the knowledge of mathematics is an abstract science. I do not think it's enough.
T51	:	I think it's enough for some time. The student is helping me with difficulties in understanding ready-made problems. Of course, I should be trained to learn about this topic in my mind.
T23	:	I think that the problems in our framework are mathematical and pedagogical. But always good is better.
T35	:	Yes, I think for the questions that have no basic teachings. But not for questions involving very specific rules in mathematics.
T22	:	I think. I make every effort to produce mathematical love to produce new questions for the increase of the learning desire to improve the existence of agelessness from the difficulties of solving the problems and to raising those who are willing and willing to solve those produced.
T14	:	I do not think it's enough. So I take care to build a problem by taking advantage of a source.
T42	:	I think it is aimed mathematically at the desired goal. Pedagogically, I think that the high school curriculum can also be done by a medium-level student of intelligence. The problem is that the teacher does not update the teacher and the students do not evaluate the process well. I do not find a high note and correct chic finding pedagogically correct.

Discussion

It is considered that the problem formulation (NCTM, 2000; Silver, 1994), which is one of the most important skills in mathematics education with this research, will give more widespread and important to our country. Problem posing is as important as problem-solving. Nakano ver is an important opportunity for problem posing students to improve their mathematics education-related aspects, as it has been pointed out by others. Higher education institutions have lessons with problem-solving skills, but no special studies are conducted with problem posing. It is thought that teacher education will affect such incomplete mathematics teaching and learning. Also, the research aimed at evaluating the opinions of mathematics teachers by establishing problems and disseminating their results.

There were 55% of the mathematics teachers who participated in the survey stated that they had ready problems, 19% had problems themselves, and 26% were both ready, and they had established themselves. When the researches are examined, it is found out that many of the teachers have used ready-made problems and even mixed the concepts of problem and practice (Çömlekoğlu & Ersoy, 2002, Korkmaz & Gür, 2006, Özreçberoğlu & Çağanağa, 2018). When teachers' opinions are examined, they stated that using ready problems is due to insufficient knowledge of the field, due to necessity, because of the subject, because of the student's level, inadequate situations and meaningfulness. They said that the teachers who set up the problems themselves were doing because of student level, exam form, question level, originality and suitability. Teachers who use both have expressed that they are doing things to enjoy convenience in subject, situation, student level and meaning.

Among how teachers see their problems is to use information, to understand, to establish a relationship, question level, according to the acquisition, daily life, experience, readiness, living by doing, student level, conformity, the inclusion of different representations, approach. The most common viewpoints are state of achievement, student level, use of knowledge, daily life, different representation. When the opinions of some teachers were examined, they stated that mathematics teachers in problem posing activities took care of the problems related to the ways they watched, prepared the problems forcibly from the easiest, daily and daily life.

They stated that teachers take into account certain criteria in their narratives or in the problems that they have established at the end of the topic. Some teachers have specified more than one criterion. When the opinions about the most important cases in the problem building criteria are examined, the correlation with daily life, acquisition suitability, student level and the problem are up to date.

Teachers think that they have multiple solutions to problems in problem posing process. Considering the explanations of the teachers in their problem, considering the explanations, the reasons for using the multiple solutions most are the student's point of view, the conceptual learning, the development of creative ideas, attracting attention, establishing the connection between the topics, increasing the skill of interpreting, and centralized examinations. The reasons for those who do not reflect the problems they have created are expressed as the confusion of the students' heads and the difficulty of preparing the problems. It has been seen that teachers take into account the problem-solving stages. It is important to note that students should be able to use the convenience of their lives, establish relationships, sorting between given and desired, improving interpreting skills, and facilitating centralized examinations.

It turns out that mathematics teachers are challenged in problem posing. The challenges and the solution proposal are as follows; level (problem determination, teacher handbook, repeat, current example, information renewal, level classes), class situation (level determination), problem diversity-quality (question variety, number change, teacher manual, (use of materials, daily life and reality), concretization (drama), measurement and evaluation (measurement evaluation criteria), and feedback (individual education, peer review)), levels of experience-social experiences (taken into account) and abstraction (material use).

In the problem posing process of mathematics teachers, the student level of the problems is among the most important ones. In this context, the problem is that the students stated that they determined the students' level in Kurdish by taking into consideration the behaviours of the students, observing them, observing readiness, curiosity-willingness, pre and post evaluation, comprehension ability, performance, solving ability, perceiving skill, It has also been found that mathematics teachers take into account the expressiveness of problems created in the problem posing process by multiple representations. When the opinions of teachers about why they pay attention to multiple representations are examined, they pointed out that increasing the interest-motivations and linking-associating are beneficial also in descending to the multi-representative student level, in concrete-thinking, in understanding-understandability, in using and transferring information.

It turns out that the problems that mathematics teachers have established are not enough regarding mathematical and pedagogical aspects. Teachers believed to be competent have stated that mathematical competence is more than pedagogical competence.

CONCLUSION

This research has come up with the findings of researching how important problem-posing is regarding mathematics education. Mathematics teachers produce different problems from the problems in the books. Teachers should focus on problem posing, finding answers to established problems, and recording all thoughts about the problem. The transfer of the benefits to teachers regarding problem formulation and the students will be a great contribution to the teaching of mathematics. In this context, education faculties that develop mathematics education should include activities of mathematics teaching with problem solving-solving approach, and if necessary, a new and compulsory course should be added to the curriculum.

REFERENCES

Akay, H. (2006). *The examination of the effect of mathematics instruction with problem-posing approach on students' academic achievement, problem-solving ability and creativity*. Ankara: Gazi University.

Barlow, A. T., & Cates, J. M. (2006). The impact of problem posing on elementary teachers' beliefs about mathematics and mathematics teaching. *School Science and Mathematics, 106*(2), 64-73.

Bilgin, N. (2006). *Sosyal bilimlerde içerik analizi: Teknikler ve örnek çalışmalar (2 bs)*. Ankara: Siyasal Kitabevi.

Bonotto, C. (2006). Extending students' understanding of decimal numbers via realistic mathematical modeling and problem posing. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics* (pp. 193-200). Prague: Charles University.

Brown, S. I., & Walter, M. I. (1993). Problem posing in mathematics education. In S. I. Brown & M. I. Walter (Eds.), *Problem Posing: Reflection and Applications* (pp. 16–27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *The Journal of Mathematical Behavior, 21*(4), 401-421.

Cankoy, O., & Darbaz, S. (2010). Effect of a problem posing based problem-solving instruction on understanding problem. *Hacettepe University Journal of Education, 38,* 11-24.

Çömlekoğlu, G. & Ersoy, Y. (2002). Matematik problemi ve problem çözme-I: Bazı düşünceler ve öneriler. *Matematikçiler Bülteni (Özel Sayı),* 6-9.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education, 11*(5), 395-415.

Creswell, J. W. (2009). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (3rd ed.)*. Thousand Oaks, CA: Sage Publications.

Dickerson, V. M. (1999). *The Impact of Problem Posing Instruction on the Mathematical Problem Solving Achievement of Seventh Graders.* Atlanta: Emory University.

El Sayed, R. A. E. (2002). Effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance. *Journal of Science and Mathematics Education in Southeast Asia, 25*(1), 56-69.

English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education,* 83-106.

English, L. D. (2003). Problem Posing in Elementary Curriculum. In F. Lester & R. Charles (Eds.), *Teaching Mathematics through Problem Solving* (pp. 187-198). Reston, VA: National Council of Teachers of Mathematics.

Fennema, E., & Franke, M. (2006). Teachers' Knowledge and Its Impact. In D. A. Grouws (Ed)., *Handbook of Research on Mathematics Teaching and Learning* (pp. 147-164). Reston, VA: National Council of Teachers of Mathematics.

Gonzales, N. A. (1998). A blueprint for problem posing. *School Science and Mathematics, 98*(8), 448-456.

Işık, C., Kar, T., Işık, A., & Güler, G. (2012). Skills of pre-service elementary mathematics teachers for determining errors in problems posing related to addition operation with fractions. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 3*(3), 161-182.

Johnson, B., & Christensen, L. (2008). *Educational Research: Quantitative, Qualitative, and Mixed Approaches*. Thousand Oaks, CA: Sage Publication.

Kılıç, Ç. (2014). Determination of primary teachers' perception forms related to problem posing. *Kastamonu Education Journal, 22*(1), 203-214.

Knott, L. (2010). Problem posing from the foundations of mathematics. *The Mathematics Enthusiast, 7*(2), 413-432.

Korkmaz, E., & Gür, H. (2016). Öğretmen adaylarının problem kurma becerilerinin belirlenmesi. *Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, *8*(1), 64-74.

Kulm, G. (2008). Teachers' Mathematics Knowledge. *School Science and Mathematics, 108*(1), 2-4.

Lavy, I., & Shriki, A. (2007). Problem posing as a means for developing mathematical knowledge of prospective teachers. *Paper presented at the meeting of 31st Conference of the International Group for the Psychology of Mathematics Education*. Seoul: International Group for the Psychology of Mathematics Education.

Leeuw, E. (2008). Self-administered questionnaires and standardized interviews. In P. Alasuutari, L. Bickman, & J. Brannen (Eds.), *The sage handbook of social research methods* (pp. 311-327). Thousand Oaks, CA: Sage Publications.

Lin, P. J. (2004). Supporting teachers on designing problem-posing tasks as a tool of assessment to understand students' mathematical learning. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (pp. 257-264). Athens: International Group for the Psychology of Mathematics Education.

Lin, K. M. & Leng, L. W. (2008). Using problem-posing as an assessment tool. *Paper presented at the 10th Asia-Pacific Conference on Giftedness*. Singapore: APCG.

Lodico, M. G., Spaulding, D. T., & Voegtle, K. H. (2010). *Methods in Educational Research: From Theory to Practice*. Hoboken, NJ: John Wiley & Sons.

McMillan, J. H. (2008). *Educational Research: Fundamentals for the Consumer (5th ed.)*. Boston: Pearson Education.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Moses, B. E., Bjork, E., & Goldenberg, P. E. (1990). Beyond Problem Solving: Problem Posing. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and Learning Mathematics in the 1990's* (pp. 82–91). Reston, VA: National Council of Teachers of Mathematics.

NCTM. (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nakano, A., Murakami, N., Hirashima, T., & Takeuchi, A. (2000). A Learning Environment for Problem Posing in Simple Arithmetical Word Problem. *Proceedings of International Conference on Computers in Education: ICCE 2000, 14*, 91-98.

Nardone, C. F., & Lee, R. G. (2010). Critical inquiry across the disciplines: Strategies for student-generated problem posing. *College Teaching, 59*(1), 13-22.

Nixon-Ponder, S. (1995). Using problem-posing dialogue: In adult literacy education. *Adult Learning, 7*(2), 10-12.

Özreçberoğlu, N., & Çağanağa, Ç. K. (2018). Making it count: Strategies for improving problem-solving skills in mathematics for students and teachers' classroom management. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(4), 1253-1261.

Patton, M. Q. (1987). *How to Use Qualitative Methods in Evaluation* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Pirie, S. E. B. (2002). Problem posing: What can it tell us about students' mathematical understanding. *Proceedings of the 24th Annual Meeting North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 925-958). Athens: International Group for the Psychology of Mathematics Education.

Richards, L. (1990). Measuring things in words: Language for learning mathematics. *Language Arts, 67*(1), 14-25.

Rizvi, N. F., & Lawson, M. J. (2007). Prospective Teachers' Knowledge: Concept of Division. *International Education Journal, 8*(2), 377-392.

Shuk-kwan, S. L. (1997). On the role of creative thinking in problem posing. *ZDM, 29*(3), 81-85.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 29*(3), 75-80.

Silver, E. A., & Cai, J. (2005). Assessing students' mathematical problem posing. *Teaching Children Mathematics, 12*(3), 129.

Ticha, M., & Hošpesová, A. (2009). Problem posing and development of pedagogical content knowledge in pre-service teacher training. *Paper presented at the meeting of CERME 6*. Lyon: CERME.

Toluk-Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25*(1), 166-175.

Yuan, X., & Sriraman, B. (2011). An exploratory study of relationshıps between students' creativity and mathematical problem-posing abilities. In B. Sriraman, & K. Lee (Eds.), *The Elements of Creativity and Giftedness in Mathematics*. Rotterdam: Sense Publisher.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 21-36

STUDENTS' COGNITIVE PROCESSES IN SOLVING PROBLEM RELATED TO THE CONCEPT OF AREA CONSERVATION

Rooselyna Ekawati, Ahmad Wachidul Kohar, Elly Matul Imah, Siti Maghfirotun Amin, Shofan Fiangga

Universitas Negeri Surabaya, Jl. Ketintang Gedung D1 Surabaya 60231, Indonesia
Email: rooselynaekawati@unesa.ac.id

Abstract

This study aimed to determine the cognitive process employed in problem-solving related to the concept of area conservation for seventh graders. Two students with different mathematical ability were chosen to be the subjects of this research. Each of them was the representative of high achievers and low achievers based on a set of area conservation test. Results indicate that both samples performed more cyclic processes on formulating solution planning, regulating solution part and detecting and correcting error during the problem-solving. However, it was found that the high achiever student performed some processes than those of low achiever. Also, while the high achiever student did not predict any outcomes of his formulated strategies, the low achiever did not carry out the thought process after detecting errors of the initial solution gained. About the concept of area conservation, the finding also reveals that within the samples' cognitive processes, the use of area formula come first before students decided to look for another strategy such as doing 'cut-rotate-paste' for the curved planes, which do not have any direct formula. The possible causes of the results were discussed to derive some recommendation for future studies.

Keywords: Students' cognitive processes, Area conservation, Problem-solving

Abstrak

Penelitian ini bertujuan untuk menentukan proses kognitif yang digunakan siswa dalam memecahkan masalah yang berkaitan dengan konsep konservasi luas. Dua siswa kelas VII dengan kemampuan matematika yang berbeda, yang masing-masing merupakan perwakilan dari kelompok berkemampuan matematika tinggi dan rendah berdasarkan seperangkat tes konservasi luas dipilih untuk menjadi subyek penelitian ini. Hasil menunjukkan bahwa kedua sampel melakukan lebih banyak proses siklik dalam merumuskan perencanaan solusi, melaksanakan rencana solusi, serta mendeteksi dan mengoreksi kesalahan selama menyelesaikan masalah. Namun, ditemukan bahwa, siswa berkemampuan tinggi melakukan lebih banyak proses siklik daripada yang berkemampuan rendah. Juga, sementara siswa berkemampuan tinggi tidak memprediksi hasil dari strategi yang diformulasikan, siswa berkemampuan rendah tidak melakukan proses pemikiran lanjut *(thought process)* setelah mendeteksi kesalahan dari solusi awal yang diperoleh. Dalam kaitannya dengan konsep konservasi luas, temuan ini juga mengungkapkan bahwa dalam proses kognitif subjek yang diteliti, penggunaan rumus luas datang terlebih dahulu sebelum siswa memutuskan untuk mencari strategi lain seperti melakukan 'potong-putar-tempel' untuk bangun datar melengkung, yang tidak memiliki rumus yang tetap. Kemungkinan penyebab hasil dalam penelitian ini didiskusikan untuk memperoleh beberapa rekomendasi untuk penelitian selanjutnya.

Kata kunci: Proses kognitif siswa, Konservasi luas, Pemecahan masalah

How to Cite: Ekawati, R., Kohar, A.W., Imah, E.M., Amin, S.M., & Fiangga, S. (2019). Students' Cognitive Processes in Solving Problem Related to the Concept of Area Conservation. *Journal on Mathematics Education, 10*(1), 21-36.

Area measurement is one of fundamental topics in mathematics. The measurement of the area of plane figure invite to the study of further mathematics and its application. The measurement becomes the essential competence that build upon scientific knowledge for all fields and careers (John et. al. 2011). Therefore, the idea of area measurement should be taught carefully such that the students could gain a well conceptual understanding of it. Kordaki (2003) asserts that in understanding the concept of area, students need to integrate three interrelated aspects: concept of area measurement, area formulae, and area conservation. However, the latter concept is often isolated from the first two concepts when students solve an area-related problem (Kordaki, 2003). In addition, in the teaching of area measurement topic, the teacher tend to focus only on the use of formula. Unfortunately, most of the area measurement teaching gives the area formula too early for the students (e.g. Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). In fact, the idea of area conservation is deeper than finding the relation of area formula. The use of the formula in measuring the area of plane figure is considered as procedural algorithm only (Fauzan, 2002). In fact, relating to the area conservation skills, most of the pupils have difficulties in decomposing problems (Kordaki & Balomenou, 2006). They are unable to see that decomposing shape into another form would make the area of the figure invariant. Therefore, the students decide to the shortcut by only interested in the formula from which a non-meaningful learning is resulted. This fact results that the students understanding on area measurement is limited to procedural only. In fact, remembering the formula is becoming the main problem on students learning not only in mathematics subject but also other science subjects. Therefore, the topic of area conservation plays an important role in the development of students reasoning on area measurement.

Area conservation can be defined as quantitative value of a certain area of figure remains unchanged after the figure is altered (Smith et al., 2011). Piaget, Inhelder & Szeminska (1960) stated that the term "conservation" means the invariance of the quantity value of the area of a plane while the plane may be transformed into a qualitatively different one. For example, students need to understand that when a shape is divided into several parts and these parts are re-arranged, the area remains the same. To state how wide the area of the figure is, a unit is selected and integrated until shape of the figure is fully covered. When arranging units into rows and columns, students can understand the area depending on the number of rows and the number of columns that there is a multiplicative relationship between these numbers. The studies of students' performance regarding the concept of area conservation have been reported by previous research with regard to some point of interest, such as students' error and misconception (Sisman & Aksu, 2016), students' solution strategies (Kospentaris, 2011), links between students' performance on the problems related to non-measurement and calculation tasks in area measurement (Tumová & Vondrová, 2017). The findings of studies is considered by many scholars as the preliminary step in understanding students' adequate mastering of area measurement (Clements & Stephans, 2004; Kospentaris et al, 2011). However, limited studies found to concern on how students perform their cognitive processes when solving area conservation-related problem. Therefore, In this study, we stress the need for the investigation into the nature of students' abilities by exploring their cognitive processes required for the improvement of students' performance on the topic of area conservation.

Cognitive processes may be described as online mental activities that are proactive in nature (the "to do" strategies) (Montague, Krawec, Enders, & Dietz, 2014). In a similar vein, cognitive processes are defined as the mental processes of an individual, with particular relation to a view that argues that the mind has internal mental states (such as beliefs, desires and intentions) and can be understood in terms of information processing, especially when a lot of abstraction or concretization is involved, or processes such as involving knowledge, expertise or learning.

Some scholars have derived some stages of cognitive processes. For example, Montague, Warger and

Morgan (2000) through the cognitive strategy instruction: *Solve It!* Believes that cognitive processes incorporate the activities of reading (identifying relevant/ irrelevant information), paraphrasing (rewording the information of the problem without changing the problem meaning), visualizing (transforming problem information to a representation that shows the relationships among problem parts), hypothesizing (setting up a plan to solve the problem by deciding on the type and order of operations), estimating (predicting the outcome based on the question/goal), computing (conducting the basic operations needed for solution), and checking (reviewing the accuracy of the process, procedures, and computation). Another cognitive process was offered by Montague (2002). The processes incorporates some stages: comprehending linguistic and numerical information in the problem, translating and transforming that information into mathematical notations, algorithms, and equations, observing relationships among the elements of the problem, formulating a plan to solve the problem, predicting the outcome, regulating the solution path as it is executed, and detecting and correcting errors during problem solution.

In this regard, the cognitive processes can be traced along the way how a learner process his/her thinking based on the types of reasoning mainly demanded by the tasks, i.e. non-measurement reasoning or measurement reasoning. Since in this study, we focus on measurement reasoning, the cognitive processes were measured following the stages of Battista (2007) from the use of numbers which not connected to unit iteration, the employment of unit iteration and enumeration which includes units properly located only along the sides/edges, the operation of numerical measurement, and the integration of measurement and non-measurement reasoning, such as understanding formulas for non-rectangular or composite shapes or determining the value of particular shapes based on a quantitative context inherent in the problem being solved . In fact, according to Montague (2002), students simply may not know "what" to do or even "how" to think about beginning the problem. In addition, if students are not asked how they solve a particular problem and if the work and explanations that accompany their answers are not observed properly, a researcher learns a little about students' understanding and misunderstanding of mathematical ideas (Stylianou et.al, 2000).

Thus, this study took a part of carrying out an in-depth investigation of what students were thinking while they performed their cognitive processes on the problem related to area conservation. The cognitive processes model guiding this investigation is based on modification of Montugue's model of cognitive process, in which thought process and extending problem of Mason's (2015) model are added in the model.

METHOD

Sample of Research

Prior to selecting the student interviewees participating in the interview session, as many as 25 seventh graders with various background in terms of gender, mathematical ability, and verbal communication from a private junior high school in Surabaya city participated a test consisting of five items examining their mathematical ability particularly around the concept of area conservation on two dimensional figure. They were asked to do the test in 45 minutes. They were also informed that their work would not be graded so that they could use their own methods to solve the tasks.

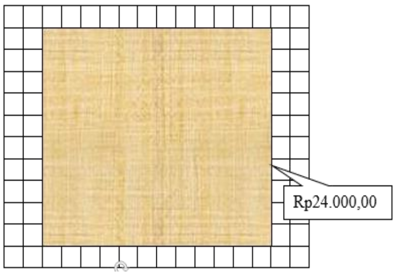
The result of the test informed that approximately half of them were in the group of high achievers (score >60 out of 100) and half in the group of low achievers (score ≤ 60 out of 100) based on their written test performance. As many as two samples were recruited from each of those two groups as the representatives by considering same gender as the control variable, ease of verbal communication based on information from their mathematics teacher and willingness to participate. Beside, to ensure the subject, we also confirm with data of students' mathematics performance. Thus, we had one male student having good score/High Achiever Student (code as HAS) and the other one male student having low score/Low Achiever Student (code as LAS). The data were analyzed qualitatively.

Instrument and Procedures

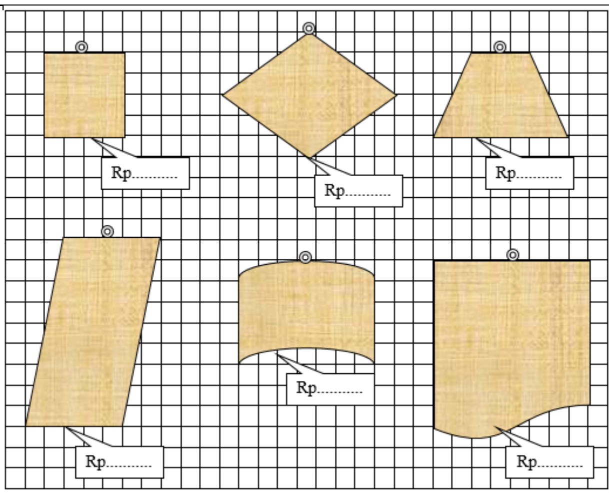
Data were collected from the samples' work on written test which is different from the test given in the initial stage of selecting samples and follow-up interviews. First, students worked on two area conservation-related tasks in 30 minutes. The first task was arranged by the authors in quantitative approach in which the real-world situation was embedded in the tasks, while the second task was developed by the authors relying on students' quantitative approach without any real-world situation. Furthermore, those two tasks were developed around the view of Euclid's elements, in which the practice of measuring area is the use of "additivity axiom", i.e. dividing one figure into some parts which rearranged would form another figure, in order to prove the area equivalence of the figures (Freudenthal, 1986). Thus, instead of only focusing *static* perspective of area measurements, the tasks also focus a *dynamic* perspective where the qualitative approach: emphasizing the conservation of area without the use of numbers (Hiebert, 1981). Those tasks were then validated by experts in terms of content, construct, and language as well as by learners, i.e. students' aside the samples to examine the practicality such as the ease of and presentation of picture and table. See those two task at Figure 1.

Task 1

Eko gets an assignment from his father to give the suitable price tag on the piece of wood he will sell. His father gave a standard price for a rectangular piece of wood measuring 12 cm x 10 cm, which is IDR 24,000

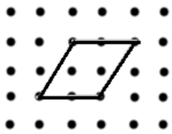


In each of the following pieces of wood, give the suitable price in the available price column.



Task 2

Look at the parallelogram below.



which of the figures below having the same area with the above parallelogram? Explain your reason.

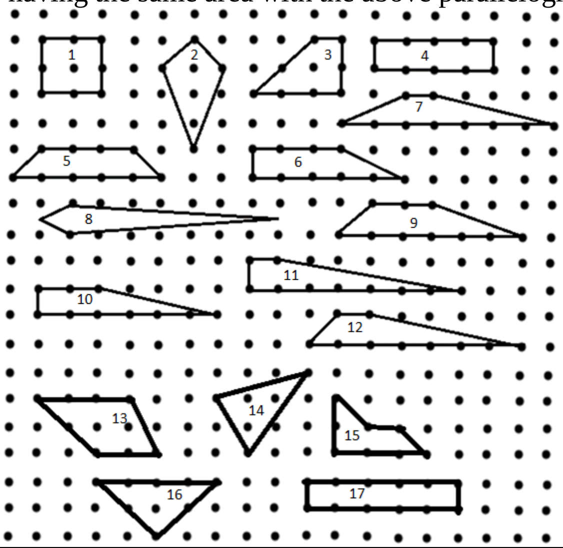


Figure 1. The dynamic area conservation tasks used in this study

In the following day, we interviewed and videotaped the two samples. Table 1 describes the interview protocol that guided the interviewer to collect data. However, this protocol does not mean to guide the interviewers used all the question items too rigidly. Rather, it plays role as the tool to confirm some particular subject's responses. This is to keep the subject reveal their thinking processes as naturally as possible. Thus, when the responses of the subject did not indicate particular cognitive process to occur during the interview, the interviewers did not ask such processes further.

Table 1. Protocol for interviewing the subjects

Cognitive process	Examples of item questions
1. Comprehending linguistic and numerical information in the problem	Please read the question to me. If you don't know certain word, say it. Tell me what the question is asking you to do. How do you understand the meaning of the particular information of this question? Which words / sentences / parts of graphics that makes you difficult to understand have you not yet identified from this problem?
2. Translating and transforming that information into mathematical notations, algorithms, and equations	Have you ever encountered this context before? In what ways? What kind of mathematics do you usually find in relation to this context?
3. Observing relationships among the elements of the problem	Which information from the sentence in the question that you think is important to use in the process of finding answers? Is there any missing information?, Can you recognize the pattern / relationship of the information provided by the problem ?
4. Formulating a plan to solve the problem, predicting the outcome	Tell me how you are going to find the answer
5. Predicting outcome	What might happen if you carry out your plan?
6. Regulating the solution path as it is executed	Show me what to do to get the answer
7. Detecting and correcting errors during problem solution	Tell me how do you convince yourself about your answer Asking about the relationship between the mathematical results obtained by the questions on the question, such as the question: "Are you sure your answer makes sense to answer the question of the problem?, Is there an image or the like that you made to strengthen your answer?
8. Thought process	Do you think there are other possible ways to find your solution?, Ask about the possibility of alternative solutions that students think about, such as through questions: do you think there are other ways to solve this problem? What is your idea? If there are, try showing how you use
9. Extending problem	Is your method of solution applicable for any cases of problem similar to this problem?

Data Analysis

Data of interview were analysed by firstly reducing data, displaying data, and finally drawing conclusions and verification (Miles & Huberman, 1994). The conclusion was sought to understand the most dominant pattern of cognitive processes performed by samples within their problem-solving activities on the tasks. To analyze data interview, we employed a modification of cognitive processes from Montugue (2002). The modification regards to the addition of one more stage as the last stage following the recommendation of Mason in which in the last stage, a solver should not only accentuate an analysis of answers, but also carry out the thought process and problem extension (Mason, 2015). Figure 2 shows the stages which possibly occur during solving a mathematics problem. The arrow direction indicated in figure 2 points out that a solver may follow a cyclic process where the solver moves back and forth, perhaps getting stuck and having to take steps back along the way (Mason, 2015). For instance, there is a possibility that a solver moves back to the stage of observing relationships among information when he/she gets stuck in formulating a solution. In addition, the cycling process can occur for more than two times depending on the degree of his/her confidence and plausibly of solution strategies obtained. Furthermore, the arrows presented in the stages in figure 2 indicate the logical progression from one process to another although it is possible for a student to skip any of these processes, or they can just jump from one process to another process when they change their solution process. For example, when trying to regulate the solution using a plan the student has derived, he/she may be directly arrive at the stage of regulating the solution path as it is executed. Thus, he/she skip any activities indicated in the stage of predicting any outcome. The model of using arrows in analysing the stages that might occur on student's cognitive process are proven as a helpful tool for keep track student's behaviors (Yeo & Yeap, 2010). While the model of analysing students' cognitive processes employing Montugue's (2002) model has been used by Jones (2006) to track the existence of the Montugue's stages of cognitive processes, there are still lack findings reporting the Montugue's stages which consider both the existence and the order of process of the stages. Thus, in this study, the modified Montugue's model in terms of the dynamic processes which might occur during student solution process indicated by the arrows of the stages were used as a tool of analysis shown in Figure 2.

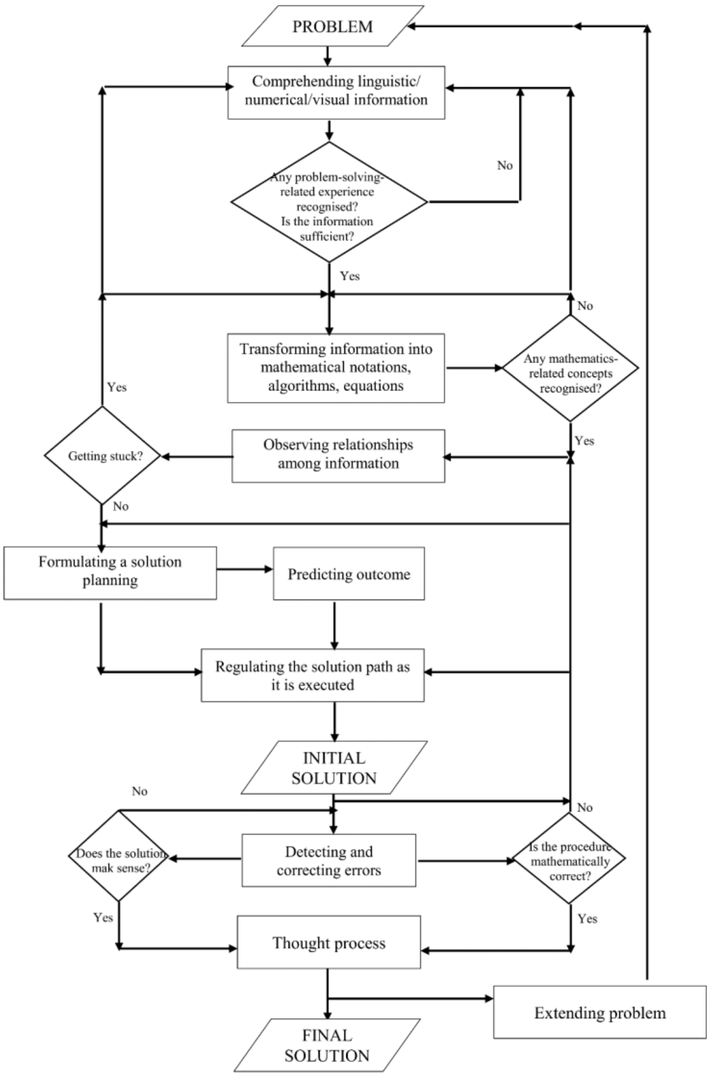


Figure 2. Framework of analysing students' cognitive processes

RESULT AND DISCUSSION

The Cognitive Processes of HAS

On the first task, HAS begun his cognitive processes by claiming to have encountered similar problems, namely about the area of land in various forms of two dimensional figures. He then mentioned information that is known: rectangular pieces of wood whose size is limited and called unknown but necessary information: no price per ‘cm’. These all activities are included in the process of comprehending linguistic, numerical, a spatial information. In the stage of translating and transforming that information into mathematical notations, algorithms, and equations, HAS redrew the figure in the problem to show the intended part per ‘cm^2', which refers to the area of the rectangle. He then observed relationships among the elements of the problem by mentioning that to find the price of the piece of wood in the problem, he needs to find the price per cm^2 or the price per square cm first. He continued his processes by starting to formulate a plan to solve the problem. He said, “To find prices per cm^2, it is necessary to divide the total price by the area of the rectangle. Then, I need to determine the price of each piece of wood by multiplying the price per ‘cm^2’ with the area of each of planes that has been calculated using the formula of plane area I ever studied”. When asked which planes he worked at first, he intended to determine the order of plane (i.e. (1) square, (2) trapezoid, and (3) rhombus) based on the formula of area. However, he had no idea for the curved planes and skipped the process of predicting outcome and continued with regulating the solution path as it is executed. Within this process, he calculated the price per ‘cm^2', obtaining 24000: (12x10) = 24000: 120 = 2000 rupiahs, then calculated the price of square pieces of wood, namely 4x4x2000 = 32,000 rupiahs.

Furthermore, detecting and correcting errors were executed by being aware of his incorrect calculation, namely the price per cm^2 should be 200 rupiahs, instead of 2000 rupiahs. That is why he repeated the process of regulating the solution path as it is executed by recalculating the price of the square–shaped wood, finding 4x4x200= 3,200 rupiahs. Likewise, on the trapezoid-shaped wood, he found the price is ((7+3) x 4)/2 x 200 = 4000 rupiahs. However, he was not sure of the formula he used for finding the price of parallelogram-shaped wood because he forgot whether to use the base x high formula to determine the area of the parallelogram although he finally used the formulate. Therefore, he reformulated his plan to solve the problem by reviewing the method he used. According to him, he needs to find other ways that do not rely on memorization of the plane area formula. In this regard, he said, “I start thinking of looking back at the parallelogram-shaped wood, then drawing line (altitude) from the upper left corner of the parallelogram. Thus, I found a triangular shape that if moved and pasted to the right side will form a rectangle.” In this case, he used ‘cut-paste’ technique. Since he thought this technique was successful, he then used it to solve other quadrilateral-shaped woods. He got stuck on using this technique for rhombus-shaped wood. He observed, “I need to get what I call as ‘a target plane’, that is a plane that becomes final object after cut-paste method is employed, so it was necessary to find out how to cut the existing plane and change it to the target plane.” Such an observation led him reformulated his plan by rotating the cut parts of initial plane, then uniting all such parts become the target plane (cut-rotate-paste). This method was then used for the rhombus-shaped wood and got success. When he applied this method for the last curved plane, however, he got stuck. He said, "It's difficult to apply this method for this plane, when the cut parts are rotated, they do not match each other."

Before completing his processes, HAS he evaluated his steps by executing thought process by comparing formula he remembered and the methods he used. In summary, on the basis of the framework of analyzing students' cognitive process in figure 1, the cognitive process in solving area conservation problem of HAS can be derived logically in figure 3a.

On the second task, HAS started his cognitive process by observing the shape of the two-dimensional plane at the figure given as the information of the task. He said, "I never found this kind of task before, the task that ask me to compare the area without any numerical information given. But, I know that I need to find the area of the parallelogram in this figure and find which figures among these choices [the two-dimensional figures presented in the task]" having the same area with the parallelogram." In this regard, HAS identified the crucial thing of the task that need to be found for the subsequent steps of solving the task. To that, he translated the missing information about the area of the parallelogram by introducing the formula of finding area of a parallelogram, which is base x height, and obtained the area is 2 x 2 = 4. When observing the relationship among the elements of the problem, he admit the some figures have similar shapes, in which the shapes consist of at least three groups: figures without hypotenuse, figures with one hypotenuse, and figures with more than one hypotenuses. He said, "I found similar shapes like a group of 1, 14, and 17, then a group of 3, 6, and 11, and maybe a group of 7, 9, and 14, the very likely difficult ones since the shape looks have more hypotenuses." This observation led him to formulate a plan to solve the task by working out the group of figures he thought was the easiest first, namely group of figures without hypotenuse. The plan, we observed, was around the use of formula for the group of figures without any hypotenuse, while keeping no idea for the other two groups.

Without predicting any outcomes of the plan he formulated, he directly executed his idea of using quadrilateral formula to find the area of figures no 1, 4, 17 and simply found that the area of figure no 1 and 4 is same with the area of the parallelogram, while the area of figure no 17 is not same with the area of the parallelogram since he said, "the area of this is 5 x 1 = 5, not same with the parallelogram, which is 4." Being aware of the weakness of the method of using plane area formula he experienced in the first task, he then tried to use his 'cut-rotate-paste' method to solve the second and the third group of figures. Thus, he reformulated his plane. Interestingly, he did not put the parallelogram figure in the information as the target plane. Rather, he put the square (figure no 1) as the target one. He argued, "I think I need to bring all the remaining figures into the shape of this square instead of the parallelogram because this is likely easier". In this regard, he predicted the more likely easier method, instead of any outcome resulted from such a method. He regulated the plan by firstly choosing figure no 3 for the first trial. In this case, he said, "I imagine this part is cut off and put the cut part into this part, I get a square".

With the same way, he get a square for figure no 10 and 11. For the third group of figures, when he tried examining figure no 7 and 9, he found it was easy and decided that these two figures also have the same area with. He argued, "Same with figure no 6, I cut off these two parts and paste them so that the figure becomes a square. "However, when he examined figure no 2, which is a kite-shaped plane, he got stuck. It was observed that he found difficulties in determining the place where he should cut the figure in his mind. After for more than 1 minute, he finally revealed that he need to cut the plane twice, rotated the cut plane, and paste it so that it forms a square. The only figure that he did not any idea to solve is figure no 14. He said, "It is very difficult to find where to cut off this plane, it might have another method, and I don't know."

To complete his solution process, he was asked to compare which method he should use when finding a similar problem in the future. He argued, "I found some difficulties when using a formula since I sometimes forget with the formula. Therefore, I have to use another method such as by cutting particular parts of the figure and move the cut parts to the other part of the figure so that it becomes a square." In brief, his cognitive processes are illustrated in Figure 3b.

Figure 3 compares the cognitive processes of HAS when solving two area conservation tasks. It indicates that there are some repeating processes done for the two tasks, primarily from the process of determining relationship among element of problems to detecting and correcting errors of the solution resulted.

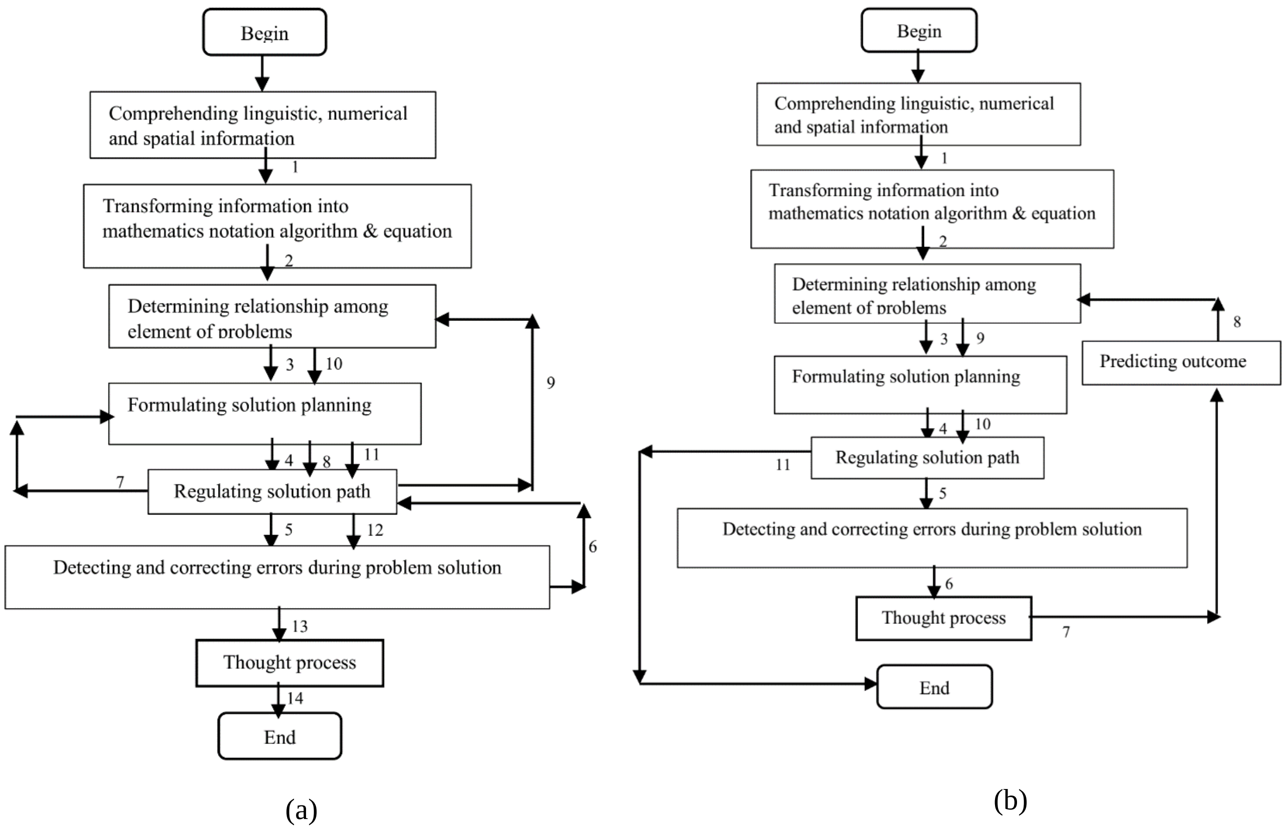


Figure 3. The cognitive process of HAS

Interestingly, HAS carried out the thought process on both the two tasks, which are proven as important processes to convince the correctness and the most effective method to derive the solution. Also, for HAS, the process of predicting any outcome does not seem likely becomes a crucial process in the initial cognitive processes. As evidence, this stage was not carried out in the first task, while this was carried out in the second task, but after the thought process, instead of between the process of determining relationship among information and formulating solution planning.

The Cognitive Processes of LAS

On the first task, LAS began his cognitive processes by claiming that he ever faced similar problem and mentioned the information either known or unknown. He then re-drew the information from the problem to show the meaning of magnitude notation as circumference of rectangle as part of the process of 'transforming information into mathematics notation, algorithm and equation. Afterwards, he observed the relationship among element of the problem by finding the unit price as basis to find total price. He continued with formulating solution planning. He determined three plans such as 1) divided the given price of pieces of wood to explore the price for the unit magnitude, 2) calculating the circumference of each plane by counting the number of square that were covered by the explored plane, 3) determining the price of each piece of wood by multiplying the price of unit magnitude with the circumference of each plane. He skipped doing predicting outcome and continued with regulating the solution path as is it executed by calculating the price of each 'cm'. However, LAS could not sure about the result of his calculation but he did not prove his prediction. This process was coded as predicting outcome process. After he did prediction, he formulated a plan again to solve the problem by doing revision on the plan of determining unit price and delete his initial idea of calculating the rectangle circumference. He tend to doing multiplication 2 (12x10) = 240 cm. Furthermore, another solution path was regulated such as by calculating the unit price of every 'cm', though he did a little error in calculation. In determining the price of unit piece of woods, a unique order of work on plane was as follows: square – rhombus – parallelogram – trapezoid –rectangle with arc modification – trapezoid with arc modification. For all planes, he counted the number of 'box/square' that cover the planes.

The cognitive process continued with comparing the price of wood in the form of parallelogram and rectangle. By this, he shared his uncertainty and did checking since he found for bigger form of wood is cheaper than those smaller one. This process was coded as detecting and correcting error during problem solution. Afterwards, another activity of formulating a plan to solve the problem was executed by revising the unit price. As a result, regulating the solution path as it is executed process appeared again by repeating the calculation for the price of each. However, LAS found difficulties in determining the price for rectangle with arc modification and trapezoid with arc modification. He felt uncertain with his strategy to solve the problem. Therefore, the following process were not performed. By considering the framework of analyzing students' cognitive process, the cognitive process in solving are conservation problem of LAS can be derived logically in figure 4a.

On the second task, LAS started his cognitive processes by admitting that he never experience with the task. He claimed that the types of mathematical questions related to area he ever work out is finding the area of a particular plane with some numerical information given. Nevertheless, he tried to understand what the task actually ask him to do. He said, "I need to find the area of the parallelogram first, then find which of the figures [figures in the options] having the same area with the parallelogram. Thus, I found there are three dots for the base and three other dots for the height." Subsequently, he argued that the area of the parallelogram can be found by using the plane area formula for parallelogram. He continued his explanation, "because the area of parallelogram is base x height, then I found the area is 3x3 = 9". In this stage, LAS transformed the information by recognizing relevant formula of area of parallelogram, although he selected irrelevant information, which is the number of dots, instead of the length of the height and the base of the parallelogram. This irrelevant information then become one of the causes of LAS' failure in the subsequent stages of his problem solution process. As the evidence, when he formulated his plan to solve the problem of the task, he used the area of the parallelogram he found, which is 9, as the criterion for finding the planes which also have the area of 9 square units.

Such the above mistakes continues until the stage of regulating solution path. For example, when he examined whether figure no 4, he explained, "this rectangle has the area of 10, because it comes from 5x2 [five dots for the length and 2 dots for the width]", even though the actual area of figure no 4 is same with the area of the parallelogram, which is 4 square units. Another example was indicated by figure no 1, in which this figure was assumed to have the same area with the parallelogram, i.e. 9 square units. Interestingly, LAS remember all the relevant formula to find the area of each of quadrilateral-shaped figures given in the task. However, he found no formula fit with the non-quadrilateral-shaped figure such as figure no 14, 15, and 16. Thus, he got stuck on these figures and did not continue his work. Finally, he also did not carry out the crucial stages of cognitive processes, i.e. detecting errors and thought process within his problem solution process. In summary, LAS' cognitive processes in task 2 is given in Figure 4b.

Figure 4 illustrates the comparison of the cognitive processes of LAS on task 1 and task 2. While these figures points out that LAS carried out the first five cognitive processes of Montague (2002), LAS did not perform the process of detecting and correcting errors. Also, he did not perform any thought process within his solution process. In this case, it is clear that the difference between the cognitive process of HAS and LAS is the existence of thought process during their solution processes. The same characteristics of HAS and LAS, however, is indicated from the dynamic process proven by the repeated processes primarily within the process of formulating solution planning to regulating solution path done by them.

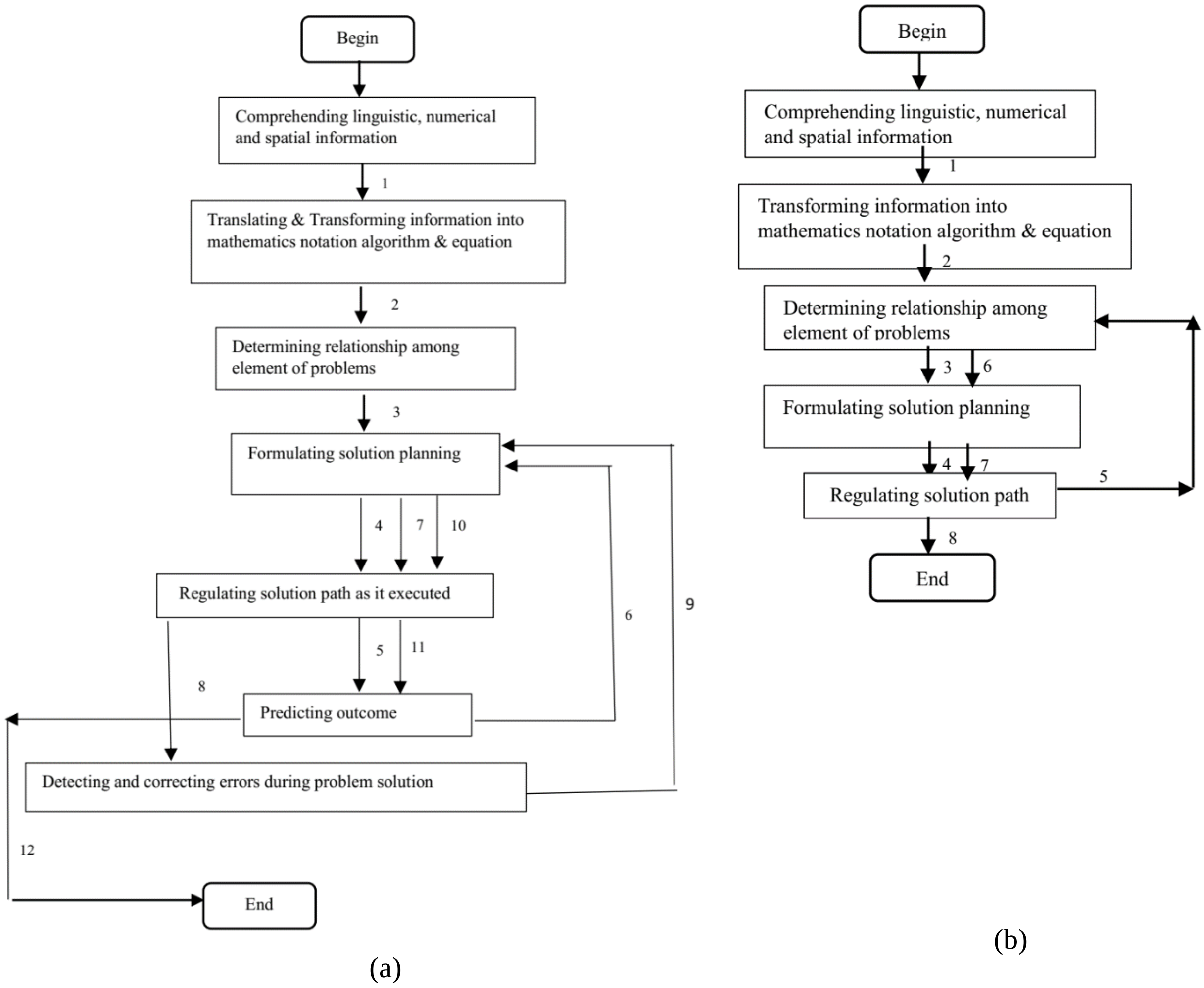


Figure 4. The cognitive process of LAS

Discussion

The present study investigated the two students' with different mathematics ability cognitive processes in solving problem of area conservation. With regard a modification of cognitive processes from Montague (2002), the two students managed to obtain mathematics' solution and describing their process. The common feature finding was those students follow the hierarchical step of cognitive process except *the thought* that was not elaborated by Low Achiever Students (LAS) . The initial strategy used to find the price was by elaborating 'unit price'. Cramer et.al (1993) suggested that unit rate approach was the most popular strategy and responsible for the largest number of correct answers. However, LAS got stuck after detected errors and repeating the cyclic of formulating solution planning and regulating solution path. He still fell uncertain with his used strategy. On the contrary, High Achiever Students (HAS) performed more cyclic cognitive process on the task with quantitative approach. The cyclic started after he detected errors and he tried to revise his errors. He turned back the process until he re-determined several elements of the problem, re-formulating solution planning and re-regulating solution path. In addition, the thought process were elaborated by HAS to arrive to his final solution. These findings in line with previous research that suggested that high academic achievers and low academic achievers have significant differences in the integration of cognitive structures and the usage of information processing strategies. (Bischoff & Anderson, 1998, 2001; Tsai, 1998, 1999; Tsai & Huang, 2001). As has been illustrated, the difficulty of LAS in determining the solution was caused by the inability of students to establish the crucial relationship between the representation in the problem and the data he found. To be more specific, Stillman (1996) hypothesized the contributing factors to an unsuccessful solution were unsatisfactory comprehension skills, lack of understanding of mathematical concept and inhibit impulsive responses to the problem.

CONCLUSION

In performing analysis of students' cognitive process in solving area conservation problem quantitatively, three crucial findings must be taken into account. The first two cognitive process namely comprehending linguistic, numerical and spatial information; Transforming information into mathematics notation algorithm & equation; and determining relationship among element of problems were two crucial starting process in solving mathematics problem related to area conservation in this study. More specifically, the primarily process which encountered by students as a continuous cyclic were formulating solution planning, regulating solution part and detecting and correcting error during problem solution. Furthermore, the predicting outcome process was elaborated by low achiever student but not for high achiever student. On the contrary, the thought process was elaborated by high achiever student but not for low achiever student.

Thus, the entry and goal setting phase of problem solving model by Mason et.al (1985) play crucial rule in solving problem. The following cognitive process phase were influenced by students' ability in activating their mathematics' thinking. The first implication of this study findings is that the characterization and the activation of cognitive process may inform teachers on the teaching strategy that can be applied. The second implication is to inform research that the seven cognitive process are elaborated by students with continuous cyclic for several processes. The process may be useful to lead teachers develop process for their students. This study focus on the solving problem quantitatively, thus, it is still possible to explore more on the cognitive process in the eyes of visually.

ACKNOWLEDGMENTS

We would like to thank the Ministry of higher education and research on PDUPT grant in the second year 2018, the Rector of Universitas Negeri Surabaya (Unesa), and all the participants involved in this study. Our thanks also to the reviewers for the useful feedback.

REFERENCES

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843–908). Charlotte, NC: Information Age Publishing Inc.

Bischoff, P. J., & Anderson, O. R. (1998). A case study analysis of the development of knowledge schema, ideational network, and higher cognitive operations among high school students who studied ecology. *School Science and Mathematics, 98*(5), 228-237.

Cramer, K., Post, T., & Currier, S. (1993). Learning and Teaching Ratio and Proportion: Research Implications. In D. Owens (Ed.), *Research Ideas For the Classroom* (pp. 159-178). NY: Macmillan Publishing Company

Freudenthal, H. (1986). *Didactical phenomenology of mathematical structures* (Vol. 1). Springer Science & Business Media. John P. Smith III, Marja van den Heuvel-Panhuizen, Anne R. Teppo. (2011). Learning, teaching, and using measurement: introduction to the issue. ZDM Mathematics Education (2011) 43:617–620 DOI 10.1007/s11858-011-0369-7 FIZ Karlsruhe

Hiebert, J. (1981). Units of measure: Results and implications from national assessment. *The Arithmetic Teacher*, *28*(6), 38-43.

Jones, V. O. (2006). *Cognitive processes during problem solving of middle school students with different levels of mathematics anxiety and self-esteem: case studies*. A Published dissertation of Florida State University

Kordaki, M., & Balomenou, A. (2006). Challenging students to view the concept of area in triangles in a broad context: Exploiting the features of Cabri-II. *International Journal of Computers for Mathematical Learning*, *11*(1), 99-135.

Kospentaris, G., Spyrou, P., & Lappas, D. (2011). Exploring students' strategies in area conservation geometrical tasks. *Educational Studies in Mathematics*, *77*(1), 105-127.

Kordaki, M., & Potari, D. (2002). The effect of tools of area measurement on students strategies: The case of a computer micro world. *International Journal of Computers for Mathematical Learning, 7*(1), 65-100.

Mason, J., Burton, L., & Stacey, K. (1985). *Thinking mathematically* (Rev. Ed.). Wokingham, UK: Addison-Wesley

Mason, J. (2015). On being stuck on a mathematical problem: What does it mean to have something come-to-mind?. *LUMAT (2013-2015 Issues)*, *3*(1), 101-121.

Miles & Huberman. (1994). *Qualitative Data Analysis: An Exposed Sourcebook 2nd*. London: SAGE Publication Ltd.

Montague, M. (2002). Mathematical problem solving instruction: Components, procedures, and materials. In M. Montague, & C. Warger (Eds.), *Afterschool extensions: Including students with disabilities in afterschool programs.* Reston, Va.: Exceptional Innovations.

Montague, M., Krawec, J., Enders, C., & Dietz, S. (2014). The effects of cognitive strategy instruction on math problem solving of middle-school students of varying ability. *Journal of Educational Psychology*, *106*(2), 469.

Montague, M., Warger, C., & Morgan, T. H. (2000). Solve it! Strategy instruction to improve mathematical problem solving. *Learning Disabilities Research & Practice*, *15*(2), 110-116.

Papadopoulous, I. (2010). Irregular Plane Figures: From the Eighteenth Century to the Modern Classroom. *International Journal of Science and Mathematics Education, 8,* 869-890.

Sisman, G. T., & Aksu, M. (2016). A study on sixth grade students' misconceptions and errors in spatial measurement: length, area, and volume. *International Journal of Science and Mathematics Education*, *14*(7), 1293-1319.

Stillman, G. (1996). Mathematical Prospective and Cognitive Demand in Problem Solving. *Mathematics Education Research Journal, 8*(2), 174-197.

Stylianou, D. A., Kenney, P. A., Silver, E.A., & Alacaci, C. (2000). Gaining Insight into Students' Thinking through Assessment Tasks. *Mathematics Teaching in the Middle School, 6*(2), 136-144

Tsai, C.-C. (1998). An analysis of Taiwanese eighth graders' science achievement, scientific epistemological beliefs and cognitive structure outcomes after learning basic atomic theory. *International Journal of Science Education, 20*, 413-425

Tsai, C.-C. (1999). Content analysis of Taiwanese 14 year olds' information processing operations shown in cognitive structures following physics instruction, with relations to science attainment and scientific epistemological beliefs. *Research in Science & Technological Education, 17,* 125–138

Tsai, C.-C., & Huang, C.-M. (2001). Development of cognitive structures and information processing strategies of elementary school students learning about biological reproduction. *Journal of Biological Education, 36*, 21-26.

Tumová, V., & Vondrová, N. (2017). Links between Success in Non-Measurement and Calculation Tasks in Area and Volume Measurement and Pupils' Problems. *Scientia in Education*, *8*(2), 100-129.

Yeo, J. B., & Yeap, B. H. (2010). Characterizing the cognitive processes in mathematical investigation. Accessed from http://www.cimt.plymouth.ac.uk/%20journal/jbwyeo.pdf, 10 October 2018.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 37-46

DEVELOPING PISA-LIKE MATHEMATICS PROBLEMS ON UNCERTAINTY AND DATA USING ASIAN GAMES FOOTBALL CONTEXT

Dedi Yansen[1], Ratu Ilma Indra Putri[2], Zulkardi[2], Siti Fatimah[3]

[1]Sekolah Tinggi Ilmu Teknik Prabumulih, Jl. Patra No.50, Prabumulih, Indonesia
[2]Sriwijaya University, Jalan Srijaya Negara, Bukit Besar, Palembang, Indonesia
[3]SMA N 3 Prabumulih, Prabumulih, Indonesia
Email: dediyansen23@gmail.com

Abstract

This study aims to generate the valid and practical PISA-like mathematics problems on uncertainty and data using the football game context of Asian Games. It also aims to see the potential effects of the problems towards the high school students' mathematical ability. This study used a design research methodology with the type of development study which was divided into two stages, namely preliminary stage and formative evaluation stage. The preliminary stage covered the analysis of research subjects, curriculum analysis, PISA framework and creating a question instrument. The formative evaluation stage included the one to one consisting of three students together with expert review consisting of two validators, small group consisting of six students, and then the field test. This study produced valid and practical PISA-like mathematics problems on uncertainty and data content which have a potential effect on the senior high school students' mathematical literacy ability. The validity was determined based on the expert review, while the practicality was determined based on one-to-one and small group assessment. The students' mathematical literacy abilities appeared during the research were communication, reasoning and argument, and devising strategies for problem-solving.

Keywords: Asian Games, Design research, Mathematical literacy, PISA-like.

Abstrak

Penelitian ini bertujuan untuk menghasilkan soal matematika tipe PISA konten *uncertainty and data* dengan konteks cabang olahraga sepak bola pada Asian Games yang valid dan praktis, serta melihat efek potensial dari soal yang dibuat terhadap kemampuan literasi matematis siswa kelas X SMA. Metodologi penelitian yang digunakan adalah *design research* dengan tipe *development study*, yang terbagi menjadi dua tahap yaitu *preliminary* dan *formative evaluation.* Tahap *preliminary* meliputi analisis subjek penelitian, analisis kurikulum, *framework* PISA dan membuat instrumen soal. Tahap *formative evaluation* meliputi *one to one* yang terdiri dari tiga orang siswa bersamaan dengan *expert review* yang terdiri dari dua orang validator. Selanjutnya, tahap *small group* melibatkan enam orang siswa dan kemudian *field test.* Penelitian ini menghasilkan soal matematika tipe PISA konten *uncertainty and data* yang valid dan praktis, dan memiliki efek potensial terhadap kemampuan literasi matematis siswa SMA. Valid berdasarkan pada *expert review*, sedangkan praktis berdasarkan pada *one to one* dan *small group*. Kemampuan literasi matematis siswa yang muncul yaitu kemampuan komunikasi, penalaran dan argument serta kemampuan memilih strategi untuk memecahkan masalah.

Kata Kunci: *Asian Games*, *Design research*, Kemampuan literasi matematis, Soal tipe PISA.

How to Cite: Yansen, D., Putri, R. I. I., Zulkardi, & Fatimah, S. (2019). Developing PISA-like mathematics problems on uncertainty and data using Asian Games football context. *Journal on Mathematics Education, 10*(1), 37-46.

Programme for International Student Assessment (PISA) is an international study to measure the skills and abilities of 15-year-olds. PISA 2015 problems were developed based on four contents, including change and relationship, space and shape, quantity, and uncertainty and data (OECD, 2016). According to Jurnaidi & Zulkardi (2014), many of the weaknesses of Indonesian students' mathematical skills were revealed in the results of the PISA study. The low result of PISA Indonesian students can be caused by several factors such as the use of rote learning method, students are not accustomed to solving PISA-like problems, and students are accustomed to working on the same problems as provided by the teacher. It fits very well with PISA's three-year results. Based on data announced by the Organization for Economic Cooperation and Development (OECD), in PISA 2012, Indonesia ranks 64th out of 65 participating countries (OECD, 2013), while in PISA 2015, Indonesia ranks 62th out of 70 countries (OECD, 2016).

One of the PISA content related to the subject of statistics and probability is the uncertainty and data. Uncertainty and data are the phenomena at the heart of mathematical analysis of many problem situations including the theories of probability and statistics as a technique of data representation and description (OECD, 2016). According to Zuhra (2015), based on the results of the PISA study in 2012 on uncertainty and data content, students were only able to be in the level 2 of 6 PISA levels. The low PISA result of Indonesian students resulted from the lack of problem-solving abilities of non-routine or high order thinking problems. Moreover, the evaluation system in Indonesia still used low-level questions, and the students were accustomed to acquiring and using formal mathematics knowledge in the classroom (Stacey, 2010; OECD, 2014).

Mathematical literacy is defined as the ability of a person to formulate, employ and interpret mathematics in various contexts, including the ability to do mathematical reasoning and use concepts, procedures, and facts to describe, explain or estimate phenomena/events (OECD, 2016). Mathematical literacy involves the seven basic skills that students must possess and underlies the process of student mathematical literacy (OECD, 2016), namely (1) communication, (2) mathematization, (3) representation, (4) reasoning and argument, (5) devising strategies for problem-solving, (6) using symbolic, formal, and engineering, and operations, and (7) using mathematical tools.

According to Johar (2012) and Hendroanto *et al* (2018), the success of Indonesian students in solving the problems of PISA was determined by the evaluation system and the ability of teachers in developing students' mathematical literacy. Putri (2009) states that in the framework of improving the quality of education, efforts to increase competence in the education process was one aspect of substantive of education management that had to be realized through various operational programs. With the improvement of these competencies, teachers were expected to be competent in the process of teaching and learning activities.

One approach that can be used in mathematics learning is the PMRI approach, where the mathematics should be close to the student and relevant to the daily life situation of the students (Putri, 2014). According to Marpaung & Julie (2011), in PMRI, learning began by presenting a contextual/realistic problem. The context was a situation or phenomenon / natural occurrence associated with the concept of mathematics being studied. One context that could be used is the Asian Games sport. Some research used the context of the Asian Games 2018 sports in designing mathematics learning. Roni, Putri, & Zulkardi (2017) used sprint context for division of fractions material. Putri & Zulkardi (2017) developed fraction problem using shot-put context. Rahayu, Putri,

& Zulkardi (2017) developed multiplication of fraction using hurdles context. Gunawan, Putri, & Zulkardi (2017) used swimming context to develop fractions problem. And finally, Nasution, Putri, & Zulkardi (2017) used *dayung* context for fraction problem, too.

Based on the description above, the formulation of the problem in this study comprised (1) how were the PISA-like mathematics problems on uncertainty and data content which was valid and practical; and (2) how was the potential effect of PISA-like mathematics problems on uncertainty and data content developed towards the ability of mathematical literacy of grade X high school students. The objectives of the study were (1) to generate mathematics problem of PISA-like on uncertainty and data content that was valid and practical (2) to find out the potential effect of mathematics problem of PISA-like on uncertainty and data content developed to the ability of mathematical literacy of tenth-grade high school students.

METHOD

The method used in this study is design research with the type of development studies. This study aims to generate the valid and practical PISA-like mathematics problems for the tenth-grade enrichment program and to see the potential effect of the problems made on the mathematical literacy ability of tenth-grade students of high school. This development research consisted of two stages: the preliminary stage and formative evaluation stage. Preliminary stages include student analysis, curriculum analysis, and analysis of PISA questions. After that, researchers designed the developed device. The formative evaluation stage included self-evaluation, expert reviews and one-to-one, small group and field tests (Tessmer, 1993; Ahyan, Zulkardi, & Darmawijoyo, 2014).

In this study, the PISA-like developed was said to be good if it met the three criteria of validity, practicality, and having the potential effect on students' mathematical literacy ability. Validity was based on the expert review comments. Practicality was based on the student comments and results on the one-to-one and small group. Practicality means easy to use without much difficulty. Potential effects were based on the test results of the skills and interviews in the field test stage.

The subject of this study was the tenth-grade students of senior high school. This study was conducted from October 16 to November 29, 2017. The data were collected using (1) Documentation; the document used was the Curriculum 2013 for high school, PISA framework, and problems of PISA-like; (2) Walkthrough, conducted at the expert review stage. Experts provided feedback, comments or suggestions regarding the content, constructs, and languages; (3) Interview, conducted in the one-to-one, small group, and field test. The results of the interviews in the one-to-one and small group stages were used as revision material on the prototype. Later, the interview results in the field test stage were used to see the potential effects on students' mathematical problem-solving abilities; (4) Observation, conducted to find out students 'difficulties in doing one-to-one question, knowing practicality about when small group stage, and to see the potential effect on students' mathematical literacy ability in field test stage.

The data collected were analyzed using descriptive analysis method. First, walkthrough analysis derived from the expert comments at expert review stage to get validity from the question. Second, the results of interviews at the one-to-one and small group stages were used to see the practicality of the problem. Furthermore, the interviews in the field test stage were used to see the potential effect on students' mathematical problem-solving abilities. Third, the results of the observation analysis at one-to-one were used to find out the difficulties of students in problem-solving, the practicality in the small group stage, and the potential effect to the student's mathematical literacy on field test stage.

RESULT AND DISCUSSION

This study produced 12 units of PISA-like mathematics problems consisting of 21 questions with the context of aquatic sports, athletics, and games of the Asian Games 2018. The discussion referred to Unit 8 (Football) consisting of two questions. In Unit 8, some students answered using different strategies.

Preliminary Stage

At this stage, we performed student analysis, curriculum analysis, and analysis of PISA problems. The student analysis aimed to find out the 15-year-old student and the students who had the low, medium and high ability. The curriculum analysis aimed to find out the competence standard and basic competence of statistics and probability materials in the curriculum of 2013. The analysis of PISA problems aimed to develop PISA-like mathematics problems on uncertainty and data content based on the 2015 PISA framework. In the preliminary stage, we designed 12 units of mathematics problems of PISA-like on uncertainty and data comprising 21 questions.

Formative Evaluation Stage

Self-Evaluation

At this stage, the researcher evaluated and reviewed the design results about the PISA-like problems on uncertainty and data content created at the preliminary stage. The results obtained at this stage were called Prototype 1.

Expert Reviews

At this stage, the design results on prototype one were developed by self-evaluation given to experts to test the validity of the content, navigation, construct and language validity. Two validators were validating the prototype 1, namely (1) Kaye Stacey (The University of Melbourn, Australia); and (2) Shahibul Ahyan (Hamzanwadi university, Indonesia). In addition to the expert review stage, we also conducted a panel discussion. It was conducted at the Sriwijaya University and attended by three lecturers as well as nine students of math education mathematics of Sriwijaya University. Suggestions from validators and panel discussions were used to revise Prototype 1.

One-to-One

At this stage, the problems developed in prototype one were tested to three students of tenth-grade senior high school with high, medium and low ability level. The three students were EM, AN, and NA. The results of the interview and student's answer observation were used to revise the design of PISA-like problems made by the researcher. The expert review comments, panel discussions, and one to one on the unit 8 problem are presented in Table 1.

Table 1. Comments expert review, panel discussion and one-to-one on Unit 8 problem

Category	Feedback	Follow up
Validation by an expert: Kaye Stacey	I think Question 8.1 might be easier than level 4, but only data can decide. Yes, the Question 8.2 level is easier than the Question 8.1 level. I wonder if this question is too easy. It is important to have some easy items in an assessment here. There is a wide variety of student abilities. So its value will depend on your target audience.	- We have fixed the level of the question.
Validation by an expert: Shahibul Ahyan	In the Title diagram, the goal is not created but scored. The country's name and percentage make it brighter and clearer	- Changing the word "made" to "scored." - Clarify country name and percentage on the diagram
Panel Discussion	The level on Question 8.2 instead of level 3 remains lower than that level.	
One to one	Because it is deceptive, language and hints of the matter are clear. It takes precision in answering questions. Based on the answer, Student 1 and Student 2 can answer all questions correctly. While Student 3 can answer Question 8.2 correctly while for Question 8.1 error occurred in the calculation.	

Based on the expert review comments, panel discussions, and one-to-one exercises in parallel, the question on prototype one was then revised. The revised question was named Prototype 2.

Small Group

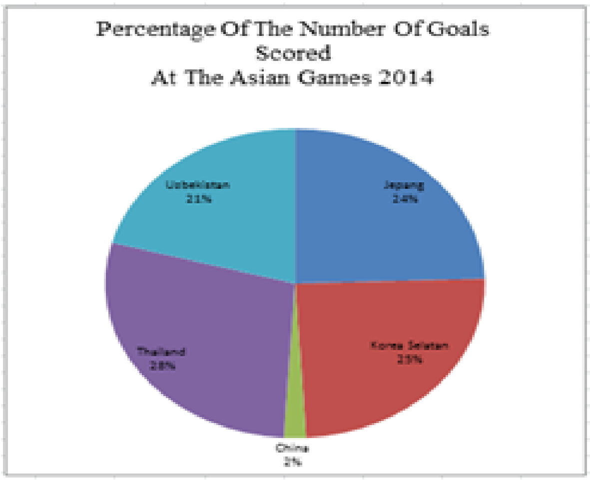
At the small group stage, a revised question based on panel discussions, expert reviews and one-to-one was called Prototype 2. It was tested on small group non-research subjects consisting of six students with different abilities, i.e., two students with high ability, two students with medium ability, and two students with low ability. We also interacted directly with the students to find out the location of the difficulties that might still occur on the PISA-like problems on Prototype 2. Small group stage looked at the practicality of the PISA-like problems. Students could work on the PISA-like problem according to the instructions. This phenomenon indicated that the PISA-like problem was practical or could be used by the students. Furthermore, the problem of PISA-like was rectified after receiving comments from the students at the small group stage.

Based on the results of the interviews and observation of student answers on Unit 8, the explanation or clue on the problem was clear, because it could be understood, most students were able to answer the problem correctly. So there was no revision on Unit 8. Based on the process at the small group stage, Prototype 3 was produced as shown in Figure 1.

UNIT 8.: Football

List of goals made by Japan, South Korea, China, Thailand, and Uzbekistan national team at the Asian Games 1998 to 2014

Year	Number of goals
1998	81
2002	51
2006	33
2010	52
2014	53



Question 8.1.: Number of goals

From the data showed above, how many goals created by South Korea national team?

Question 8.2.: Number of goals

Which country do you think made the most goals among the 5 countries stated above? Give your reasons.

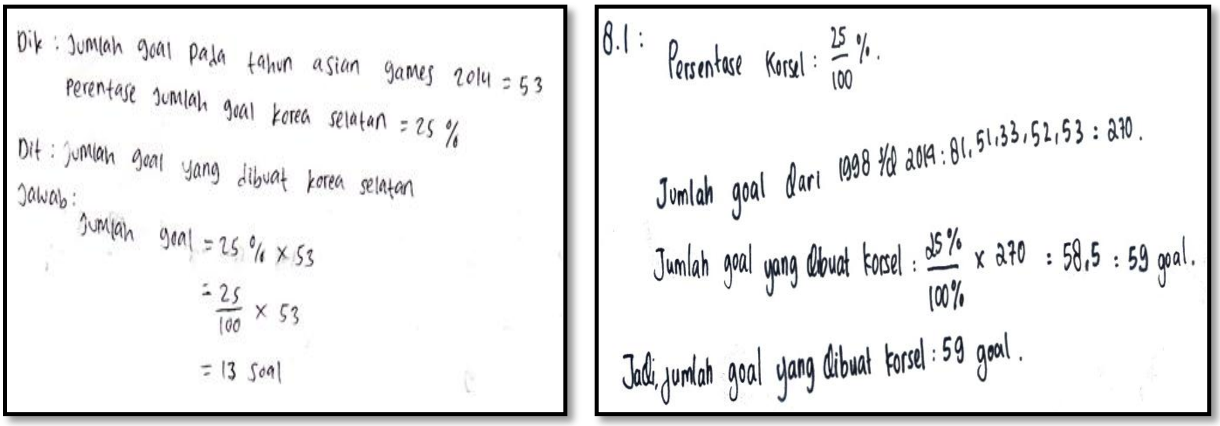
Figure 1. Prototype 3

Field Test

In the field test stage, the result of the revision of the small group stage was called Prototype 3. It was tested on tenth-grade of senior high school students involving 31 students. The situation is illustrated in Figure 2. The product to be tested in the field test stage had to meet the criteria quality. According to van den Akker (1999) a learning device developed in this case the problem was said to be good if it met three valid, practical, and effective criteria.

Figure 2. Students at the *Field Test*

Here is the student's answer to the question on Unit 8 problem in Figure 3.



(a) (b)

Figure 3. Student answers Question 8.1

Based on the answer, Figure 2(a) showed that students wrote the information first in reaching the solution completely and then determined the strategy to answer the questions given to the problems. The answer to Figure 2(a) included the category of full credit. As for the answer on Figure 2(b) students used the right steps and were able to use a good strategy, but the students were less careful in reading the instructions. Students used the number of goals in Asian Games 1998 to 2014. While on the instructions about, the number of goals that were used only in the Asian Games 2014 so that the answer included the category of partial credit. Judging from the analysis of the students' answers of a and b, the literacy ability that emerged was the ability in devising Strategies for Solving Problems. A total of 21 out of 31 students could define a strategy for reshaping mathematical contextual problems completely and correctly, as shown in Figure 4.

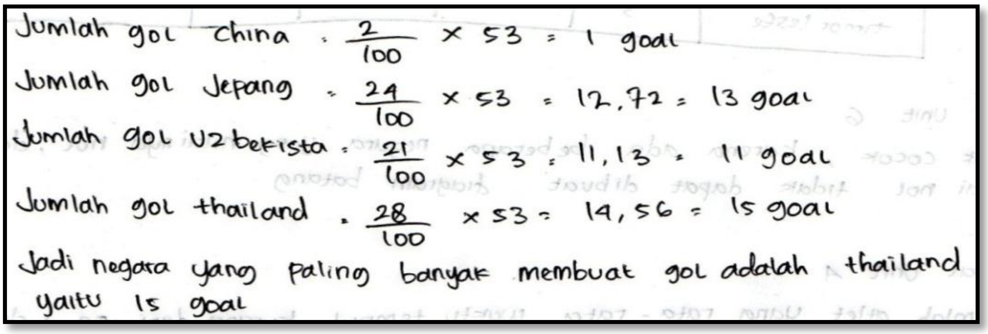


Figure 4. The answer of student 1 Question 8.2

Based on the students' answers one on Problem 8.2, students first calculated the number of goals for each country. After that, they concluded which country scored the most goals. The ability of the mathematical literacy that arose from the student's answer was the communication ability. A total of 25 out of 31 students could write the process in reaching the solution completely and correctly. One of the example is shown in Figure 5.

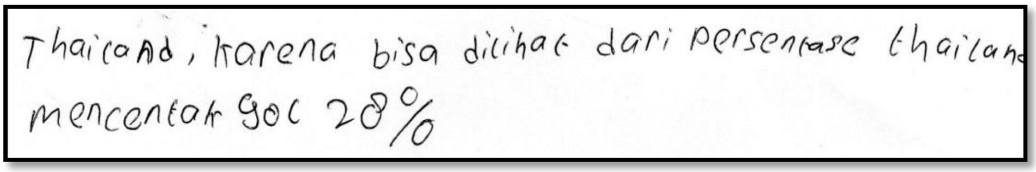


Figure 5. The answer of student 2 Question 8.2

Based on the student's answer in Figure 5, the student used different strategies by directly providing a reason for the answer in writing without having to perform the calculation process. By the PISA indicator, the emerged literacy abilities were reasoning and argument. A total of 4 out of 31 students could explain the full justification for the representation of the identified or designed real-world situation. From the students' answers of 1 and 2 to Question 8.2, students were able to apply communication skills and reasoning abilities and arguments to solve problems in Unit 8.2.

CONCLUSION

This study produced mathematics problems of PISA-like on uncertainty and data contents of 1 unit of questions consisting of 2 questions with the valid and practical context of football sport of the Asian Games 2018. Validity was based on one to one and expert reviews stages. Practicality was based on the interviews and student work on the small group stage. Also, the potential effects were obtained from the test results of students' skills and interviews in the field test stage. The results of the interviews showed that the developed mathematics problems of PISA-like were quite challenging, motivating, helping in using the ability of math in everyday life, training students to work on the mathematics problems of PISA-like, and students were happy to do the given questions. The emerged ability of students' mathematical literacy was communication, reasoning and argument ability, and ability in devising strategies for solving problems.

ACKNOWLEDGEMENT

The authors would like to express his gratitude to `the teacher (Siti Fatimah) and the tenth-grade students of Senior High School who have helped a lot in this research. Then, the authors would like to express gratitude to the Directorate General of Higher Education Indonesia -who has funded post-grant research in 2018.

REFERENCES

Ahyan, S., Zulkardi, & Darmawijoyo. (2014). Developing Mathematics Problems Based on PISA Level of Change and Relationships Content. *Journal on Mathematics Education*, *5*(1), 47-56.

Gunawan, M. S., Putri, R. I. I., & Zulkardi. (2017). Learning fractions through swimming context for elementary school students. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017* (pp. 61-65). Paris: Atlantis Press.

Hendroanto, A., Istiandaru, A., Syakrina, N., Setyawan, F., Prahmana, R. C. I., & Hidayat, A. S. E. (2018). How Students Solves PISA Tasks: An Overview of Students' Mathematical Literacy. *International Journal on Emerging Mathematics Education*, *2*(2), 129-138.

Johar, R. (2012). Domain soal PISA untuk literasi matematika. *Jurnal Peluang*, *1*(1), 30-41.

Jurnaidi, J., & Zulkardi. (2014). Pengembangan soal model PISA konten change and relationship untuk mengetahui kemampuan penalaran matematis siswa Sekolah Menengah Pertama. *Jurnal Pendidikan Matematika*, *8*(1), 25-42.

Marpaung, Y., & Julie, H. (2011). PMRI dan PISA: Suatu usaha peningkatan mutu pendidikan matematika di Indonesia. *Widya Dharma: Jurnal Kependidikan*, *23*(1), 1-11.

Nasution, M. F., Putri, R. I. I., & Zulkardi. (2017). Dayung context in fraction. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017* (pp. 1-6). Paris: Atlantis Press.

OECD. (2013). *PISA 2012 Results: What Students Know and Can Do, Student Performance in Mathematics, Reading and Science (Volume 1).* Paris: OECD Publishing.

OECD. (2014). *PISA 2012 Results in Focus: What 15-year-old know and what they can do with what they know*. Paris: OECD Publishing.

OECD. (2016). *PISA 2015 Results: Excellence and Equity in Education (Volume 1).* Paris: OECD Publishing.

Putri, R. I. I. (2009). Efek potensial pelatihan PMRI terhadap guru-guru di Palembang. *Jurnal Pendidikan Matematika, 3*(2), 65-71.

Putri, R. I. I. (2014). Evaluasi Program Pelatihan Pendidikan Matematika Realistik Indonesia (PMRI) Bagi Guru Matematika Sumatera Selatan. *Prosiding Seminar Nasional Evaluasi Pendidikan, 1*(1), 522-527.

Putri, R. I. I., & Zulkardi. (2017). Fraction in Shot-Put: A Learning Trajectory. *The 4th International Conference on Research, Implementation, and Education of Mathematics and Science (4th ICRIEMS) 2017* (pp. 1-8). Yogyakarta: Universitas Negeri Yogyakarta.

Rahayu, C., Putri, R. I. I., & Zulkardi. (2017). Multiplication of Fraction With Natural Number by Using Hurdles. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017*, (pp. 43-47). Paris: Atlantis Press.

Roni, A., Zulkardi., & Putri, R. I. I. (2017). Sprint Context of Asian Games in the Division of Fractions. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017*, (pp. *22-29*). Paris: Atlantis Press.

Stacey, K. (2010). Mathematical and Scientific Literacy around the World. *Journal of Science and Mathematics, 33*(1), 1-16.

Tessmer, M. (1993). *Planning and Conducting Formative Evaluations*. London: Kogan Page.

van den Akker, J. (1999). Principles and methods of development research. In J. van den Akker, R. Branch, K. Gustafson, N. Nieveen, & T. Plomp (Eds.), *Design approaches and tools in education and training* (pp. 1-14). Dordrecht: Kluwer Academic Publishers.

Zuhra, R. (2015). *Analisis Strategi Siswa dalam Menyelesaikan Soal PISA* (*Programme For International Student Assesment*) *Konten Uncertainty and Data pada Siswa Kelas VIII MTSN Model Banda Aceh*. Banda Aceh: Universitas Syiah Kuala.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 47-58

ETHNOMATHEMATICS IN SASAKNESE ARCHITECTURE

Sri Supiyati, Farida Hanum, Jailani

Universitas Negeri Yogyakarta, Jl. Colombo No. 1 Yogyakarta, 55281, Indonesia
Email: supiyatisambada@gmail.com

Abstract

This research is aimed to explore the cultural elements especially in *Sasak* architectures in Lombok Island such as a home living (Bale), worship building (masjid), lumbung (*sambi*) in ethnomathematics point of view. This research uses qualitative approach with cognitive (anthropology cognitive). The interviewees are the experienced custom leader and humanists who has conducted the research on *Sasak* society. Meanwhile, the data collection method used the participant, observation and documentation. Data analysis on this research is not only based on the researcher is interpretation but also the idea structure of the society. This research shows the evidence of sensitivity on the use of numbers practiced by *Sasak* ancestor long time ago in doing the measurement using their anthropometric ability (Ethnomathematics). The architecture products of *sasak* society also describe that *sasak* society is more focus on the process than the final products itself which describes the consistency in conducting role and cultural device which control the desire of individual construction.

Keywords: Ethnomathematics, Architecture, *Sasak* Society

Abstrak

Kajian ini bertujuan untuk mengekplorasi unsur-unsur budaya khususnya pada arsitektur masyarakat suku *sasak* di Lombok seperti rumah hunian (*bale*), bangunan tempat ibadah (*masjid*) dan lumbung padi (*sambi)* dalam sudut pandang etnomatematika.Kajian ini menggunakan pendekatan kualitatif dengan model etnografi (*anthropology cognitvef).*informan yang digunakan adalah para pemangku adat dan budayawan-budayawan yang sudah memiliki pengalaman dalam melakukan penelitian-penelitian tentang budaya masyarakat *sasak*. Sedangkan teknik pengambilan data menggunakan pengamatan berperan serta (*participant observation)* dan dekomentasi. Analisis data dalam penelitian ini tidak didasarkan semata-mata pada interpretasi peneliti tetapi merupakan susunan pikiran dari anggota masyarakat yang dikorek keluar oleh peneliti. Kajian ini mengungkap bukti kepekaan terhadap angka yang dipraktikkan oleh nenek moyang masyarakat suku *sasak* sejak dahulu, dalam hal melakukan pengukuran yang menggunakan kemampuan antropometris (etnomatematika). Produk-produk arsitektur tradisional masyarakat suku *sasak* ini juga menggambarkan bahwa tradisi masyarakat *sasak* itu lebih mementingkan suatu proses diatas produk akhir yang ingin dihasilkan yakni tergambarnya masyarakat yang selalu konsisten dalam menjalankan peran dan piranti adat yang mengatur hajat membangun dari masing-masing individu didalamnya.

Kata kunci: Etnomatematika, Arsitektur, Masyarakat

How to Cite: Supiyati, S., Hanum, F., & Jailani. (2019). Ethnomathematics in Sasaknese Architecture. *Journal on Mathematics Education, 10*(1), 47-58.

Culture is a typical way for humans to adapt themselves to the environment, while mathematics is realized because of human activities. It is in line with the phrase Freudenthal, "mathematics as a human activities" (Gravemeijer & Terwel, 2000). Next, Suhirman (2001) defines mathematics as the science of form, structure, quantity and other related concepts with large numbers divided into three fields, namely algebra, analysis and geometry. Meanwhile, according to Abdurrahman (2003) states that mathematics is a direction to find answers to problems faced by human beings, a way of using information, using knowledge of form and size, using knowledge about counting and thinking in man himself view and use relationships.

Mathematics and Cultural Products

Mathematical and cultural integration means contextual and realistic mathematics. Various cultural products of our ancestors reveal artistic creativity that contains mathematics, for example in carved ornaments and architectural forms in traditional houses containing three-dimensional geometric formations (Kucuk, 2013). Likewise found by Nasir and Cobb (2007) and Orey (2000) that mathematical contextualization has been described as identification of mathematics that is practiced and developed in different cultural groups. So also stated by (Rius, 2000; Rosa & Orey, 2007) stating that if mathematics is seen as a cultural construct, then it is cultural development.

Orey (2009) stated that mathematics in various cultural contexts confirms this as a demand. Cultural products in the form of artifacts such as traditional buildings are given the opportunity to be developed through mathematical thinking. Mathematical creative thinking that is integrated with culture can also arise in economic behavior. The concept of calculating mathematics through a linear program to determine the critical point as well as meeting several variables becomes a solution when many needs must be met but funds are limited. Mathematical calculations here are an alternative problem solving. Humans appear to fulfill their creativity needs with limited funds. For that it is clear that mathematics is the product of the human mind.

Ethnomathematics

Ethnomathematics is a field study. D Ambrosio (1990) defined ethnomathematics as a very broad term that refers to the socio-cultural context including language, jargon, symbols, behavior and myths. In line with this definition, Orey (2003) explained ethnomathematics to explain, and understand the world to manage problems faced by people, a larger humanitarian problem. More specific again D Ambrosio (1993) stated that the mission of the ethnomathematics program is to recognize that there are different ways of doing mathematics where different cultures can negotiate in practicing mathematics. Furthermore, Borba (1997) described ethnomathematics as a way in which people use certain cultures in mathematical concepts in dealing with relational and spatial aspects in their lives. Thus, ethnomathematics emerges as a new category in the conceptual discourse of mathematics education and as an interaction between mathematics and culture.

One of the objectives of mathematics learning is to prepare students to use mathematics and mathematical mindset in everyday life (Soedjadi, 2000). According to Barta and Shockey (2006) ethnomathemathic is a complex and dynamic representation that describes the influence of cultural use of mathematics in its application. In mathematics learning in schools so far students still consider that mathematics is difficult, too abstract, many formulas and so on, this is because the mathematics learning in school is too formal and not in accordance with what is experienced by students in everyday life. Bandeira and Lucena (2004) investigated mathematical ideas and practices obtained by members of the vegetable farming community in the Northeast region of Brazil. They learn the mathematical concepts used by farmers to harvest, produce and commercialize vegetables. They found that the specific mathematical knowledge produced by farmers differed from the mathematical knowledge obtained in academic settings. Therefore, more researches are needed to explore the cultures that exist around us to find concrete mathematical concepts so that the concepts can be applied in school math learning.

From some of these opinions, it appears that in math learning at school there is a tendency that to explore the initial knowledge of students should begin by associating formal mathematics with the experience of students in everyday life. Therefore, in this study focused on exploring and exploring more about the customs (customs) of *sasak* community related to the calculation, measurement, modeling, designing especially related to the traditional buildings of Sasaknese society. According to Rosa and Orey (2016), ethnomathematics is the application of math skills, ideas, procedures, and practices applied in the past by members of certain cultural groups in different contexts, which are often used today in today's context. It is hoped that based on the findings of this study can be a reference for teachers and observers of mathematics learning in order to create textbooks and use local culture as a source of learning mathematics in schools, so that learning math is more interesting and meaningful for students.

METHOD

The research method used in this study was a qualitative approach with the model of ethnography (anthropology cognitive) (Spradley, 1987). A new ethnography that focuses its efforts to discover how people organize their customs in mind and then use them in life. Informants used are the custom leader and cultural observers who already have experience in conducting research on the culture of the *sasak* community. While the data retrieval technique using participant observation and documentation. The analysis in this study is not only based solely on the interpretation of the researcher but also the composition of the idea of members of the community who are scraped out by the researchers, who then use four stages in analyzing the data such as an anarchic domain, taxonomic analysis, component analysis and theme determination.

RESULTS AND DISCUSSION

Architecture is an art and technique of designing and building and can be associated with construction (Irawan, 2016). The basic concept of *sasak* community building is a residential building called "bale" (house of residence), building of worship place "mesigit" (ancient mosque), and building of food store "granary" (sambi) is basically the same as Austronesia building in general which has a construction with poles and wooden beams both on the coast to the mountains. This spread leads to the emergence of vernacular structures that incorporate mound patterns with the construction of poles and beams according to the natural challenges encountered.

- House of Residence (*bale langgak*)

The house in the *sasak* language called "bale" is a space that is occupied as a residence settled hereditary in the hope of obtaining serenity, tranquility and welfare of the birth and the mind. Home is one of the embodiment of cultural products and the development of human civilization (Fathurrahman, 2016). Spatial and home structures characterize the sociological dimension that reflects the social fabric of its supporting society. Classes and social functions in society are two elements that affect the shape, structure, and naming of space and home. *Gedeng* mention for the house of the nobles, cottage (*bale jamaq*) for ordinary people and bale bonter and mountain bale rate occupied by the religious teacher, village chief and the nobles. As for the concept of *sasak* architecture development always depart from *pemole* and *semaiq* concept which is then applied based on *sasak* custom system in example system of value, social, ritual and technical system.

- *Bale Jamaq/Bale Tani*

The concept of *bale tani* is built in accordance with the environment generally on the slopes of the hill; this bale has a roof of the back and side almost touching the ground and made with a slope that can launch the fall of rain water. Framework of the roof is made higher because basically this roof also functions to form space, because generally the bale tani wall is relatively low in example about 2 meters while the floor (floor) the *dalem* is higher than the *ampik*.

Space and function of this farm bale is divided into three spaces namely *bale dalem*, *dalem bale* and *ampik*. *Bale dalem* serves for the bedding of girls and *dalem bale* to store food and *ampik* divided into 2 *ampik belo* (long) and *ampe konteq* (short) which each function long *ampik* to place doing productive things like weaving and weaving while the short *ampik* is used for the boys resting place. All shapes can be seen in Figure 1.

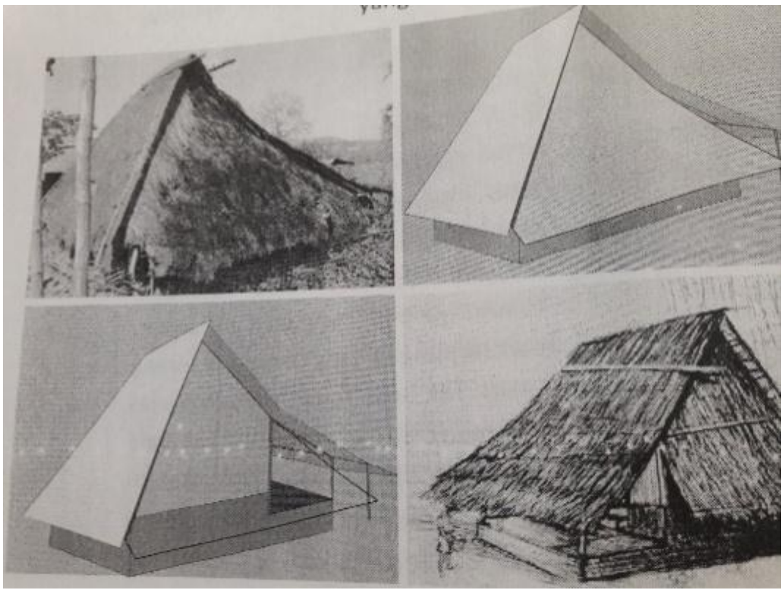


Figure 1. *Bale Jamaq/Bale Tani* (Photo Source: *Museum Provinsi NTB*)

- *Bale Bonter*

The concept of *bale bonter* is a residence for the citizens who have the function and status of the middle nobility, this building is made to be more authoritative than *bale jamaq* with symbols such as peduncles and the use of *jejait* on the frame. *Bale bonter* construction is more robust compared to other bale-bale poles and blander-blander and its doors use carvings. *Bantaran* (foundation) *bale bonter* built rather high so that built steps (steps) made of soil. In general, this type of house is equipped with a six (*berugaq bertiang* six) or *bale jajar* also called *bencingah*. Space and *bale bonter* functions have in common with the *bale jamaq* that is applying the concept of *dalem bale*, *dalem bale* and *ampik* that can be seen in Figure 2.

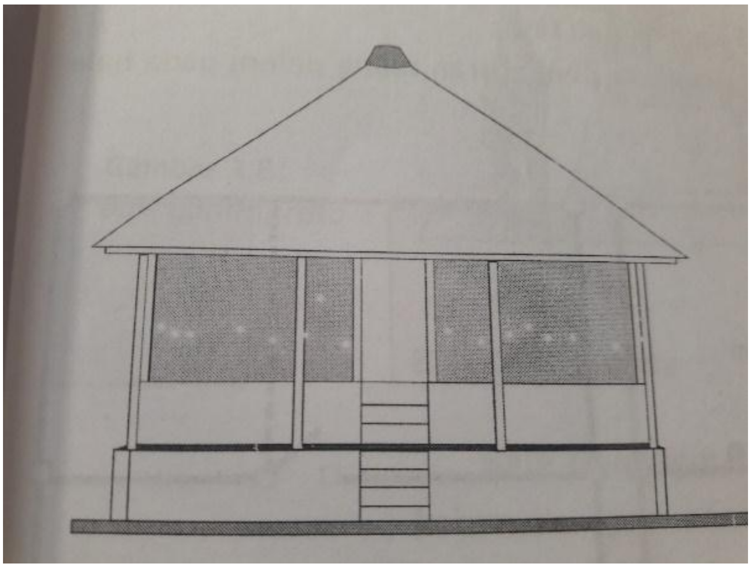


Figure 2. *Bale Bonter* (Photo Source: *Museum Provinsi NTB*)

- *Bale Gunungrate*

Bale gunungrate does not vary much with the *bale bonter*, both materials and construction. There is little difference in the shape of the roof. Construction of bale mountain roof rate using *tunjeng* and *bumbungan* generally decorated with carvings this hole usually has a length of 2 meters is used as a pedestal four pieces *ujuk-ujuk* same length. This type of building has a wall of yard so everyone who wants to enter it does not apply haphazardly. They will maintain the attitude or etiquette, especially because of the gate to the yard. The *Bale Gunungrate* and its sketch can be seen in Figure 3.

Figure 3. *Bale Gunungrate* and *Bale Gunungrate* sketch (Photo Source: *Museum Provinsi NTB*)

- Building for worship *(Mensigit)*

One component of traditional architecture that has a long history in Lombok is the architecture of the mosque. Not because the quantity has reached hundreds of thousands, until Lombok dubbed the island of a thousand mosques but because of the historical aspect and function of the mosque as a traditional icon, almost certainly there is a site of a mosque or ancient mosque that serves as a center of civilization.

The building of the mosque uses the pattern of the mound foundation with the low pillars and walls and the towering roof extends downward as if it were almost touching the ground, so everyone who will enter must be in a downcast position. The size of the customary mosque is generally 7 x 7 meters or 9 x 9 meters on the floor, in the middle of the mosque space, stands 4 main pillars that directly into a crown pole or cone. Around the wall is built 20 poles tied with *gelampar* components and *jejait* using a system of *purus* (knockdown) and wooden pegs of roofing materials are usually made of reeds and some are using bamboo, and the roof is mounted through the wall about one meter above the floor, so people enter the mosque should bow down. Downward attitude is interpreted as a form of homage to the mosque or the people in it. Mosque buildings are usually placed in separate locations separated from housing or settlements (Figure 4).

Figure 4. Ancient mosque of *bayan*

- Granary /*sambi*

Setia Sopandi (2013) identifies *sasak* granary building technology in Lombok dating from 3500 years BC. *Banguanan* barn consists of two parts, namely the top and bottom which each has its own function of the top serves as a place to store the rice so that it is made closed. This upper space is formed by the roof with a symmetrical dome pattern if viewed from the front and rear. At the bottom consists of four poles with the arrangement in a clockwise direction, ranging from *nyake*, teacher, *pendite*, and *kire*. The illustration can be seen in Figure 5.

Above the four pillars are formed *jelepeng* which serves to prevent rats rose into the barn through the pole, in addition to increase the beauty. Above *jelepeng* is placed coral reef-shaped place where laying *gelampar* bottom that serves as a base with foot footprint women (*selampak nine*). The form of a granary developed by the *sasak* community is called "*mekadal meteng*" which in *sasak* society symbolizes the frugal attitude (frugal). The location of the granary in the yard is always in the right position of the house, as a sign of appreciation for the rice that is the fortune of the God.

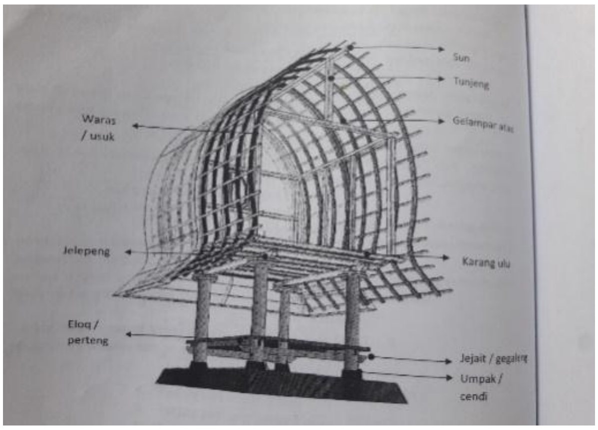


Figure 5. Granary *(sambi)*

The concept of mathematics in the cultural architecture of the sasak society

There are some concept of mathematics in the cultural architecture of the *sasak* society, such as measuring and measurement, designing, and geometric form. The explanation of the concept explained in the following.

- Measuring and measurement

In forming *sasak* community building structures, there is a feature known as "sikut/elbow" which means size, but in this case size is not related to length, width and height (numerical), but also relates to elbow time, elbow space and elbows ramon. Elbow length corresponds to the length to the power (north), width towards the andang and mudi (front - behind), high elbows associated with the nature of the building is determined by the size of the owner of the building while the elbow time related to the start building. All of these elephants are termed anthropometry (a measure that refers to the size of a human body).

The use of the concept of size (elbow) in determining the size of buildings in the community *sasak* use some terms in accordance with what will be measured.

1. *Elbow size*: the size of the construction of a house that refers to the size of the human body known as anthropometry, this measure consists of *seperunjung*, *sedepa*, *sejengkak*, *sekepal*, *sehasta*, and *senyari* (Figure 6).
2. *Elbow andang*: measurement to ensure the accuracy of direction that refers to the environmental layout, the accuracy of the determination of the direction of doors and windows tailored to the comfort and security of the inhabitants of the house.
3. *Time elbow*: precise time planning measurements to construct buildings such as houses, mosques, barns or other buildings.
4. *Elbow of space*: measurements related to the position of the planned room and determine the proper position to be made the core room in accordance with the traditional buildings that apply in the community.
5. *Elbow of ramon*: This elbow is used to select the right building material and placed in the right position especially wood and bamboo materials.

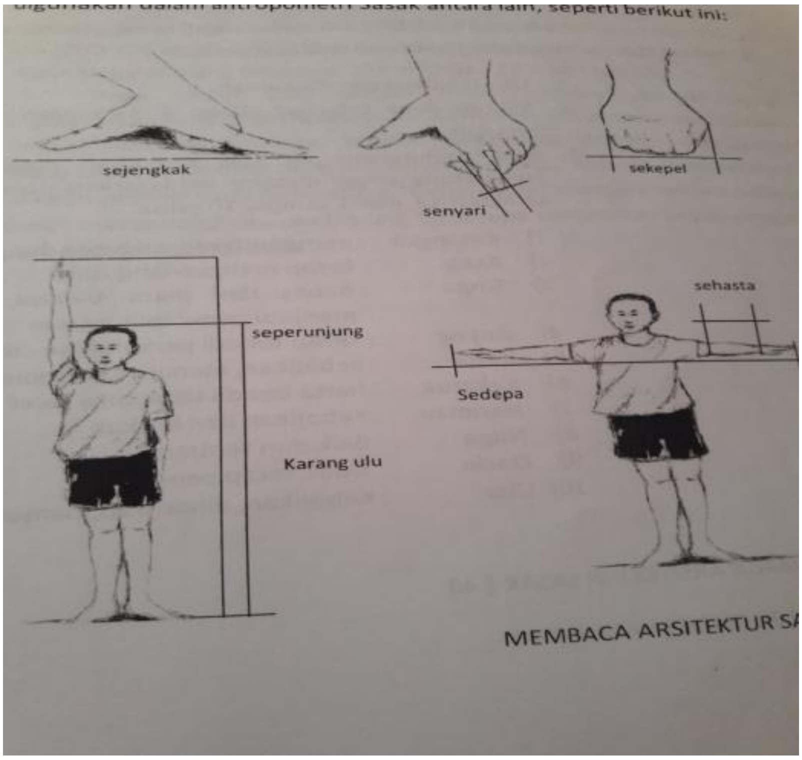


Figure 6. *Sasak* anthropometry size.

- Designing

Sasak community activities related to the designing in the activities of building design such as establishing houses of residence, places of worship, and buildings related to the social life. In implementing the values that must be upheld in designing or planning development, the *sasak* community realize the four basic concepts those are:

1. *Semaiq*, not excessive in accordance with the needs so that the main size used is the size of the owner.
2. *Hardened* (sturdy), logical in applying the correct structure concept
3. *Pantes* (ethical), meets standards of decency
4. *Solah* (aesthetic) satisfies the owner's heart and others who see it.

- Geometric form

Geometry form that exists in the construction of buildings founded by *Sasak* community. It can be seen in Table 1.

Table 1. Mathematics concept and its application in *Sasak'* building ornament

No.	Mathemacics Concept	Building Ornament
1	Angle	Angle is at every house, mosque, and *lumbung*
2	Triangle	Pieces of houses and mosques
3	Square/rectangle	There are on the walls of houses and base od *lumbung* .
4	Symmetry	Spaces of *dalem bale*, domes of *lumbung*.
5	Reflection	There are carvings on the door of the house and the gate of *rumah gunung rate*
6	Trapezium	Walls of *rumah gunung rate*
7	Circle	Baseboard/barn beam
8	Prismatic	Roof of houses/*bale tani*, *bale bonter* and *bale gunung rate*.
9	Parallel	Walls of house and mosque
10	Cylinder, beam, and cut cone	*Jelepeng lumbung*, pillar of house and lumbung
11	Wave shaped sinusoide $Y= a \cos (x - \alpha)$ $Y= a \sin (x - \alpha)$	Roof of *lumbung*
12	Pyramid	Palate of mosque

CONCLUSION

The *sasak* architecture has different construction and ornamentation designs to present a functioning and beautiful building. In addition, the shape of a unique model is geometrically shaped. Inadvertently the *sasak* community has been practicing mathematics in everyday life, visible from the shape and model of the building which has the pattern, angle, building space, angle, triangle, square, cylinder, prism and circle. In addition the form and function of various kinds of buildings and public spaces that contain noble values. This study shows that the architecture of the *sasak* community has a taste, finesse and awareness of servant hood in maintaining the cultural customs inherited by their ancestors, and it is evident that the *sasak* people have sensitivity to the numbers and mathematical calculations practiced by ancestors since time immemorial.

The ethnomathematics practiced by the *sasak* community is evident from the methods of measuring, designing and building forms that exist in *sasak* culture. Therefore, through studies more focused on extracting data related to customs that contain mathematical concepts so that the findings obtained can be implemented in learning mathematics in schools, so that school mathematics learning is more interesting, fun, and not too abstract because the context is directly related to everyday life found in their own culture.

ACKNOWLEDGMENTS

The researcher would like to express his gratitude to all informants who have provided in-depth information for the needs of this study.

REFERENCES

Abdurrahman, M. (2003).*Pendidikan Bagi Anak Berkesulitan Belajar*. Jakarta: Rineka Cipta.

Adam, S., Alangui, W., & Barton, B. (2003). A comment on Rowlands and Carson: Where would formal academic mathematics stand in a curriculum informed by ethnomathematics? A critical review. *Educational Studies in Mathematics, 52*(3), 327-335.

Barta, J., & Shockey, T. (2006). The mathematical ways of an aboriginal people. The Northern Ute. *Journal of Mathematics and Culture*, *1*(1), 79-89.

Bandeira, F. A., & Lucena, I. C. R. (2004*). Etnomatemática e práticas sociais* [Ethnomathematics and social practices]. Coleção Introdução à Etnomatemática [Introduction to Ethnomathematics Collection]. Natal, RN, Brazil: UFRN.

D'Ambrosio, U. (1993). Etnomatemática: Um programa [Ethnomathematics: A program]. *A Educação Matemática em Revista, 1*(1), 5-11.

D'Ambrosio, U. (2001). What is Ethnomathematics and how can it help children in schools? *Teaching Children Mathematics, 7*(6), 308-310.

D'Ambrosio, U. (2006). Ethnomathematics: Link between traditions and modernity**.** *ZDM, 40*(6), 1033-1034.

Hartoyo, A. (2011). Etnomatematika Pada Budaya Masyarakat Dayak Perbatasan Indonesia-Malaysia. *Jurnal UPI, 12*(1).

Fathurrahman, L. A. (2016). *Membaca Arsitektur Sasak. Mataram*. Nusa Tenggara Barat: Penerbit Genius.

Gravemijer, K., & Terwel (2000). *Hans Freudenthal a mathematician on didactics and curriculum theory*. *Journal of Curriculum Studies, 32*(6), 777-796.

Orey, D. C. (2000). The ethnomathematics of the Sioux tipi and cone. In H. Selin (Ed.), *Mathematics across culture: the History of non-Western mathematics* (pp.239-252). Dordrecht: Kulwer Academic Publishers.

Kucuk, A. (2013) Ethnomathematics in Anatolia (In Turkey). Mathematical Thoughts in Multiculturalism. *RevistaLatinoamericana de Etnomatematica, 7*(1), 171-184.

Nasir, N. S., & Cobb, P. (2007). *Equity in students' access to significant mathematical ideas*. New York: Teachers College Press.

Rosa, M., & Orey, D. C. (2003). Vinho e queijo: Etnomatemática e Modelagem! [Wine and cheese: Ethnomathematics and modelling!]. *BOLEMA, 16*(20), 1-16.

Rosa, M., & Orey, D. C. (2007). Cultural assertions and challenges towards pedagogical action of an ethnomathematics program. *For the Learning of Mathematics, 27*(1), 10-16.

Rosa, M., & Orey, D. C. (2009). Challenges faced by multicultural and multilingual schools in the United States: The case of mathematics. *La Salle - Revista de Educação, Ciência e Cultura, 14*(1), 29-44.

Rosa, M., & Orey, D. C. (2010). Ethnomodeling: A Pedagogical Action for Uncovering Ethnomathematical Practices. *Journal of Mathematical Modelling and Application, 1*(3), 58-67.

Rosa, M., & Orey, D. C. (2016). Humanizing Mathematics through Ethnomodelling. Gerais Brasil. *Journal of Humanistic Mathematics*, *6*(2).

Soedjadi, R. (2000). *Kiat Pendidikan Matematika di Indonesia.* Jakarta: Direktorat Jenderal Pendidikan Nasional.

Sopandi. (2007). *Sejarah Arsitektur Sebuah Pengantar*. Jakarta: Gramedia.

Spradley, J.P. (1979). *The Ethnograpihic interview*. California: Wadsworth Publishing Company.

Suherman, E., *et al*. (2001). *Strategi Pembelajaran Matematika Kontemporer*. Bandung: JICA.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 59-68

DEVELOPING INTERACTIVE LEARNING MEDIA FOR SCHOOL LEVEL MATHEMATICS THROUGH OPEN-ENDED APPROACH AIDED BY VISUAL BASIC APPLICATION FOR EXCEL

Euis Eti Rohaeti, Martin Bernard, Restu Bias Primandhika

IKIP Siliwangi, Jalan Terusan Jendral Sudirman Cimahi
Email: e2rht@ikipsiliwangi.ac.id

Abstract

This research is a merged descriptive and quasi-experiment research aimed to describe students' creativity in developing interactive learning media with Visual Basic Application for Excel. Besides that, it had done to review students' ability which learning used open-ended Visual Basic Application for Excel compared with the other students' which learning used the expository method with Visual Basic Application for Excel based. The subject contained 35 students in experiment class and 33 students in the control class, where the creativity of their learning media is reviewed and is analysed using SPSS. The research result shows that: (1) Students' creativity in creating interactive learning media that the learning used open-ended Visual Basic Application for Excel based is better than the one with expository method of Visual Basic Application for Excel found; (2) Students creativity in creating interactive learning media is the combination of their mastery towards school level mathematics and the ability in applying Visual Basic Application for Excel.

Keywords: Creative thinking ability, Open-Ended approach, Visual Basic application for Excel

Abstrak

Penelitian ini merupakan gabungan penelitian deskriptif dan kuasi eksperimen yang bertujuan untuk mendeskripsikan kreativitas mahasiswa dalam mengembangkan media pembelajaran interaktif berbasis Visual Basic Application for Excel. Selain itu juga untuk menelaah kemampuan mahasiswa yang pembelajarannya menggunakan pendekatan open ended berbasis Visual Basic Application for Excel dibandingkan dengan yang menggunakan metode ekspositori berbasis Visual Basic Application for Excel. Subjek penelitian terdiri dari 35 mahasiswa kelas eksperimen dan 33 mahasiswa kelas kontrol, di mana media pembelajaran yang dihasilkan ditelaah kreativitasnya dan dianalisis dengan menggunakan SPSS. Hasil penelitian menunjukkan bahwa : (1) Kreativitas mahasiswa dalam membuat media pembelajaran interaktif yang menggunakan pendekatan open ended berbasis Visual Basic Application for Excel lebih baik daripada yang menggunakan metode ekspositori berbasis Visual Basic Application for Excel; (2) Kreativitas mahasiswa dalam membuat media pembelajaran interaktif merupakan kombinasi dari penguasaannya terhadap materi matematika sekolah dan keterampilannya dalam mengaplikasikan Visual Basic Application for Excel.

Kata Kunci: Kemampuan Berpikir Kreatif, Pendekatan Open-Ended, *Visual Basic Application for Excel*

How to Cite: Rohaeti, E.E., Bernard, M., & Primandhika, R.B. (2019). Developing interactive learning media for school level mathematics through open-ended approach aided by Visual Basic application for Excel. *Journal on Mathematics Education, 10*(1), 59-68.

School level mathematics material is still elementary but is an essential concept as the basic as the prerequisites of higher mathematical concept mastery. It is by the mathematics fact as a structured knowledge in which according to Tim MKPBM (2001), in mathematics contains some topic or concept as a prerequisite for the basic to understand the next topic or concept. It is like constructing a multi-level building; there will be no second and subsequent manifestations if the foundations of the future have not been well conditioned.

As a structured-concept science that has an order that learns about the regularity of pattern, organized, structure, and mathematical concept served from the simplest to the most complex. The mastery of a topic is the basic and prerequisites to understand another idea. So as to school mathematics, the topics are chosen and adjusted in a particular way with students' intellectual stage and are expected to be able to stimulate the developing of students' thinking ability stated the nature and characteristic of school mathematics as: (1) the activity of searching patterns and relation, implication in learning gives a chance to student to discover and investigate the pattern to determine relation; (2) the activity that gives student to do some trial in their own way so that they could explore creativity that requires imagination and intuition. The implication of this activity in a learning context is to motivate the student initiative and to give the chance to think in their way differently. (3) The solving-problem activity which implied in the learning is to motivate the student to think logically, consistently, systematically to develop documentation system (Depdikbud, 2013).

In fact, some research found that the mathematical ability of student especially in comprehension, communication, connection, problem-solving and students' mathematical reasoning is not at the optimal result (Bernard & Rohaeti, 2016; Fatah, Suryadi, Sabandar, & Turmudi, 2016; Hendriana, Rohaeti, & Hidayat, 2017; Kadir, Lucyana, & Satriawati, 2017; Rohaeti, Budiyanto, & Sumarmo, 2014; Siswono, 2010; Sumarmo, Kusnadi, & Maya, 2018). Other than that, based on the research results from Hendriana, et al. (2017) towards students in some remote areas in West Java, shows that the low mastery of mathematics students on the concepts of mathematical essence led to low mathematical abilities.

The studies reported the superiority of innovative learning than conventional teaching on improving students' ability (Happy & Widjajanti, 2014; Shahrill, Putri, Zulkardi, & Prahmana, 2018; Sumarmo, Hidayat, Zukarnaen, Hamidah, & Sariningsih, 2012). Mathematics learning media is a tool for the student to deliver mathematical materials to a real form with the basic form to understand the use of mathematics, which it is not merely to memorize. Something to be considered, for mathematics education students as a prospective teacher, regarding the media is to be able to give clear depiction from the most abstract form. It is according to Bernard & Rohaeti (2016) said that related to the main function of the learning media of mathematics using props must adapt to its purpose. One of the mathematical learning media is that gives effectively and more economical is the one with ICT aid. Meanwhile, the constraints from students are to use new mathematical software and the technique to use animation cannot be mastered yet. Nevertheless, Microsoft Excel is conventional software that most people use to process data and is related to the utilization of mathematical function.

The relation between mathematics learning with *Microsoft Excel* needs students' creativity to create some ways, one of those is the achievement for the student to develop mathematical programs to understand elementary school students about numbers, operation, and geometry (Chotimah, Bernard, & Wulandari, 2018). *Visual Basic for Excel* is a programming language from Microsoft Excel that can utilize a mathematical function that set figural objects, called "shape," to be more interactive. Students can bring up various ideas come to their mind using open-ended approach. The approach is useful when troubleshooting students about the images already provided in the "Forms" menu in Microsoft Excel. The first step of this learning started from the students. They had to ask the question from the specified picture and determine what to make about mathematical learning props as an initial creation planning. Secondly, students proposed solutions to make mathematical games. Lastly, students proposed solution after problem-solving with making new props creation plan or developing props (Silver & Cai, 1996).

METHOD

This research is a merged descriptive and quasi-experiment research aimed to describe prospective-teacher students' creativity in creating interactive learning media with Visual Basic Application for Excel. Besides that, the data was also obtained from two classes that had through different treatment in which the first class used Open Ended Visual Basic for Excel based (experiment class), involving 35 students. The other one used expository Visual Basic for Excel based (control class) including 33 students. Learning media produced by both classes is processed with SPSS to compare which classes have better creativity. The subjects in this research are the classes that got the subject ICT Application for mathematics learning.

RESULTS AND DISCUSSION

From the essential mathematical concept mastery test obtained from 68 students, the result shows 73.91% the students understand about arithmetical operation such as addition, subtraction, division, and multiplication. There are also 60.86% of students following the example picture or not creating on their own. 8.69% of students formed the other mathematical mastery such as geometry field, fraction, roots, exponents, any different than arithmetical operation resulted in 17.39% and calculator application related to the interactive pictures. These results describe that: (1) the making of arithmetical operation is more easily understood by students; yet, some errors found where students were not paying attention to the division. (2) Students could not be able to develop their creation or manifesting new ideas (3) Most of the students had difficulties in relating mathematical function to shapes form.

After the learning, some of the results shown students creative thinking ability at mathematical mastery and the other findings from students' new ideas from mathematics learning using *Visual Basic Application for Excel* through open-ended. 40% of students found to be mastered about the arithmetic operation, 45.7 % of students were able to relate and apply for the numbers in *Microsoft Excel* cells. 60% of students were able to use shapes to be given a command to run the result of the code they made using VBA for Excel. 20% of students can make games in *Microsoft Excel*. 51.43% of students were able to explain to create props using *Microsoft Excel*. 42.8% of the student was able to create props that related to geometry material. 28.57% of students were able to create props that related to arithmetic. 31.43% of students were able to develop interactive prop with new findings, and 20% of students create animation-based props.

As we can see from the data that from the beginning of the introduction of ICT Application in elementary school mathematics for elementary school, 73.91% of students preferred arithmetic operation after knowing *Microsoft Excel* function. 40% of them like arithmetic operation but students could relate 42.8% geometry like shapes and solids and 28.57% development of arithmetic concepts like fraction, exponents, and roots. There were also found the gain of creativity of their creation that initially 8.69% and 60.86% of students still depending on example picture and the making of VBA for Excel means 39.14% still depending on the example of *VBA for Excel*. After the learning, 31.43% of students could create new ideas and made their creation without the following the example, and 20% of students made their interactive animation.

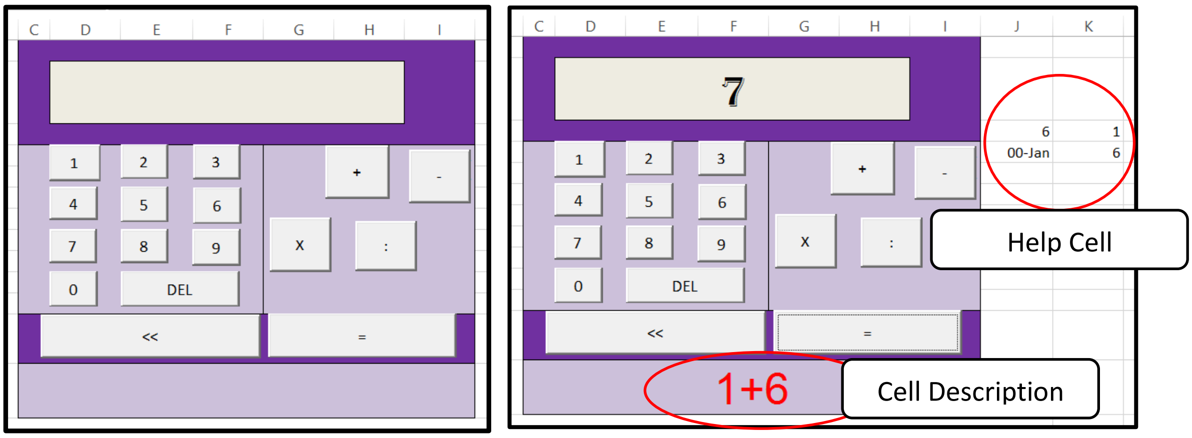
From the creation of making learning media between students with open-ended learning with ICT Application based (experiment class) and the one used ICT application only, it can be obtained that the creativity of students in making learning media shown as Table 1.

Table 1. Homogeneity Test and t-test Experimental and control classes

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Score	Equal variances assumed	.211	.647	3.941	60	.000	6.046	1.534	2.977	9.115
	Equal variances not assumed			3.917	57.143	.000	6.046	1.544	2.955	9.137

From the result of homogeneity test, it can be obtained the value of sig Levene's Test 0,647 greater than 0.05 which means the two homogenous data continue with the difference of two averages is using t-test and obtained the value of sig. 0,000 (2-tail) and $\frac{0,000}{2} = 0,000$ (1-tail) are less than 0.05 which means the creation of experiment class in making learning media is better than control class.

The implementation of making learning media with Visual Basic Application for Excel aims for students so that they have prerequisites to develop simple props. Students firstly must understand about numbers such as natural numbers, odd numbers, even numbers and prime numbers (Bernard & Rohaeti, 2016). After that, students are given the basic knowledge of making a simple calculator. Some of learning media made by the group of students can be described in Figure 1.



(a) (b)

Figure 1. The making of simple calculator using *Command Button* (a) and the result of calculator (b)

The student aided by cells J3 and J5 in calculator making process using *Visual Basic Application for Excel*. The reason behind this is to avoid the errors at subtraction operation. When students put the number of 9-6 in cells, it was detected as a date that shows "6 September," so that the label in the description box should be cleared in format to be set as a general format. However, the primary result showed no error. There could also be described that the students thinking process can be depicted in Figure 2.

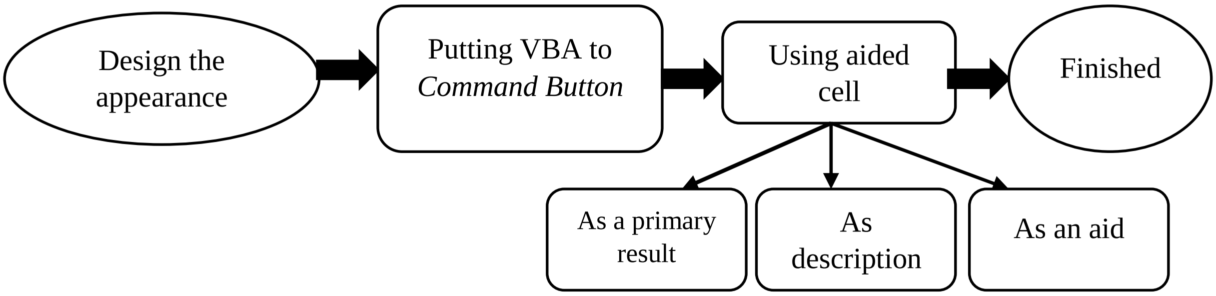
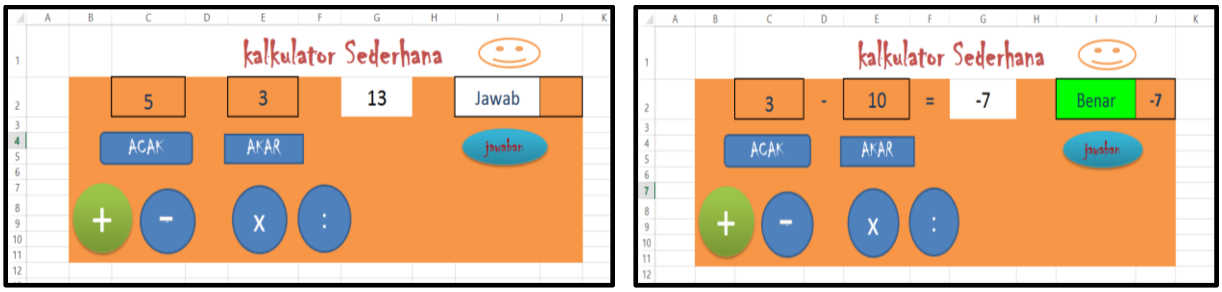


Figure 2. The steps of the first group's thinking process

The second group of students made the calculator using shapes form the "INSERT" menu. Students put their ideas to design VBA *for Excel* using *Shapes* or picture to *Microsoft Excel* for Arithmetical operation games. The result can be seen in Figure 3.



(a) **(b)**

Figure 3. The making of calculator using *Shapes* (a) and showing the result of the second student (b)

The calculator made by the second students' group is different with connecting VBA for Excel through *Macro* tool, but still, they used *DEVELOPER* tab menu. The way of connecting between *shapes* and VBA for Excel is to use *Assign Macro*. There were some flaws found in the second group's calculator that is the random button function was still not function properly as it was not correctly integrated to Randomize function. Consequently, when the random button was pressed, the numbers the number back into initial number repeatedly. The second weakness came when random button clicked the answer was not cleared from the cell. But the strength of this calculator was the interaction when students answer correctly or incorrectly. The second students' group thinking process can be depicted in Figure 4.

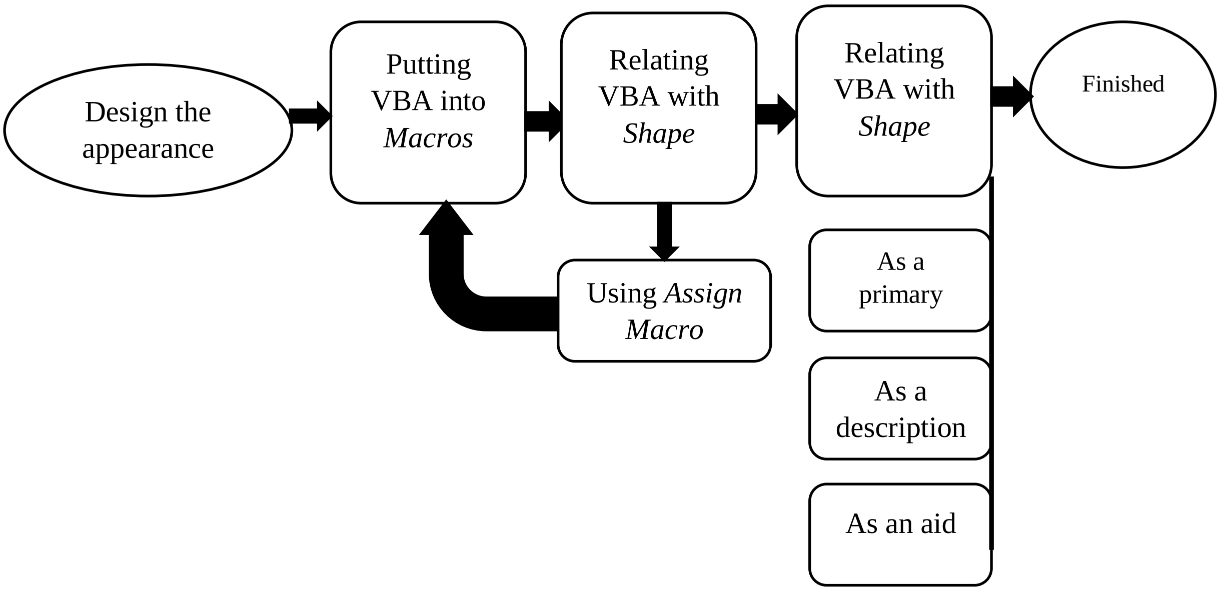


Figure 4. The steps of the second group's thinking process

The third group made a calculator using the relation of two positive numbers multiplication size with *Shape* as the area. The question from the third student was about how the result of the product determined interactively by a picture. Then, the third student made a design for the media and related it to VBA *for Excel*. The result of the third student can be seen in Figure 5.

(a) **(b)**

Figure 5. Making a command and to be commanded using *Shapes* (a) and the result of interactive *Shapes* using VBA *for Excel* (b)

The third student calculator tried to make the command button to give a command for being commanded button with putting the name in the first place in the *shapes* using *Selection Pane* from HOME menu and select. There is a command that relates the name to *Visual Basic Application for Microsoft Excel*. The weakness of this calculator was seen when the shape size was not changing if the value entered to the input cell. The third student's creative thinking process can be seen in Figure 6.

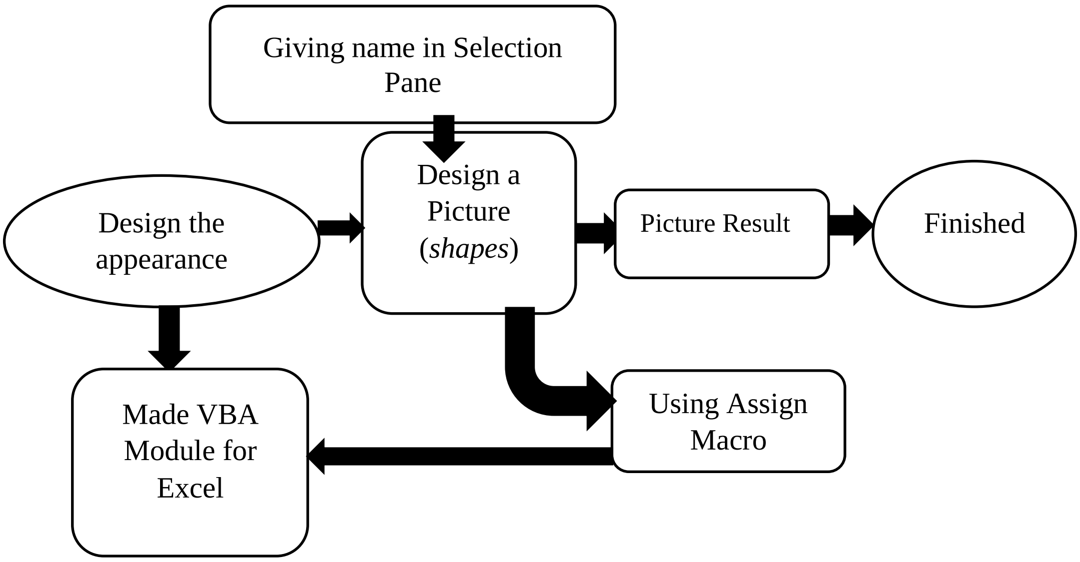


Figure 6. The steps of the third group's thinking process

The fourth students' group made a learning media using *jarimatika* (fingers of arithmetic) that can be described in Figure 7.

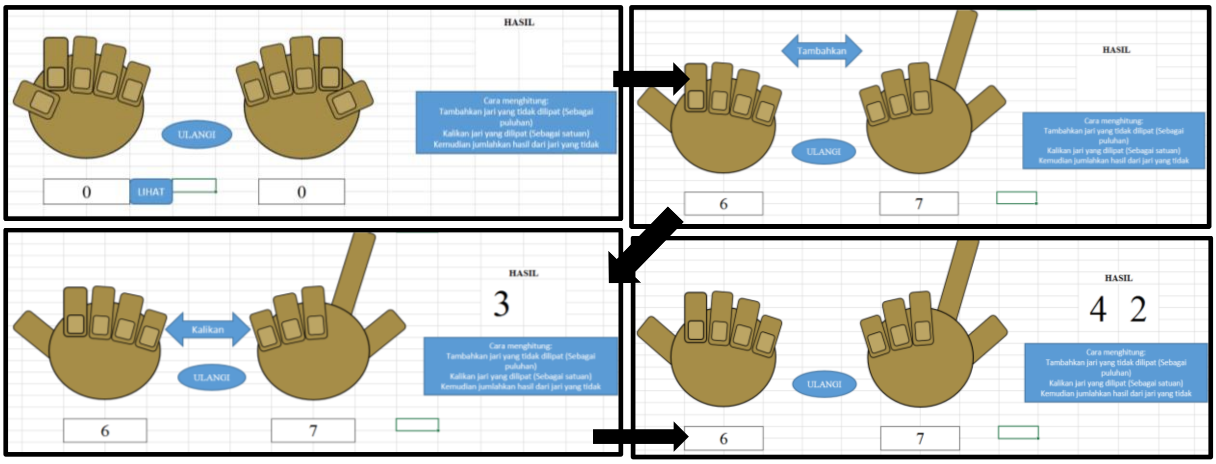


Figure 7. Students creative results of making *Jarimatika* props

The making of *jarimatika* props by the students are made to make multiplication easier that illustrated by fingers used in both two hands and are useful especially for second-grader students where the utilization of picture media in *Microsoft Excel* as a prop to master the multiplication. The sixth group of students used *Wayang Golek* (traditional puppet) Game to operate addition and subtraction of integers. The steps of making this *Wayang Golek* Games Excel (WAGGE) started by giving addition or subtraction sign of integers. Students were asked to click "-" sign. The second step is to press "2" button. The third step is to click "+" sign. The fourth step is to click "4." The fifth step is to look at the result on the black box. Student answered by typing numbers in *Microsoft Excel* cell. Then student choose one of the four Wayangs that they like while seeing the name of *Wayang* in the box (Figure 8).

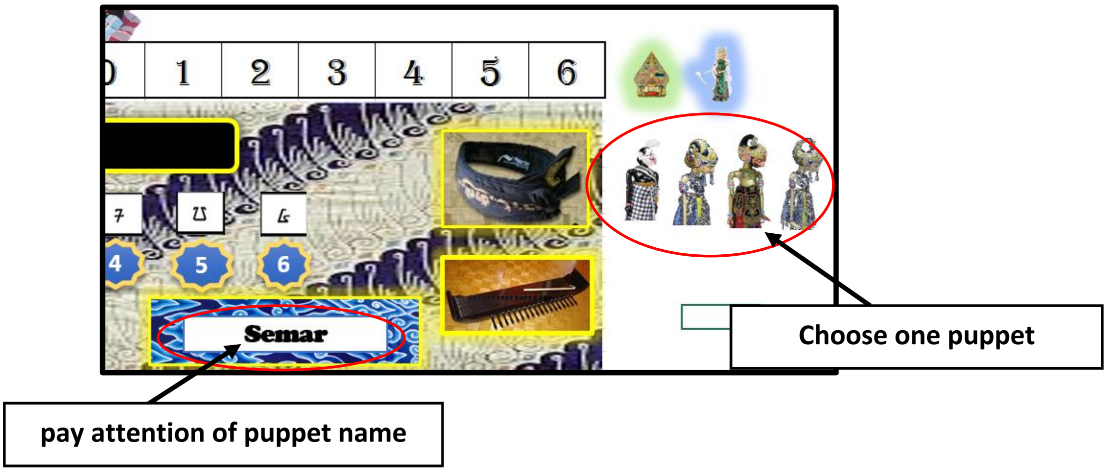


Figure 8. The preview of *Wayang Golek* Games Excel learning media

And then they started the game with the rules 1) when the number is positive, the *wayang* would face right side, if negative, the *wayang* turned to the left side. If it comes to addition, the *wayang* would move forward, and it would move backward if it is subtracted. The process is done the image that determines the answer was correct or incorrect appeared. It is shown in Figure 9.

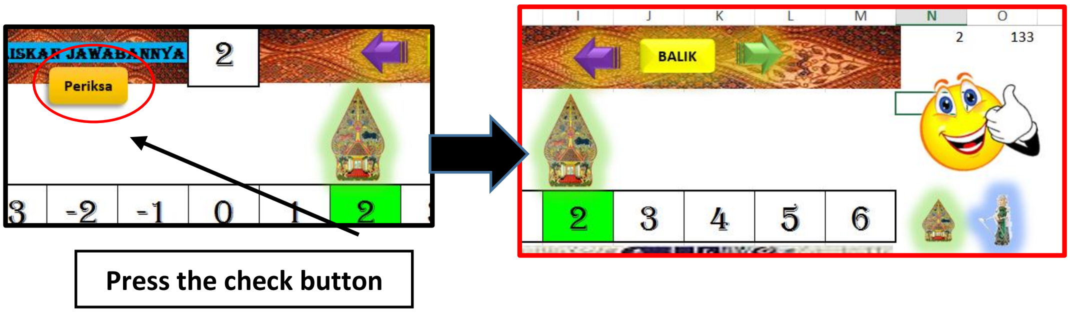


Figure 9. The final page of WAGGE

From the analysis result towards the learning media created by the group of students, there are various ways to produce the application results understanding of mathematical material into *Microsoft Excel* with the aid of *Visual Basic for Excel* to be a usable prop for students in school. The combination of students' mastery towards school mathematics and the ability to make a learning media with the aid of *Visual Basic Application for Excel* is a good creative thinking process. The creative thinking is a process to make something involving elements and experiences that currently exist to be processed in the brain to create brand new things (Bernard & Rohaeti, 2016; Fatah, et al., 2016; Hendriana, et al., 2017; Kadir, et al., 2017; Rohaeti, et al., 2014; Siswono, 2010; Sumarmo, et al., 2018).

Coleman & Hammen (1990) that stated that the supporting factors of one's thinking ability besides ones' cognitive are the open attitude towards the internal and external stimulus, free attitude in thinking and expressing, the independent thinking that has no bound towards authority and existing social convention and most importantly is the believe of oneself.

CONCLUSION

The creativity of students using interactive learning media using open-ended learning with Visual Basic Application for Excel based is better than the one using expository Visual Basic Application for Excel based. Furthermore, the creativity of students in creating learning media is a combination of their school mathematical mastery and their creativity in applying Visual Basic Application for Excel.

REFERENCES

Bernard, M., & Rohaeti, E. E. (2016). Meningkatkan Kemampuan Penalaran dan Disposisi Matematik Siswa Melaui Pembelajaran Kontekstual Berbantuan Game Adobe Flash CS 4.0 (STL-GAF). *Edusentris*, *3*(1), 85-94.

Chotimah, S., Bernard, M., & Wulandari, S. M. (2018). Contextual approach using VBA learning media to improve students' mathematical displacement and disposition ability. *Journal of Physics: Conference Series, 948*(1), 012025.

Coleman, J. J. C., & Hammen, C. L. (1960). *Contemporary psychology and effective behavior*. Scott Foresman.

Depdikbud (2013). Kurikulum Sekolah Menengah tahun 2013. Jakarta: Depdikbud

Fatah, A., Suryadi, D., Sabandar, J., & Turmudi, T. (2016). Open-ended approach: an effort in cultivating students' mathematical creative thinking ability and self-esteem in mathematics. *Journal on Mathematics Education*, *7*(1), 9-18.

Happy, N., & Widjajanti, D. B. (2014). Keefektifan PBL ditinjau dari kemampuan berpikir kritis dan kreatif matematis, serta self-esteem siswa SMP. *Jurnal Riset Pendidikan Matematika*, *1*(1), 48-57.

Hendriana, H., Rohaeti, E. E., & Hidayat, W. (2017). Metaphorical Thinking Learning and Junior High School Teachers' Mathematical Questioning Ability. *Journal on Mathematics Education*, *8*(1), 55-64.

Kadir, K., Lucyana, L., & Satriawati, G. (2017). The implementation of open-inquiry approach to improve students' learning activities, responses, and mathematical creative thinking skills. *Journal on Mathematics Education*, *8*(1), 103-114.

Rohaeti, E. E., Budiyanto, A. M., & Sumarmo, U. (2014). Enhancing students'mathematical logical thinking ability and self-regulated learning through problem-based learning. *International Journal of Education*, *8*(1), 54-63.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education*, *27*(5), 521-539.

Shahrill, M., Putri, R. I. I., Zulkardi, & Prahmana, R. C. I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings*, *1952*(1), 020019.

Siswono, T. Y. E. (2010). Leveling students’ creative thinking in solving and posing mathematical problem. *Journal on Mathematics Education*, *1*(1), 17-40.

Sumarmo, U., Hidayat, W., Zukarnaen, R., Hamidah, M., & Sariningsih, R. (2012). Kemampuan dan disposisi berpikir logis, kritis, dan kreatif matematik (eksperimen terhadap siswa sma menggunakan pembelajaran berbasis masalah dan strategi think-talk-write). *Jurnal Pengajaran MIPA*, *17*(1), 17-33.

Sumarmo, U., Kusnadi, A., & Maya, R. (2018). Mathematical critical thinking ability and disposition (Experiment with vocational high school students using scientific approach). *Journal of Educational Experts (JEE)*, *1*(2).

Tim MKPBM. (2001). *Strategi Pembelajaran Matematika Kontemporer*. UPI: JICA.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 69-80

MATHEMATICAL CONNECTION OF ELEMENTARY SCHOOL STUDENTS TO SOLVE MATHEMATICAL PROBLEMS

Ary Kiswanto Kenedi, Yullys Helsa, Yetti Ariani, Melva Zainil, Sherlyane Hendri

Universitas Negeri Padang, Jl. Prof. Dr. Hamka Air Tawar, Padang 25171, Indonesia
Email: arykenedi@fip.unp.ac.id

Abstract

This study aims to determine the mathematical connection ability of elementary school students in solving mathematical problems. This research uses the qualitative approach with descriptive method. We used observation, interview, and test to collect data. This study shows that 6.67% of the samples scored between 60 and 69 in the fair category. A total of 98 students or 81.67% scored between 45 and 59 who belonged to the below average category, and 14 students or 11.67% scored between 0 and 44 which were classified as a poor category. This result proves that the mathematical connection ability of elementary school students in solving mathematics problems is still low.

Keywords: Ability, Elementary school, Mathematical connection, Mathematical problems.

Abstrak

Penelitian ini bertujuan untuk mengetahui kemampuan koneksi matematis siswa sekolah dasar dalam memecahkan masalah matematika. Penelitian ini menggunakan pendekatan deskriptif kualitatif. Metode pengumpulan data dalam penelitian ini adalah observasi, wawancara, dan tes. Penelitian ini menunjukkan bahwa sebanyak 8 orang siswa atau 6,67% dari sampel penelitian mempunyai nilai di antara 60 dan 69 yang termasuk kategori cukup. Sebanyak 98 siswa atau 81,67% mempunyai nilai di antara 45 dan 59 yang termasuk kategori kurang. Sedangkan 14 siswa atau 11,67% mempunyai nilai di antara 0 dan 44 yang termasuk kategori sangat kurang. Hal ini membuktikan bahwa kemampuan koneksi matematika siswa sekolah dasar dalam memecahkan masalah matematika masih rendah.

Kata kunci: Kemampuan, Koneksi matematis, Masalah matematika, Sekolah dasar.

How to Cite: Kenedi, A. K., Helsa, Y., Ariani, Y., Zainil, M., & Hendri, S. (2019). Mathematical connection of elementary school students to solve mathematical problems. *Journal on Mathematics Education*, *10*(1), 69-80.

Mathematics is a compulsory subject in elementary school. Mathematics learning in primary schools has a useful purpose for the life of a country. The Indonesian Ministry of National Education (MONE) argues that mathematics learning can equip students with logical, analytical, systematic, and creative thinking skills (BSNP, 2006). Moreover, the MONE states that the purpose of mathematics learning in elementary school is to enable students to understand the concept of mathematics. Students are expected to explain the relationship among one concept with the others, to use the right concept to solve problems, to make sense of patterns and traits, to manipulate mathematics in making reports, and to explain mathematical ideas (BSNP, 2006).

Based on the purpose of mathematics learning in elementary school, we can conclude that mathematics learning is very important for elementary school students since it is closely related to the daily life (Hamidah & Chotimah, 2015). The elementary school mathematics learning can equip elementary school students to think logically, analytically, systematically, critically and can cooperate with others. Therefore, it is necessary to qualified mathematics learning such that the students could not only memorize or use the existing formula but also connect the material learned at school with the real life or vice versa. Mathematics learning should avoid the use of traditional learning methods that lead to convergent thinking in which students only remember mathematical theorems and rules to solve problems. What often happens in mathematics learning is students are given the closed problem to be solved (Maharani, Sukestiyarno, & Waluya, 2017).

Mathematics is a subject that links concepts (Nurhasanah, Kusumah, & Sabandar, 2017). Students' ability to understand the concept is the ability to not only know or remember some concepts learned but also able to express again in other forms that are easy to understand, provide interpretation of data, and able to apply the concept with its cognitive structure (Ulia, 2015; Istihapsari, 2017). Concepts in learning mathematics are interconnected with each other. When the students learn a concept, then they need to learn the other ones. This state is called a mathematical connection, the ability of students in connecting a concept with the other concepts.

The mathematical connection is the ability to associate students' mathematical knowledge with other mathematical skills and real life (Bahr & DeGarcia, 2008). Mathematical connections are part of a network of interconnected knowledge with other knowledge composed of critical concepts to understand and develop relationships between mathematical ideas, concepts, and procedures. The ability of students to connect mathematically is one of the essential things that must be achieved by students in the learning process because if students know the relationship between the concepts, they will quickly understand the mathematics itself and open opportunities for students to develop their mathematical skills.

The National Council of Teachers of Mathematics states that there are five standards of mathematics learning, namely communication, reasoning and proof, representation, connections, problem-solving (Rickard, 2005). Thus, mathematical connections need to be taken into consideration in the mathematics learning process especially in elementary schools. In mathematics learning, the mathematical connection should be developed since elementary school age. The mathematical connection ability emerges when the students can connect between one material to the others. Students can communicate the concepts that they learn because they have mastered the prerequisite materials related to daily life. If the students can connect the material which they learn from the previous subject or with other subjects, then the learning of mathematics becomes more meaningful (Linto, 2012).

The mathematical connection is vital to be developed in the students because it will help students in understanding a concept and can improve their understanding of other science by connecting the concept of mathematical concepts with other concepts (Hendriana, 2014). Also, the ability of the mathematical connection needs to be widely developed as it can increase students' cognitive by remembering a concept, understanding and applying the concept in daily life, without which students will find it challenging to learn mathematical concepts (Siregar & Surya, 2017). Thus, the mathematical connection should be developed since elementary school age.

However, many argue that the mathematical connections of elementary school students are still low. Hermawan & Prabawanto (2015) found that the mathematical connection ability of elementary school students is still low with a score of 10.87 from a maximum value of 24.00. It is caused by the assumption of elementary school students that learning mathematics as problematic and unpleasant learning and also this is caused by teachers who carry out the learning process using the lecture method (Hermawan & Prabawanto, 2015). Also, the ability of mathematical connections of elementary school students using conventional methods is still in the low category; this proves that students' mathematical communication skills need to be improved (Putri, Rahayu, Saptini, & Misnarti, 2016). The research proves that the mathematical connections of elementary school students in Indonesia are still low.

The lack of mathematical connections of elementary school students influences the ability to solve mathematics problems in primary schools. The process of solving the problem requires an effort to establish a connection between the stages of problem-solving (Tasni & Susanti, 2017). Based on this idea, the mathematical connections of students cannot be separated from the attempt to solve mathematical problems. The process of solving mathematical problems is the activity of students who can build mathematical connections of students; this happens because in solving the problems of mathematics students must have the ability to find the linkage of concepts or theorems used to solve a problem (Siregar & Surya, 2017). Our analysis of the above opinion results that the ability of mathematical connections has a relationship in solving mathematical problems related to everyday life. Mathematical connections can improve thinking skills in solving mathematics problems.

To know the conditions in the field about the ability of mathematical connections in primary school, we conducted interviews with elementary school teachers in X Koto Singkarak District. A teacher said that he did not know about the connections of the five basic skills in mathematics learning, especially regarding mathematical connections. The teacher only knows the problem solving while the other skills the teacher does not know. The teacher only gave mathematics problems when there are basic competencies related to problem-solving. Many teachers implement the learning process by using the direct method. This phenomenon is due to lack of media for the learning process of mathematics. We also interviewed a teacher from a different school who claim that the term of mathematical connection is not familiar, only the term problem-solving is familiar to the common teacher. Teachers rarely provide material related to solving mathematics problems. Teachers occasionally use demonstration methods as an effort to improve the quality of mathematics learning. Teachers also stated that the low learning outcomes of students.

The results of our analysis of the interview are the lack of information by teachers on basic skills in learning mathematics in primary schools, while in improving the quality of mathematics learning should master the skill of the skill. Teachers do not know the benefits of mathematical connections, while the benefits of mathematical connection ability one of them can improve the learning of mathematics, both cognitive, affective, and psychomotor. Also, the teacher lack strategies for improving the quality of mathematics learning in primary schools, especially in improving mathematical connections. Teachers do not connect the learning materials into the student's life. There is no effort to create a learning process that uses mathematical connection skills in solving mathematical problems. Based on the problems, we are interested in analyzing the mathematical connection ability of elementary students in X Koto Singkarak District of Solok regency in problem-solving.

METHOD

This research uses a qualitative approach. The method used was descriptive research using observation, interview, and test. Descriptive method is a method of research by collecting actual data. After the data collected, the data are prepared, processed, and analyzed to be able to provide an overview of existing problems. The population in this research is the third-grade students of public elementary schools in X Koto Singkarak District, Indonesia, in the 2017/2018 academic year. The school is grouped into five clusters by region. Each region is taken one elementary school to represent the region. Sampling on this research is using cluster sampling technique.

This research is qualitative, so the analysis used is a descriptive and interpretative analysis done since data collection is done. Analysis of test results is used to determine and obtain the level of mathematical connection ability of students in solving mathematical problems. To simplify the research, we undertook the research steps: (1) designing the research; (2) determining the location and subject to be studied; (3) contacting the research site; (4) preparing the research instruments; (5) validating instruments and revising them; (6) collecting data by using observation and test, (7) analyzing the data obtained and summing up the results of research and presenting the data.

RESULT AND DISCUSSION

The first step in this research is designing the research include determining methodology used is using the qualitative approach with observation, interview, and test. After designing the research, the next step is to determine the sample and location of the subject under study. Samples were collected using cluster sampling technique representing each cluster in elementary schools located in X Koto Singkarak district, namely SDN 03 Tikalak, SDN 10 Saning Bakar, SDN 13 Sumani, SDN 04 Tanjung Alai, and SDN 15 Kacang.

The next step is to design the research instrument. The design of the problem to measure the ability of mathematical connections is tailored to the learning materials that are taught in the third grade of elementary school. The item was designed to measure success in achieving learning objectives. This statement is in line with the opinion of Adedoyin (2010) that teachers' questions are of significant values for many instructional purposes, eliciting student reflection and challenging students’ understanding deeper and engagement in the classroom.

The problem is matched to the mathematical connection indicator that students should be able to recognize and utilize the relationship between ideas in mathematics. Students also need to understand how ideas in mathematics interconnect and underlie each other to produce a coherent unity, and apply mathematics in daily life and adjusted to the scope of subjects on elementary school education units namely numbers, geometry and measurement. Churchill states that there are two ways to propose an item. First, determine the question that must be determined its validity and second is to scale the item by way of reviewing the literature (Rahi, 2017). Therefore, the designed instrument is tailored to the literature review and validated by an expert mathematician of elementary school and an expert in the oral language of elementary school to be corrected whether the question is worth to be used in measuring the mathematical connection ability in solving mathematical problems. To see the validation results from mathematics expert, we can see Table 1.

Table 1. Comments Expert of Mathematics of Elementary School

Comment	Revision
The problems designed to represent the learning materials have been in line with the mathematical connection indicators. However, there were some questions such as the number x and x not used because it is not relevant to the indicator mathematics connection, namely understanding how ideas in mathematics interconnected and underlie one another to produce a coherent unity.	We follow the expert's advice not to include problems Number 3 and Number 7 because they were not relevant to the second indicator. Deleting these two problems do not affect other problems because some other problems already represented the second indicator.

Meanwhile, we also addressed the validation to the language expert. It is important to make sure that the message in the problem could be delivered properly to the students. The result of validation can be seen in Table 2.

Table 2. Comments Expert Language of Elementary School

Comment	Revision
So that the problem of 2, 4 and six the language is more simplified because the problem used less concrete terminology	Simplify the sentence of the problem

The next step was giving the question to the school that has been selected. The school has 120 students. We administered some tests in 60 minutes to the students. In the test, two questions represent two questions represent one indicator of mathematical connection. Furthermore, the data is processed and analyzed based on the assessment rubric.

The first problem is adapted to the mathematical connection indicator that students should be able to recognize and utilize the relationship between ideas in mathematics. The problem is written in Problem 1.

Today Andy and family will be recreation. Andy's mother brought eight packets of sweets to consume during the trip. Each candy wrap contains nine candy bars., Brother Andy brought 15 candies. How many candies are brought by Andy's family?

In Problem 1, the student is asked to be able to recognize and utilize the relationship between ideas in learning mathematics. Problem 1 requires some mathematical concepts and links between the concepts to answer the question. Students are required to be able to understand and recognize problems, to try to relate those problems to some known mathematical concepts. Problem 1 requires knowledge of students in recognizing a problem. After that, students are required to associate the multiplication concept and addition. Based on the analysis result, from 120 student samples, only 21 students or 17,5% get score 3, 74 students or 61,67% student get score 2, and 21 students or 17,5% get score 1, four students or 3.34% earned score 0.

From the results, we can conclude that elementary school students are still low in recognizing and utilizing the relationship between ideas in learning mathematics. NCTM states that when students can connect mathematical ideas, students' understanding becomes more profound and more enduring, (Rismawati & Irawan, 2016). In addition to learning by knowing relationships mathematically, students will better understand mathematics and also give them greater mathematical power (Romli, 2016). Therefore, this problem is expected in elementary school students can use ideas and concepts in mathematics to solve problems.

The next question represents mathematical communication indicators in solving mathematical problems of understanding how ideas in mathematics interconnect and underlie each other to produce a coherent unity. The problem is written in Problem 2.

David experimented on the growth of green beans. On the first day of green beans grown along 1 centimeter, David's second day measured to 3 centimeters, on the third day to 5 centimeters. What is the estimated growth on the fourth day?

In Problem 2, the students are asked to be able to connect with other concepts that result in a coherent unity. Problem 2 contains the meaning that the learning of mathematics is associated with science learning so that it becomes a whole and coherent matter so that students can associate the concept of mathematics in other learning without separating a mathematical concept with other concepts. In this problem, no students get scores 3. There are 23 students, or 19.17% of students get a score of 2, 15 students or 12.5% of students get a score of 1, and 82 students or 68.34% of students get a score of 0. The question proves that the weakness of elementary school students in connecting mathematical material into another fundamental science as a coherent unity.

Mathematics is the source of other sciences, that is to say, the many sciences that discovery and development depend on mathematics so that mathematics courses are useful for learners as the virgin for application in other fields (Sholihah, 2015). Besides, mathematics as the science of logic about form, structure, quantity, and concepts related to each other. Concepts in mathematics related to concepts outside mathematics. Many theories and other branches of science are found through the concept of mathematics. Thus, learning mathematics is related to other learning. Therefore, if the students could connect one concept in mathematics with the concept beyond mathematics, then they could connect mathematics well. However, based on the results of research that elementary school students are still weak in connecting the concept in mathematics and outside mathematics in a comprehensive way.

Problem 3 represents the indicators of recognizing and applying mathematics in daily life. The problem is:

My family and I leave at 7.30 from Solok to Pekan Baru. After walking three and a half hours, we rest for lunch for 30 minutes in Bukittinggi. Then we continue the trip for 3 hours. At what time did we get to Pekanbaru?

In Problem 3, students are asked to be able to recognize mathematical problems and apply them in everyday life. In Problem 3, students are required to understand the immediate problems in their life and can solve the problem in the mathematical model. In this problem, as many as 34 students or 28.34% of students get a score of 3, as many as 56 students or 46.67%. Problem 3 illustrated that the ability of students in solving math problems in everyday life is still low. Mathematics learning is closely related to daily life. Mathematics is one of the essential lessons mastered by elementary school students because of its many uses in everyday life. Mathematics was not only arithmetic but also contribute to life value which was necessary for people life (Soeprianto, 2009).

Problem 3 can prove that the application of primary school students' mathematics learning is still low. Whereas helping students learn to make connections between various forms of mathematical knowledge, as well as between mathematics and real-life experience, is increasingly recognized as integral to effective mathematics learning and teaching with students able to apply the learning mathematics in everyday life. Mathematics is one of the disciplines that can improve thinking ability and contribute to daily problem solving and in the world of work and provide support in the development of science and technology (Susanto, 2013).

After the analysis of the grading problem as a whole, there were eight students or about 6.67% of the sample scored between 60 to 69 which belongs to the fair category. A total of 98 students or 81.67% scored between 45 and 59 who belonged to the below average category, and 14 students or 11.67% scored between 0 and 44 who were classified as a poor category. These results prove that the highest student presentation is in a low category. Thus, the mathematical connection ability of elementary school students in solving math problems is still low.

We conducted interviews with elementary school students. We questioned a mathematical case like " *Wilda bought a book for Rp. 7,500.00. He pays by using a sheet of five thousand and two thousand two. How much change will Wilda accept* ?". When we asked that question, elementary school students answered with hesitation, " *maybe the answer is Rp. 1000,00 sir* ". Our analysis of the answers is that students are still unable and accustomed to associate ideas in mathematics learning in primary schools.

We conducted interviews with other students, giving the following questions "*Deni experiments on ping pong balls and golf balls. Both balls have almost the same shape. However, when the ball is dropped to the ground, the first ball to the ground is a golf ball. To determine the cause of Deni try to observe from the shape and weight. What tool does Deni need to know the weight of both balls?*". The question was answered by the student "*using a scales maybe, sir*." Our analysis of the student's answer is that the students are still unable to understand the problem and have not been able to link the learning of mathematics with other learning.

The next sample interview is with different students. We asked "*if Garden Mr Alfi be a square shape with an area of 100 meters. How long the length of the side of the garden*?". Then the students start thinking and start looking for papers to do the calculations. After a long time the students answered "*25 sirs*", then we asked again. "*25 what*?". Students answer "*25 sirs*". Our analysis of the process is that students have been able to recognize and apply mathematics in their daily life but still unable to answer correctly.

From the analysis we showed that elementary school students are still weak in relating ideas in mathematics learning in primary schools, understanding how ideas in mathematics interconnect and underlie each other to produce a coherent unity as well recognize and apply mathematics in everyday life.

Based on the observation we have done to the SDN 03 Tikalak when the teacher performs the learning process, the teacher only taught mathematics by explaining the material on the blackboard after which the students were asked to do the exercises. When we did observations in SDN 10 Saniang Baka, we also found the same problem that was the teacher explains the material by providing training to students. We also do observations in SDN 13 Sumani found the teacher explains the concept of mathematics by using learning media but the teacher only limited to explain the media without any attempt to associate the learning with mathematical concepts and concepts outside mathematics. In SDN 04 Tanjung Alai also found that teacher did not attempt to improve the ability of mathematical connections of their students, to embed mathematics concept but still used conventional methods. Observations that we did in the SDN 15 Kacang find things that were not much different than teachers teach using the lecture method and still provide practice to testing the understanding of the concept.

Mathematical connections in solving mathematical problems need to be developed since elementary school age. The ability of mathematical connections is important because it has the same properties with the systematic and structured science that contains concepts related to each other (Hendriana, Slamet, & Sumarmo, 2014). Besides, mathematical connections also help students in seeing the relationships of math with everyday life. Mathematical connections can make students understand a concept and assist students in improving the understanding of mathematical concepts. Also, mathematical connections help students in providing mathematical models that connect between concepts, data, and situations (Agustini, Suryadi, & Jupri, 2017).

However, what happened in the field is the mathematical connection ability of students in solving math problems is still low. The low ability of mathematical connection causes a lack of meaning and interest of students to learn mathematics. It can be seen from great fear and the lack of attention of students in learning mathematics (Kartikasari & Widjajanti, 2017). The low mathematical connections of elementary school students in solving mathematical problems will have an impact on the learning process and outcomes.

CONCLUSION

In this research, the mathematical connection ability of elementary school students in solving mathematical problems is measured through the problems made based on mathematical connection indicator which is adjusted with the learning material in class III. These indicators include recognizing and exploiting the relationship between ideas in mathematics, understanding how ideas in mathematics interconnect and underlie each other to produce a coherent unity, and recognize and apply mathematics in everyday life. From the results of the study, we conclude that the mathematical connection ability of elementary school students is categorized as low.

ACKNOWLEDGEMENTS

Thanks to Universitas Negeri Padang which has supported this research and all research participants.

REFERENCES

Adedoyin, O. (2010). An investigation of the effects of teachers' classroom questions on the achievements of students in mathematics: Case study of Botswana community junior secondary schools. *European Journal of Educational Studies*, *2*(3).

Agustini, R. Y., Suryadi, D., & Jupri, A. (2017). Construction of open-ended problems for assessing elementary student mathematical connection ability on plane geometry. *Journal of Physics: Conference Series*, *895*(1), 012148.

Bahr, D. L., & DeGarcia, L. A. (2008). Elementary mathematics is anything but elementary: Content and methods from a developmental perspective. Belmont, CA: Cengage Learning.

BSNP. (2006). *Kurikulum tingkat satuan pendidikan*. Jakarta: BSNP Departemen Pendidikan Nasional.

Hamidah & Chotimah, S. (2015). Pengaruh model pembelajaran Van Hiele terhadap kemampuan koneksi matematis siswa SMP. *Jurnal Ilmiah UPT P2M STKIP Siliwangi*, *2*(2), 203-208.

Hendriana, H., Slamet, U. R., & Sumarmo, U. (2014). Mathematical connection ability and self-confidence (An experiment on junior high school students through contextual teaching and learning with mathematical manipulative). *International Journal of Education*, *8*(1), 1-11.

Hermawan, D., & Prabawanto, S. (2016). Pengaruh penerapan model pembelajaran problem based learning berbantuan media teknologi informasi dan komunikasi terhadap kemampuan koneksi matematis siswa sekolah dasar. *Eduhumaniora: Jurnal Pendidikan Dasar*, *7*(1).

Istihapsari, V. (2017). Meningkatkan pemahaman konsep materi matematika SMP menggunakan model pembelajaran kooperatif tipe Jigsaw pada mahasiswa prodi pendidikan matematika UAD. *AdMathEdu: Jurnal Ilmiah Pendidikan Matematika, Ilmu Matematika dan Matematika Terapan*, *7*(1), 83-98.

Kartikasari, A., & Widjajanti, D. B. (2017). The effectiveness of problem-based learning approach based on multiple intelligences regarding student’s achievement, mathematical connection ability, and self-esteem. *Journal of Physics: Conference Series*, *812*(1), 01209.

Linto, R. L. (2012). Kemampuan koneksi matematis dan metode pembelajaran quantum teaching dengan peta pikiran. *Jurnal Pendidikan Matematika*, *1*(1), 83-87.

Maharani, H. R., Sukestiyarno, S., & Waluya, B. (2017). Creative thinking process based on Wallas model in solving mathematics problem. *International Journal on Emerging Mathematics Education*, *1*(2), 177-184.

Nurhasanah, F., Kusumah, Y. S., & Sabandar, J. (2017). Concept of triangle: Examples of mathematical abstraction in two different contexts. *International Journal on Emerging Mathematics Education*, *1*(1), 53-70.

Putri, H. E., Rahayu, P., Saptini, R. D., & Misnarti, M. (2016). Keterkaitan penerapan pendekatan CPA dan peningkatan kemampuan koneksi matematis siswa sekolah dasar. *Metodik Didaktik*, *11*(1), 41-49.

Rahi, S. (2017). Research design and methods: A systematic review of research paradigms, sampling issues and instruments development. *International Journal of Economics & Management Sciences*, *6*(2), 1-4

Rickard, A. (2005). Constant perimeter, varying area: A case study of teaching and learning mathematics to design a fish rack. *Journal of American Indian Education*, *44*(3), 80-100.

Rismawati, M., & Irawan, E. (2016). Analisis kesalahan koneksi matematis sisa pada materi sistem persamaan linier dua variabel. *Proceedings of the National Conference on Research on Mathematics and Learning (KNPMP I)*. Surakarta: Universitas Muhammadiyah Surakarta.

Romli, M. (2016). Profil koneksi matematis siswa perempuan SMA dengan kemampuan matematika tinggi dalam menyelesaikan masalah matematika. *MUST: Journal of Mathematics Education, Science, and Technology*, *1*(2), 144-163.

Sholihah, D. A., & Mahmudi, A. (2015). Keefektifan experiential learning pembelajaran matematika MTs materi bangun ruang sisi datar. *Jurnal Riset Pendidikan Matematika*, *2*(2), 175-185.

Siregar, N. D., & Surya, E. (2017). Analysis of students’ junior high school mathematical connection ability. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, *33*(2), 309-320.

Soeprianto, H. (2009). Penerapan pembelajaran nilai-nilai yang terintegrasi dalam mata pelajaran matematika. *Educatio*, 4(2), 28-37.

Susanto, A. (2013). *Teori Belajar & Pembelajaran di Sekolah Dasar*. Kencana: Jakarta.

Tasni, N., & Susanti, E. (2017). Membangun koneksi matematis siswa dalam pemecahan masalah verbal. *Beta Jurnal Tadris Matematika*, *10*(1), 103-116.

Ulia, N. (2015). Peningkatan pemahaman konsep matematika materi bangun datar dengan pembelajaran kooperatif tipe group investigation dengan pendekatan saintifik di SD. *Jurnal Tunas Bangsa*, *1*(1), 55-68.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 81-92

LONG JUMP IN ASIAN GAMES: CONTEXT OF PISA-LIKE MATHEMATICS PROBLEMS

Ika Pratiwi, Ratu Ilma Indra Putri, Zulkardi

Universitas Sriwijaya, Jalan Srijaya Negara, Bukit Besar, Palembang, Indonesia
Email: ika06pratiwi.ip@gmail.com

Abstract

This study aimed to produce a set of PISA-like mathematics problems content of uncertainty and data using long jump context in Asian Games that were valid, practical and had potential effects on the ability of the students' mathematical literacy. This study used the design research method of development studies type in two stages, preliminary and formative evaluation. The subject of this study involved 34 students of Senior High School. The result of this study was a set of PISA-like mathematics problems using long jump context in Asian Games which were valid, practical, and had potential effects. The result of students' answer analysis showed some strategies and arguments used by students. In question 1, 3 out of 34 students were involved in representation, reasoning and argument ability, 31 out of 34 students were involved in reasoning and argument ability. In question 2, 30 out of 34 students were involved in representation and reasoning and argument ability, and 4 out of 34 students were involved in communication, representation, reasoning and argument ability. Also, using the PISA-like mathematics problems with long jump context made students more interested and active during the learning process.

Keywords: Design research, PISA-like mathematics problems, Asian Games

Abstrak

Penelitian ini bertujuan untuk menghasilkan seperangkat soal matematika tipe PISA konten ketidakpastian dan data menggunakan konteks lompat jauh di Asian Games yang valid, praktis, dan memiliki efek potensial terhadap kemampuan literasi matematika siswa. Metode penelitian ini adalah *design research* tipe *development studies*yang terdiri dari dua tahap, yaitu tahap *preliminary* dan tahap *formative evaluation*. Subjek penelitian ini melibatkan 34 siswa Sekolah Menengah Atas. Hasil penelitian ini adalah seperangkat soal matematika tipe PISA menggunakan konteks lompat jauh di *Asian Games* yang valid dan praktis serta memiliki efek potensial. Hasil analisis siswa menunjukkan beberapa strategi dan argumen yang digunakan siswa. Pada soal 1, terdapat 3 dari 34 siswa yang melibatkan kemampuan representasi dan penalaran dan argumen dan 31 dari 34 siswa hanya menggunakan kemampuan penalaran dan argumen. Pada soal 2, terdapat 30 dari 34 siswa melibatkan kemampuan representasi dan penalaran dan argumen, dan 4 dari 34 siswa melibatkan kemampuan komunikasi, representasi, penalaran dan argumen. Selain itu, penggunaan soal matematika tipe PISA dengan konteks lompat jauh membuat siswa lebih tertarik dan aktif selama pembelajaran.

Kata Kunci: *Design research*, Soal matematika tipe PISA, *Asian Games*

How to Cite: Pratiwi, I., Putri, R.I.I., & Zulkardi. (2019). Long jump in Asian Games: Contexts of PISA-Like mathematic problems. *Journal on Mathematics Education, 10*(1), 81-92.

Uncertainty and data is one of the content in Programme for International Student Assesment (PISA) for the domain of mathematical literacy and used in various fields of life especially in sports, business, industry, politics, weather, forecasts, population and so forth (Bakker, 2004; Hannah Muhsetyo, & Sisworo, 2016; Yanti, Nusantara, & Qohar, 2016). Students should have a good understanding and mathematical literacy ability on uncertainty and data content so they could solve the problems in the various situation (Johar, 2012). Despite the fact, the mathematical literacy achievement of Indonesian student in PISA on the uncertainty and data content was still low and always at the lower level compared to other countries. Indonesia's score on uncertainty and data content in 2003 PISA was 385 with the rank of 38 out of 40 countries, while in 2012 PISA only 384 of the

OECD average score, 493, with the rank of 63 out of 65 countries (OECD, 2004; OECD, 2014). This was because Indonesian students were not used to solving context-based problems such as PISA problems in learning process and evaluation, especially for the high-level problems (Dewantara, Zulkardi, & Darmawijoyo, 2015; Novita, Zulkardi, & Hartono, 2012; Ahyan, Zulkardi, & Darmawijoyo, 2014).

PISA results were the basis of the 2013 curriculum development where learning in Indonesia had to be adjusted to the PISA problems (Kemendikbud, 2014), so that teachers and collegers in Indonesia should be able to design a learning and evaluation problems in accordance to PISA characteristics such as using context that has been known by students in daily life (Kohar, Zulkardi, & Darmawijoyo, 2014; Zulkardi & Putri, 2006). PMRI approach is one of the learning that fit the objectives of the 2013 curriculum and emphasizes on the contexts familiarized by the students as the starting point of the learning for the formation of models, concepts, and mathematics motivation so that it will make the learning process becomes more meaningful for students (Putri, 2014; Zulkardi & Putri, 2006; Putri, 2011). From the situation, students will be required to be able to find their facts, build concepts, and new values in life during the learning process(Kemendikbud, 2013).

One of the contexts that can be used in learning of uncertainty and data and has been known by students is sport especially long jump sport in Asian Games which has currently been a public concern in Indonesia. Even the president of Indonesia, the governor and the head of the Palembang national education office instructed to the entire society, especially teachers in Palembang, to socialize the 2018 Asian Games in elementary and middle school by using Asian Games as a situation to get more interesting and meaningful learning for students (Instruksi Presiden, 2016; Sumselpostonline, 2015; Sumselpotonline, 2017). Also, the previous researchers have shown that sports in Asian Games as a learning context could help students to understand the materials, enlarge their thinking, and provide more meaningful experience and fun learning (Roni, Zulkardi, & Putri, 2017; Nasution, Putri, & Zulkardi, 2017).

This study aimed to produce PISA like mathematics problems on uncertainty and data content using the context of the long jump in Asian Games that were valid, practical and had a potential effect on the ability of the mathematical literacy students. Based on the 2015 PISA frameworks, the ability of the mathematical literacy students is an individual’s ability to formulate, apply, and interpret mathematics that underlined by the seven fundamental mathematical capabilities: communication, mathematising, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal, and technical language and operations, and using mathematical tools (OECD, 2016).

METHOD

This study is design research with development studies type Akker, 2006). This study was done by involving the tenth-grade students of Senior High School at the odd semester of the academic year of 2017/2018. This study was conducted in two stages, preliminary evaluation, and formative evaluation. The preliminary stage covered the analysis of research preparation and the design of PISA like mathematics problems based on 2015 PISA framework. Meanwhile, the formative evaluation stage consisted of self-evaluation, expert reviews, one-to-one, small group, and field test

(Tessmer,1999; Zulkardi, 2006).

At the self-evaluation, the researchers evaluated the PISA like mathematics problems by herself and revised the items, so that prototype one was obtained. Then at the expert reviews, prototype one was consulted to the experts and evaluated by the validation criteria of content, construct, and language. Simultaneously, the one-to-one was undertaken where prototype one was tested to three students at this stage. The result of expert reviews and one-to-one was used to revise prototype one so that valid prototype two was obtained. Then at the small group, prototype two was tested to six students in a learning process that fit the lesson plan (RPP) made. The result of the students' answer analysis and their suggestion for the problems were used to revise the prototype two so that the valid and practical of protoype three was obtained. Furthermore, in the field test, prototype three was tested on the research subject through the learning process to find out the potential effects of the PISA-like mathematics problems on the ability of students' mathematical literacy. The data were gathered using walkthrough at the expert reviews, observation of the video recording, interview during the learning process, and students' answer.

RESULT AND DISCUSSION

In this study, there were five units consisting of ten items of PISA-like mathematics problems on uncertainty and data using games and athletics in Asian Games contexts. However, only one unit, using long jump in athletics context, was discussed in this study because there were various students' strategies and arguments in this unit. The steps undertaken in the development process were shown below.

Preliminary Stage

In the early stage of the development, the researchers analyzed the characteristic of the PISA problems, curriculum, and students' analysis. In analyzing the characteristics of the PISA problems, the researchers analyzed the characteristics of the content, context, competence of process capability, and the level of the mathematical ability in PISA framework so that the characteristics of PISA problems in mathematics domain and PISA items on uncertainty and data content were obtained and used for the developing problems. The researchers also analyzed the uncertainty and data content in the curriculum used by the research school, curriculum 2013, and found that the uncertainty and data content was taught at tenth-grade students in probability and statistics subject. While in the student analysis, the researchers identified students who would play a role in the one-to-one and small group phase, and the research subject in a field test with the help of the teachers.

In the design preparation stage, the researchers designed the instruments of the research consisting the question grids, problems card, the assessment rubric, and lesson plan. At this stage, the researchers designed an initial prototype of four units PISA problems on uncertainty the and data content developed based on 2015 PISA framework into five items of PISA like mathematics problems using the context of football, Asian Games sports, sprint, and Indonesian medals at Asian Games.

One of the problems used was 2006 PISA items with M505 code about garbage. M505 PISA problems used the context of the timeframe of the decomposition process. From the various kinds of garbage and timeframe needed for the decomposition process, students were asked to give a reason why the data was not suitable to be displayed in the bar graph. From the PISA M505 problem, the researchers designed new problems by changing the context using sports and the year of the Asian Games when held. Students were asked to decide wether the data was suitable to be displayed in a bar graph first and gave supporting reasons. The initial problems that designed using long jump context were shown in Figure 1.

Cabang Olahraga *Asian Games*

Sejak diadakan pertama kali pada tahun 1951 hingga tahun 2018 yang akan datang, terdapat 51 cabang olahraga yang diselenggarakan di *Asian Games*. Cabang olahraga tersebut mencakup 39 disiplin ilmu olahraga yang berbeda dan terbagi menjadi sekitar 400 acara/*event*. Namun, tidak semua cabang olahraga tersebut dilombakan pada setiap penyelenggaraan *Asian Games*. Ada beberapa cabang olahraga yang hanya dilombakan selama 2 kali penyelenggaraan *Asian Games*, hingga cabang olahraga yang hanya ada pada penyelenggaraan *Asian Games* pada tahun tertentu saja. Berikut ini data penyelenggaraan beberapa cabang olahraga di *Asian Games* sampai tahun 2018:

Cabang Olahraga	Tahun
Binaraga	2002–2006
Bowling	1978, 1986, dan sejak 1994
Catur	2006–2010
Panahan	Sejak 1978
Pencak Silat	2018
Sepak Bola	Sejak 1951
Sepatu Roda	2010

(Sumber: *www. en.wikipedia.org/wiki/Asian_Games*)

Apakah data pada tabel di atas dapat disajikan dalam bentuk diagram batang? Mengapa? Berikan alasanmu.

Figure1. Problem using long jump context at the design stage

Formative Evaluation Stage

a. Self Evaluation

At the self-evaluation, the researchers re-evaluated the research instruments that had been designed in the previous stage. The initial prototype was evaluated regarding content, constructs, and languages. Based on the result of the evaluation, the researchers decided to make some revisions such as adding new questions, changing the context of Asian Games which were still general as in the Asian Games sports and the Indonesian's medal to becoming more specific to the games and athletics sports, and revise the sentences used in the problems.

At the Asian Games sports unit, the researchers decided to change the context of the problems to be the data of the women's long jump record in the 2014 Asian Games so that it would be more appropriate to the sport and athletics in Asian Games context. In question 1, the researchers changed the question of the suitability of the displaying data in a bar graph into a pie graph. Also, the researchers also added one item in this unit where in question, students were asked to determine the winner of long jump based on the available data and criteria.

b. Expert Reviews and One-to-One

Expert reviews and one-to-one were done in parallel to determine the validity of prototype one based on predefined criteria. At the expert reviews, prototype one was consulted to the experts as validators. Then this prototype one was analyzed and evaluated qualitatively by the validators regarding content, constructs, and languages. The validation process in this stage was done in three ways (Tessmer, 1999), that were (1) mail reviews with Masitah Sharill, an assistant senior professor and lecturer at the Hassanal Bolkiah Institute of Education (SHBIE), Brunei Darussalam University (UBD), (2) panel reviews with the mathematics education lecturer in Sriwijaya University and master's students of mathematics education of Sriwijaya University with the research focus on developing PISA like mathematics problems, (3) face-to-face reviews with Idda Mawaddah, a mathematics teacher. Table 1 showed the experts' comments and suggestion to the long jump unit.

Table 1. The validator's coments and suggestions at the expert reviews

Validator	Comments and Suggestions
Masitah Shahrill	a. The context of the chosen questions (regarding long jump for the female athletes) showed good and interesting connections to real-life knowledge and experiences. And so far, it showed fairness about the distribution of gender questions. b. Please try not to use the exclamation mark (!) for any statements or questions. It reads as if you are forceful.
Elika Kurniadi	a. Add information of the empty table in the description. b. Add information of Asian Games data in the description of the problems if it is the original data of Asian Games
Riya Dhotul Jannah	a. In the data table, you should add a column for the name of the participating country, and the flag in the name of the athletes should be removed b. Give the unit of distance used in the data table c. Describe the empty table data

While at the one-to-one, prototype one was tested to three students of Senior High School with high, medium, and low ability. This stage was to look at the students' thinking on the prototype one problems and to know the students' difficulties in understanding and solving the problems on prototype 1. From the analysis result of students' answer, it was found that most students had a little difficulty in understanding the empty data on the table and the criteria of long jump winner regarding the furthest jump achieved by the athlete in question 2. After the students were given a little direction about the meaning of the furthest jump criteria, they were able to solve the problem well.

Based on the result of the expert reviews and one-to-one, the researchers decided to make several revisions to prototype 1. The result of the revisions was prototype two that had been valid. At the long jump unit, the researchers revised the problems by adding the information that the given data on the table was the data from the women's long jump result at the 2014 Asian Games. The researchers also added the information that empty data on the table means that the athlete did not jump. While in question 2, the researchers changed the sentence on the criteria of the winner of the long jump event. So that the students could identify the winner easily.

c. *Small Group*

At this stage, prototype two was tested to six students of Senior High School with high, medium, and low ability in three meetings, two meetings for learning and one for the final test. Based on the result of small groups, it was found out that most students were able to understand the problems well, could use the table or diagrams contained in the problem, and understood the context used in the problems.

At the long jump unit of the question 2, the researchers decided to change the sentence on the first criteria of the long jump winner of “the furthest jump achieved by the competing athletes from the sixth of jumping trials” to the “the furthest jump in all trials from the competing athlete.” Besides revising the words that were still difficult to understand, the researchers also decided to make one meting for learning and one meeting for the final test in the next stage. It was done to save the time because according to the researchers, the time used in the previous learning process should be made more efficient.

d. *Field Test*

At field test, the researchers tested the prototype 3 to the research subject, students at the tenth grade of Senior High School consisting of 34 students. At the field test, the researchers tested prototype 3 at one learning process and one final test with the classroom teacher, Idda Mawaddah, as a model in the learning process. Figure 2 showed the long jump unit in prototype 3.

Lompat Jauh

Pada *Asian Games*, lompat jauh termasuk dalam cabang olahraga atletik. Pada pelaksanaannya, masing-masing atlet diberikan kesempatan sebanyak 6 kali untuk melompat. Berikut ini catatan jarak yang dicapai oleh para atlet putri dalam even lompat jauh di *Asian Games* 2014:

Nama Atlet	Percobaan ke-					
	1	2	3	4	5	6
Jung Soon-ok (KOR)	X	X	6.26	6.34	6.20	6.12
Lu Minjia (CHN)	6.24	6.22	6.28	6.21	6.22	6.03
Marestella Torres (PHI)	X	X	X			
Keshari Chaudhari (NEP)	4.71	4.76	X			
Yanfei Jiang (CHN)	X	6.24	6.34	6.24	6.19	6.29
Bae Chanmi (KOR)	5.67	6.34	6.14	X	X	5.89
Mayookha Johny (IND)	6.04	6.12	6.02			
Darya Reznichenko (UZB)	6.25	6.04	6.01	6.11	6.05	6.10
Bashair Obaid (QAT)	4.07	X	4.34			
Maria Natalia Londa (INA)	X	6.19	X	6.40	X	6.55
M.A Prajusha (IND)	6.23	X	X	6.17	X	X
Bui Thi Thu (VIE)	6.35	X	6.44	6.31	6.30	6.33
Yulia Tarasova (UZB)						

(Sumber: *en.wikipedia.org*)

Catatan : X : Gagal
Kosong : Tidak melakukan lompatan

Soal 1

Apakah data pada tabel di atas sesuai untuk disajikan dalam bentuk diagram lingkaran? Berikan alasanmu.

Soal 2

Pemenang dalam even lompat jauh ditentukan oleh :

1. Lompatan dengan jarak terjauh pada seluruh percobaan dari para atlet yang bertanding
2. Jika terdapat jarak lompatan yang sama, maka atlet dengan kegagalan lebih sedikit dalam percobaan lompatannya akan lebih unggul daripada atlet yang lain

Berdasarkan data di atas, identifikasi atlet yang berhasil memperoleh medali emas, perak, dan perunggu. Isi tabel berikut ini dengan nama atlet yang menjadi peraih medali dan catatan jarak lompatan terjauhnya. Berikan alasanmu.

Medali	Atlet	Jarak Lompatan (meter)
Emas		
Perak		
Perunggu		

Figure 2. Long jump unit in prototype 3

At the beginning of the learning process, the model teacher gave two situations to students: football and sprint in Asian Games. Then students were given time to understand and tried to solve individual problems first. Then students were asked to discuss in groups that had been previously divided, a group of 4-5 students with high, medium, and low ability. After the discussion finished, the model teacher asked one of the groups to present the results of their discussion. The model teacher acting as a facilitator looked at the students' interaction and contribution during the process.

In the learning using the PISA like mathematics problems with long jump context, it showed that students were more interested in the situation used in the learning and more interactive during the discussion. It was inline with the study conducted by Gunawan, Putri, and Zulkardi (2017) saying that students were more enthusiastic and active during the learning that used the sports context in Asian Games. Also, the learning that used the sports context in Asian Games could be a bridge of students' thinking and helped students to understand the materials with more meaningful experience and fun learning (Nasution, Putri, & Zulkardi, 2018; Rahayu, Putri, & Zulkardi, 2017).

At the final test, the teacher gave three units of PISA-like mathematics problems on prototype 3 consisting five items to students and asked them to solve it individually within two lesson hours. The model teacher kept watching the final test so that students could not discuss the problem with others. Then, the students' answers were analyzed to look at the potential effects of the PISA-like mathematics problems on the ability of the students' mathematical literacy. It could be seen from the basic math skills used by the students in solving the problems. The following were some of students answers on the long jump unit.

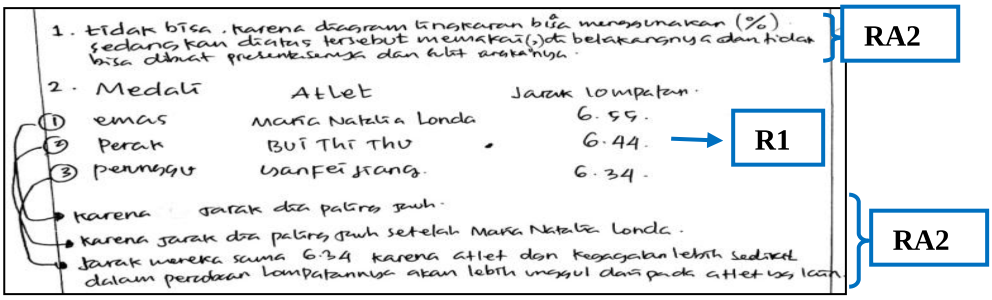


Figure 3. Student's anwer in field test

Figure 3 showed that in question 1, students were able to involve the ability of the reasoning and argument of the indicator to make the explanation that supports the solution (RA2) by suggesting the reason of why the data were not suitable to be displayed in a pie graph because the data was difficult to be made in percentage form as in the pie graph. While in question 2, students were able to involve the ability of representation of the indicator to interpret the result in a form of representation (R1) by making the table of the result of the medal identification and the ability of the reasoning and argument of the indicator to make the explanation that supports the solution (RA2) by arguing the athlete able to win the medal.

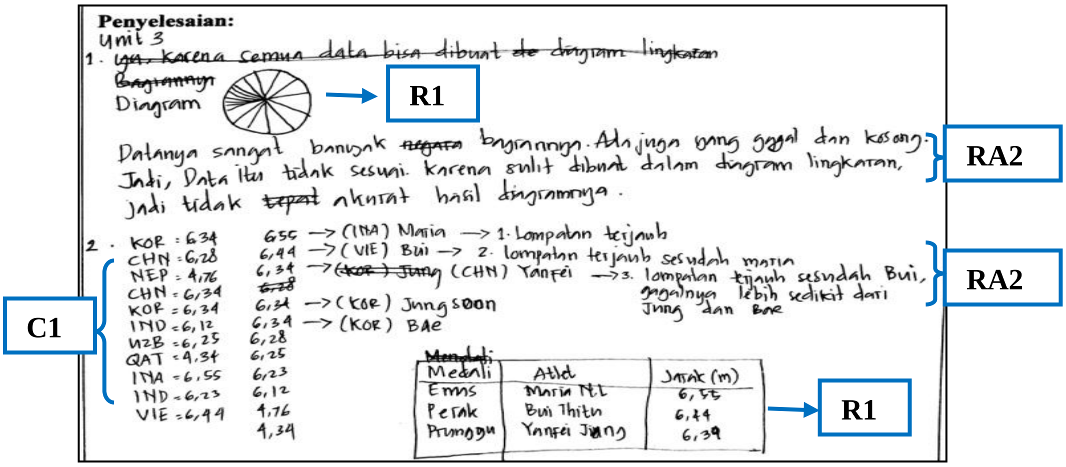


Figure 4. Student answer in field test

Figure 4 showed that in question 1, students were able to involve the representation ability of the indicator to interpret the result in the form of representation (R1) by trying to make a model of the pie graph from the data on the table. Then student involved the ability of the reasoning and arguments of the indicator to make explanations that supports the solution (RA2) by suggesting the reason of the data that were not suitable to be displayed in a pie graph because of the large number of the data so it would be inaccurate when it was made in pie graph. While in question 2, the students were able to involve the ability of communication of the indicator to make an explanation in the context of the problem (C1) by writing the sequence of the furthest jumps achieved in each athlete, then involved the ability of reasoning and arguments of the indicator to make explanations that supports the solution (RA2) by writing the reason for the athlete obtaining the medal, and the ability of the representation of the indicator to interpret the result in a form of representation (R1) by creating a table of the result of the medal identification.

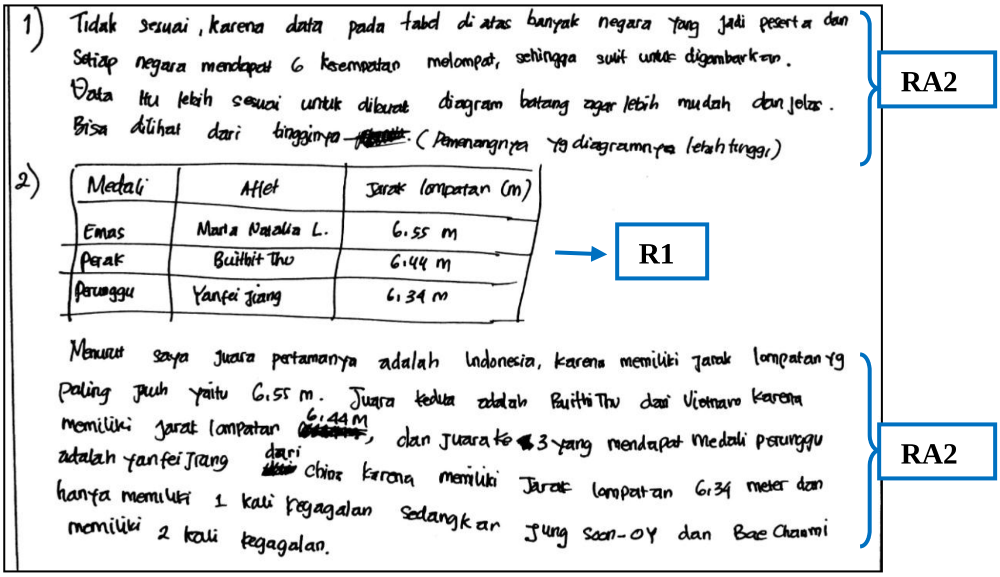


Figure 5. Student answer at field test

Figure 5 showed that in question 1, the students were able to involve the ability of the reasoning and argument of the indicator to make the explanation that supports the solution (RA2) by suggesting the reason of why the data were not suitable to be displayed in a pie graph because the data had many categories, so it was easier to be displayed in a bar graph. While in question 2, the students were able to involve the ability of representation of the indicator to interpret the result in a form of representation (R1) by making the table of the result of the medal identification and the ability of the reasoning and argument of the indicator to make the explanation that supports the solution (RA2) by arguing that the athlete was able to win the medal.

In question 1, from the result of the analysis, it was found out that 3 out of 34 students involved the ability of representation and the reasoning and argument where two students were able to answer correctly and completely, and a student answered correctly but not complete. While 31 out of 34 students involved only the ability of the reasoning and argument where nine students were able to answer correctly and completely, 18 students were able to answer correctly but not complete, and four students' answers were wrong.

In question 2, from the result of the analysis, it was found out that 30 out of 34 students involved the ability of representation and the reasoning and argument where 10 students were able to answer correctly and completely, 10 students were able to answer correctly but not complete, and 9 students' answers were wrong. While 4 out of 34 students involved the ability of communication, representation, and reasoning and argument where two students were able to answer correctly and completely, a student was able to answer correctly but not complete, and a student's answer was wrong.

The result of the students' answers analysis showed that most of the students were able to solve the problems by involving some of the fundamental mathematical capabilities such as the ability of communication, representation, and reasoning and argument. Some students still had difficulty to solve the problems because they were not accustomed to PISA-like mathematics problems. It was in line with the research of the Mujulifah, Sugiatno, and Hamdani (2015) saying that students had not been trained and accustomed to solving PISA-like mathematics problem that required the ability of understanding, employing, and reasoning. Also, there were still some students who were unable to make an explanation and argument that supports the solution correctly and completely. This was in line with the research of the Mardhiyanti, Putri, and Kesumawati (2013) that students were not accustomed to solving PISA-like mathematics problems that required students to communicate their answer in writing along with the explanations and reasons.

CONCLUSION

This study produced a set of PISA like mathematics problems on uncertainty and data content using the context of the long jump in Asian Games that were valid, practical, and had a potential effect on the ability of the students' mathematical literacy. Valid criteria for this problems was seen regarding content, constructs, and languages at the expert reviews and one-to-one. While the practicality criteria were seen from the result of the small groups where the problems using the context recognizable by students could be understood and applied in the learning. Potential effects of this PISA-like mathematics problems using long jump context on the ability of the students mathematical literacy were seen from the result of the students answer analysis where in question 1 students could show the ability of representation and the reasoning and argument, and in question 2 students could show the ability of the communication, representation, and the reasoning and argument. Also, the use of PISA-like mathematics problems with a long jump in Asian Games context made students more interested and active during discussions in the learning process.

ACKNOWLEDGMENTS

The researchers would like to express gratitude to the Direktorat Jendral Pendidikan Tinggi Indonesia who has funded 'Hibah Pasca' research in 2018, and to Mrs. Idda Mawaddah and her students for participating in this research. As well as those who have helped the researchers in developing the research and writing this article.

REFERENCES

Ahyan, S., Zulkardi, & Darmawijoyo. (2014). Developing Mathematics Problems Based on PISA Level. *Journal on Mathematics Education, 5*(1), 47-56.

Akker, J.V. (2006). *Introducing Educational Design Research: Educational Research*. Newyork. Routledge. (Online), (http://www.fisme.science.uu.nl/ublicities/literatur/ EducationDesignResear ch.pdf), retrieved on September 15, 2017.

Anisah, Zulkardi, & Darmawijoyo. (2011). Pengembangan Soal Matematika Tipe PISA Pada Konten Quantity untuk Mengukur Kemampuan Penalaran Matematis Siswa Sekolah Menengah Pertama. *Jurnal Pendidikan Matematika, 5*(1).

Bakker. (2004). Design Research in Statistics Education on Symbolizing And Computer Tools. *Publised Doctoral Dissertation*. Wilco, Amersfoorts/Freudenthal Institute, Utrecht.

Dewantara, A.H., Zulkardi, & Darmawijoyo. (2015). Assesing Seventh Graders' Mathematical Literacy in Solving PISA-Like Task. *Journal on Mathematics Education,*6(2), 39-49.

Gunawan, M.S., Putri, R.I.I., & Zulkardi. (2017). Learning Fractions through Swimming Context for Elementary School Students. *Proceeding in The Fifth South East Asia Design/Development Research (SEA-DR) International Conference 2017* (pp. 61-65). Banjarmasin: Universitas Lambung Mangkurat.

Hanah, R., Mushetyo, G., & Sisworo. (2016). Penggunaan Bahan Manipulatif untuk Memahamkan Materi Peluang pada Siswa SMP dengan Pendekatan Pendidikan Matematika Realistik. *Jurnal Pendidikan: Teori, Penelitian, dan Pengembangan, 1*(5), 927-939.

Instruksi Presiden. (2016). *Instruksi Presiden Republik Indonesia Nomor 2 Tahun 2016 Tentang Dukungan Penyelenggaraan Asian Games XVIII Tahun 2018*. Sekretariat Negara. Jakarta.

Johar, R. (2012). Domain Soal PISA untuk Literasi Matematika. *Jurnal Peluang, 1*(1), 30-41.

Kemendikbud. (2014). *Paparan Menteri Pendidikan dan Kebudayaan RI, Press Workshop: Implementasi Kurikulum 2013*. Jakarta: Kementerian Pendidikan dan Kebudayaan.

Kohar, A.W., Zulkardi, & Darmawijoyo. (2014). Developing PISA-Like Mathematics Task to Promote Student's Mathematical Literacy. *Proceeding in The Second South East Asia Design/Development Research (SEA-DR) Conference 2014* (pp. 14-26). Palembang: Sriwijaya University.

Mardhiyanti, D., Putri, R.I.I., & Kesumawati, N. (2013). Pengembangan Soal Matematika Model PISA untuk Mengukur Kemampuan Komunikasi Matematis Siswa Sekolah Dasar. *Jurnal Pendidikan Matematika, 5*(1).

Mujulifah, F., Sugiatno, & Hamdani.(2015). Literasi Matematis Siswa Dalam Menyederhanakan Ekspresi Aljabar. *Jurnal Pendidikan dan Pembelajaran, 4*(1) 1-12.

Nasution, M.F., Putri, R.I.I., & Zulkardi. (2017). Dayung Context in Fraction. *Proceeding in The Fifth South East Asia Design/Development Research (SEA-DR) International Conference 2017* (pp. 1-6). Banjarmasin: Universitas Lambung Mangkurat.

Nasution, M.F., Putri, R.I.I., & Zulkardi. (2018). Rowing Sport in Learning Fractions of The Fourth Grade Students. *Journal on Mathematics Education, 9*(1), 69-80.

Novita, R., Zulkardi, Hartono, Y. (2012). Exploring Primary Student's Problem-Solving Abillity. *Journal on Mathematics Education, 3*(2), 133-150.

OECD. (2004). *Learning for Tomorrow's World: First Results from PISA 2000*. Paris: OECD Publishing

OECD. (2014). *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science* (Volume I, Revised edition, February 2014). Paris: OECD Publishing.

OECD. (2016). *PISA 2015 Assessment and Analytical Framework: Science, Reading, Mathematic and Financial Literacy*. Paris: OECD Publishing.

Putri, R.I.I. (2011). Pembelajaran Materi Bangun Datar Melalui Cerita Menggunakan Pendidikan Matematika Realistik Indonesia (PMRI) di Sekolah Dasar. *Jurnal Pendidikan dan Pembelajaran, 18*(2), 234-239.

Putri, R. I.I. (2014). Evaluasi Program Pelatihan Pendidikan Matematika Realistik Indonesia (PMRI) Bagi Guru Matematika Sumatera Selatan. *Prosiding Seminar Nasional Evaluasi Pendidikan 2014* (pp. 522-527). Jakarta: Universitas Negeri Jakarta.

Rahayu, C., Putri, R.I.I., & Zulkardi. (2017). Multiplication of Fraction With Natural Number by Using Hurdles. *Proceeding in The Fifth South East Asia Design/Development Research (SEA-DR) International Conference 2017* (pp. 43-37). Banjarmasin: Universitas Lambung Mangkurat.

Roni, A., Zulkardi, & Putri, R.I.I. (2017). Sprint Context of Asian Games in the Division of Fractions. *Proceeding in The Fifth South East Asia Design/Development Research (SEA-DR) International Conference 2017* (pp. 22-29). Banjarmasin: Universitas Lambung Mangkurat.

Sumsel Post Online. (29 Desember 2015). Ciptakan Pembelajaran Matematika dengan Asian Games. (Online),(http://sumselpostonline.com/ciptakan-pembelajaran-matematika-dengan-asean-games/) retrieved on September 12, 2017.

Sumsel Post Online. (2017). Gubernur Ajak Guru Sukseskan Asian Games 2018. (Online), (http://sumselpostonline.com/gubernur-ajak-guru-sukseskan-asian-games-2018/), rerieved on September 12, 2017.

Tessmer, M. (1999). *Planning and Conducting Formative Evaluation:Improving The Quality of Education and Training*. London, Philadelphia: Kogan Page.

Yanti, W., Nusantara, T., & Qohar, A. (2016). Analisis Kesalahan Dalam Menyelesaikan Soal Pada Materi Permutasi dan Kombinasi. *Prosiding Seminar Nasional Pendidikan Matematika 2016* (pp. 97-104). Malang: Universitas Negeri Malang.

Zulkardi. 2006. *Formative Evaluation: What, Why, When, and How*. (Online), (http://www.oocities.org/zulkardi/books.html), retrieved on September 10, 2017 .

Zulkardi, & Putri, R.I.I. (2006). Mendesain Sendiri Soal Kontekstual Matematika. *Prosiding Konferensi Nasional Matematika ke-13 (KNM13) 2006* (pp. 1-7). Semarang: Universitas Negeri Semarang.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 93-102

CREATING MANIPULATIVES: IMPROVING STUDENTS' CREATIVITY THROUGH PROJECT-BASED LEARNING

Siti Khoiruli Ummah, Akhsanul In'am, Rizal Dian Azmi

University of Muhammadiyah Malang, Jalan Raya Tlogomas 246 Malang, Indonesia
Email: khoiruliummah@umm.ac.id

Abstract

The purpose of this study is describing the implementation of project-based learning model and analysing the student's creativity improvement in the subject of learning media. We describe the implementation of project-based learning based on our note of the learning syntax during one semester. Then, compared the students' product to measure their creativity improvement in the aspects of originality, novelty, and flexibility. This study used descriptive-qualitative method and involved the students who took the learning media course as its subjects. The results showed that there was an improvement of the students' creativity in the project-based learning implementation. The improvement was measured in the making of the manipulatives. The students' flexibility and novelty were good, while their originality was fair.

Keywords: Creativity, Mathematics manipulatives, Project-based learning.

Abstrak

Penelitian ini bertujuan untuk menggambarkan implementasi pembelajaran berbasis proyek dan menganalisis peningkatan kreativitas mahasiswa dalam pembuatan media pembelajaran. Deskripsi tentang pelaksanaan model pembelajaran berbasis proyek didasarkan pada sintaks yang diperoleh melalui catatan lapangan selama satu semester. Kemudian, peningkatan kreativitas mahasiswa berkaitan dengan aspek orisinalitas, kebaruan dan fleksibilitas yang diperoleh dengan membandingkan proyek akhir yang dibuat. Metode penelitian yang digunakan adalah deskriptif kualitatif. Subjek penelitian ini adalah mahasiswa matematika yang mengambil mata kuliah sumber belajar dan media pembelajaran matematika. Hasil penelitian ini menunjukkan peningkatan kreativitas melalui pembelajaran berbasis proyek. Meningkatnya kretivitas ini didasarkan pada proyek pembuatan media pembelajaran matematika manipulatif. Peningkatan kreativitas juga didasarkan pada dipenuhinya aspek fleksibilitas dan kebaruan dalam kriteria baik, sedangkan aspek orisinalitas memenuhi kriteria cukup baik.

Kata kunci: Kreativitas, Media pembelajaran matematika manipulatif, Pembelajaran berbasis proyek.

How to Cite: Ummah, S. K., In'am, A., & Azmi, R. D. (2019). Creating manipulatives: improving students' creativity through project-based learning. *Journal on Mathematics Education, 10*(1), 93-102.

Learning media is one of the compulsory subjects for mathematics education students. It becomes compulsory because the students who are going to become professional pre-service teachers are required to be creative in making and using learning media. Based on interviewing students, mathematics learning at schools often uses worksheet and whiteboard to explain the mathematics material.

Project-Based Learning (PjBL) is one of learning models appropriate for learning with certain product as an output, for example, creating learning media. This model is appropriate for students from elementary school level to university level (Jacques, 2017; Burlbaw, 2013). PjBL facilitates students to collaborate in conceptual understanding, to apply prior knowledge, and to gain skills. It can integrate several disciplines to create a project (Capraro & Slough, 2013). Furthermore, the other benefits of PjBL are enabling students to demonstrate higher ability (Crowley, 2015), improving students' achievement (Ali, Akhter, Shahzad, Sultana, & Ramzan, 2011), challenging students to solve real problems, becoming a good collaborator (Roberts, 2011), motivating students (Liu, 2010), improving content knowledge, and meeting the needs of students with variety of skills and learning styles (Coyne, Hollas, & Potter, 2016).

One of the benefits of PjBL is showing student's higher ability. By using PjBL, we could observe the student's creativity through various projects of school mathematics learning media. Creativity is a process that produces novelty, which is useful, sustainable, or satisfying for people (Sousa, 2012). In mathematics learning, creativity has an important role (Levenson, 2011). One of the benefits of students' creativity in mathematics learning is improving students' knowledge. The students' knowledge is not limited to mathematical knowledge but also the knowledge associated with daily life (Bolden, 2010). This idea is in line with the implementation of PjBL that can improve students' skills, especially the creativity.

We could measure creativity in several aspects, including fluency, flexibility, originality, and novelty (Adams, 2005). Fluency can be measured through many answers. Flexibility can be measured using many variations of ideas to answer the problem. Originality is measured using the question of whether the answer made is different from the other students or whether the answer produced is an unusual answer. Novelty is a measured on how innovative answers made by the students (Guilford, 1959; Torrance, 1974).

The importance of mathematical creativity in the learning media course can be seen in conceptual and constructive activities in making learning media. Students are required to construct the concept independently to apply the making of mathematics learning media by utilising the tools and materials around the student's environment (Tienken, 2010). The media is something connect the source of information to the recipient (Sulisworo & Permprayoon, 2018). Learning media can be defined as a software or a model to deliver educational message or information presented by using a teaching aid (Yazgan-Sag & Emre-Akdoğan, 2016). In mathematics learning, manipulative learning media can develop an understanding of the students towards the abstract concept (Hidayah, Dwijanto, & Istiandaru, 2018). Manipulative media could help students represent abstract concepts in the form of concrete learning media (Yazgan-Sag & Emre-Akdoğan, 2016; Ozel, 2014).

The ability to create and use manipulative learning media in mathematics learning is important for pre-service teacher. Creation of manipulative media that requires a long process and time can be adjusted to the model of PjBL (Cope, 2015). It could happen because each syntax will be able to analyse and to describe the results.

METHOD

The approach used in this research was descriptive qualitative. Qualitative approach was used to describe and to express the implementation of project-based learning based on the instruments we have prepared. Then, we analyse the students' creative ability and the product of manipulative learning media. The subjects in this study were students of the mathematics education department of University of Muhammadiyah Malang who took the course of learning media in the third semester of 2016/2017 academic year.

Research Steps

This study follows some steps, namely: (1) initial observation and interview, (2) studying the relevant literature, (3) preparing instruments, (4) implementing project-based learning, (5) addressing basic question, (6) designing the learning media, (7) discussing the schedule, (8) monitoring, (9) examining the result, (10) evaluation, (11) interview, (12) data reduction, and (13) analyzing the data. First, we observed and interviewed the mathematics teachers and students to find out the prominent problem in mathematics learning media. We also reviewed previous studies related to the problem and preparing the instrument to collect data. Once the instruments were ready, we started to implement the project-based learning in the classroom.

Regarding the manipulative project, we started by addressing basic questions about the project, e.g. the main subject, the core competence, the basic competence, the learning indicators, and the idea of learning media. Furthermore, we provided a storyboard format for the students to design the manipulatives and then discussed the timeline of the making of manipulatives. Once the timeline agreed, the students started to make the manipulatives and we monitored the progress of the project. When the students finished their projects, we asked them to present the products in front of the class. After that, we conducted an exhibition and interview to evaluate the manipulatives. Finally, we analyze the data using data reduction and comparing the data with the relevant literature.

Data Collection and Technique and Instruments

We used observation and interview to collect data. The observation was done to look at the manipulative learning media produced by the students, while the interview tried to look at the creative aspects of flexibility, originality, and novelty of the media. The instruments used in this study include field note format, project assessment, and interview guidelines. Instruments were prepared based on supporting theory obtained from the literature study. We also prepared storyboard contained: (1) basic questions, (2) tools and materials, (3) estimated time of manufacture, (4) mode of use, (5) estimated time of media use in class, and (6) sketch of learning media to help the students designing the media.

In the monitoring steps, we asked the students filling out logbook then guided them during the manufacture of media. While in the evaluation step, we analyzed the weaknesses and advantages of learning media. The presentation of the results of the data presentation was reinforced through interviews in the making of learning media. Through this way, we could know how the students' creativity in making mathematics learning media built. The written conclusion was based on the concept of improving the creativity of mathematics education students through project-based learning in the learning resources and learning media.

RESULT AND DISCUSSION

The implementation of the project-based learning took place during the odd semester of the 2016/2017 academic year. The participants were students of mathematics education department. They got the material about the concept of learning resources and learning media, especially in mathematics. This study focused on the manipulative learning media. The students got the material of manipulative media in mathematics in each meeting.

Basic Questions

Earlier, the implementation of project-based learning was started by determining the fundamental questions. They were about: (1) main material, (2) core competence, (3) basic competence, (4) learning indicators, and (5) idea of learning media. The topic, core competences, and basic competences are determined based on the 2013 curriculum at junior high school level. The ideas to make learning media were obtained through browsing, reading research journals, and the development of learning media products. The result of this first step was that the students chose the mathematical topic of planar geometry. The students also had an idea to make learning media in the form of wall magazines or stickers, and to create a learning media by changing the shape and colour of learning media that the other researchers have ever developed. All of these results were obtained from the student's presentation in the classroom. We asked questions based on the five fundamental questions. The student's creativity is visible in the flexibility aspect, i.e. students can develop ideas freely and various ideas of making learning media. Aspects of flexibility are increased due to changes and the addition of colour and shape compared to the previous study.

Project Schedule

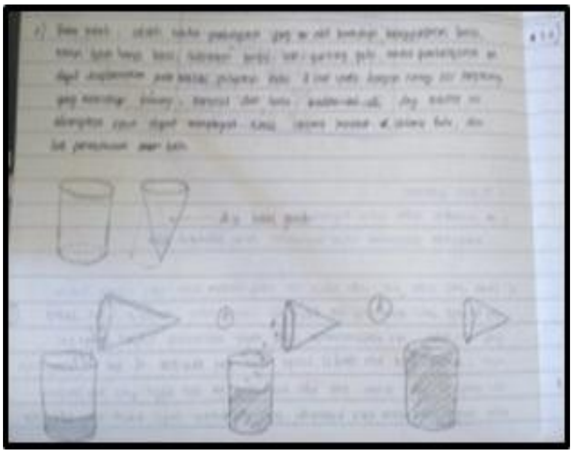
The second step is obtaining the schedule. The schedule is discussed in the class and can be seen in Table 1.

Table 1. The schedule of project-based learning

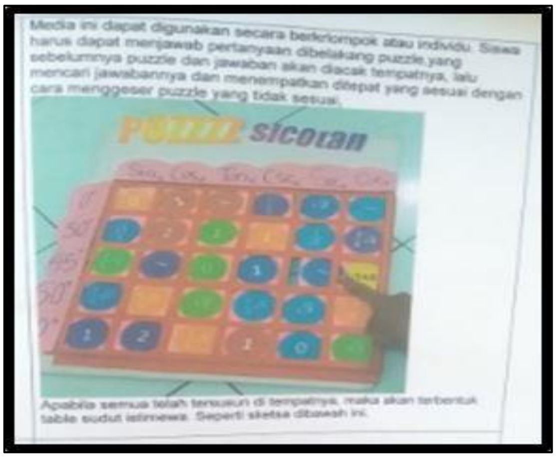
No	Time	Activity	Place
1	Week 1 of October 2016	Obtaining the basic idea to answer the fundamental questions	Common building, Room 3.06
2	Week 2 of October 2016	Obtaining schedule	Common building, Room 3.06
3	Week 3 of October 2016	Designing project planning, completing the storyboard, and submitting the plan	Common building, Room 3.06
4	Week 4 of October 2016	Monitoring: storyboard presentation	Common building, Room 3.06
5	Week 2 of November to Week 4 of December 2016	Testing the project result: Presenting the learning media	Common building, Room 3.06
6	Week 1 of January 2017	Evaluating: Outdoor exhibition of learning media	Park

Planning the Project

The designed activities of project planning were done by creating and presenting the storyboard. Storyboard made referring to the following format: (1) main material, (2) core competence, (3) basic competence, (4) learning indicators, (5) tools and materials, (6) estimated time of manufacture, (7) how to use, (8) estimation of media usage time in class, and (9) sketch of learning media. Through the activities of storyboard, the students have been able to design the learning media according to the ideas that have been stated previously. A total of 23 students changed their idea different to the first plan. Based on the interview, the students stated that they had additional ideas based on reading literature. This phenomenon can be seen through the design changes, colours, and ways of using media. For example, students named LR had an idea to make learning media in the form of the concept of space geometry, but when designing the storyboard, LR changed the media into trigonometric puzzles. Storyboard drawings and early sketches can be seen in Figure 1.



(a) First sketch



(b) Storyboard

Figure 1. Mathematics manipulatives 1

Student creativity looks through aspects of originality and flexibility. The originality aspect is said to increase because of the idea of doing a puzzle that is not owned by other students as well as the selection of different puzzle material with the puzzle that has been developed previously. An increased aspect of flexibility can be seen that the student LR easily replace the material along with customized learning media.

Monitoring

The fourth step of project-based learning activity is monitoring. Monitoring is not done by filling the logbook but presenting the storyboard that has been made then do the revision on the storyboard and the media created (Cope, 2015). Through the storyboard, presentation activities can be seen that the students make changes to the learning media on the grounds to avoid similarities with the media that is owned by other students. Students also revised learning media based on inputs and suggestions from lecturers and other students so that it can be concluded that students can decide changes in learning media according to the learning objectives that have been set themselves (Cope, 2015; Ozel, 2014).

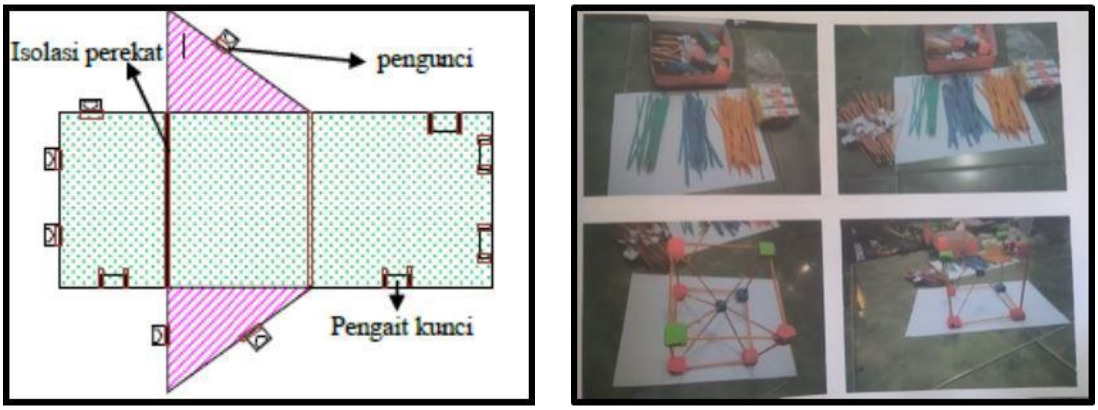
The existence of statistical learning media evidenced the aspect of sufficient originality with the topic of presenting data in the form of graphs. Student EN used paper beam material (See Figure 2a) while Student HN used the pipe (See Figure 2b) to create a bar chart. The weakness is seen in bar charts using paper because it is not durable or quickly damaged, but it is more interesting and more interactive than just drawing on a piece of paper (Ozel, 2014).

(a) Bar chart using pipes (b) Bar chart using paper beam

Figure 2. Mathematics manipulatives 2

Results Examination

In the fifth steps, each student was asked to present interactive and manipulative learning media. Learning media that have been made has increased significantly. In the aspect of flexibility, students can create a variety of learning media. For example, in the learning media used in the geometry material using paper media (Yulistyarini & Mahmudi, 2015) that formed the nets of solid figures using double-tape. In this study, the resulted geometry products were developed using bamboo rods as ribs and clay as the vertices. Student PI showed the novelty aspect was in a fair category because the student could adapt the cube framework using different tools and materials. The weakness of the media was the absence of the faces of the cube. The aspect of originality was in a good category because the learning media of solid figure was only a question card that was made to resemble wall magazines or crossword puzzles. It means that the wall magazines or crossword puzzles have increased creativity on aspects of excellent originality. The different between the design and the realization of the media can be seen in Figure 3.



(a) Sketch of prism (b) PI's Media

Figure 3. Mathematics manipulatives 3

This phenomenon also can be seen in Figures 4 which show wall magazines and crossword puzzles adapting from previous research by changing the rules of crossword puzzles into issues of space geometry.

(a) Wall magazines (b) Crossword puzzles

Figure 4. Mathematics manipulatives 4

Enhancement of student creativity through the making of manipulative learning media can be seen clearly from the existence of learning media on algebra, matrix, statistic, linear equation of one variable, and two linear equation system. It satisfies the excellent aspects of flexibility, originality, and novelty. It could happen because the learning media has not been available before. The various learning media that has not been available before can be seen in Figure 5.

(a) Stacko algebra to learn linear equation

(b) Cube of matrices to learn the type of matrices

Figure 5. Mathematics manipulatives 5

The improvement of student creativity through project-based learning is also seen based on the flexibility aspect. In this aspect, the learning media has made very interesting, diverse and used a variety of used materials (Tienken, 2010). Students can utilise in using materials as raw materials of learning media. Aspects of flexibility have been seen in various forms of learning media such as stacko, wall magazine concepts, scales, lottery game concept, monopoly, snake ladder, puzzle, pop-up book, a miniature of space geometry, graphics miniature, plot board, and an electrical circuit.

Evaluation

The final step of the implementation of project-based learning is the evaluation of the project in the form of an exhibition of learning media. The exhibition was organised by displaying various manipulative media from three different classes. Each student is in charge of assessing the media of other friend's media based on the assessment sheet provided by the researcher. The learning media exhibition lasted for five hours in a gallery. It enabled visitors to freely enter the gazebo and find out various kinds of mathematics learning media.

CONCLUSION

Based on the description of the results of the implementation of project-based learning to improve the creativity of students, it can be concluded that there was an improvement of the student's creativity in completing the project of making mathematics learning media based on the aspects of flexibility which met very good category. It was evidenced by the change of learning media by comparing the storyboard with the final project result. Besides, the flexibility aspect was good based on the wide variety of learning media created on various mathematics learning materials of junior high schools. There was also an improvement of students' creativity in completing the project of making the mathematics learning media based on the aspect of originality, though some students still developed learning media with the similar rules and form with the previous developed learning media. The last conclusion is an improvement of students' creativity in completing the project of making mathematics learning media based on novelty aspect meets the good category. It is because some of the manipulative media were original, though the rest of the others still adapted from the previous studies.

ACKNOWLEDGEMENT

We thank the Directorate of Research and Community Service of the University of Muhammadiyah Malang who has provided the funding support for this research.

REFERENCES

Adams, K. (2005). *The Sources of Innovation and Creativity*. New York: National Center on Education and The Economy.

Ali, R., Akhter, A., Shahzad, S., Sultana, N., & Ramzan, M. (2011). The impact of motivation on students' academic achievement in problem based learning environment. *International Journal of Academic Research, 3*(1), 306-309.

Bolden, D. S. (2010). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics, 73*(2), 245-265.

Burlbaw, L. (2013). *STEM Project Based Learning: An Integrated Science, Technology, Engineering, and Mathematics (STEM) Approach*. Rotterdam: Sense Publishers.

Capraro, R., & Slough, S. (2013). *Why PBL? Why STEM? Why now?* Rotterdam: Sense Publisher.

Cope, L. (2015). Math Manipulatives: Making The Abstract Tangible. *Delta Journal of Education*, *5*(1), 10-19.

Coyne, J., Hollas, T., & Potter, J. P. (2016). Jumping in: Redefining teaching and learning in physical education through project-based learning. *Strategies*, *29*(1), 43-46.

Crowley, B. M. (2015). The Effects of Problem-Based Learning on Mathematics Achievement of Elementary Students Across Time. *Thesis*. Bowling Green: Western Kentucky University.

Guilford, J. P. (1959). Three faces of intellect. *American psychologist*, *14*(8), 469.

Hidayah, I., Dwijanto, & Istiandaru, A. (2018). Manipulatives and Question Series for Elementary School Mathematics Teaching on Solid Geometry. *International Journal of Instruction*, *11*(3), 649-662.

Jacques, L. A. (2017). What does Project-based Learning (PBL) look like in the mathematics classroom? *American Journal of Educational Research*, *5*(4), 428-433.

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *Journal of Creative Behavior*, *45*(3), 215-234.

Liu, Y. (2010). A case study of online project-based learning: The beer king project. *International Journal of Technology in Teaching and Learning*, *6*(1), 43-57.

Ozel, S. (2014). Effectiveness of an Online Manipulative Tool and Students' Technology Acceptances. International Journal of Educational Studies in Mathematics, 1(1), 1-15.

Roberts, J. (2011). Jazzed about Javits—GEMS data showing impact on student learning in math and science. *Proceeding of the NAGC 58th Annual Convention*. New Orleans, LA: National Association for Gifted Children.

Sousa, F. C. (2012). Creativity, Innovation and Collaborative Organizations. *International Journal of Organizational Innovation*, *5*(1), 1-39.

Suliswоro, D., & Permprayoon, K. (2018). What is the Better Social Media for Mathematics Learning? A Case Study at A Rural School in Yogyakarta Indonesia. *International Journal on Emerging Mathematics Education*, *2*(1), 39-48.

Tienken, C. H. (2010). Common Core State Standards: I Wonder? *Kappa Delta Pi Record*, *47*(1), 14-17.

Torrance, E. (1974). *Torrance tests of creative thinking: Norms-technical manual*. Lexinton: Personnel Press/Ginn and Company.

Yazgan-Sag, G., & Emre-Akdoğan. (2016). Creativity from Two Perspectives: Prospective Mathematics Teachers and Mathematician. *Australian Journal of Teacher Education*, *41*(12), 25-40.

Yulistiyarini, H., & Mahmudi, A. (2015). Pengembangan perangkat pembelajaran materi geometri ruang SMP dengan memanfaatkan alat peraga manipulatif dan lingkungan. *Pythagoras: Jurnal Pendidikan Matematika*, *10*(2), 155-167.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 103-116

COMBINING GOOGLE SKETCHUP AND ISPRING SUITE 8: A BREAKTHROUGH TO DEVELOP GEOMETRY LEARNING MEDIA

Ani Nurwijayanti, Budiyono, Laila Fitriana

Sebelas Maret University, Jl. Ir. Sutami No. 36A, Jebres, Surakarta 57126, Indonesia
Email: aniinurwijayanti@gmail.com

Abstract

This study aims to develop geometry learning media on curved-solid objects using Ispring Suite 8 with 3D effects supported by Google SketchUp. It also aims to find the effectiveness of the media towards the basic geometry skills and the learning result of 9th-grade students of junior high school. This study is a development study which refers to Budiyono's development model that includes four stages. These stages are the preliminary, product development, product trial on its effectiveness, dissemination and product implementation. The entire stage was imposed on three different schools in Karanganyar, one of a district in Indonesia, by using stratified cluster random sampling. Within the three schools, we took seven classes to join the effectiveness test through the assessment questionnaire and the before-after test, as well as the efficacy test after the expert's judgement using the validation sheet. The result shows that the geometry learning media is valid based on the experts' validation judgement, and also practical based on the teacher and students' judgement in the trial of the product. The students' basic geometry skills and learning result are improved after getting the treatment with the media. Finally, we can conclude that this media is effective and able to be used further in the junior high school level.

Keywords: Geometry learning media, Google SketchUp, Ispring Suite 8.

Abstrak

Penelitian ini bertujuan untuk mengembangkan dan menghasilkan media geometri bangun ruang sisi lengkung menggunakan Ispring Suite 8 dengan efek 3D berbantuan program Google SketchUp yang valid dan praktis. Penelitian ini juga bertujuan untuk mengetahui efektifitas media yang dikembangkan terhadap kemampuan dasar geometri dan hasil belajar geometri siswa SMP kelas 9. Model pengembangan yang digunakan mengacu pada model pengembangan menurut Budiyono dengan empat tahapan, yaitu studi pendahuluan, pengembangan produk, pengujian keampuhan produk, serta diseminasi dan implementasi produk. Keseluruhan tahap dikenakan pada tiga sekolah yang berbeda di Karanganyar, Indonesia, dengan pemilihan sekolah menggunakan teknik *stratified cluster random sampling*. Dari tiga sekolah tersebut, diambil tujuh kelas yang terlibat dalam uji coba keefektifan melalui angket penilaian dan tes sebelum dan sesudah perlakuan, serta uji keampuhan setelah dilakukan penilaian ahli menggunakan lembar validasi. Hasil dari penelitian pengembangan ini yaitu media pembelajaran geometri berupa Ispring Suite 8 didukung program Google SketchUp sebagai fitur visualisasi 3D yang valid berdasar penilaian validasi pakar, dan praktis berdasar penilaian guru dan siswa pada saat proses uji coba produk. Kemampuan dasar geometri dan hasil belajar geometri siswa meningkat setelah pembelajaran menggunakan media tersebut. Hal ini menunjukan bahwa media tersebut efektif untuk digunakan lebih lanjut, khususnya pada jenjang SMP.

Kata kunci: Google SketchUp, Ispring Suite 8, Media pembelajaran Geometri.

How to Cite: Nurwijayanti, A., Budiyono, & Fitriana, L. (2019). Combining Google SketchUp and Ispring Suite 8: A breakthrough to develop geometry learning media. *Journal on Mathematics Education, 10*(1), 103-116.

The research and development in computer-based media become the focus of the mathematics experts and researchers to improve the teaching and learning quality. Many studies of the implementation of technology in teaching and learning emphasize the effectiveness of technology (Aliasgari, Riahinia, & Mojdehavar, 2010; Bennison & Gooos, 2010; Leung, 2017; Saadati, Tarmizi, & Ayub, 2014). Much earlier, the use of technology in mathematics teaching and learning had been recommended by the National Council of Teachers of Mathematics (NCTM, 2000). UNESCO also recommended the use of technology in teaching and learning in 2002. The reason behind it is that since mathematics is an abstract subject which will not be easily observed by the senses. Therefore, it is understandable if most students find it difficult to understand it, and requires learning media. Moreover, the advanced technology also affects the learning.

Considering that the interactive multimedia can stimulate the students' mathematical thinking, technology has an important role in teaching and learning. Students use it to find pattern, to solve problem, to communicate the mathematics idea into a picture, and to use the connection among mathematics topics in problem-solving (Kariadinata, 2007). By using a learning media, teacher could explain the material with more proportional thinking, starting with the connection into possibilities (Borovcnik & Kapadia, 2009). Kirkwood & Price (2014) explains that the technology-enhanced learning was increasingly used in Europe as it can help the teaching and learning process. Zhang (2005) states that students joining the interactive multimedia-based learning will produce higher learning result compared to the less interactive or traditional learning. Bennison & Gooos (2010) also confirmed that the use of technology is one of the ways to improve learning.

Currently, there are a lot of media to support and improve teaching and learning quality. Ispring Suite 8 presents an application software which can help the teacher to explain the lesson material. Further, this software is changes the presentation file that is compatible with powerpoint into flash format. Ispring Suite 8 also provides the learning material in a slide containing picture, animation, video, and audio becomes more interesting, practical, and ideal. It has a positive impact in learning process (Suprapti, 2016; Wijayanto, Utaya, & Astina, 2017; Sasahan, Oktova, & Oktavia, 2017).

In mathematics, particularly in geometry, there are numerous developed media used in the class. For examples, there are Cabri 3D, Wingeom, Geogebra, and Google SketchUp. Many studies find out the effectiveness of these media towards the geometry learning and how the media improves the students' spatial and visual skills (Nurwijayanti, Budiyono, & Fitriana, 2018a; Uygan & Kurtulus, 2010; Uygan & Kurtulus, 2016; Toptas, Celik, & Karaca, 2012; Hamdunah *et al.*, 2016; Nisiyatussani *et al.*, 2018; Zengin, 2017; Chou & Wu, 2014; Turğut & Uygan, 2014). The results show that the dynamic 3D software gives more effective spatial skill compared to conventional learning (Baki, Kosa, & Guven, 2011). Another study promotes the use of technology in geometry learning (González & Herbst, 2009). The study analyzed how the students solve the geometrical problem. When the students use the dynamic geometrical software, they manage to get the mathematical ideas better compared to when they are not using it.

Google SketchUp is a software developed by Google which combines several advanced tools in 3D graphic design on the computer screen. Panorkou & Pratt (2016) explore how this software can help the students experience through dimension. Abu, Ali, & Hock (2012) claim that learning uses Google SketchUp emphasizes the importance of geometrical structure in mathematics which is related to the visualization activity using the Google SketchUp feature. This activity also represents a concept that involves various geometrical representation such as a diagram, picture, and abstract visualization.

Abu *et al.* (2012) also states that Google SketchUp helps the students to visualize, construct the geometry concept, associate, and develop their thinking process. Therefore, they can recognize the geometry from the simplest form into the complex ones. But the study only helps students to construct and visualize geometrical objects in 3 dimensions without involving the development of the students' geometric concepts as a whole.

The students' low achievement in learning mathematics becomes one of the concerns in most countries (Peker & Mirasyedioglu, 2008). It means that there are still many barriers and obstacles in the mathematics teaching and learning process, including Indonesia. Moreover, based on the national exam result on junior high school level in 2015/2016 academics year, the topic of geometry has the lowest score compared to other material. Thus, geometry is still considered a difficult topic.

Connoly (2010) reveals that the geometry lesson is one of the most difficult to understand and also the most disliked material for students. Adolphus (2011) claims that geometry is the most feared and difficult mathematics element, which the students also acknowledge this fact. Further, the result from Nurwijayanti *et al.* (2018b) proves these two opinions based on the theory stated by Hoffer (1981) about the five basic geometry skills (visual, verbal, drawing, logical, and application). The study also shows that the students are only able to reach the basic visual skill.

On the verbal skill, they still have difficulty in linking the connection between solids, so they still lack in their ability to build the geometrical concept. Meanwhile, the verbal and visual skills have to develop the basic logical reasoning (Connoly, 2010). Hoffer (1981) states that those basic skills of geometry are crucial and must be balanced with the development of proving and problem-solving. These require the integration of those five basic skills. The structured and broader geometrical knowledge has a positive correlation with better problem-solving work. However, referring to the interview result, geometry is the hardest element to understand. The reason is that it requires the students to draw and then calculate it.

According to the earlier findings for the media's importance to improve the learning result and to find the students' difficulty with geometry, some action in developing and producing a learning media is necessary. Therefore, this study will focus on the development a geometry media using the Ispring Suite 8 software combined with the 3D effects in Google SketchUp feature.

METHOD

This study used the research and development design which refers to the development model of Budiyono (2017). It consists of four stages, which are the preliminary, product development, product trial on its effectivity, dissemination and product implementation. However, the analysis of this study is only up to the product trial phase only. The four steps can be illustrated in Figure 1.

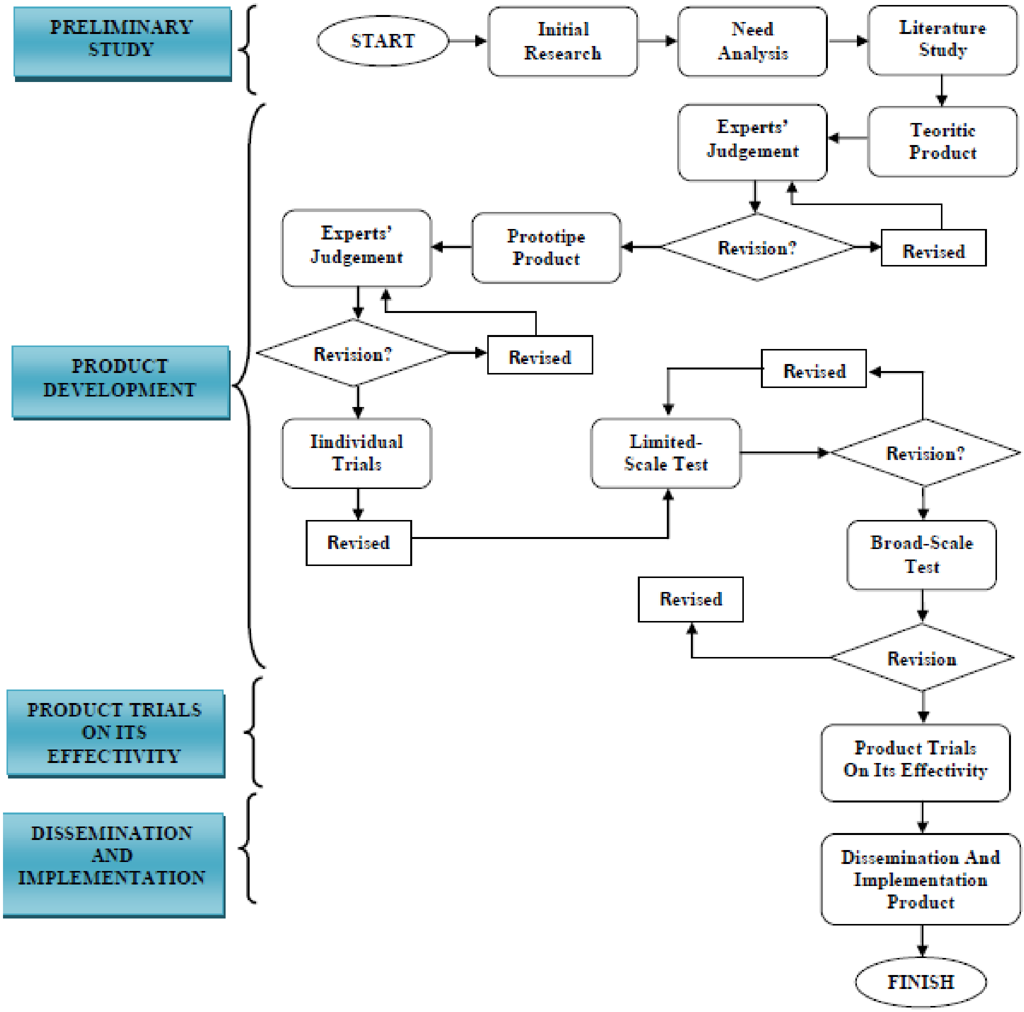


Figure 1. Research and development process according to Budiyono (2017)

The preliminary stage included the initial research, the needs analysis, and literature study. This stage was done on the target school by using the observation sheet, pre-test, the needs analysis questionnaire, and an interview with teachers and students. The product development stage included the experts' validation, individual, limited-scale, and broad-scale trials. This stage was done to the validator by using the media and material experts' validation sheet, as well as to the three classes using a media assessment questionnaire, the learning activities observation sheet, also the geometry basic skills test which is conducted on the broad-scale trial. The product trials stage used the experimental research with the static group comparison model, subjected to four classes consisting of two control and two experimental classes selected with stratified cluster random sampling. These four classes are subjected to a test of geometry after the implementation of the learning media, then analyzed following the experimental using stage using hypothesis analysis. The media itself is the Ispring Suite 8 application. On the teaching and learning activities, the uses of it combined with the 3D feature of the Google SketchUp program.

RESULT AND DISCUSSION

Preliminary Study

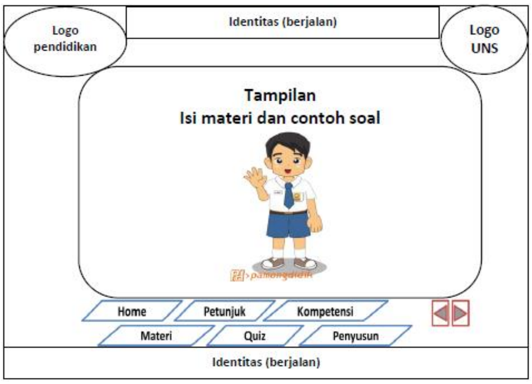
On this stage, we found that the teachers' activity only supported by text-book. They delivered the lesson without any visual model. The illustration was merely drawn on the whiteboard. Also, the interview result with several students showed that they tended to memorize the formula, but not to understand it. Moreover, the teachers rarely gave the varied teaching method. The use of learning media was also limited. It used models for some subjects only. Whereas, according to students the use of media on the learning activity could help them to understand the subject better. The pre-test result for the basic geometry skills to some students revealed that their ability was limited to visual skill only. On the verbal skill, they still struggled to link the connection between geometry which obstructed them to build the geometry concept. It caused their lack of logical process and geometry concept implementation for daily matters.

The next step was filling the needs analysis questionnaire by the teachers and students also interview. The results from this phase are as follows: (1) In the process of mathematics learning, teachers only used a textbook as the main source of learning. To get a wider range of material, the students demanded a learning source that was not only from the textbook, for instance, from the internet; (2) The students still struggled to visualize the geometry shapes. It was particularly the curved-face-three-dimensional objects. It led to their lack of ability to link the connection between it and troubled them to build the geometrical concept; (3) The media that often used in learning geometry was the geometrical framework. It showed through the demonstration method. To get a deeper understanding, the students wanted an alternative media as the additional source, such as using computer-based learning media. The use of interactive media as a supporting source of learning has not been implemented to the fullest by the teachers. Also, it was necessarily needed for the students and teachers as the alternative learning method. Moreover, it could help the students to have a broader knowledge of the related subject.

Then, we also conducted a literature study to find the syllabus information for the material referred to the basic competence. It was to find the surface geometry area and curved-solid objects (cylinder, cone, and sphere) volume formulas as well as to solve the related contextual problem. Then, these materials were implemented in the learning media. Further, the media also contained the materials from various sources, and one of them was the teacher book in the 2013 curriculum.

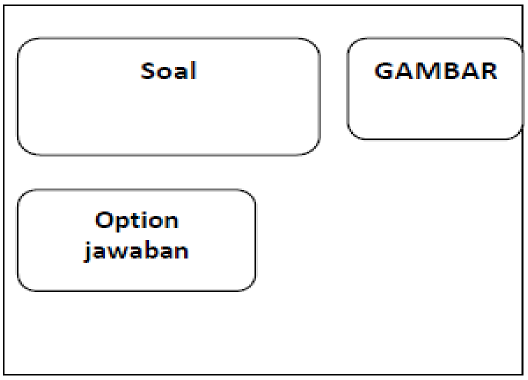
Product Development

The development stage started with composing the media draft. It included the homepage, the sub-materials page of curved-solid objects, and the quiz page that made by Ispring quiz maker. Then, we developed this draft into an intact media. Figure 2 were the result of the developed media draft.

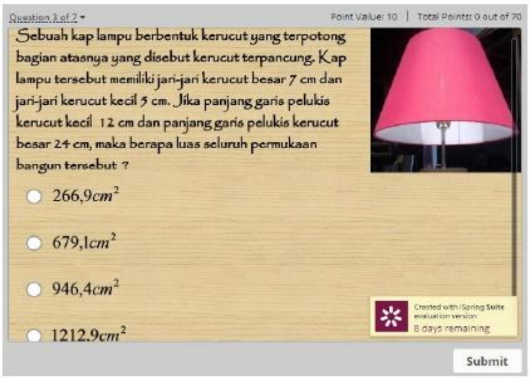


(a) Content page storyboard

(b) Content page preview



(c) Quiz page storyboard



(d) Quiz page preview

Figure 2. The design and the realization of the learning media

Figure 2a is the storyboard of the media design on the content page, while Figure 2b is the development result. Figure 2c is the storyboard for quiz design and then developed using Ispring quiz maker shown in Figure 2d.

After the development of the media draft, the next step was media and material experts' validation. The first and second media validation results showed in Figure 3.

Media Experts' Judgement

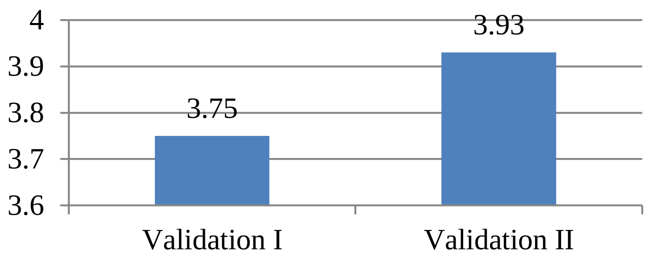


Figure 3. Media experts' judgement results

The media first validation average score was 3.75, which is in the good category. However, there were suggestions and feedbacks from the validators. They suggested fixing the system in the Ispring Suite 8 media to link with Google SketchUp. It was to make the quiz link and system directly connected to the media. Based on that, we made the improvements and refined the lack of the media. Further, the polished media was validated again. The score for the second validation was 3.93 with the good category. Thus, the geometry media of Ispring Suite 8 supported by Google SketchUp could be used in the learning activities. The second validation result was presented in Figure 4.

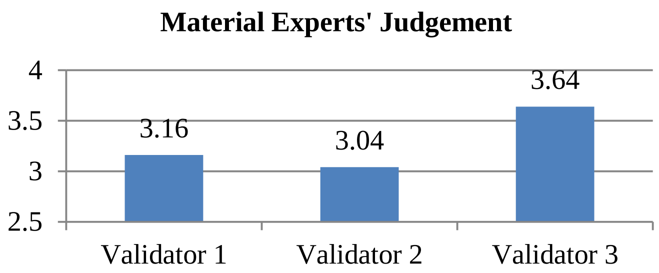


Figure 4. Material Experts' Judgement Results

The average score from the first validator was 3.16 and the second validator was 3.04 both were in fair category, and the third validator was 3.64 in a good category. However, there were several suggestions and feedbacks from the validators. First, to add geometry illustration is on the quiz and test. It was to make the students able to observe visually. Then, made the test questions are closer to students' daily life. After making the media text more colourful and adjusted with its background, we served the images accordingly to avoid the ambiguous material concepts. Based on the validators' suggestions and feedbacks, the researchers made the improvements and refined the lacking materials in the media then validated it again. According to the second validation, the score was 3.80 which is in a good category. Therefore, the geometry media Ispring Suite 8 supported by Google SketchUp generally could be used in the learning activity.

After the product was said to be valid, then it was ready to be tested. The first trial was the individual test. This test was performed to seven students who randomly chosen from one of the schools. It used a questionnaire as the instrument of the test. The total scoring score for this test was 894 with its average 3.55. This score was in the strongly agree category if the media used further. Moreover, there was a suggestion and feedback as well from the students. It was to make the material deeper so they could understand it in more details. Then, we also observed the run of the program. The result of it revealed several barriers. These were the incompatibility of the media with the computer's capacity, due to some laptop or computer encountered an error when used it. These input suggestions and the barrier were used as the reference for the researchers to make the improvements. Also, the detailed result of the individual trial presented in Figure 5.

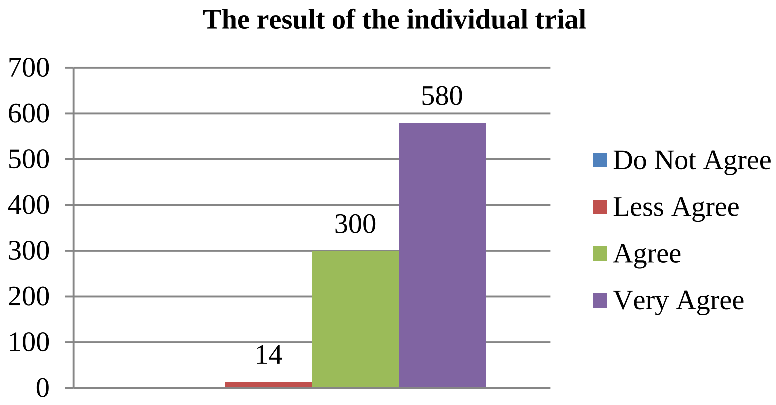
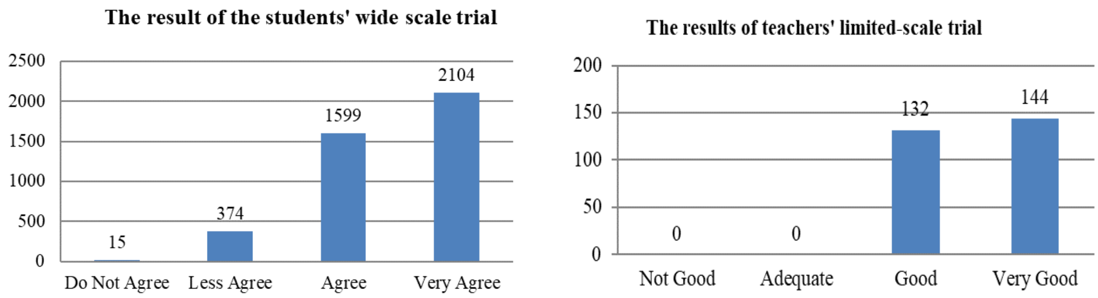


Figure 5. The appraisal question of media on individual trial

The second trial was the limited-scale test. It was conducted to one class consisted of 35 students and teachers by using questionnaire. The result from the students indicated a total score of 4122 with an average of 3.27. This score was –along with the individual test—included in the strongly agree category if the media used further. Further, the scoring on this test was included the teachers as the users. The result of their valuation was 141 with the average score of 3.52 in the good category. Aside from the questionnaire, the scoring is also performed by observation. The observation results presented the conclusion that from the media or the learning activities aspects were already run well and appropriate. As for the obstacle happened when used the media was: the video on the media could not be displayed, so it had to be shown separately. We used this constraint as the basis to refine the media. The students and teachers scoring results displayed in Figure 6.



(a) The media scoring result from students (b) The media scoring result from teachers

Figure 6. The results of the trial

The third trial was the broad-scale test. It was performed by using the stratified cluster random sampling to two classes from two different schools. These two classes were from SMP 1 Karanganyar (9J) and SMP 1 Jaten (9A). The before-after experimental design used in this test. Later, it used the t-test to analyze the data. Then, the data collection technique also used before and after the media used in the class tests, for the students' result of the basic geometry skills on the curved-solid objects. Table 1 presents the result of the basic geometry skills test.

Table 1. The before-after geometry basic skills test results

Group	The Number of Students	Highest Score	Lowest Score	Mean
Before	85	8	1	4.48
After	82	10	1.25	6.66

Before running the t-test, it has to fulfil the normality and the homogeneity tests first, as presented in in Table 2.

Table 2. The normality test result of the before-after tests

Group	The Number of Students	L_{obs}	L_{tabel}	Hypothesis Result	Description
Before	85	0.0507	0.0961	H_0 accepted	The sample was from the normal distribution population
After	82	0.0608	0.0978	H_0 accepted	The sample was from the normal distribution population

The average score from the group before treatment was 4.48 and the after treatment was 6.66. In Table 2, the normality test showed decent results from both before and after groups which means that both samples were from the normal distribution population. Based on the homogeneity test calculation, the variance result was $\chi^2_{obs} = 0.0001$ with $\chi^2_{0,05;1} = 3.841$. According to the critical region criterion $CR = \{\chi^2 | \chi^2. > \chi^2_{\alpha,k-1}\}$ then $\chi^2_{obs} = 0.0001 \notin CR$. Thus, the null hypothesis was accepted since it signified that both population variances were homogeneous or equal.

After meeting both requirements (normal and homogeneous), we could conduct the *t*-test. From the *t*-test, the calculation result revealed that $t_{obs} = 7.364$ with the critical region for $\alpha = 0.05$ was $CR = \{t | t > 1.65\}$, so $t_{obs} = 7.364 \in CR$. Therefore, the test decision for the null hypothesis was rejected. Both groups (before and after) average scores were different. On the other words, the result from the after group was better compared to the before group. It established that the learning used the geometry media Ispring Suite 8 supported by Google SketchUp on the curved-solid objects was effective to use.

To find the effect (before and after) in applying media for the students' basic geometry skills, we took some students' answers as the samples for deeper analysis. It covered the visual, verbal, drawing, logical, and application which referred to Hoffer (1981). From the analysis results, it showed these skills improved after the implementation of the media in the class. According to those average scores explanation and the basic skills analysis above, we could conclude that geometry media Ispring Suite 8 supported by Google SketchUp was effective to use especially in junior high school level.

Product Trials on Its Effectiveness

The product trials on its effectiveness performed in four classes from two different schools as the stratified random sampling. From the first school, there were class 9E as the control class and 9I as the experiment class in SMP 1 Karanganyar. Another school was SMP 1 Jaten with class 9B as the control class and 9C as the experiment class. The trials itself had done by using the Static Group Comparison experiment draft then analyzed by the *t*-test. The data is presented in Table 3.

Table 3. Test result data

Group	The Number of Students	Highest Score	Lowest Score	Mean
Experiment	57	90	45	67.63
Control	59	85	35	63.22

Before using the *t*-test, it required fulfilling the normality and homogeneity tests first, as presented in Table 4.

Table 4. Normality test result of the test result data

Group	The Number of Students	L_{obs}	L_{table}	Hypothesis Result	Description
Experiment	57	0.0781	0.1174	H_0 accepted	The sample was from the normal distribution population
Control	59	0.0865	0.1153	H_0 accepted	The sample was from the normal distribution population

The average score of experiment class was 67.63 and control class was 63.22. In Table 4, the normality test showed for both classes, these samples came from the normally distributed population. Then, the variance according to the homogeneity test acquired the result $\chi^2_{obs} = 0.1872$ with $\chi^2_{0,05;1} = 3.841$. Since the critical region criterion was $CR = \left\{\chi^2 \middle| \chi^2. > \chi^2_{\alpha,k-1}\right\}$ then $\chi^2_{obs} = 0.1872 \notin$ *CR*. Therefore, the null hypothesis was accepted since it signified that both population variances were homogeneous or equal.

The hypothesis test used the *t*-test was performed after passed the requirement for both normality and homogeneity. From the *t*-test result, the $t_{obs} = 2.037$ with the critical region criterion for $\alpha = 0.05$ was $= \{t|t > \ 1.65\}$, so $t_{obs} = 2.037 \in CR$. Then, the null hypothesis was rejected. In addition, for both experiment and control groups had a different mean score. On the other hands, the learning used the geometry media Ispring Suite 8 supported by Google SketchUp had a better achievement compared to the one without it.

CONCLUSION

Geometry learning media using Ispring Suite 8 combined with the 3D effect of Google SketchUp program was valid according to the experts. It was also practical to use as the students and teachers' response on the product trials process. Further, the students' geometry basic skills was improved after the implementation of the media. It showed that it was effective to use the media. Moreover, from the development series, we also concluded that this media was valid, practical, and effective to be used in the geometry teaching and learning process.

ACKNOWLEDGEMENT

I thank Budiyono and Laila Fitriana as mentors in writing this article and the Master Program of Mathematics Education, Faculty of Teacher Training and Education, Sebelas Maret University.

REFERENCES

Abu, M. S., Ali, M. B., & Hock, T. T. (2012). Assisting primary school children to progress through their Van Hiele's levels of geometry thinking using Google Sketchup. *Procedia-Social and Behavioral Sciences*, *64*, 75-84.

Adolphus, T. (2011). Problems of teaching and learning of geometry in secondary schools in Rivers State Nigeria. *International Journal of Emerging Sciences*, *1*(2), 143-152.

Aliasgari, M., Riahinia, N., & Mojdehavar, F. (2010). Computer-assisted instruction and student attitudes towards learning mathematics. *Education, Business, and Society: Contemporary Middle Eastern Issues*, *3*(1), 6-14.

Baki, A., Kosa, T., & Guven, B. (2011). A comparative study of the effects of using dynamic geometry software and physical manipulatives on the spatial visualisation skills of pre- service mathematics teachers. *British Journal of Educational Technology*, *42*(2), 291-310.

Bennison, A., & Goos, M. (2010). Learning to teach mathematics with technology: A survey of professional development needs, experiences, and impacts. *Mathematics Education Research Journal*, *22*(1), 31-56.

Borovcnik, M., & Kapadia, R. (2009). Research and developments in probability education. *IEJME-Mathematics Education*, *4*(3), 111-130.

Budiyono. (2017). *Pengantar Metodologi Penelitian Pendidikan*. Surakarta: UNS Press.

Chou, P. N., & Wu, C. Y. (2014). Integrating 3D visualization tools into teaching surface area in elementary school classrooms: An example of Google Sketchup. *Taiwan Journal of Mathematics Education*, *1*(1), 1-18.

Connoly, S. (2010). The impact of Van Hiele-based geometry instruction on student understanding. *Thesis*. Pittsford: St. John Fisher College.

González, G., & Herbst, P. G. (2009). Students' conceptions of congruency through the use of dynamic geometry software. *International Journal of Computers for Mathematical Learning*, *14*(2), 153-182.

Hamdunah, H., Yunita, A., Zulkardi, Z., & Muhafzan, M. (2016). Development a constructivist module and web on circle and sphere material with Wingeom Software. *Journal on Mathematics Education*, *7*(2), 109-116.

Hoffer, A. (1981). Geometry is more than proof. *Mathematics Teacher*, *74*(1), 11-18.

Kariadinata, R. (2007). Desain dan pengembangan perangkat lunak (software) pembelajaran matematika berbasis multimedia. *Jurnal Pendidikan dan Kebudayaan, 69*(13), 1055-1080.

Kirkwood, A., & Price, L. (2014). Technology-enhanced learning and teaching in higher education: What is 'enhanced' and how do we know? A critical literature review. *Learning, Media, and Technology*, *39*(1), 6-36.

Uygan, C., & Kurtuluş, A. (2010). The effects of Google Sketchup-based geometry activities and projects on spatial visualization ability of student mathematics teachers. *Procedia-Social and Behavioral Sciences*, *9,* 384-389.

Uygan, C., & Kurtuluş, A. (2016). Effects of teaching activities via Google Sketchup and concrete models on spatial skills of preservice mathematics teachers 1. *Turkish Journal of Computer and Mathematics Education*, *7*(3), 510-535.

Leung, A. (2017). Exploring techno-pedagogic task design in the mathematics classroom. *Digital Technologies in Designing Mathematics Education Tasks*, *8,* 3-16.

NCTM. (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nisiyatussani, N., Ayuningtyas, V., Fathurrohman, M., & Anriani, N. (2018). GeoGebra applets design and development for junior high school students to learn quadrilateral mathematics concepts. *Journal on Mathematics Education*, *9*(1), 27-40.

Nurwijayanti, A., Budiyono & Fitriana, L. (2018a). The geometry ability of junior high school students in Karanganyar based on the Hoffer's theory. *Journal of Physics: Conference Series*, *983*(1), 012085.

Nurwijayanti, A., Budiyono & Fitriana, L. (2018b). The use of interactive media ispring suite 8 supported by Google SketchUp to improve students' geometry skills based on Hoffer's Theory. *Journal of Physics: Conference Series*, *1008*(1), 012075.

Panorkou, N., & Pratt, D. (2016). Using Google SketchUp to develop students' experiences of dimension in geometry. *Digital Experiences in Mathematics Education*, *2*(3), 199-227.

Peker, M., & Mirasyedioğlu, Ş. (2008). Pre-service elementary school teachers' learning styles and attitudes towards mathematics. *Eurasia Journal of Mathematics, Science & Technology Education*, *4*(1), 21-26.

Saadati, F., Tarmizi, R. A., & Ayub, A. F. M. (2014). Utilization of Information and communication technologies in mathematics learning. *Journal on Mathematics Education*, *5*(2), 138-147.

Sasahan, E. Y., Oktova, R., & Oktavia, O. O. (2017). Pengembangan media pembelajaran interaktif tentang optika berbasis android menggunakan perangkat lunak Ispring Suite 7.0 untuk mahasiswa S-1 Pendidikan Fisika pada pokok bahasan interferensi cahaya. *Prosiding SNFA (Seminar Nasional Fisika dan Aplikasinya)*, *2*, 52-61.

Suprapti, E. (2016). Pengembangan perangkat pembelajaran matematika model kooperatif tipe STAD dengan media powerpoint Ispring pada materi jajargenjang, layang-layang dan trapesium di kelas VII SMP. *Journal of Mathematics Education, Science and Technology*, *1*(1), 57-68.

Toptas, V., Celik, S., & Karaca, E. T. (2012). Improving 8th grades spatial thinking abilities through a 3D modeling program. *TOJET: The Turkish Online Journal of Educational Technology*, *11*(2), 128-134.

Turğut, M. & Uygan, C. (2014). Spatial ability training for undergraduate mathematics education students: Designing tasks with Sketchup. *Electronic Journal of Mathematics and Technology, 8*(1), 53–65.

Wijayanto, P. A., Utaya, S. & Astina I. K. (2017). Increasing student's motivation and geography learning outcome using active debate method assisted by Ispring Suite. *International Journal of Social Sciences and Management*, *4*(4), 240-247.

Zengin, Y. (2017). The effects of Geogebra software on pre-service mathematics teachersí attitudes and views toward proof and proving. *International Journal of Mathematical Education in Science and Technology*, *48*(7), 1-21.

Zhang, D. (2005). Interactive multimedia-based e-learning: A study of effectiveness. *The American Journal of Distance Education*, *19*(3), 149-162.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 117-126

MATHEMATICAL PROOF: THE LEARNING OBSTACLES OF PRE-SERVICE MATHEMATICS TEACHERS ON TRANSFORMATION GEOMETRY

Muchamad Subali Noto[1,2], Nanang Priatna[2], Jarnawi Afgani Dahlan[2]

[1]Universitas Swadaya Gunung Djati, Jl. Perjuangan No.1 Cirebon, Indonesia
[2]Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229 Bandung 40154, Indonesia
E-mail: msubalinoto@student.upi.edu

Abstract

Several studies related to mathematical proof have been done by many researchers on high-level materials, but not yet examined on the material of transformation geometry. The aim of this research is identification learning obstacles pre-service teachers on transformation geometry. This study is qualitative research; data were collected from interview sheets and test. There were four problems given to 9 pre-service mathematics teachers. The results of this research were as follows: learning obstacles related to the difficulty in applying the concept; related to visualize the geometry object; related to obstacles in determining principle; related to understanding the problem and related obstacles in mathematical proofs such as not understanding and unable to express a definition, not knowing to use the definition to construct the proof, not understanding the use of language and mathematical notation, not knowing how to start the proof.

Keywords: Mathematical proof, Pre-service mathematics teachers, Geometry.

Abstrak

Beberapa penelitian terkait pembuktian matematis telah dilakukan oleh banyak peneliti pada materi-materi tingkat tinggi, namun belum ada yang meneliti pada materi geometri transformasi. Tujuan dari penelitian ini adalah mengidentifikasi hambatan belajar calon guru pada transformasi geometri. Penelitian ini merupakan penelitian kualitatif, data dikumpulkan dari lembar wawancara dan tes. Ada 4 masalah yang diberikan kepada 9 calon guru matematika. Hasil penelitian adalah sebagai berikut: hambatan belajar terkait kesulitan dalam mengaplikasikan konsep; terkait dengan memvisualisasikan objek geometri; terkait dengan kesulitan dalam memahami prinsip-prinsip; terkait dengan memahami masalah dan terkait dengan kesulitan dalam pembuktian matematis seperti tidak memahami dan tidak dapat mengungkapkan definisi, tidak tahu untuk menggunakan definisi dalam mengkonstruksi bukti, tidak memahami penggunaan bahasa dan notasi matematika, tidak tahu bagaimana memulai pembuktian dan tidak tahu bagaimana memvisualisasikan masalah yang diberikan.

Kata Kunci: Pembuktian matematis, Calon guru matematika, Geometri.

How to Cite: Noto, M.S., Priatna, N., & Dahlan, J.A. (2019). Mathematical proof: Learning obstacles pre-service teachers on transformation geometry. *Journal on Mathematics Education, 10*(1), 117-126.

Geometry is an integral part of the learning of mathematics (Fachrudin, Putri, & Darmawijoyo, 2014; Sukirwan, Darhim, Herman, & Prahmana, 2018; Ahamad, Li, Shahrill, & Prahmana, 2018). However, the development of learning geometry at this time is less developed. One reason is the difficulty in forming a real construction student carefully and accurately, the notion that to paint geometry requires precision in the measurement and requires a long time, and not infrequently students experiencing obstacles in the process of evidence (Rizkianto, Zulkardi, & Darmawijaya, 2013; Novita, Prahmana, Fajri, & Putra, 2018). Meanwhile, the painting plays an essential role in teaching geometry at school for painting geometric connection between physical space and theory. If further investigation of the link between the objects with the abstract geometry student obstacles in learning geometry, it will be alleged that in fact there is a problem in teaching geometry at school relates to the formation of abstract concepts. Learn abstract concepts cannot be done only with the transfer of information, but it takes a process of formation of concepts through a series of activities experienced directly by students (Nurhasanah, Kusumah, & Sabandar, 2017). The series of abstract concept formation activity of these are referred to the process of abstraction.

Studying mathematics meant to be also studied branches of mathematics is the science of geometry. Everything in this universe is geometry so that the branches of mathematics through geometry learn about the concepts embodied in the objects that exist in nature through geometric concepts. Thus, assessment of learning geometry must continue to be developed so that each learner can analyze the geometry of objects into a concept of geometry and can construct geometry knowledge with formal proofs. Mata-pereira & Ponte (2017) say that a proof is a connected sequence of assertions that includes a set of accepted statements, forms of reasoning and modes of representing arguments. Stefanowicz & Kyle (2014) say that a proof is a sequence of logical statements, one implying another, which explains why a given statement is true.

However, mathematical proofs in geometry material lately become an obstacle that they seem poorly developed. Difficulties analyzing geometric properties are realized in the form of theorems to create a concept widely experienced by the students. Proof becomes a severe matter in determining the school curriculum in every different country. This is what makes reasoning and proof NCTM enters into one of the standard processes. This means that in every lesson a teacher must enter the elements in each classroom. Maya & Sumarmo (2011) state that possessing mathematical proving ability was certainty ability because it is an essential ability that should be possessed by all students who learn mathematics. Komatsu (2017) state also that proof and proving to play a crucial role in the discipline of mathematics and should be an essential component of mathematical learning.

Several studies have been conducted regarding evidence learning in secondary school (Harel & Sowder, 1998; Mariotti, 2006). Several research methods have also been conducted in proof-learning. Duval's research (1991) has identified the arguments and evidence that explain the difficulties students experience in understanding and making proof. Other researchers such as Balacheff, 1988; Harel & Sowder, 1998; Marrades & Gutiérrez, 2000 focuses more on identifying the types of empirical and deductive evidence generated by students that enable student progress in learning to prove.

Some researchers identify and explain the reasons why students are unwilling or unable to complete deductive evidence from their allegations (Arzarello, Micheletti, Olivero, & Robutti, 1998). Researchers also focus on analyzing learning especially on deductive evidence (Antonini, 2003; Stylianides, Stylianides, & Philippou, 2007; Antonini & Mariotti, 2008). More details can be found in Mariotti's research (2006), Reid and Knipping (2010), and Hanna and De Villiers (2012).

In traditional learning, mathematical proof is only used as a means to eliminate the doubts of students on the concepts taught. However, the evidence is not used as a means of increasing the higher mathematical ability. Just as revealed by Hana (Christou, et al. 2004) that the function of evidence and proof are: verification, explanatory, systematization, invention, communication, construction, exploration, and incorporation. Verification of proving and the proof is regarded as the most fundamental functions in the proof because both are products of the process of the development of mathematical thinking very mature. Verification refers to the truth of the statement while explanations provide insight into why this is true.

As for the role played proof in mathematics, namely: 1) to verify that a statement is true, 2) to explain why a statement can be said to be true, 3) to establish communication mathematics, 4) to find or create new math and 5) To make systematic statement in an axiomatic system (Knuth, 2002). Hanna (Stylianides, Stylianides, and Philippou, 2007) say that there are three main reasons why the ability to prove the need to be improved. First, the proof is crucial to learn to explore mathematics. Second, the ability of students in the proof can improve their math skills more broadly, because the evidence "involved in all situations where the conclusion must be reached in the making of decisions to be made", and the third, the difficulties experienced by high school students and college students will affect their ability to perform mathematical proofs on a broader level again, so it is crucial for students to learn mathematical proof on the level of previous education.

METHOD

This study was conducted to analyze student learning obstacles, especially regarding the difficulty students epistemology regarding materials, both presented in the form of materials or materials in lectures. This research method is descriptive qualitative research that aims to describe obstacles regarding epistemology student learning mathematical proofs related to the subject of transformation geometry. Subjects were nine student teachers Unswagati contracting mathematics courses transformation geometry consisting of 3 students with high prior knowledge mathematically, three students with medium prior knowledge of the mathematically and three students with low prior knowledge of mathematically. The beginning of knowledge is based on the acquisition of student achievement index in the previous semester. For students learning obstacles regarding epistemology student at transformation material using five indicators: concept, visualization, principles, understand the problem, and mathematical proofs.

RESULTS AND DISCUSSION

This research resulted in qualitative data. Learning obstacles students understand the concepts in terms of epistemology transformation geometry in working on the geometry transformation is divided into 5 types, it is in terms of indicators in assessing learning obstacles, namely: a) learning obstacles related to the difficulty in applying the concept; b) learning obstacles related to visualize the geometry object; c) learning obstacles related to obstacles in determining principle; d) learning obstacles related to understanding the problem and e) related obstacles in mathematical proofs.

Related understanding and applying the concept of learning difficulty is the difficulty experienced by students in understanding and applying the concept by the command matter. Examples of these obstacles one student does not understand the concept, students cannot mention the definition of transformation. It happened at the beginning of the mathematical knowledge of students with high, medium or low. Here is one example of the questions and responses of students experiencing barriers to learning. For example, on the following question: write the definition of a transformation in the field of V.

Learning obstacle students with high prior knowledge is described as follows. S1 had trouble with the concept, to define the transformation; these students do not write domain/codomain of a function called transformation. S5 can write correctly and complete the definition of transformation. S8 experienced obstacle of the concept, to define the transformation, these students do not write domain/ codomain of a function called transformation.

Learning obstacle students with prior knowledge is being described as follows. S3 having trouble against the concept, it cannot define the transformation correctly; these students mention that the transformation is a bijective function, but do not write domain /codomain of these functions. S4 in defining transformation by merely mentioning that a transformation is injective functions only. S9 can write the definition of transformation correctly and completely. Obstacles students with low initial knowledge are described as follows. S2 no difficulty is in writing the definition of transformation. S6 and S7 to write the definition of transformation are not complete. Both of these students do not write domain/codomain of a function is the transformation in the field of S6 V. It also found one in writing notation.

Related Learning Obstacles Visualize learning obstacles related visualize the geometry object. The point is that students have obstacles regarding describing the line of the transformation result. Examples of these obstacles include the inability of students in painting properly and appropriately. Here is one example of the questions and responses of students who experience learning obstacles. For example, in Question 2 is as follows: Draw the line $g' = M_h(g)$ if h = {(x,y) |y = x + 1} and $g = \{(x, y)| y = -x\}$.

Learning obstacle students with high prior knowledge is described as follows. S1 is having trouble visualize right lines g and h so that images reflection created false images. S5 can visualize by what is known and questioned, but the images are still made without a ruler. S8 can paint lines mirroring the results appropriately. Obstacles students with prior knowledge are being described as follows.S3 had difficulty in visualizing the line h so that the reflection is illustrated one. S4 in the paint did not use a ruler, but the results are correct reflection depicted. S9 cannot describe all that is known. Obstacles students with low initial knowledge are described as follows. S2 can paint reflection results correctly. S6 and S7 cannot describe all that is known so that the reflection nothing, other than that, the two students were not drawing Cartesian coordinates as a first step in painting a line mirroring results.

Learning obstacles related to the principle of. Learning obstacles is the difficulty experienced by students in terms solves the problem by defining the principles to be used in solving the problem of transformation. Examples of this difficulty are the inability of students in the mentioned properties of isometry, so it cannot be a member of reasons of the questions in the matter. Here is one example of the questions and responses of students who have difficulty learning. For example, the following problem: Given: T and S isometry. Determine the statements below True or False? Give your reason.

a. If *g* is a line, then *g*'= (TS)*(g)* is also a line.
b. If g // h and g'= (TS)(g), h'= (TS)(h) then g'// h'.
c. If S is a reflection of the S is involutory.

Learning obstacle students with high prior knowledge is described as follows. S1 can answer correctly, but the reasons expressed by one. S5 can be answered correctly and the reasons for appropriately. S8 can mention the definition of isometry correctly, but cannot answer questions related to the principle of isometry, so that reason used improperly. Obstacles students with prior knowledge are being described as follows. S3 was having difficulty writing down the definition of isometric and members wrong reasons related to statements given. S4 can write for the right reasons, but they are notational wrong. S9 is important to specify the definition of isometry, giving the wrong reasons. Obstacles students with low initial knowledge are described as follows. S2 is important to specify the definition, and the reason given was also incorrect. S6 and S7 wrong in writing down the definition and does not include the reason (no answer).

Learning obstacles related to understanding the problem. Is the difficulty of learning obstacles experienced by students regarding understanding the problem to solve the problem by using the steps in the completion of the write down what is known and asked about the matter. Examples of this difficulty are the inability of students to solve problems by the steps to completion. Here is one example of the questions and responses of students who have difficulty learning. For example, on the following question: define a line equation $g' = M_h(g)$ if h = {(x,y) |y = x + 1} and $g = \{(x, y) | y = -x\}$.

Learning obstacle students with high prior knowledge is described as follows. S1 was having trouble determining what is known and troubleshooting procedures are still wrong. S5 can understand the problem, find out what is known and asked, can solve the problem by the settlement procedures. S8 can understand the problem and solve it according to the procedure. Obstacles students with prior knowledge are being described as follows. S3 can understand the problem and solve the problem with proper procedures, but there are errors in arithmetic operations. S4 can understand the problem and solve the problem according to the procedure. S9 cannot understand the problem and cannot solve it. Obstacles students with low initial knowledge are described as follows. S2 can understand the problem but cannot finish the correct procedure. S6 and S7 are not able to understand the problems and did not finish.

The difficulty is in proving mathematical. Learning difficulty is the difficulty experienced by students in constructing the proof of the matter. Here is one example of the questions and responses of students who have difficulty learning. For example, the following problem: to prove that the reflection on the line g is an isometry.

Learning obstacle students with high prior knowledge is described as follows. S1 can construct evidence correctly, but there is still incorrect notation. S5 can construct evidence properly. S8 can construct proofs in part, at the end of the part that is wrong in giving reasons. Obstacles students with prior knowledge are being described as follows. S3 and S4 obstacles for constructing proofs can't use the existing definition. S9 trouble is to begin constructing proofs. Obstacles students with low initial knowledge are described as follows. S2 cannot use a definition for constructing proofs. S6 and S7 begin constructing the evidence about be proved, difficulty in starting the construction of the evidence and not be able to use the definition for constructing proofs.

Some mathematical proofs related research shows some of the things that are essential mathematical proofs. Activities considered difficult by students to learn and teachers to teach include justification or proof (Suryadi, 2007). Research studies conducted Dryfus (Jones and Rood, 2001) showed that students always fail to look at the adequacy of the evidence because they are too often asked to prove things that are obvious to them. Students also fail to distinguish between the different forms of mathematical reasoning such as heuristic or argument, explanation or proof. A significant gap in the research literature is still at least main set students "because it looks right" instead of "because he worked on issues" for the argument that believed.

The research result Knuth (2002) showed that teachers recognize the many roles of the play proof in mathematics, in learning the role of evidence should not be abandoned, and the evidence as a tool for learning mathematics. The results also show that many teachers still have limitations in determining the nature of the evidence used in the study of mathematics.

Heinze and Reiss (2003) research results showed that some students found with some of the answers wrong though ideas about the solution are proving correct. This occurs in the empirical argument. Many errors occur experienced by students is related to aspects of the structure of evidence. Most of the students interviewed were mostly already know that the argument does not form empirical evidence.

Study interviews in this study also showed that the three aspects of methodological knowledge of relevant proof when assessing the evidence. It seems that on this aspect conclusion chain is not problematic, because the proof is right mostly depicted as true. However, some cases it was not clear whether the students understand every step of the evidence compiled. The problem with this aspect of the scheme of evidence, in particular, an inductive argument, often trained using inductive argument in elementary school. Students have difficulty bridging the gap between empirical arguments to formal arguments. This is confirmed by a study by Lin (Heinze and Reiss, 2003) showed that Taiwanese students a different problem that is wrong or improper argument transformation in improving formal arguments were observed.

Mariotti study (2001) revealed the geometry construction is an essential part of the experience of students who should be organized. Results of the study Mariotti showed that if the geometry is just a pencil and paper geometry theory perspective, it is difficult to understand. When students draw on paper students can only focus on the images being constructed and can't manipulate it.

CONCLUSION

Average-ability students, especially in mathematical constructing proofs of 11.33 (maximum score 28) with details as follows: for students with high mathematical prior knowledge gained an average of 19.67; students with early mathematical knowledge have gained an average of 9.33, and with low prior knowledge mathematical earned an average of 5.00. There are five kinds of difficulties related to the students regarding epistemology on geometry transformations, namely a) learning difficulties related to the difficulty in applying the concept; b) associated learning difficulties visualizing the geometry object; c) learning difficulties related to difficulties in determining principle; d. Related learning difficulties to understand the problem and e. Related difficulties in mathematical proofs. Specialized in mathematical proofs, students have difficulty, among others: do not know how to start the construction of the evidence, can't use the definition (concept) and the principle already known, and are likely to begin construction of the evidence with what must be proved.

ACKNOWLEDGMENTS

Author would say thank to KEMENRISTEK DIKTI that giving research and doctoral dissertation grant.

REFERENCES

Ahamad, S. N. S. H., Li, H. C., Shahrill, M., & Prahmana, R. C. I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series, 943*(1), 012008.

Antonini, S., & Mariotti, M. A. (2008). Indirect proof: What is specific to this way of proving? *ZDM International Journal on Mathematics Education, 40*(3), 401–412.

Arzarello, F., Micheletti, C., Olivero, F., & Robutti, O. (1998). A model for analyzing the transition to formal proofs in geometry. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd PME Conference,* 24–31. Stellenbosch, Republic of South Africa: PME.

Balacheff, N. (1988). Aspects of proof in pupils' practice of school mathematics. In D. Pimm (Ed.), *Mathematics, teachers and children,* 216–235. London: Hodder & Stoughton.

Christou, C., Mousoulides, N., Pittalis, M and Pantazi, D.P. (2004). Proofs Through Exploration In Dynamic Geometry Environments. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* 2004, 215–222.

Duval, R. (1991). Structure du raisonnement déductif et apprentissage de la démonstration. *Educational Studies in Mathematics, 22*(3), 233–261.

Fachrudin, A. D., Putri, R. I. I., & Darmawijoyo. (2014). Building Students' Understanding of Quadratic Equation Concept Using Naïve Geometry. *Journal on Mathematics Education, 5*(2), 192-202.

Hanna, G., & de Villiers, M. (Eds.). (2012). *Proof and proving in mathematics education*. Dordrecht, The Netherlands: Springer.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education*, 234–283. Providence, RI: American Mathematical Society.

Heinze, A., & Reiss, K. (2003). Reasoning and proof: Methodological knowledge as a component of proof competence. *Proceedings of the third conference of the European Society for Research in mathematics education*, 1-10.

Jones, K & Rodd, M. (2001). Geometry and Proof. *Proceedings of the British Society for Research into Learning Mathematics 21*(1), 95-100.

Komatsu, K. (2017). Fostering empirical examination after proof construction in secondary school geometry. *Educational Studies in Mathematics, 96*(2), 129–144.

Knuth, E.J. (2002). Teachers' Conception of Proof in the Context of Secondary School Mathematics. *Journal of Mathematics Teacher Education 5*(1), 61–88.

Marrades, R., & Gutiérrez, A. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational Studies in Mathematics, 44(2)*, 87–125.

Mariotti, M. A. (2001). Introduction To Proof: The Mediation Of A Dynamic Software Environment. *Educational Studies in Mathematics 44*(1), 25–53.

Mariotti, M. A. (2006). *Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.), Handbook of research on the psychology of mathematics education. Rotterdam*. The Netherlands: Sense.

Mata-pereira, J., & Ponte, J. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics, 96*(2), 169–186.

Maya, R., & Sumarmo, U. (2014). Mathematical Understanding and Proving Abilities: Experiment With Undergraduate Student By Using Modified Moore Learning

Approach. *Journal on Mathematics Education, 2*(2), 231-250

Novita, R., Prahmana, R. C. I., Fajri, N., & Putra, M. (2018). Penyebab kesulitan belajar geometri dimensi tiga. *Jurnal Riset Pendidikan Matematika*, *5*(1), 18-29.

Nurhasanah, F., Kusumah, Y. S., & Sabandar, J. (2017). Concept of triangle: Examples of mathematical abstraction in two different contexts. *International Journal on Emerging Mathematics Education*, *1*(1), 53-70.

Rizkianto, I., Zulkardi, & Darmawijaya. (2013). Constructing Geometric Properties of Rectangle, Square, and Triangle in the Third Grade of Indonesian Primary Schools. *Journal on Mathematics Education*, *4*(2), 160-171.

Reid, D. A., & Knipping, C. (2010). *Proof in mathematics education*. Rotterdam, The Netherlands: Sense.

Stefanowicz, A., & Kyle, J. (2014). Proofs and Mathematical Reasoning. University of Birmingham. Retrieved from https://www.birmingham.ac.uk/

Stylianides, G. J., Stylianides, A. J., & Philippou, G. N. (2007). Preservice teachers' knowledge of proof by mathematical induction. *Journal of Mathematics Teacher Education, 10*(3), 145–166.

Sukirwan, Darhim, Herman, T., & Prahmana, R. C. I. (2018). The students' mathematical argumentation in geometry. *Journal of Physics: Conference Series*, *943*(1), 012026.

Suryadi, D. (2007). Model Bahan Ajar dan Kerangka-Kerja Pedagogis Matematika Untuk Menumbuhkembangkan Kemampuan Berpikir Matematik Tingkat Tinggi [Model of Teaching Materials and Framework of Mathematical Pedagogy for Developing Higher Mathematical Thinking Ability]. Unpublished Research Report. Bandung: SPS UPI.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 127-142

LEARNING OF DIVISION OPERATION FOR MENTAL RETARDATIONS' STUDENT THROUGH MATH GASING

Laila Fatika Nuari[1], Rully Charitas Indra Prahmana[1], Irma Fatmawati[2]

[1]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Yogyakarta, 55161, Indonesia
[2]SLB Bhakti Kencana I Berbah, Krikilan, Tegaltirto, Berbah, Sleman Yogyakarta, 55573, Indonesia
Email: rully.indra@mpmat.uad.ac.id

Abstract

This study aims to look at tenth-grade a mental retardation student in solving the problem of a division operation. The method used is Single Subject Research (SSR) with learning outcomes as variables that are measured and describe student learning activities in solving problems using Math GASING. The data collection technique is done by video recorder, documentation and test questions. The instruments are a video that is to see student activities during the study, photos to see the results of student work, and work the answer to see student answers to the questions given. Analysis of this research data is analyzed in conditions and between conditions. The results of the research carried out obtained that student experienced an increase in solving the division questions and the student gave a good response to the behavior carried out with Math GASING.

Keywords: Math GASING, Mentally retarded, Division operation, Single subject research.

Abstrak

Penelitian ini bertujuan untuk melihat kemampuan siswa berkebutuhan khusus kelas sepuluh dalam menyelesaikan permasalahan operasi pembagian. Metode penelitian yang digunakan adalah Single Subject Research (SSR) dengan hasil belajar sebagai variabel yang diukur dan menggambarkan aktivitas belajar siswa dalam menyelesaikan masalah yang diberikan menggunakan pembelajaran Matematika GASING. Teknik pengumpulan data dilakukan dengan video, dokumentasi, dan pertanyaan tes. Instrumen penelitiannya adalah video yaitu untuk melihat aktivitas siswa selama belajar, foto untuk melihat hasil kerja siswa, dan mengerjakan jawaban untuk melihat jawaban siswa atas pertanyaan yang diberikan. Analisis data penelitian ini dianalisis dalam kondisi dan antar kondisi. Hasil penelitian menunjukkan bahwa siswa mengalami peningkatan dalam menyelesaikan soal pembagian dan siswa memberikan respon yang baik terhadap perilaku yang dilakukan selama proses pembelajaran dengan Matematika GASING.

Kata kunci: Matematika GASING, Retardasi mental, Operasi pembagian, Penelitian subjek tunggal.

How to Cite: Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. *Journal on Mathematics Education, 10*(1), 127-142.

In public school and mathematics is one of the subjects that are always there (Cooper & Carsenty, 2016), but there are still many students who are afraid to learn mathematics because the material is difficult and very complex (Laurens, et al., 2018). Abstract of mathematics causes students to find it difficult to learn, whereas teachers less attention to providing learning to students (Widodo, et al., 2018). Other researchers also mentioned the results of his research that students experience difficulties in number operations (Prahmana, 2013). So, there are still many students who find it difficult to learn mathematics that has abstract and complex.

Children with mental retardation can be measured or seen from IQ possessed by making a mistake in determining arithmetic abilities (Koshy, 2017). The intellectual ability of mentally retarded children is usually only around 51 - 70 (Rejokirono & Dewi, 2018). Mild mental retardation experienced by students can cause students difficulties in developing the knowledge and skills they have even though they are capable students (Kuswardhana, et al., 2017). Other difficulties experienced by mentally retarded students are in measuring and estimating (Yankova & Yanina, 2010). The ability to think is limited, low memory, and the difficulty of abstract thinking of the students is the reason students have difficulties in academic fields such as number division skills (Putri, et al., 2017). Therefore, the mentally retarded students often experience difficulties due to their learning activities, especially in mathematics.

A study states that mathematics is very important to improve students' high thinking skills (Laurens, et al., 2018). Mathematics is also important in solving problems encountered, exploring around us, will be an interesting object if studied (Reis, et al., 2010; Dong, 2018). For adults with mental retardation, learning mathematics can help to be able to interact with their groups and reduce the risk due to inability to count (Prendergast, et al., 2017). For example, counting money and estimating payments is very important to learn in solving mathematical problems (Root, et al., 2018). Counting operations on the most basic mathematical learning, namely addition, subtraction, multiplication, and division need to be studied to do more complex calculations (Juliana & Hao, 2018; Prendergast, et al., 2017). Seeing the importance of mathematics, it is highly recommended to learn mathematics to solve daily problems for a mentally retarded student, especially in counting number.

Mathematics learning, especially for mentally retarded students, can use a variety of methods previously mentioned, but a lot of maturity and time are needed (Sigh & Agarwal, 2013). The use of assistive devices and technology can help mentally retarded students overcome difficulties in measuring (Yankova & Yanina, 2010; Kuswardhana, Hasegawa, & Juhanaini, 2017) and increase student motivation (Alabdulaziz & Higgins, 2017). Besides the use of tools and concrete objects or can be seen highly recommended for mentally retarded students to solve mathematical problems (Prendergast, et al., 2017). Concrete objects are objects that can be seen, held, and explored by students (Prahmana, 2013). These objects should be found by students in everyday life (Soylu, et al., 2017). Such as, the use of newspapers for mathematics learning in operating material is for secondary school students (Root, et al., 2018). Therefore, learning mathematics should use concrete objects so that mentally retarded students are easier to understand and can solve mathematical problems.

Learning material for abstract mathematical concepts makes students feel difficult if not done correctly (Multu & Akgun, 2018). A researcher makes learning design division operations using Math GASING by converting something concrete towards an abstract thing (Prahmana & Suwasti, 2014). Math GASING can be used as an intermediary in teaching the concept of division to students (Prahmana, 2013). The learning outcomes of class X students on physics subjects using Math GASING can increase (Nurfathoanah, 2017). In addition, Math GASING can be applied to help understand students about addition operations (Siregar, et al., 2014). Seeing many researchers who use Math GASING to teach mathematics, Math GASING is the right method to make it easier for students to learn mathematics in the division operation.

This study uses Math GASING to see the learning outcomes of mentally retarded students in material number distribution operations and see student responses. The Math GASING shows students about the process of converting concrete things into abstracts and drawing conclusions made by the students themselves (Prahmana, 2015). GASING is an abbreviation of easy (*GAmpang*), fun (*ASyIk*), and enjoyable (*menyenaNGkan*). Researchers conducted research on mentally retarded students because students still experience difficulties in division operations. In addition, students are less focused, less accurate in counting, and easy to forget. This statement is supported by the research that's been done before that students have difficulty in operating numbers, particularly the operations division (Nuari & Prahmana, 2018). So that researchers hope that students can solve the mathematical problems he faces using the concepts he got from Math GASING and solve the mathematics problems.

METHOD

The type of research used is descriptive research using the Single Subject Research (SSR) research method. Researchers use the SSR to describe or explain students' behavior in solving natural number division questions and observe students in solving problems when given treatment. The design used is A-B design with 1 baseline condition (A) and 1 intervention condition (B). SSR research was conducted on a retarded class X high school student with the initials A. The subject of this study was male and 17 years old. Blood loss is experienced by students from birth. At the time of childbirth the student's head is squeezed too long, then students lack oxygen. This results in a disturbance in the student's brain namely intellectual limitations.

The data collection techniques used in this study is video recordings, documentation, and written tests. The instrument used is based on data collection techniques, namely videos, photos, and test results. The video is used to describe the learning process of students when working on a problem or when an intervention is carried out by the researcher. Photos are used to document the results of student work and as material for analysis and research evidence. The student's written test sheet contains the student's answer in solving the questions given by the researcher with each question validated by the validator lecturer. These instruments are used to see an increase in learning outcomes or influences that occur after the research is conducted.

Data analysis techniques are performed on changes in conditions. First, the length of conditions that state the number of sessions or meetings conducted during the study in the baseline and intervention conditions. Second, the direct tendency is used to see the description of the behavior of the subject being studied. Third, the stability tendency is used to see the stability of each condition. Researchers used a stability tendency of 15%. Fourth, the data trace or trend traces of each measurement condition are used to see whether the data can be reduced (-), up (+) or horizontal (=). Fifth, the level of stability and range is done to see how big or small the range of data groups are in the baseline condition or intervention. Sixth, the level of change that shows the amount of change in data in one condition. Furthermore, analysis techniques between conditions are almost the same as analysis in conditions. Both of them discussed the same thing. First, the number of variables changed, namely the number of dependent variables in the study. Second, changes in the direction and effect tendencies can take the data in the analysis under conditions. Third, changes in the tendency of stability comes from baseline to intervention, namely to see changes in conditions before and after the intervention based on an analysis in conditions. Fourth, changes in levels are used to see changes that occur based on the difference in data points. Fifth, the percentage of overlap to see changes in the better or worse the influence of intervention on the target behavior.

RESULT AND DISCUSSION

The baseline condition is the measurement of the target behavior (behavior) with no previous treatment, while the intervention is the measurement of the target behavior after treatment. The researcher made observations on the A's condition for 3 days and B conditions 12 days, with duration of about 90 minutes per session. In this study, the dependent variable in this study is the ability of students to solve the problem of the operation of the division of natural numbers (learning outcomes). And the independent variable is the use of Math GASING learning to see student learning outcomes.

Table 1. Student Result

CONDITIONS	DATE	SCORE
BASELINE (A)	4 April 2018	13.75
	5 April 2018	15
	6 April 2018	15.63
INTERVENTION (B)	9 April 2018	12.5
	10 April 2018	36.25
	11 April 2018	32.5
	23 April 2018	31.25
	24 April 2018	28.75
	25 April 2018	33.75
	26 April 2018	33.75
	30 April 2018	32.5
	31 April 2018	33.75
	3 May 2018	33.75
	7 May 2018	33.75
	9 May 2018	30

Table 1 shows the results obtained by a student in solving the distribution operation problem. It is seen that the initial conditions or baseline results obtained are very low, while the conditions of student intervention increase. Student scores are presented in a graph form as in Figure 1.

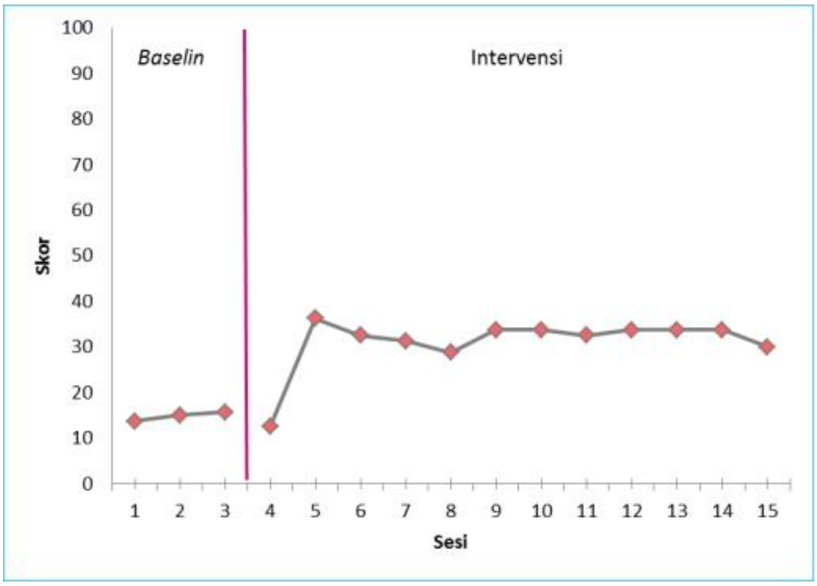


Figure 1. Visual Data of Baseline Conditions and Intervention Conditions

Furthermore, the data obtained is analyzed, namely:

1. The analysis in Conditions
   a. Length of condition

   Figure 1 displays a Figure of student learning outcomes using the A-B research design. Length of measurement conditions 3 sessions for condition A and 12 sessions for condition B.

   b. Direction

   Figure 2 shows the direction trends obtained through the intersection of vertical lines which divide the same parts in each condition with Figures.

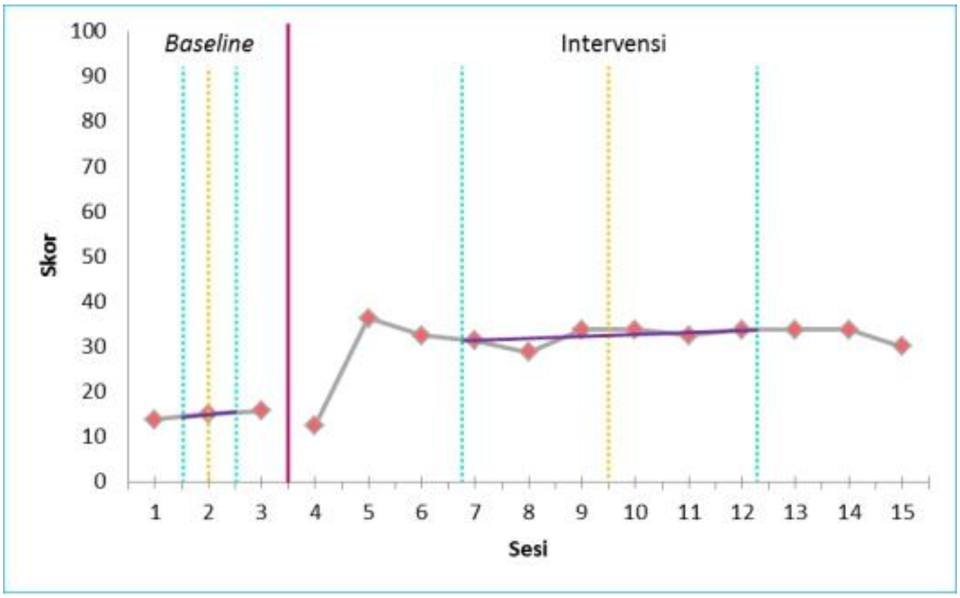


Figure 2. Direction Trends

c. Stability Trends

The stability criteria used to determine the trend of stability is 15%. Stability criteria are used to determine the stability range, upper limit, and lower limit of each condition. The upper boundary, lower boundary and mean level (blue) can be seen in Figure 3.

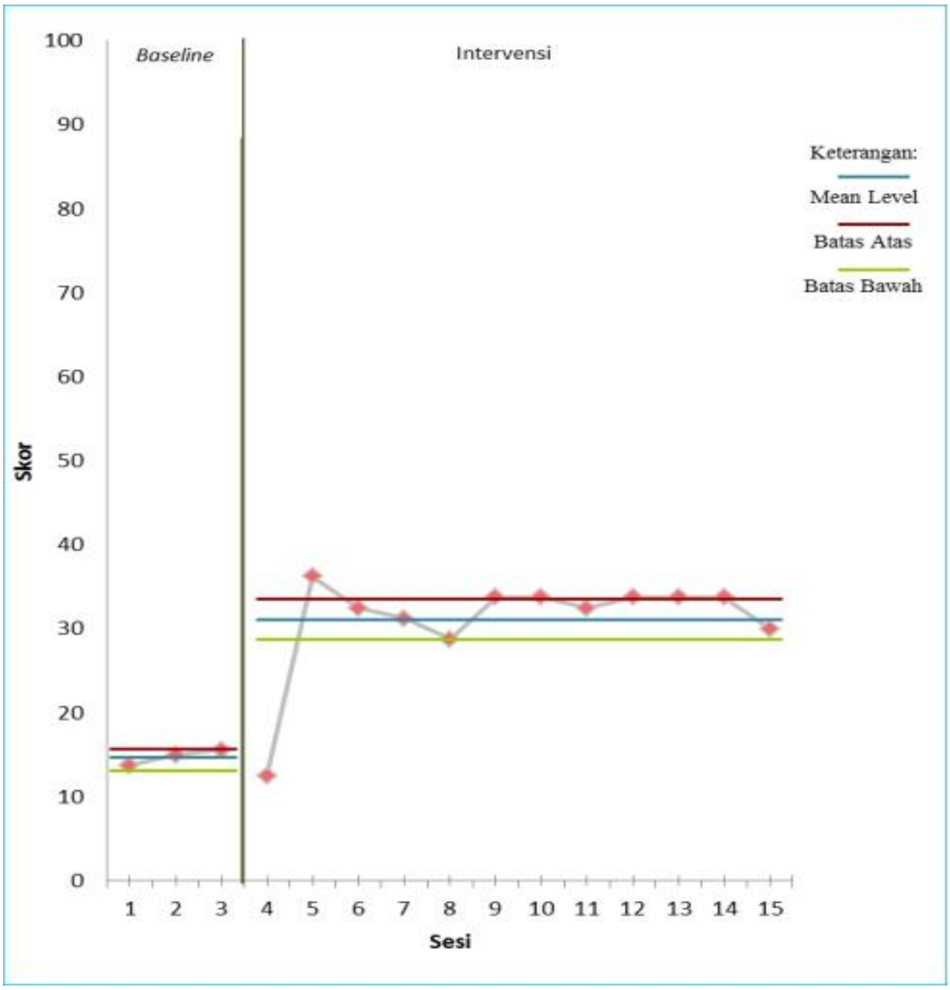


Figure 3. Upper and Lower Boundary Baseline Conditions and Intervention Conditions

Figure 3 shows that the data points for baseline conditions (B) that are in the upper boundary range (red) and lower boundary (green) are 3. The first session data point scores of the student are far below the lower boundary and the smallest among the other scores. The second session data point is also outside the range, greater than the upper boundary and this has an effect on data stability. The percentage of data points on baseline conditions that are in the range of stability is 100%, the data is declared stable. Looking at Figure 3 and through calculations on intervention conditions, there are 10 data points in the upper boundary range (red) and lower boundary (green). The percentage of data points for intervention conditions is 83%, so the data can be said to be stable because the data range is at intervals of 80% - 100%.

d. Data trace or trace tendency

Both conditions show a tendency to flatten due to improved but less visible changes.

e. Stability Level

The calculation of the level of stability of the data can be seen in the calculation of stability trends. The condition of A data is stable with a range of 13.75 - 15.63 stable data intervention conditions with a range of 12.50 - 36.25.

f. Rate of change

In condition A, the difference of 1.88 is obtained, which means there is a change and the intervention condition with a difference of 17.50 shows a change (improved). All components that have been calculated can be summarized as in Table 2.

Table 2. Summary of Visual Analysis Results in Conditions

No	Conditions	A	B
1	Length of Condition	3	12
2	Directions		
3	Stability Trends	Stable (100%)	Stable (83%)
4	Data trace or trace tendency	(=)	(=)
5	Stability Level	Stable 13.75 – 15.63	Stable 12.50 – 36.25
6	Rate of change	15.63 – 13.75 (+ 1.88)	30 – 12.50 (+17.50)

2. Visual Analysis Between Conditions

Inter-condition analysis in this study began by comparing conditions (B) with conditions (A), which is 2: 1, which means that the code for the baseline condition is 1 and the intervention condition code is 2. In the analysis of the conditions of this study carried out in several stages, namely:

a. Number of variables

The variables that were changed in this study were student learning outcomes in mathematical problems. In Table 5 the number 1 is written which means that the variable is changed to only one.

b. Changes in direction trends

Changes in direction trends in the analysis between conditions can be determined by taking data from the analysis under conditions. Writing changes in direction trends similar to analysis in conditions, both of which have good effects (+).

c. Changes in Stability Trends

Changes in direction trends can also be determined by looking at data on the tendency for the Stability of analysis in conditions. In this study, the changes that occur in both conditions are stable towards the stable.

d. Level change

The last session data point of the baseline condition was 15.63 and the first session data point for the intervention condition was 12.50. Then disputed to obtain 3.13 for a comparison of conditions B: A. Sign (-) means that it has decreased from the previous data.

e. Percentage of overlap

The percentage of overlap data in the comparison of baseline conditions with intervention conditions was 8.33%. The smaller the percentage overlap the better the influence of intervention on the target behavior.

A summary of all data analysis components between conditions can be seen in Table 3.

Table 3. Summary of Inter-Condition Analysis

No	**Comparison of Conditions**	**B:A (2:1)**
1	Numbers of Variables	1
2	Changes in Direction and Effect	 (=) (=) Positive
3	Changes in Stability Trends	Stable to stable
4	Level change	(15.63 – 12.50) (–) 3.13
5	Percentage of Overlap	8.33

Based on the results of the research that has been carried out there is an increase in student learning outcomes in calculating division operations by using Math GASING. Changes that occur can be observed in the graph and summary analysis table above which includes visual analysis, the analysis in conditions, and analysis between conditions. To be clearer, researchers discuss the results of research on each condition, namely:

1. Baseline condition (A)

In condition "A" the first session the student gets a low score, this is because the student has never worked on the same problem before. Whereas in the second and third sessions the value of student begins to increase because the student is already getting used to the forms of questions that they are working on. This increase in value is not much, ranging from 1-5 points. The measurement of the baseline conditions results and the location of the errors is almost the same.

This shows that student experience difficulties in certain parts namely repeated reduction. A student can make deductions, but when doing repeated deductions the numbers used to subtract are not subtracted. Instead, the numbers will be subtracted, as seen in Figure 4.

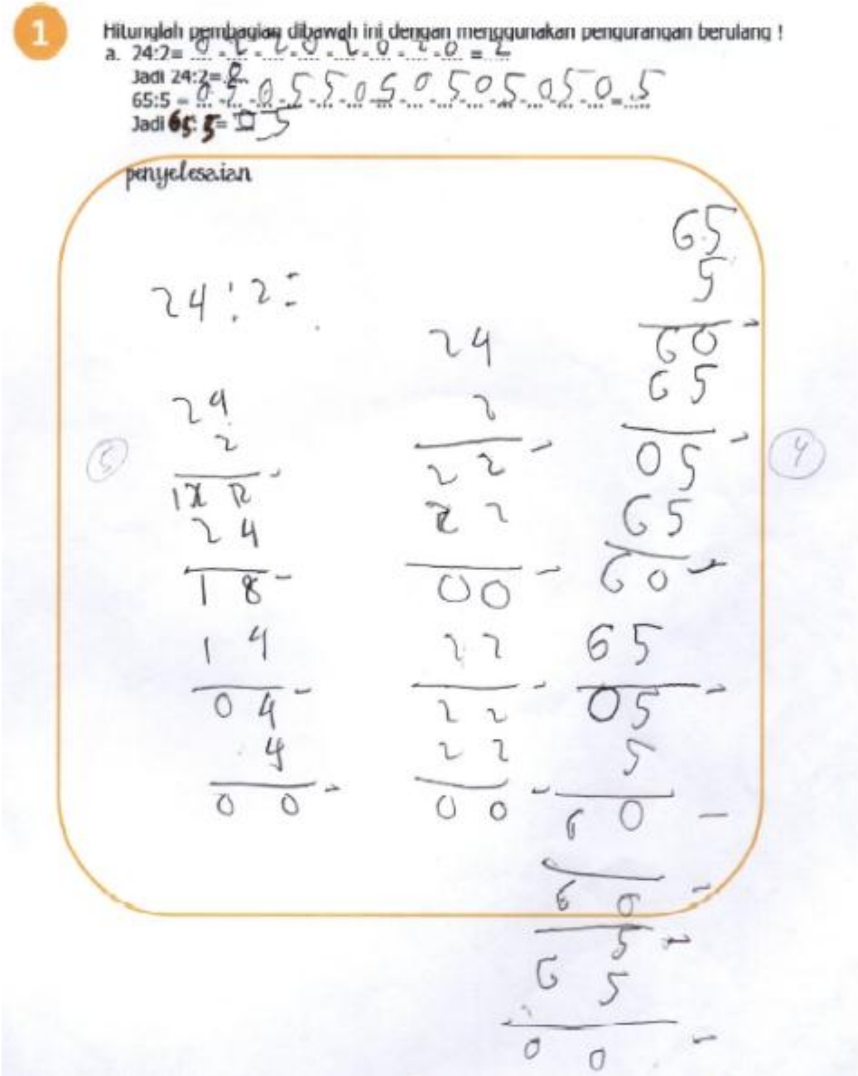


Figure 4. Student Calculation Results in Baseline Conditions

In addition to recurrent subtraction downward, the mentally retarded student also find it difficult to write down the reduction horizontally. A student writes down the deduction based on the subtraction that he did before. Because the reduction made is wrong, the writing of the reduction recurs horizontally is also still wrong and there is a remainder of the reduction. Whereas a researcher with Math GASING states that division is a reduction done repeatedly with the same number until the remaining reduction cannot be deducted again (Surya, 2007: 88). The results of the division operation are of two kinds, namely the number of deductions formed is called the quotient and the subtraction result is called the remainder of the division (Weaver, 2012: 30). Then the result of the division operation is the number of repetitions carried out repeatedly to produce a residual reduction that cannot be subtracted again.

2. Intervention condition (B)

   In the intervention conditions, researchers used Math GASING to provide treatment to the student. Learning with Math GASING begins with introducing the concept of division by using real objects. Then divert the use of concrete objects with semi-concrete like the picture. Furthermore, the student is given learning by using residual subtraction until repetitive reduction which in the end students can determine the results of the division of the repeated reduction. This study uses candy as a tool for the student to calculate division questions. The first and second sessions of intervention conditions student are still confused in counting using candy. A student can't conclude the results of the division using the grouping of sweets. As in the second session baseline condition measurement activities is with the distribution of 12: 3. The student is still confused to distinguish the results of the division by division. Measurement activities are shown in Figure 5.

Figure 5. Calculating Distribution Operations with Candies

Measuring the condition of intervention in the third session the researcher introduced how to calculate subdivision operations with subtraction. However, the student still find it difficult to do recurring reductions arranged down with the problem 56: 3. At the beginning of the reduction process can be done well, but on the third subtraction student make mistakes. Student write down the results of zero reduction with three is three, and the fifth reduction in students subtracts the results of the previous reduction by four. The student should subtract the number three according to the distribution problem given.

In the fourth session, the researchers asked a student not to use candies but instead used circle images or candy drawings on paper to calculate division questions. Researchers tried to use candy images so student practice not always using real objects when counting. The images are grouped with members of each group as many as the dividing numbers. The group formed is as a result of the division, as shown in Figure 6 as a result of the fourth session evaluation questions.

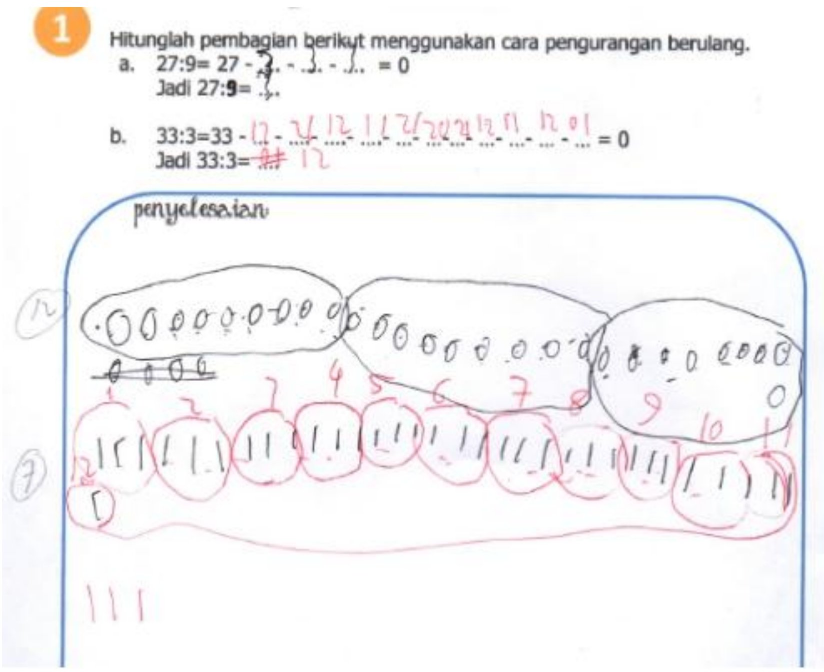


Figure 6. Results of Work on Student Evaluation Questions

A student can follow the directions of the researcher well. At the end of the fourth session, the researchers asked the student about which was easier if counting with candy (concrete) or candies pictures (semi-concrete). Students say that they prefer to use pictures rather than using real candy. The fifth session of the students began to modify by using a rectangular image analogous to the image of folding paper, a triangle as a cake, a circle as money, and a stick, as a tool to calculate the division. However, student experience errors when working using images. The student is sometimes less precise when grouping pictures into one group if the next image is in the second row. This causes the results of student calculations to be incorrect, shown in Figure 7.

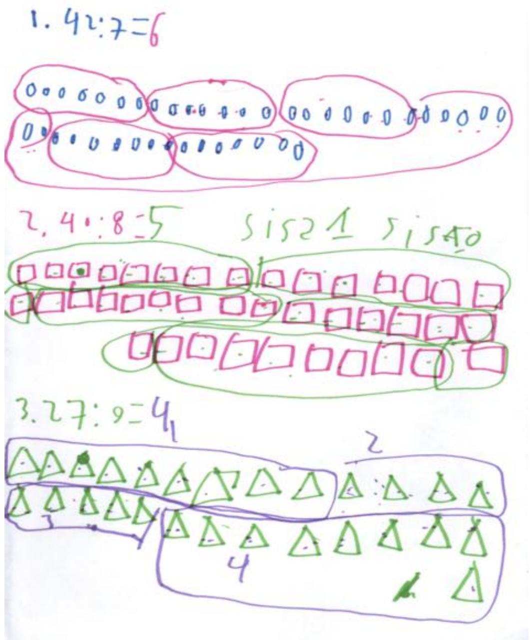


Figure 7. Calculation of Distribution Operations with Pictures

In the sixth and seventh sessions, there were not many changes in values obtained by the student. A student begins to understand how to calculate the division operation problem, which is using images or with repeated subtraction. When compared with the baseline condition the value achieved by a student is not far afloat but has increased. Researchers tried to teach again about repetitive reduction to calculate division questions so student reduces the use of images and switch to numbers that are abstract in nature. In addition to the values, there is a change in attitude that is shown by a student during the measurement of intervention conditions. This change has occurred since the fourth session, the student begins to enjoy learning to use games or practice to remember the previous material.

In accordance with behavior modification, this study brings changes in student behavior to good things. A student feels happier when learning by using games or giving rewards in the form of snacks when they succeed in working on the questions until they are finished. This is in accordance with Math GASING which teaches mathematics material with a fun method so that students feel happy while studying. Strengthened by the opinion of Halyadi, et al. (2016), which states that learning using Math GASING makes students feel easy because it starts with something tangible or concrete, fun because they use games, and fun because students are not forced during learning. It was proven when the researcher asked the students about the opinion on the calculation of the compilation of ways and divisions that were taught by the researcher, such as the Dialogue 1 in session sixth.

*Dialogue 1

Researcher	*: Sukanya yang gimana?* [What are you like?]
Student	*: Koyo ngene* [Like this] *(sambil memperagakan menggambar garis)* [While draw the lines]
Researcher	*: Yang pake apa?* [What is used?]
Student	*: Garis-garis itu* [The lines]
Researcher	*: Pake garis-garis. Kalo nggak pake garis-garis pake apalagi?* [Using the lines. What the else?]
Student	*: Pake yang bulet-bulet. Trus sama segitiga.* [Using the circles. Then a triangle]
	..……..
Student	*: Ho o. Kalo porogapit agak angel (siswa menjelaskan pembagian dengan bersusun lebih sulit)* [Sure. If it using the old way more difficult]. Student explains the compiled division more difficult.

The activity that student often do when intervening is telling things that students like. Researchers give time to tell stories so the student does not feel bored when the learning process takes place. Social interactions outside the learning process are very important especially for building students' self-esteem and interest which impacts on good learning outcomes (Aro & Ahoen, 2011). Student learning outcomes that increase can be seen from the scores achieved by students while working on the evaluation of the intervention conditions and it is proven that students want additional questions in the eighth session.

The student has been able to distinguish the results of the division and the rest of the division with repeated reductions. It is clear that student experience changes when compared to the results of measuring baseline conditions. Interventions conducted by researchers to see changes in student learning outcomes. Increased learning outcomes are also influenced by students' willingness or awareness to learn. Indirectly the intervention carried out had a good impact on the student who was initially less interested in learning mathematics. This is also supported by student class teacher statements delivered in Dialogue 2.

*Dialogue 2

Teacher : *Ya memang kita kan menyesuaikan kemampuan anak mbak. Cuman kan sebenarnya kayak A itu menurut saya bisa diubah karakternya lho. Kalo dia bisa merubah karakternya kan kemungkinan kemampuannya bisa dioptimalkan. [The teacher only adjusts students' abilities. It's just that students like A can be changed in character. If A can change its character, it's likely that its capabilities can be optimized].*

Researcher : *Iya bu. [Yes, Miss]*

Interventions conducted by researchers also adjust to students' abilities. Starting from concrete objects in the form of candies to semi-concrete objects and abstract images that use repeated reduction techniques.

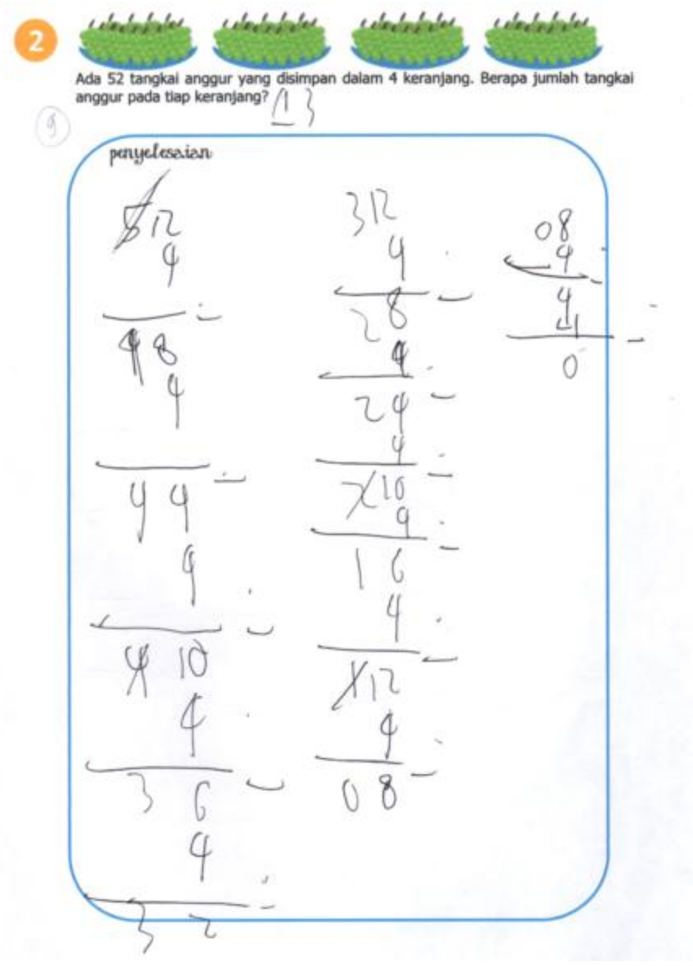


Figure 8. Distribution Results Using Repeat Reduction

The calculation of students using compounded reduction can be seen in Figure 8 which calculates the distribution problem of 52: 4. A student can make deductions correctly so that the remaining reduction is zero. In addition, the student can calculate the results of the recurring subtraction correctly, namely 13. The complete question in Figure 8 is, "There are 52 stalks of grape stored in four baskets. How many grapes stalk per basket? "

In accordance with previous researchers that learning operations sharing using Math GASING always starts from something concrete towards something abstract (Prahmana & Suwasti, 2014). The student has succeeded in using candy as a counting tool, drawing candy or bread in lieu of the actual candy to do repetitive cuts that have leftovers. That is, students have been able to pass the critical point of division as stated by Prahmana (2013) that the critical point of the distribution operation using GASING Mathematics is that students can make a reduction in the remainder, so a student can learn variations in distribution easily. Based on this study Math GASING can help students to improve operating learning outcomes in the distribution of mentally retarded student and provide other positive influences in the form of increased learning interest.

CONCLUSION

Learning the division of operations on student mentally retarded using Math GASING can improve student learning outcomes and provide a good influence on student. The student feel happy to learn to use Math GASING and can be one of the solutions for learning division operations for other mental retardations' students.

ACKNOWLEDGMENTS

Firstly, we would like to thank Universitas Ahmad Dahlan for providing the opportunity to develop research and have facilitated until this research is completed. Then, we thank for SLB Bhakti Kencana 1 Berbah and their teacher for allowing us to conduct the research.

REFERENCES

Alabdulaziz, M., & Higgins, S. (2017). Understanding Technology Use and Constructivist Strategies When Addressing Saudi Primary Students' Mathematics Difficulties. *International Journal of Innovative Research in Science*, *6*(1), 1111-1118.

Copper, J., & Karsenty, R. (2016). Can teachers and mathematicians communicate productively? The case of division with remainder. *Journal of Mathematics Teacher Education*, *21*(3), 237-261.

Dong, L. (2018). The Investigation of Educational Reform for Economic Mathematics Combined with Financial Characters. *In 2nd International Conference on Education, Economics and Management Research (ICEEMR 2018)*, 605-607.

Halyadi, H., Agustianie, D., Handayani, T., & Windria, H. (2016). Penggunaan Kobesi dalam Matematika Gasing untuk Meningkatkan Pemahaman Materi Perkalian Siswa SD. *Suska Journal of Mathematics Education, 2*(2), 81-88.

Juliana & Hao, L.C. (2018). Effects of Using The Japanese Abacus Method Upon The Addition and Multiplication Performance of Grade 3 Indonesian Students. *International Journal of Indonesian Education and Teaching, 2*(1). 47-59.

Koshy, B., Thomas, H.M.T., Samuel, P., Sarkar, R., Kendall, S., & Kang. G. (2017). Seguin Form Board as an Intelligence Tool for Young Children in an Indian Urban Slum. *Family Medicine and Community Health, 5*(4), 275–281.

Kuswardhana, D., Hasegawa, S., & Juhanaini. (2017). The Instructional Thematic Game for Children with Mild Mental Retardation: For Enhancement of Left-Right Recognition Skill. *International Journal of Electrical and Computer Engineering (IJECE), 7*(1), 469-478.

Laurens, T., Batlolona, F.A., Batlolona, J.R., & Leasa, M. (2018). How Does Realistic Mathematics Education (RME) Improve Students' Mathematics Cognitive Achievement?. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(2), 569-578.

Mutlu, Y., & Akgun, L. (2018). Using computer for developing arithmetical skills of students with mathematics learning difficulties. *International Journal of Research in Education and Science (IJRES), 5*(1), 237-251.

Nuari, L.F. & Prahmana, R.C.I. (2018). Kemampuan Operasi Hitung Perkalian dan Pembagian Siswa Tunagrahita Kelas X SMA. *Journal of Songke Math, 1*(1), 12-25.

Nurfathoanah. (2017). Implementasi Metode Pembelajaran GASING (Gampang, Asyik dan Menyenangkan) Terhadap Hasil Belajar Fisika Peserta Didik Kelas X SMA Negeri 3 Polongbangkeng Utara. *Jurnal Pendidikan Fisika, 5*(3), 351-361.

Prahmana, R.C.I. (2013). Designing Division Operation Learning in The Mathematics of Gasing. In *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013* (pp. 391-398).

Prahmana, R.C.I. (2017). The Hypothetical Learning Trajectory on Addition in Mathematics GASING. *Southeast Asian Mathematics Education Journal, 5*(1), 49-61.

Prahmana, R.C.I., & Suwasti, P. (2014). Local Instruction Theory on Division in Mathematics GASING: The Case of Rural Area's Student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26.

Prendergast, M., Spassiani, N.A., & Roche, J. (2017). Developing a Mathematics Module for Students with Intellectual Disability in Higher Education. *International Journal of Higher Education, 6*(3), 169-177.

Putri, N.D.A., Salim, A., & Sunardi. (2017). The Effectiveness of The Use of Course Review Horay (CRH) Methods to Improve Numeracy Division Skill of Children With Mild Mental Retardation in SLB Negeri Surakarta, Indonesia Year 2016/2017. *European Journal of Special Eduacation Research, 2*(3), 32-42.

Reis, M.G.A.D., Cabral, L., Peres, E., Bessa, M., Valente, A., Morais, R., …. & Bulas-Cruz, J.A. (2010). Using Information Technology Based Exercises in Primary Mathematics Teaching of Children with cerebral Palsy and Mental Retardation: A case study. *TOJET: The Turkish Online Journal of Educational Technology, 9*(3), 106-118.

Rejokirono & Dewi, S.R. (2018). Skills Learning Model for Children with Mild Mental Retardation: Best Practice in Vocational Education Management of Mild Mentally Impaired Student. *In Proceeding International Seminar on Education* (pp. 127-137).

Root, J.R., Cox, S.K., Hammons, N., Saunders, A.F., Gilley, D. (2018). Contextualizing Mathematics: Teaching Problem Solving to Secondary Students with Intellectual and Developmental Disabilities. *Intellectual and Developmental Disabilities, 56*(6), 442-457.

Singh, Y.P., & Agarwal, A. (2013). Teaching mathematics to children with mental retardation using computer games. *Educationia Confab, 2*(1), 44-58.

Siregar, J. H., Wiyanti, W., Wakhyuningsih, N. S., & Godjali, A. (2014). Learning The Critical Points for Addition in Matematika GASING. *Journal on Mathematics Education, 5*(2), 160-169.

Soylu, F., Raymond, D., Gutierrez, A., & Newman, S.D. (2017). The Differential Relationship Between Finger Gnosis, and Addition and Subtraction: An fMRI Study. *Journal of Numerical Cognition, 3*(3), 694-715.

Surya, Y. (2007). *Matematika itu Asyik 2B*. Jakarta: Armandelta Selaras.

Weaver, A. (2012). *Arithmetic: A Textbook for Math 01*. Bronx: Department of Mathematics and Computer Science.

Widodo, S.A., Darhim, & Ikhwanudin, T. (2018). Improving Mathematical Problem Solving Skills Through Visual Media. *J. Phys.: Conf. Ser.* 948 012004.

Zankova, Z., & Yanina, A. (2010). Assistive Devices and Technology in Education of Children and Students with Mental Retardation. *Trakia Journal of Sciences, 8*(3), 273-277.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 143-156

EXTERNAL REPRESENTATION FLEXIBILITY OF DOMAIN AND RANGE OF FUNCTION

Tian Abdul Aziz, Meyta Dwi Kurniasih

Universitas Muhammadiyah Prof. Dr. HAMKA, Jakarta, Indonesia
E-mail: tian_aziz@uhamka.ac.id

Abstract

This study attempts to analyze pre-service secondary mathematics teachers' flexibility of external representations of domain and range of functions. To reach the purpose, a task consisted of thirty question items were designed. Participants of the study were thirty-eight Indonesian pre-service secondary mathematics teachers attending mathematics education department at one private university in Jakarta, Indonesia. Based on the analysis participants written responses, this paper revealed participants' difficulties in providing a proper and consistent definition of the concept of domain and range of functions. We also disclosed the participants' lack of flexibility in doing translation among representations under the concept of domain and range of function. In general, participants written responses to the task did not provide evidence of a solid understanding of domain and range. There are several implications of these findings offered for secondary mathematics teacher education's program.

Keywords: Domain, Flexibility, Function, Pre-service mathematics teachers, Range.

Abstrak

Penelitian ini menganalisis fleksibilitas representasi eksternal domain dan range fungsi mahasiswa calon guru matematika di tingkat sekolah menengah. Untuk itu, sebuah tes yang terdiri dari tiga puluh pertanyaan telah didesain. Penelitian ini melibatkan 38 mahasiswa calon guru matematika yang sedang menempuh studi di jurusan pendidikan matematika di salah satu universitas di Jakarta, Indonesia. Berdasarkan analisis terhadap hasil pekerjaan tulis mereka, artikel ini mengungkap kesulitan mahasiswa dalam mengekspresikan domain dan range fungsi secara benar dan konsisten. Kami juga menemukan kelemahan mahasiswa dalam fleksibilitas antar representasi domain dan range fungsi. Selain itu, mereka juga kesulitan dalam menemukan domain dan range fungsi-fungsi yang diberikan dalam tes. Secara umum, berdasarkan jawaban terhadap soal yang diberikan, mahasiswa calon guru matematika ini tidak memiliki pemahaman yang kuat tentang domain dan range fungsi. Terdapat implikasi dari hasil penelitian ini untuk program pendidikan guru matematika tingkat menengah.

Kata kunci: Calon guru matematika, Domain, Fleksibilitas, Fungsi, Range.

How to Cite: Aziz, T. A., & Kurniasih, M. D. (2019). External representation flexibility of domain and range of functions. *Journal on Mathematics Education, 10*(1), 143-156.

A great majority of researchers in mathematics education has agreed that the concept of functions is the most powerful notion, the basic concepts, the heart of mathematics (Clement, 2001), and plays a substantial role in all level of mathematics curriculum. At the beginning of instruction associated with the function, teacher usually presents the concept of domain and range of the function. The domain is defined as the set of meaningful inputs x, whereas the range is described as the set of corresponding outputs y (Rockswold, 2012). These concepts become critically important in learning function or further mathematical concepts as it leads students to generalize ideas. Students' accurate understanding of domain and range function could assist them in comprehending linear transformation (Dorko & Weber, 2014) and inverse function (Arnold, 2004).

The importance of the concept of domain and range function, unfortunately, is not converged with students' comprehension about the concept. There are several previous studies devoted to exploring students' understanding of domain and range function. The studies reported that a large majority of students have difficulty in determining the domain of multivariable functions (Martínez-Planell & Gaisman, 2012), the domain of a composite function (Neger & Frame, 2005; Özkan & Ünal, 2009), and the domain of square root function (Drlik, 2015). Also, even though students are taught the way how to determine the domain and range of a function, they encounter difficulty when dealing with various types of problems. The reason might lay in the fact that understanding associated with domain and range of functions is likely to be overlooked by most instructors in university (Dorko & Weber, 2014) or high school level (Arnold, 2004). In other words, there is a lack of awareness of teachers in presenting the topic of domain and range function. Instead of having students catch on this topic comprehensively, most teachers are likely to present it briefly and focus on the operation of the function.

Another possible reason is that within the context of the topic of functions, the presentation tends to emphasize on single representation instead of considering various representations. The salient aspects of function concepts are the diversity regarding representations and interpretations (Sajka, 2003). Students focus heavily on algebraic symbol impedes them to possess a comprehensive understanding of multi-representation. Martínez-Planell & Gaisman (2009) found that when students were not exposed to distinct representations, they demonstrated a weak understanding of domain and range of functions. Elia & Spyrou (2006) revealed three factors might contribute to students' acquisition in determining domain and range of a function, one of which is the ability to employ various modes of representations.

The function could be expressed in various ways such as a table, ordered pairs, algebraic symbol, and graphics. Generally, students are likely to have a narrow view about function in which function deals solely with algebraic formula (Clement, 2001). Many researchers put emphasizes on the importance of various representations to help students grasp the concept of function. Therefore, for students to gain a comprehensive understanding of domain and range function, teachers are necessary to present various representations of function.

Goldin & Steingold (2001) distinguished two facets of representations, namely internal and external representations. Internal representation refers to the images a person generate in his/her brain for mathematical objects and operations (Cuoco & Curcio, 2001) or cognitive processes to mathematical ideas (Yilmaz, Durmus, & Yaman, 2018). In this respect, internal representations of an individual could not be observed directly or abstract. Meanwhile, external representation could be observed physically as the forms of it are an algebraic expression, real number line, Cartesian coordinate, diagrams, and so forth (Goldin & Steingold, 2001). Sierpinska (1992) added that making the connection among different representations of functions is another challenge for students.

The ability of students to do translation among representations is characterized as flexibility or translation (Bannister, 2014). Dufour-Janvier, Bednarz, & Belanger (1987) argue that the psychological processes involved in the translation process. Comprehending the concept of multiple representations and moving from one mode of representation to another are important aspects as it demonstrates students' understanding of function (Moschkovich, Schoenfeld, & Arcavi, 1993). Gagatsis & Shiakalli (2004) added that this ability could enhance students' success in problem-solving particularly and mathematics education generally.

A sheer number of studies and attention are devoted to understanding the concept of function and its teaching strategies as well, yet specifically, domain and range function receive little to no attention in the research.

As understanding of domain and range function might contribute to a comprehensive understanding of function, thus perhaps this time should be spent talking about domain and range function. Through a review of the literature, there are several studies conducted to investigate students understanding of the concept of domain and range function. Most of the previous studies in the same field concentrated solely on single external representation, such as graphical representation (Cho & Moore-Russo, 2014; Cho, 2013; Martínez-Planell, Gaisman, & McGee, 2015) or symbolic representation (Dorko & Weber, 2014; Özkan & Ünal, 2009). However, a study concerning the flexibility of external representations of domain and range of functions is not yet investigated. Even though both internal and external representations interact and important to effective mathematics teaching and learning, in this study we focus on the external one. Besides, what is new in this study is that it involved pre-service secondary mathematics teachers. Understanding their flexibility of external representation of domain and range of the function is of value as it might help teacher educators to make an effort to refine pre-service secondary mathematics teachers' mistakes and misconception about the concept of domain and range of function.

METHOD

As the present study set out to investigate pre-service secondary mathematics teachers' flexibility of external representations of domain and range function, we collected data quantitatively using test administration. The participants of the study consisted of the thirty-eight pre-service secondary mathematics education department in one private university in Jakarta, Indonesia. They were selected conveniently for the study as they have taken a course on the concept under study, that is, differential calculus. As the course is offered during the first semester, hence they were those who were in the second, fourth, and sixth semester of their four-year secondary mathematics teacher education program. Twenty-three of them were females, and seven were males.

We developed a test of domain and range of functions understanding using multiple representations. The test consisted of thirty items, i.e. five items were statements' analysis, nine items were multiple-choice questions, and the rest were essay questions. Most of the items were developed specifically for this study by the researchers, and several of them were taken from items used in the previous study (Cho, 2013). Content and face validity of the instruments was confirmed by expert opinions. Two experts in mathematics education took part as validators.

It measured three aspects related to flexibility external representations of domain and range functions. The first aspect is composed of seven items assessed students' understanding of the concept of domain and range function. The first two items requested participants to explain the definition of domain and range of function using their own words. The other five items asked students to analyze and respond to presented statements regarding the concept of domain and range function. Participants could choose among 'I don't know', 'Incorrect', 'I doubt', and 'Correct' options. The second aspect consists of two items measured students' knowledge about interval. It asked students to translate information from line number into interval notation and vice versa. The reason behind involving knowledge of interval in the test was related to the fact that this knowledge plays a significant role in facilitating students in determining domain and range of function. The third aspects consisted of twenty-one items assessing students' translation ability across various representations of domain and range of function. One item is requested participants to translate information from the graph into interval notation. Five items are requested the participant to translate information from graph to set notation. One item asked participants to translate information from the graph into a two-set arrow diagram. Six items are about translating information from algebraic expressions into set notation. One item requires participants to select presented graphs of functions whose domain and range are the same. Two items are about the restriction of domain and range of function. Three items focus on determining possible values of domain and range of function. One item is devoted to having students translate information from algebraic expression into the ordered set. Finally, one item asked students to determine domain and range of function from two-set arrow diagrams. All participants were instructed to complete the task in 100 minutes.

RESULT AND DISCUSSION

The main purpose of the paper is to draw attention to investigate pre-service secondary mathematics teachers' flexibility of external representations in understanding domain and range of functions. This study highlighted conception held and difficulties encountered by pre-service secondary mathematics teachers as they attempted to address tasks related to domain and range of functions. The findings of this study were mainly based on the analysis of participants written responses gathered from test administration. Subsequently, it is categorized according to themes. There are four main themes observed and classified as described in Table 1.

Table 1. Participants' responses to the definition of domain and range of a function

The domain	The range
Referring to the definition of the function	Referring to the definition of the function
Origin Set	Result set
The input of a function or the value of x	The output value
Left set in a two-set arrow diagram	Right set in a two-set arrow diagram

Students' understanding of the definition of domain and range function

Various participants' responses to the definition of domain and range were categorized into the following classifications as shown in Table 1. The first typical response is that they attempted to describe it by referring to the definition of the function. The function is defined as a rule that relates to every member of one set with a member of another set (Downing, 2009). Based on this, they argued that domain is a member of one set that will be associated with a member of another set. Conceptually it is, of course, an acceptable definition. Nevertheless, using this explanation might be hard to comprehend immediate practical concerns or cases. A question such as, "What is the domain of $f(x) = 2x$?" could not be addressed with such explanation. Besides, the definition of function was also taken into account as an attempt to delineate the meaning of range.

Second, several participants claimed that the domain is defined as origin set, whereas the range is described as the result set. Unfortunately, no further explanation was given to describe their answers in detail. Such responses are also acceptable to some extent as it is likely to be failed when attempting to understand immediate practical cases. Also, it is prevalent in most Indonesian classrooms as the teachers tend to use the terms of domain and origin set as well as range and result set interchangeably.

Third, some participants argued that the domain of a function is illustrated as the input of a function or the value of x and range of the function is the output or the value of y. This response is almost close to the desired response. However, an explanation is of which values of x or values of y satisfy all the requirements for meaningful $f(x)$ is not addressed. Therefore, the statement is unsatisfactory. Meaningful $f(x)$ seems to be unnoticed aspects in teaching domain and range of function.

Fourth, several participants refer to the position of domain and range at a two-set of diagram arrow. At the beginning of function lesson, mathematics teachers in Indonesia tend to present a two-set of diagram arrow as a way to illustrate the concept of function. It is followed by a comprehensive explanation about domain, codomain, and range. Teachers, generally speaking, set the domain at the left side, whereas they set the co-domain and range at the right side. The way of how to determine the domain, codomain, and range of a function is by looking at where a member is located.

Based on the above responses, it appears that the majority of participants were not able to express the meaning of domain and range of function properly. Set of possible values of x to make meaningful $f(x)$ is an ignored notion. The finding of this study is in line with a study conducted by Elia, Panaoura, Eracleous, & Gagatsis (2007) who uncovered students' inconsistencies in constructing the definition of function. Their understanding seems to be influenced by teachers' articulation of such a concept. Besides, teaching processes which focus heavily on algebraic manipulation might contribute to this obstacle. Presenting the formula or the rule at the beginning of mathematics lesson is likely to be prevalent in most Indonesian classrooms rather than articulating definition. Aziz, Pramudiani, & Purnomo (2017) supported this finding in which they found that mathematics teachers do not seem to have the intention to guide students to express definition of algebra concept correctly.

Furthermore, the students' responses towards statements we provided about the domain and range of the function are presented in Table 2.

Table 2. Participants' responses to the statements

Item No	Statements	DK	DA	DB	AG	NR
4	The domain and range of a function corresponding to the y and x-axis of the Cartesian coordinate system.	1	28	4	3	2
5	Any real numbers divided by zero is zero.	0	29	0	7	2
6	Among real numbers set, the square root of any negative numbers does not exist.	1	2	3	30	2
7	$\sqrt{0} = 0$	0	1	1	34	2
8	The range of a function is determined by the function as well the domain.	0	0	4	30	4

Note: DK = I don't know; IN = I disagree; DB = I doubt; AG = I agree; NR = No Response

Students' knowledge about interval

In this study participants' knowledge of interval were evaluated. Albeit relatively neglected, comprehension of intervals is prominent as it might contribute to students' success in coming to grips with determining domain and range of functions. The students' response to determining the domain and range of the function is presented in Table 3.

Table 3. Participants' Responses to Determining Domain and Range of Function

Item No	Sub-items	Correct	Incorrect	No response
3	Domain	34	0	4
	Range	28	6	4
9	A	27	7	4
	B	21	13	4
	C	24	9	5
10	A	25	8	5
	B	26	7	5
	C	22	10	6
11	Domain	26	5	7
	Range	17	13	8
12	Domain	1	14	23
	Range	1	14	23
13	Domain	1	16	21
	Range	1	16	21
14	Domain	0	22	16
	Range	3	19	16
15	Domain	16	11	11
	Range	12	14	12
16	Domain	3	13	22
	Range	2	12	24
17	-	5	26	7
18	Domain	13	17	8
	Range	4	23	11
19	Domain	13	16	9
	Range	8	19	11
20	Domain	17	11	10
	Range	14	12	12
21	Domain	2	14	12
	Range	1	24	13
22	Domain	2	24	12
	Range	3	19	16
23	Domain	14	12	12
	Range	14	12	12
24	-	3	23	12
25	-	31	2	6
26	-	30	3	5
27	-	29	3	6
28	-	10	18	10

29	-	25	6	7
30	-	18	8	12

In the task, there are two sub-categories namely; translating information from real number line to interval notation (Item 9) and translating information from interval notation to real number lines (Item 10). Table 3 shows that most of them were able to deal with the task. They appear to have a robust knowledge of interval and recognize how to represent it in another form. However, several participants made a mistake by disregarding the order of the numbers in interval notation in which it should be written from the smallest to the largest.

Students' algebraic manipulation ability

Having capability of manipulating algebraic expression is necessary for determining domain and range of a function. Therefore, in the task we provided three items consisted of knowledge about division by zero, taking the square root of negative numbers, and taking the square root of zero. As shown in Table 2, it appears that most of them do not have a serious misconception. Nevertheless, when they are presented with a task, they could not capitalize on their algebraic knowledge. In other words, their knowledge is not by their performance when solving problems. It was reported in the literature that students might possess a high level of conceptual knowledge but lack procedural skill (Keating & Crane, 1990). Therefore, the interplay between conceptual and procedural knowledge or between knowledge and practice is important to address mathematical problems successfully.

Students' understanding of domain and range of functions through various representations

Understanding of the concept of a function includes the capability of moving from one representation to another representation, flexibility of using effective representation in solving a problem, and capability of discerning multiple representations when working on a function (Eisenberg & Dreyfus, 1994). Therefore, this study discloses participants' flexibility of external representation in an attempt to determine domain and range of functions. External representations used in this study are set of ordered pairs, two-set arrow diagram, graphical representation, symbolic representation, set notations, and interval notations.

Based on participants' responses, their difficulties are observed when attempting to translate information from symbolic and graphical representation to set notation or interval notation. The observed sources of these difficulties are (1) dominance on the integers; (2) special angles ;(3) poor algebraic manipulation; (4) expressing range in x; and (5) focusing on restriction on the Cartesian coordinate. Dominance on the integers tends to be a major cause of their inability to determine domain and range of function presented symbolically or graphically. On the contrary, when they are presented with multiple-choice items in the task having them select possible values substituted to the value of x as well as f(x), they do not get in trouble as the given choices are in the form of the integers. This dominance might lead students to put real numbers aside and discern continuous curve as discrete things. It seems that it is important for them to have a robust understanding of the properties of real numbers. Several researchers also found that students' mistakes when working with algebra are in consequence of the dominance of the integers (Almog & Ilany, 2012).

Besides, most participants do not have adversities in translating information from a two-set arrow diagram and ordered set. The reason might be that it capitalizes on the integers and they are in favour of working with it instead of real numbers. In the didactic process, most mathematics teachers also tend to use the integers frequently as examples. Therefore, in this task, they do not face a big challenge in locating the domain and range.

In this study, we also find that they consider that the limit set on the Cartesian coordinate is the end of restriction to the function. The main reason might be due to their ignorance about the meaning of arrow at the end of curved lines. Even though there is considerable research indicating the importance of graphical representation in teaching and learning process as it could aid students in looking at the concept or problems in distinct ways as, students' comprehension of the use of Cartesian coordinate needs to be improved. These results are in good agreement with another study which has shown that students often seem to concentrate on observed aspects of a graph instead of seeing the graph in its entirely (P. Cho & Moore-Russo, 2014). Abdullah (2010) also revealed students adversities using Cartesian graph. Working excessively on integer numbers might also contribute to this ignorance in which they only read integer coordinates.

Among functions presented, most of the participants had trouble determining domain and range of sine function presented symbolically and graphically. Works of literature have shown that trigonometry is perceived as one of mathematics topic in which most students undergo crucial adversities in learning (Gür, 2009; Kamber & Takaci, 2018; Orhun, 2001). The reason is due to that the topic of trigonometry lacks coherence in mathematics education. Focusing on how to present the topics so that students comprehend it meaningfully becomes a challenge for mathematics teachers.

Besides, item asked to translate information from graphical representation to a two-set arrow diagram was not able to be addressed by most participants. Participants' inaccurate understanding of the graphical representation of a function in Cartesian coordinate might impede them to catch the necessary information provided and then translate it to another representation such as two-set arrow diagram. Even though working with a two-set arrow diagram seems straightforward, it becomes more challenging when necessary information is not presented directly.

The findings of the data analysis assert that generally speaking participants indicates a lack of flexibility among external representations in an attempt to determine domain and range functions. The finding of this study converges with prior research showing (Bannister, 2014) that teachers could exhibit flexible, disconnected, or constrained conception. In this study, pre-service teachers tend to possess constrained conception in which they demonstrated the construct from one perspective and did not deal with various external representations.

Determining domain and range of function from two-set arrow diagrams

There is one item that asked students to determine domain and range of function represented by two-set arrow diagram. It seems that the majority of them did not encounter difficulty in coping with this item. As elements of each set were illustrated clearly, they assign the element into domain and range easily. This success could be supported by at least two possible reasons. Firstly, the elements of both sets are integer numbers which students tend to favor. Secondly, a two-set arrow diagram is the most familiar representation of function and relation concepts. At the beginning of the function lesson, most mathematics teachers present this diagram as well as showing its domain, co-domain, and range. On the contrary, few participants showed their inability to determine the range of function. The reason may lay in the fact that they were not able to make a clear distinction between co-domain and range of function.

Translating information from graph to set notation or interval notation

Participants' ability to translate information from graphical representation into interval notation or set notation was examined by six items. The items asked participants to determine the domain and range of function presented graphically and writes it in the form of set notation or interval notation. The graph of functions used in the task, to wit, linear function (Item 11), rational function (Item 12), the Sine function (Item 13), quadratics function (Item 14), square root function (Item 15), and arbitrary function (Item 16).

Based on Table 3, it seems that most participants tend to be able to cope with determining domain and range of linear function. However, most of them provided incorrect answers when working with a rational function, the sine function, quadratics function, as well as square root function. Besides, determining the range of the function is not as easy as determining the domain of the function. It is evident that most of them failed in determining the range of function.

There are four typical mistakes made compiled. The first is dominance on the integers. Several participants do not seem to get accustomed to working with real numbers. It is obviously clear when they attempted to translate information from graphical representations to set notation or interval notation. Therefore, when determining domain and range of the function given, they solely consider the integers and ignore other numbers. It seems that the participants do not have an accurate comprehension of the properties of real numbers.

The second is expressing range in x. Several participants express a range of function in terms of x instead of y or $f(x)$. It seems that it might be due to their carelessness or ignorance. Based on their responses to item 4 evaluated their understanding concerning the relation between domain and range of function and Cartesian coordinate system, most participants are likely to have an accurate understanding.

The third is focusing on special angles. When attempting to determine the domain of Sine function, most of the participants only consider special angles such as 30^0, 45^0, 60^0, 90^0, and so forth. Therefore, the obtained ranges from these angles are limited. The curve of Sine function is not discrete. The curve is smooth or continuous as it is defined for each real values of x. The reason may lie in the fact that mathematics teachers tend to overemphasize the use these special angles on various mathematical activities. Accordingly, non-special angles are put aside.

The fourth is focusing on restriction on the Cartesian coordinate. When analyzing participants written responses to several items associated with this category, it appears that they restrict their domain or range based on graphical limitation. They are not likely to grasp the meaning of arrow at the end of the curved line. Therefore, when determining domain or range of a function, several participants restricted it by considering where the Cartesian coordinate comes to an end.

Translating information from the graph into a two-set arrow diagram

There is one item asked participant to translate information from graphical representation to two-set arrow diagram. The type of question is multiple choices, and it seems rather straightforward. Nevertheless, most participants are not able to address the task. The presented function does not continue in which it consists of several open intervals. Most participants did not pay attention to this and considered it as a continuous function. Knowledge of interval is likely to contribute to participants' success in addressing the task.

Translating information from algebraic expressions into set notation or interval notation

There are six items requested participants to determine domain and range of functions represented by algebraic expression and express it in the form of set notation or interval notation. Each item in this category has its unique characteristics. There are four different sort of functions taken into consideration, to wit, rational function (Item 18), square root function (Item 19 and Item 22), a linear function (Item 20 and Item 23), and the Sine function (Item 21) (See Table 3).

According to the table, it seems that participants have puzzlement in dealing with these items. Compared with other tasks, tasks involving linear function (Item 20 and Item 23) possessed a high tendency to answer correctly although no more than half of them succeeded in dealing with it. It appears that linear function is the most straightforward facet of function. Besides, working with trigonometric functions is still a challenge for them as only two out of them managed to address it correctly. Also, determining range seems to be more difficult than determining the domain of the function.

There are similar four typical mistakes made by them when translating information from algebraic expressions into set notation or interval notation, to wit: dominance on the integers, focusing on special angles, error in algebraic manipulation, and writing range in terms of x.

Selecting graphs of functions whose domain and range are the same

Item 24 asked participants to select three out of six graphs of functions whose domain and range are the same. Only a few of them could deal with it. Inability to catch information from the graph tends to be a major cause of their difficulty.

Restriction on the domain and range of function

There are two items included in this category. The first item (Item 28) requested participants to determine the range of function whose domain was restricted at first. The second item (Item 30) asked the participant to select one out of four graphs of functions represented the function whose domain which was restricted. Table 3 indicates participants' responses to these two items. No more than half of the participants were able to provide correct responses to these items. It seems that restriction on domain and range is an unfamiliar topic for them as it is not introduced widely in the high school mathematics curriculum. Conceptually, participants' knowledge of restriction on domain and range has been evaluated on item 8 and most of them were able to give a favorable response.

Determining possible values of domain and range of function

Three multiple-choice items were designed to ask participants determining presented values of domain and range of functions (Item 25, Item 26, and Item 27). The functions are square root functions, rational functions, and combination between rational and square root function. Based on Table 1, the majority of the participants were able to select correct choices. As the options were in the form of integer numbers, thus they do not face any adversities.

Translating information from algebraic expression into an ordered set

The last category in the task is to have participants translate information from algebraic expression into the ordered set. In other words, participants were asked to determine domain and range of function represented as algebraic expression and express it in the form of the ordered set. As the item is multiple choices, most of them can deal with it.

CONCLUSION

The objective of this study is to investigate pre-service secondary mathematics teachers' flexibility of external representations of domain and range of functions. From the study that has been carried out, it is possible to conclude that participants exhibit inconsistencies in constructing a definition of domain and range of function, lack of flexibility among external representations of approaching domain and range of function, and inability to determine domain and range of functions.

The findings of the present study have several notable implications for classroom instructions. The topic of domain and range of function should not be disregarded by teachers. As a topic of domain and range are introductory in the topic of function, review of interval algebraic manipulation including the use of multiple representations seems to be appropriate and prominent before continuing to subsequent topics. Besides, teaching and learning approach to introducing the concept of domain and range functions needs to be improved. This research was concerned with secondary pre-service mathematics teachers attending courses at university; however, the results should be applicable also to teacher educators, in-service mathematics teachers, and high school students as well. To sum up, the implications of the study's findings could be considered with the purpose to promote pre-service secondary mathematics teachers' understanding of the concept of domain and range of function.

The present study adds to the paucity of studies on pre-service secondary mathematics teachers' understanding of the domain and range in multiple representations, providing insights that correspond with the previous study on the topic. Further study of the issue is still required. In consideration of this study, it appears interesting to conduct subsequent research using an in-depth interview to gain a deep comprehension of how pre-service secondary mathematics teachers' difficulties in understanding and determining domain and range. It could enrich our knowledge about the concept examined in the present study. Besides, further research will be required to observe the way how mathematics teachers or teacher educators introduce the concept of domain and range.

REFERENCES

Abdullah, S. A. S. (2010). Comprehending the concept of functions. *Procedia-Social and Behavioral Sciences*, *8*, 281–287).

Almog, N., & Ilany, B. S. (2012). Absolute value inequalities: High school students' solutions and misconceptions. *Educational Studies in Mathematics*, *81*(3), 347–364.

Arnold, S. (2004). Investigating functions: Domains and ranges. *Australian Senior Mathematics Journal*, *18*(1), 59–64.

Aziz, T. A., Pramudiani, P., & Purnomo, Y. W. (2017). How do college students solve logarithm questions? *International Journal on Emerging Mathematics Education*, *1*(1), 25–40.

Bannister, V. R. P. (2014). Flexible conceptions of perspectives and representations: An examination of pre-service mathematics teachers' knowledge. *International Journal of Education in Mathematics, Science and Technology*, *2*(3), 223–233.

Cho, P., & Moore-Russo, D. (2014). How students come to understand the domain and range for the graphs of functions. *Proceedings of the Joint Meeting of the PME 38 and PME-NA 36* (Vol. 2, pp. 281–288). Vancouver: International Group for the Psychology of Mathematics Education.

Cho, Y. D. (2013). *College Students' Understanding of the Domain and Range of Functions on Graphs*. Buffalo: State University of New York at Buffalo.

Clement, L. L. (2001). What do students really know about functions? *Mathematics Teacher*, *94*(9), 745–748.

Cuoco, A. A., & Curcio, F. R. (2001). *The Roles of Representation in School Mathematics*. Reston: National Council of Teachers of Mathematics.

Dorko, A., & Weber, E. (2014). Generalising calculus ideas from two dimensions to three: How multivariable calculus students think about domain and range. *Research in Mathematics Education*, *16*(3), 269–287.

Downing, D. (2009). *Dictionary of Mathematics Terms* (3rd ed.). New York: Barron's.

Drlik, D. I. (2015). *Student Understanding of Function and Success in Calculus*. Boise: Boise State University.

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. *Problems of Representation in the Teaching and Learning of Mathematics*, 109–122.

Eisenberg, T., & Dreyfus, T. (1994). On understanding how students learn to visualize function transformations. *Research in Collegiate Mathematics Education*, *1*, 45–68.

Elia, I., Panaoura, A., Eracleous, A., & Gagatsis, A. (2007). Relations between secondary pupils' conceptions about functions and problem solving in different representations. *International Journal of Science and Mathematics Education*, *5*(3), 533–556.

Elia, I., & Spyrou, P. (2006). How students conceive function: A triarchic conceptual-semiotic model of the understanding of a complex concept. *The Montana Mathematics Enthusiast*, *3*(2), 256–272.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology,* 24(5), 645–657.

Goldin, G., & Steingold, N. (2001). Systems of Representations and the Development of Mathematical Concepts. *The Roles of Representation in School Mathematics*, 1–23.

Gür, H. (2009). Trigonometry learning. *New Horizons in Education*, *57*(1), 67–80.

Kamber, D., & Takaci, D. (2018). On problematic aspects in learning trigonometry. *International Journal of Mathematical Education in Science and Technology*, *49*(2), 161–175.

Keating, D. P., & Crane, L. L. (1990). Domain-general and domain-specific processes in proportional reasoning: A commentary on the "Merrill-Palmer Quarterly" special issue on cognitive development. *Merrill-Palmer Quarterly*, *36*(3), 411–424.

Martínez-Planell, R., Gaisman, M. T., & McGee, D. (2015). On students' understanding of the differential calculus of functions of two variables. *The Journal of Mathematical Behavior*, *38,* 57–86.

Martínez-Planell, R., & Gaisman, M. T. (2009). Students' ideas on functions of two variables: Domain, range, and representations. *Proceedings of the 31st PME-NA* (Vol. 5, pp. 73–80). Vancouver: International Group for the Psychology of Mathematics Education.

Martínez-Planell, R., & Gaisman, M. T. (2012). Students' understanding of the general notion of a function of two variables. *Educational Studies in Mathematics*, *81*(3), 365–384.

Moschkovich, J., Schoenfeld, A. H., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. *Integrating Research on the Graphical Representation of Functions*, 69–100.

Neger, N., & Frame, M. (2005). Visualizing the domain and range of the composition of functions. *Mathematics Teacher*, *98*(5), 306–311.

Orhun, N. (2001). Students' mistakes and misconceptions on teaching of trigonometry. *Journal of Curriculum Studies*, *32*(6), 797–820.

Özkan, E. M., & Ünal, H. (2009). Misconception in Calculus-I: Engineering students' misconceptions in the process of finding domain of functions. *Procedia - Social and Behavioral Sciences*, *1*(1), 1792–1796.

Rockswold, G. (2012). *Essentials of College Algebra with Modeling and Visualization* (4th ed.). Boston, MA: Pearson Education, Inc.

Sajka, M. (2003). A secondary school student's understanding of the concept of function - A case study. *Educational Studies in Mathematics*, *53*(3), 229–254.

Sierpinska, A. (1992). On understanding the notion of function. *The Concept of Function: Aspects of Epistemology and Pedagogy*, *25*, 23–58.

Yilmaz, Y., Durmus, S., & Yaman, H. (2018). An investigation of pattern problems posed by middle school mathematics preservice teachers using multiple representation. *International Journal of Research in Education and Science*, *4*(1), 148–164.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 157-170

SOFT TENNIS AND VOLLEYBALL CONTEXTS IN ASIAN GAMES FOR PISA-LIKE MATHEMATICS PROBLEMS

Riya Dhotul Jannah, Ratu Ilma Indra Putri, Zulkardi

Sriwijaya University, Jalan Srijaya Negara, Bukit Besar, Palembang, Indonesia
Email: riyadhotuljannah852@gmail.com

Abstract

This study aims to produce valid and practical PISA-like mathematics problems of uncertainty and data content by using soft tennis and volleyball contexts in Asian Games. It also aims to find out the potential effect of the problems on students' mathematical literacy. The method used in this study was design research with the type of development studies. The validity was viewed from expert assessment regarding the content, constructs, language, and students' comments to the clarity of the problems in the one-to-one phase. Then, the practicality was viewed in the small group phase, and it was found that the students could understand the problem well. Based on the answers from 33 tenth-grade students of senior high school, it was found that the problems had potential effects that arouse the ability to use symbolic, formal, and technical language, as well as the ability of operation, communication, and representation. With the problem of the soft tennis context, five students could apply the ability to use symbolic, formal, and technical language, and operations, 12 students could apply representation ability, and 16 students could apply the communication ability. Meanwhile, with the problem of the volleyball context, 18 students could apply the communication ability, and nine students could apply the representation ability.

Keywords: Asian Games, Design research, Mathematical literacy, PISA.

Abstrak

Penelitian ini bertujuan untuk menghasilkan soal matematika tipe PISA konten *uncertainty and data* dengan menggunakan konteks cabang olahraga *soft tennis* dan voli pada Asian Games yang valid dan praktis, serta mengetahui efek potensial soal terhadap kemampuan literasi matematis siswa. Metode yang digunakan dalam penelitian ini adalah *design research* dengan tipe *development studies.* Kevalidan soal terlihat dari hasil penilaian validator dari segi konten, konstruk, dan Bahasa, serta dari komentar siswa terhadap kejelasan/keterbacaan soal pada tahap *one-to-one*. Kemudian, kepraktisan soal terlihat pada tahap *small group*, yaitu siswa dapat memahami soal dengan baik. Berdasarkan jawaban dari 33 siswa kelas X SMA, diperoleh bahwa soal memiliki efek potensial yaitu memunculkan kemampuan menggunakan bahasa simbolik, formal, dan teknik, serta operasi, kemampuan komunikasi, dan kemampuan representasi. Pada soal dengan konteks *soft tennis*, diperoleh lima siswa dapat menerapkan kemampuan bahasa dan operasi simbolis, formal, dan teknis, 12 siswa dapat menerapkan kemampuan representasi serta 16 siswa dapat menerapkan kemampuan komunikasi. Sedangkan pada soal dengan konteks bola voli diperoleh 18 siswa dapat menerapkan kemampuan komunikasi dan 9 siswa dapat menerapkan kemampuan representasi.

Kata kunci: *Asian Games, Design Research*, Literasi matematika, PISA.

How to Cite: Jannah, R. D., Putri, R. I. I., & Zulkardi. (2019). Soft Tennis and Volleyball Context in Asian Games for PISA-like Mathematics Problems. *Journal on Mathematics Education, 10*(1), 157-170.

In this modern age, mathematical literacy is very important for everyone to face the matter encountered in everyday life. Ojose (2011) mentions that mathematical literacy is the knowledge for knowing and using basic mathematics in everyday life. Also, the main idea of mathematical literacy is the use of mathematical science in human life (Stacey & Turner, 2015). However, the importance of this mathematical literacy has not been in line with the achievements that Indonesian students got in the international assessment. One of the international assessments that assess this mathematical literacy is PISA (Program International Student Assessment). In the PISA study results in 2012, Indonesia was in ranked 64 out of 65 countries with an average score of 375 from the OECD country average score of 494 (OECD, 2013), and in the latest PISA study results in 2015, Indonesia ranked 63 out of 70 countries with an average score of 386 from OECD country average score of 490 (OECD, 2016). Based on the results of the last two periods of mathematical literacy, Indonesia is ranked at the bottom with a low average based on the OECD country average.

In mathematical literacy, it is explained that there are four contents used in PISA problems. One of the content is “uncertainty and data”. This content is the heart of mathematical analysis for many situational problems, and it is also the theory of opportunity and statistics as a technique for data representation and description (OECD, 2016). Therefore, the content is important in mathematical literacy because it can help students to analyze mathematical problems. However, the result of PISA for this content is in line with the achievement of Indonesian students' mathematical literacy, in which based on the 2012 results Indonesia got an average score of 384, which was still below the average score of other countries and the for “uncertainty and data” content was 493 points (OECD, 2014).

Various things can cause the low achievement of students in PISA. One of them is the evaluation system in Indonesia that still uses the low-level problem, and the students are accustomed to acquiring formal mathematical knowledge in the classroom so that the ability of non-routine problem solving or high-level problem solving becomes weak (Stacey, 2010: Wu, 2012; Novita, Zulkardi, & Hartono, 2012). Indonesian students are weak in solving problems that require them to give opinions or make reasoning (Firdaus, Kailani, Bakar, & Bakry, 2015). Also, Indonesian students have difficulty in solving PISA-like mathematics problems using the context and in turning them into mathematical problems (Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014).

Therefore, one of the efforts that can be done is to encourage teachers and students to plan a PISA- based learning and design a learning evaluation which is by the characteristics of PISA (Kohar, Zulkardi, Darmawijoyo, 2014). Besides, teachers are also required to be able to design the problem using a context which is close to the life of their students (Zulkardi & Putri, 2006). The learning emphasizing the context or situation is the PMRI Approach. According to Zulkardi and Putri (2006), PMRI approach is a learning approach that uses context as a starting point for students to develop the understanding of mathematics and simultaneously use the context as a source of mathematical applications. It is one of the learning approaches that will lead students to understand the concept of mathematics by their self-constructing through previous knowledge related to their daily life; which was then, it is hoped that the student learning becomes meaningful (Putri, 2011).

The game is a context that can be used. It has an entertainment side that can motivate students in learning so that there is increasing understanding of students about the concept contained in the game (Wijaya, 2008). This context can be found in one of the sports conducted in the Asian Games. The Asian Games is the Asian sports festival held every four years. The sports branches at the Asian Games can assist students to understand mathematics learning. Putri and Zulkardi's study (2017) employed the context of shot-put, Nasution, Putri, and Zulkardi's study (2017) discussed the context of dayung, Roni, Putri, and Zulkardi's study (2017) discussed the context of sprint, Gunawan, Putri, and Zulkardi's study (2017) used the context of swimming, and Rahayu, Putri, and Zulkardi's study (2017) used the context of hurdles. These studies show that the context of the Asian Games sports can attract students' interest in learning.

Various researches on the development of mathematical PISA-like problems for High School students with various focus both on content and ability have been conducted. For example, Mardhiyanti, Putri, and Kesumawati (2013) developed the PISA problem to measure students' mathematical communication ability; Kamaliyah, Zulkardi, and Darmawijoyo (2014) developed level 6 PISA problem for junior high school; and Oktiningrum, Zulkardi, and Hartono (2016) developed the PISA-like mathematics task with the context of Indonesian natural and cultural heritage.

Therefore, we were interested in developing the PISA-like mathematic problem with the sports branches conducted in Asian Games. Based on the description, this study aimed to generate valid and practical mathematical PISA-like problems of “uncertainty and data” content by using soft tennis and volleyball contexts in the Asian Games and to know the potential effect of the problems on students' mathematical literacy.

METHOD

The method used in this study was design research with the type of development studies. This study consisted of two main stages; preliminary and formative evaluation stages (Zulkardi, 2002). In the formative evaluation stage following the developing flow, the steps included self-evaluation, expert reviews, one-to-one, small group and field test (Tessmer, 1998). Moreover, the panel discussion (item panelling) was also conducted in the expert review phase.

In the preliminary stage, we determined and analyzed the place and subjects of the study. The subjects of this research were the tenth-grade students of Senior High School. Also, we conducted the analyses of curriculum and mathematical PISA-like problems based on the 2015 PISA framework. Next, we designed the problem set which included the problem grids, problem cards, and scoring rubric. Then, the next stage was the formative evaluation.

In the formative evaluation stage, the first phase was self-evaluation. It is the phase where the researcher evaluates his own instructional (Tessmer, 1998). In this phase, we evaluated and re-examined the PISA-like mathematics set prepared previously. The result was called prototype I. Then, it proceeded to the expert reviews and one-to-one phases simultaneously. Expert reviews are the phase undertaken by experienced experts to evaluate the problems. The experts assessed prototype I by evaluating it based on the content, constructs, and language. In the expert reviews phase, panel discussion was also conducted. The panel discussion is one of the important step numbers in the development of high-quality test items. The panel discussion was conducted with a lecturer of

Mathematics Education of Sriwijaya University and Mathematics colleagues of Sriwijaya University who also did researches on developing the PISA-like mathematics problems

In the same time, the one-to-one phase was also conducted. At this phase, the prototype I was tested to three students. The students employed were the tenth graders of Senior High School by having different abilities. They were one student with high ability, one student with medium ability, and one student with low ability. The selection of students in this one-to-one phase was based on the discussion with the teacher who taught them mathematics. The focus of this one-to-one was to get students' comments on the clarity/legibility of the problem’s intention, a suggestion for improvement and to investigate the reasons for the difficulty of solving the problem. We made the results and findings in the expert reviews and one-to-one phases as a consideration to revise the Prototype I. The result of the revision was called Prototype II.

Prototype II was tested in a small group phase. Six tenth-grade students of Senior High School with different abilities were involved in this phase to work on the problems and provide suggestions and comments to find out the practicality of the problems. The six students were two students with high ability, two students with medium ability, and two students with low ability. Same as the one-to-one phase, we also discussed with mathematics teachers who taught in the class in determining which students to be involved in this small group phase. The comments and findings at this small group phase were taken into consideration in revising the Prototype II. The revision of Prototype II was called Prototype III.

Prototype III was then field tested to the subjects of the study, 33 tenth-grade students of Senior High School. The result of the field test was to determine the potential effects of the problems. The potential effect was to find out which mathematical literacy ability appeared from the problems developed by analyzing the results of the student's answering strategy.

The data collection techniques in this study included documentation, walkthrough, tests, observations, and interviews. The developed problem set focused on three criteria, namely the validity, practicality, and effectiveness (Nieveen, 2007). The validity of the problems was viewed from the validation results done by experts regarding content, constructs, and language in the phase of expert reviews. Moreover, the instrument is considered to be practical if the problem can be understood by the users. For example, when looking at the problem, students do not find any difficulty in interpreting the intention, language, or how to use the instructions; even if they find one, students can get through it easily.

On the other hand, the instrument is said to be effective if the experts and practitioners based on their experience state that the instrument (problem) has a potential effect on the ability of students (Van den Akker, 1999), and in this study, it is the ability of students' mathematical literacy. Also, the effectiveness implies that the outcome of the use of a product should be consistent with the objectives of the study (Zulkardi, 2002), in which this study focused on the potential effect of the problems on students' mathematical literacy abilities. To find the emerged mathematical literacy can be by analyzing students' answering strategy based on indicators and descriptors of each mathematical literacy ability which is by the PISA 2015 framework.

RESULT AND DISCUSSION

This research produced 14 PISA-like mathematics problems with sports branch contexts in the Asian Games. However, this study focuses on two problems with the context of sports games (soft tennis and volleyball) with the content of "uncertainty and data".

Preliminary Stage

The preliminary stage involved the student analysis, curriculum analysis, as well as PISA problems and the design analyses. We designed the problem by analyzing and studying the existing mathematical PISA problems. For the problem with soft tennis context, we developed a problem by reflecting the 2006 PISA problem "choice" which asks about how many combinations of pizza with different topping that can be selected. From this problem, we designed "soft tennis players formation" mathematical problem using the context of soft tennis sports. This problem had two questions. Question 1 asked how many sets of man and woman's soft tennis players could be formed by the coach using the sample space concept, while Question 2 asked the chance of choosing one soft tennis player by using the concept of probability. The predicted level for the problem was Level 3.

In the problem of volleyball context, we were also inspired by the "choice" problem of PISA in 2006. From this problem, we designed the problem for "women's volleyball" using the volleyball sports context. This problem asked the number of matches that could occur from two groups in the preliminary round. To solve this problem, the concept of sample space can be utilized. The predicted level of the problem was Level 4. In the preliminary stage, the problem set included its problem grids, problem cards, and scoring rubrics which were called the initial prototype. After the initial prototype was designed, the next stage was the formative evaluation stage.

Formative Evaluation Stage

Formative evaluation stage involved self-evaluation, expert reviews, one-to-one, small group, and field test phase. In the self-evaluation phase, we analyzed, evaluated, and reviewed the initial prototype from the preliminary stage. We found the error or the minus of the problems by paying attention to the characteristics that became the focus of the prototype including the three characteristics: content, constructs, and language. The result obtained was called Prototype I.

The prototype I was validated at the expert review phase regarding content, constructs, and language by three experts, namely Kaye Stacey (University of Melbourne, Australia), Ross Turner (ACER, Australia), and Hongki Julie (Sanata Darma University Yogyakarta, Indonesia). The validation process was conducted through mail review with the assistance of Kaye Stacey, Ross Turner, and Hongki Julie. In the expert review phase, the panel discussion was also conducted with three lecturers of Sriwijaya University of Palembang and nine Master's students of Mathematics Education of Sriwijaya University who were also developing the mathematical PISA-like problems.

Along with expert reviews phase, Prototype I was tested to three tenth-grade students of Senior High School with different (high, medium, and low) ability. The three students were S, MA, and AG. During the process, we observed and found out how each student answered and understood the problem. Table 1 describes the comments for Prototype I of soft tennis problem.

Table 1. Comments from experts and students on the soft tennis problem

Validation	Comments/Responses	Revision
Ross Turner	- The Competencies are no longer used in PISA. - The second problem is less appropriate to use the probability because the team selection is not random, and pay attention to the meaningfulness of the questions because mentioning single player is doubtful since the problem concerns a mixed player.	- The competency is no longer used in the problem card. - Change the sentence of the first problem by simply asking the order of players on the soft tennis men's doubles. - The second problem is not used in the next stage because it is not the material for probability.
Hongkie Julie	- Problems are accepted.	
Panel discussion	- Use the first question only.	
Student	- For Problem 1, the sentence is confusing because of it concern about the sets of the team for soft tennis women's double and men's doubles players; it raises the double meaning.	

Furthermore, Table 2 describes the validation comments for the volleyball problem. It also consisted of expert judgement, panel review, and students' response.

Table 2. Comments from experts and students on the volleyball problem

Validation	Comments/Responses	Revision
Ross Turner	- This problem is okay, but you need to design a response coding scheme.	- Sentences in the questions ask about total matches until the final round.
Hongkie Julie	- Questions on the subject should not only be in the preliminary round but add up to the final round.	- Revisions on spelling; 'volli' becomes 'volleyball', and 'grub' becomes 'group'.
Panel Discussion	- The question should cover the matches until the final round.	
Student	- Errors in spelling; 'Volii' should be 'Volleyball' and 'grub' should 'group.'	

The results of the expert reviews and one-to-one were used to revise Prototype 1. The revision of Prototype 1 was called Prototype II. The valid Prototype II was then tested to six tenth-grade students of senior high school in the small group phase with two students with high mathematics abilities named RA and NM, two students with medium mathematics ability named MD and DN, and two students with low mathematics ability named IS and MAG. In the small group phase, it was found that the six students could understand the purpose of the problem, understand the instructions or questions contained in the problem, as well as see and read pictures and tables.

From this stage, there was a revision to soft tennis context. The image and the "female soft tennis" as one of the contents in the table were omitted because they were not used or did not assist students to answer problems and caused different meanings when students completed the answer on the problem. Finally, the problem was revised to produce Prototype III which was tested in the field test phase. The result of Prototype III of the soft tennis problem is illustrated in Figure 1.

SOFT TENNIS PLAYERS FORMATION

Soft Tennis is one of the sports that will be in conducted at the upcoming Asian Games 2018.

Source : https://m.tempo.co/read/news/2013/02/03/100458745/tim-piala-davis-indonesia kalah-telak-dari-jepang

Here are the names of Indonesian Soft Tennis athletes

No	Men's Soft Tennis Athlete
1	Hendri Susilo Pramono
2	Edi Kusdaryanto
3	Prima Simpatiaji
4	Ferdy Fauzi

Determine the formation of men's double soft tennis players that coach can form? write down the reasons that support your answer.

Figure 1. Prototype III on the soft tennis context

The result of Prototype III of the volleyball context is illustrated in Figure 2.

Volleyball Women

There are four rounds in the volley competition, namely preliminary round, the quarter-final round, the semi-final round, and the final round. In the 2014 Asian Games' women's volleyball matches, there were nine volleyball teams who participated in the game. The nine female volleyball teams were divided into two groups in the preliminary round as shown in the table below:

GROUP A
South Korea
Thailand
Japan
India

GROUP B
RRC
Chinese Taipei
Kazakthstan
Hongkong
Maldiv

In each preliminary round group stage, each team played with one another. There was one of the women's volleyball teams that had to be eliminated in the preliminary round to continue in the quarter-finals. In the quarter-finals, each team only competed once. Then the game was continued by using the knockout system (the losing team directly eliminated) until the winner was determined . How many total of women's volleyball matches? Write down the reasons that support your answer.

Figure 2. Prototype III on the volleyball context

Potential Effect of the Problem

The valid and practical Prototype III was then tested in the field test to see the potential effects on the students' mathematical literacy ability. The implementation of the field test was done with the 33 tenth-grade students of senior high school. The obtained results based on their answers in the field test can be seen in Figure 3.

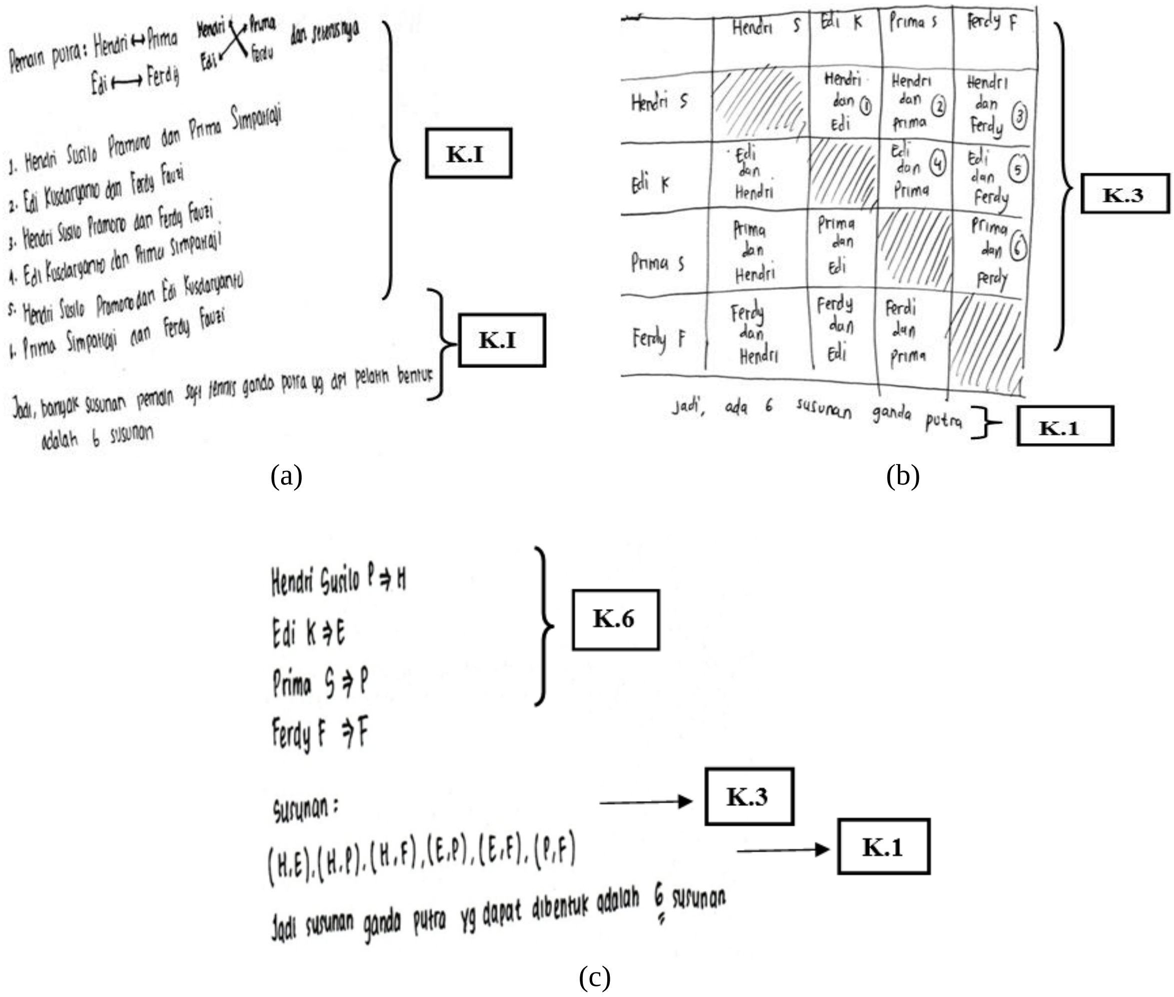
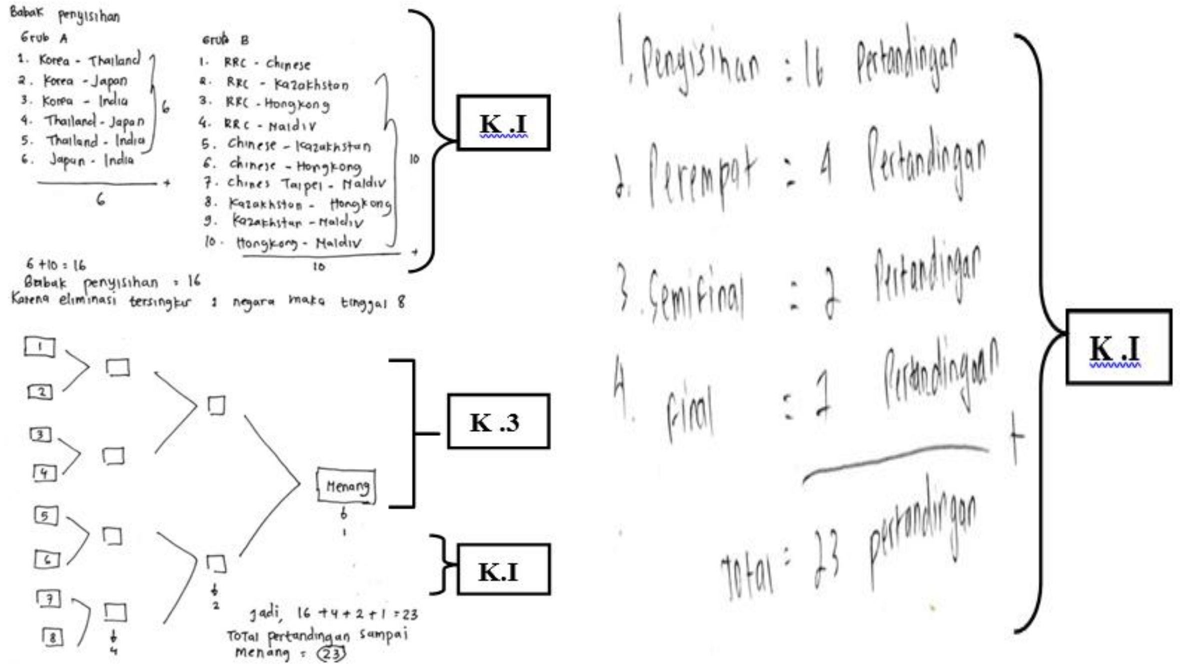


Figure 3. Students’ answer strategies for soft tennis context problem

Figure 3 shows that the students' answering strategies were different, but their answering conclusion was the same. Figure 3a shows the result of the students' answers which indicated that the problem could elicit communication ability (K.I) because they could write the process in reaching the solution and conclude the mathematical result which was the indicator of their communication ability. Students could write the process of achieving the solution because they could read the table of the man's athletes’ name then wrote the pair of men's doubles consisting of 4 athletes. At the end of the completion, the students concluded the mathematical result by stating that there were six sets of men's doubles players that could be formed. In contrast to the student's answering strategy in Figure 3b, the students' answers emerged the representation ability (K.3) because the students had used the representation in the table to solve the problem. It is an indicator of representation ability which used various representations in problem-solving.

Meanwhile, at the end of the completion, the students concluded the answer to the representation made. There were six sets of men's athlete which could be formed. This phenomenon showed that the students could conclude mathematical results, and this was one of the indicators of communication ability. Meanwhile, the students’ answering strategy in Figure 3c triggered the ability to use symbolic, formal, and technical language, and operations (K.6) because the students used variables to write men's athletes' names, this was an indicator of K.6 which used the formal form based on mathematical definitions and rules. After assuming the names of those soft tennis athletes in the form of variables, students paired each variable to form the formation of players. Students used the concept of sample space by registering. It shows their answering strategy that raised the representation ability (K.3). Then, they concluded there were six men's doubles players who could be formed, and this was an indicator of communication ability (KI), namely concluding the mathematics result. Based on the answer strategy of 33 tenth-grade students of Senior High School on the problem of soft tennis context that we found during the field test process, 5 students could apply the ability of using symbolic, formal, and technical language, and operations, 12 students could apply representation ability and 16 students could apply communication ability.



(a) (b)

Figure 4. Students' answer strategies for volleyball context problem

Figure 4 shows the two students' response strategies. Figure 4a showed the result of students' answers that emerged the communication ability (K.I). This phenomenon was based on the indicator of the communication ability, namely writing the process to get the solution. It can be seen from the students who wrote the process of getting the number of matches in the preliminary round by outlining every game in group A and group B so that the students could get 16 matches in the round. They continued creating a scheme to know the number of rounds up to the final round. Their steps in creating the scheme to show the answering strategy emerged the representation ability (K.3), with the indicator of using various representations in problem-solving. Then, the steps also emerged the communication ability because it concluded the mathematical results of summing each total match obtained up to the final round with the 23 total numbers of matches.

On the other hand, Figure 4b from the results of students' answering strategies could generate the communication ability (KI) because students could write the amount of every match in the preliminary, quarterfinal, semifinal, and final rounds. This result shows the communication ability (K.I)'s an indicator that is writing down the process of getting the solution. Then the students added up the overall match that had been implemented so that they obtained 23 games as a conclusion for the mathematical results. It was an indicator of communication ability (K.I).

From the results of 33 students' answering strategy on the problem of volleyball context which we found during field process, it was obtained that 18 students were able to apply communication ability and nine students who could apply representation ability.

CONCLUSION

This study produced PISA-like mathematics problems of uncertainty and data content by using soft tennis and volleyball contexts in Asian Games. The validity was viewed from the assessment of experts regarding content, constructs, and language, as well as from students' comments on the clarity/readability of the problems in the one-to-one phase. In addition to that, the practicality was seen from the way students understand and use mathematical PISA-like problem well enough. Information or guidance in the form of either picture or table contained in problems could be read clearly in the trial small group phase. Based on the analysis of students' answering strategies at the field test phase, it was found that the problems had potential effects that increase the ability to use symbolic, formal, technical language, and operations, communication ability, and representation ability. To sum up, it is suggested for teachers and students to use PISA-like mathematics problems with soft tennis and volleyball context to familiarize students with PISA problems.

ACKNOWLEDGEMENT

We would like to express gratitude to the Directorate General of Higher Education of Indonesia who has funded this research in 2018 through the scheme of 'Hibah Pasca', and to Ms Eriga and her students for participating in this research, as well as to those who have helped us in developing the research and writing this article.

REFERENCES

Firdaus, F., Kailani, I., Bakar, M. N. B., & Bakry, B. (2015). Developing critical thinking skills of students in mathematics learning. *Journal of Education and Learning, 9*(3), 226-236.

Gunawan, M. S., Putri, R. I. I., & Zulkardi, M. (2017). Learning Fractions through Swimming Context for Elementary School Students. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*. Paris: Atlantis Press.

Kamaliyah, K., Zulkardi, Z., & Darmawijoyo, D. (2014). Developing the sixth level of PISA-like mathematics problems for secondary school students. *Journal on Mathematics Education*, *4*(1), 9-28.

Kohar, A. W., Zulkardi, Z., & Darmawijoyo, D. (2014). Developing PISA-like mathematics task to promote student's mathematical literacy. *Proceedings of the 2nd SEA-DR (South East Asia Development Research) International Conference 2014 (SEADRIC 2014)*. Paris: Atlantis Press.

Mardhiyanti, D., Putri, R. I. I., & Kesumawati, N. (2013). Pengembangan soal matematika model pisa untuk mengukur kemampuan komunikasi matematis siswa sekolah dasar. *Jurnal Pendidikan Matematika UNSRI*, *5*(1), 16-29.

Nasution, M. F., Putri, R. I. I., & Zulkardi, Z. (2017). Dayung context in fraction. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*. Paris: Atlantis Press.

Nieveen, N. (2007). Formative evalution in educational design research. In N. Nieveen & T. Plomp (Eds.), *An Introduction to Educational Design Research* (pp. 89-102). Enschede: SLO.

Novita, R., Zulkardi, Z., & Hartono, Y. (2012). Exploring primary student's problem-solving ability by doing tasks like PISA's question. *Journal on Mathematics Education*, *3*(2), 133-150.

OECD. (2013). *PISA 2012 Results: What Students Know and Can Do, Student Performance in Mathematics, Reading and Science (Volume 1).* Paris: OECD Publishing.

OECD. (2014). *PISA 2012 Results in Focus: What 15-year-old know and what they can do with what they know*. Paris: OECD Publishing.

OECD. (2016). *PISA 2015 Results: Excellence and Equity in Education (Volume 1).* Paris: OECD Publishing.

Ojose, B. (2011). Mathematics for literacy: Are we able to put the mathematics we learn into everyday use? *Journal on Mathematics Education*, *4*(1), 89-100.

Oktiningrum, W., Zulkardi, Z., & Hartono, Y. (2016). Developing PISA-like mathematics task with indonesia natural and cultural heritage as context to assess students' mathematical literacy. *Journal on Mathematics Education*, *7*(1), 1-8.

Putri, R. I. I. (2011). Pembelajaran materi bangun datar melalui cerita menggunakan pendekatan Pendidikan Matematika Realistik Indonesia (PMRI) di sekolah dasar. *Jurnal Pendidikan dan Pembelajaran*, *18*(2), 234-239.

Putri, R. I. I., & Zulkardi, Z. (2017). Fraction in shot-put: A learning trajectory. *Proceedings of the 4th International Conference on Research, Implementation, and Education of Mathematics and Science (4th ICRIEMS) 2017*, pp. 1-8. Yogyakarta: UNY.

Rahayu, C., Putri, R. I. I., & Zulkardi, Z. (2017). Multiplication of fraction with natural number by using hurdles. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*, pp. 43-47. Paris: Atlantis Press.

Roni, A., Zulkardi, Z., & Putri, R. I. I. (2017). Sprint context of Asian Games in the division of fractions. *Proceedings of the 5th SEA-DR (South East Asia Development Research) International Conference 2017 (SEADRIC 2017)*, pp. 22-29. Paris: Atlantis Press.

Stacey, K. (2010). Mathematical and scientific literacy around the world. *Journal of Science and Mathematics*, *33*(1), 1-16.

Stacey, K., & Turner, R. (2015). The evolution and key concepts of the PISA mathematics frameworks. *Assessing mathematical literacy* (pp. 5-33). Cham: Springer.

Tessmer, M. (1998). *Planning and conducting formative evaluations: Improving the quality of education and training*. London: Kogan Page.

Van den Akker, J (Ed.). (1999). Principles and Methods of Development Research. *Design Approaches and Tools in Education and Training*. Dordrecht: Kluwer Academic Publishers.

Wijaya, A. (2008). Design Research in Mathematics Education: Indonesia Traditional Games as Means to Support Second Graders 'Learning of Linier Measurement'. *Unpublished Thesis*. Netherlands: Utrecht University.

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast*, *11*(3), 555-584.

Wu, M. (2012). Using PISA and TIMSS mathematics assessments to identify relative strengths of students in Western and Asian. *Journal of Research in Education Sciences*, *56*(1), 67-89.

Zulkardi. (2002). Developing A Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers. *Published Dissertation*. Enschede: University of Twente.

Zulkardi, Z., & Putri, R. I. I. (2006). Mendesain sendiri soal kontekstual matematika. *Proceedings of KNM 13*, pp. 1-7, Semarang: UNNES.